1a) Graph in Excel the function $f(x)$ = sin (x) — 1/1000 (sin (1000x)) in the viewing rectangle [-2π, 2π] by [-4, 4]. What slope does the graph appear to have at the origin?

Slope= (1/4) / (1/4) which appears to be 1 at the origin

b) Zoom in to the viewing window [-0.4, 0.4] by [-0.25, 0.25] in Excel and estimate the value of $f'(0)$. Does this agree with your answer from part (a)?

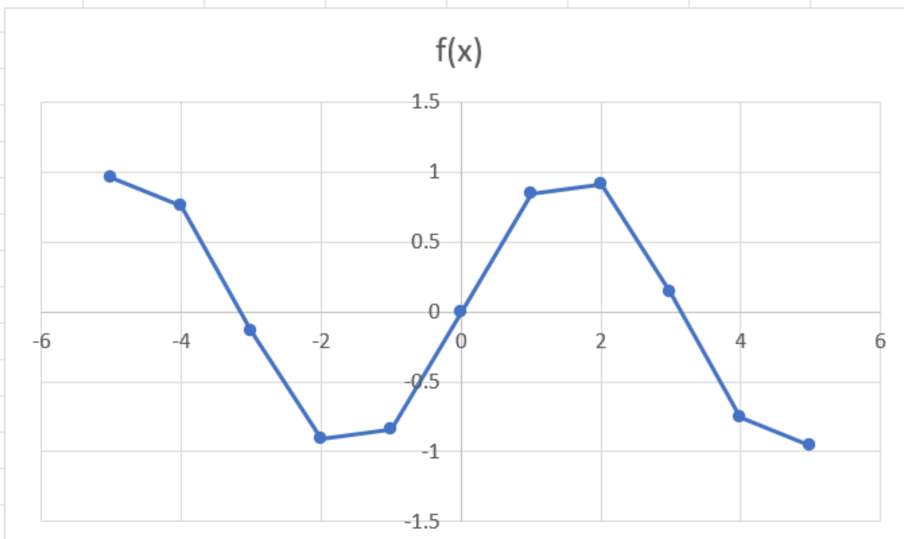
By Zooming in again, the slope appears to be 0.1/0.1 which is 1 again. Therefore, we estimate f'(0) value to be 1. This agrees with answer from part(a)

c) Now zoom in to the viewing window [-0.008, 0.008] by [-0.005, 0.005] in Excel. Do you wish to revise your estimate for $f'(0)$?

Now the graph gets much more stronger. The slope is 0.05/0.05 which again comes as equal to 1. Hence, the slope of the curve at 0 which is f'(0) appears to be strongly at 1.

Q No 1

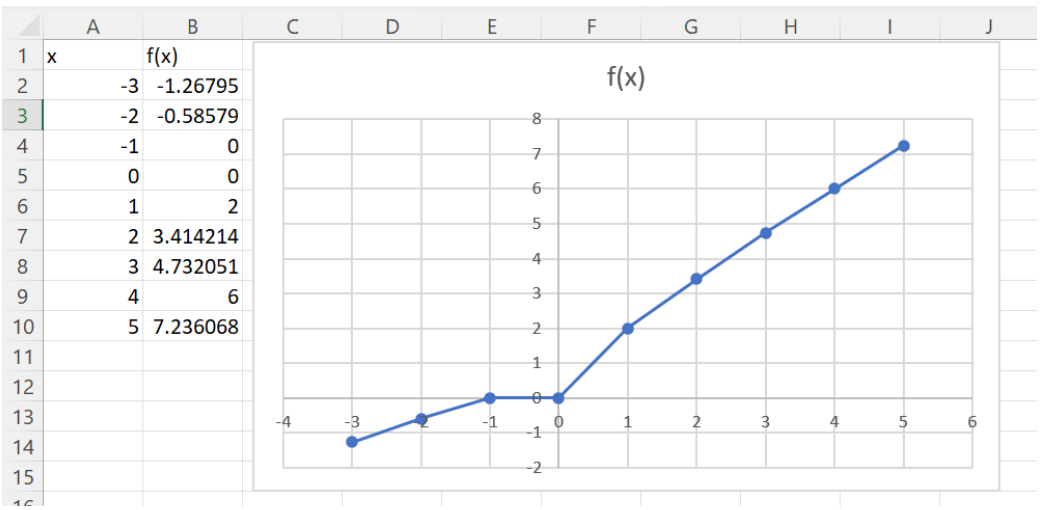
x	f(x)
-5	0.95794
-4	0.75612
-3	-0.1409
-2	-0.90837
-1	-0.84064
0	0
1	0.84064
2	0.90837
3	0.1409
4	-0.75612
5	-0.95794



2. Graph in Excel the function $f(x) = x + |x|$. Zoom in repeatedly, first toward the point (-1, 0) and then toward the origin. What is different about the behavior of f in the vicinity of these two points? What do you conclude about the differentiability of f ?

Zooming at (-1,0) f is differentiable at (-1,0) because it is smooth at (-1,0) and there are no sudden changes.

Zooming at the origin f is NOT differentiable at origin because it has a kink at origin. There are sudden changes in the slope.



x	f(x)
-3	-1.26795
-2	-0.58579
-1	0
0	0
1	2
2	3.414214
3	4.732051
4	6
5	7.236068

3. The left-hand and right-hand derivatives of f at a are defined by

$f'-(a)$ = lim $h \to 0-$ $(f(a+h) - f(a))/h$ and $f'+(a)$ = lim $h \to 0+$ $(f(a+h) - f(a))/h$

if these limits exist. Then $f'(a)$ exists if and only if these one-sided derivatives

exist and are equal.

(a) Find $f'-(4)$ and $f'+(4)$ for the function

To find f'-(4). we substitue value of 4 in the function. We get,

f'(-4) = lim $h \to 0-$ (f(4+h) - f(4))/ h [Here, if h<0, 4+h is also less than 4]

= lim $h \to 0-$ ((5- (4+h)- 1)) / h [f(4)=1 since, from definition, f(4)=1/(5-x)=1]

= lim $h \to 0-$ (5-4-h-1) /h

= lim $h \to 0-$ (-h/h)

= -1

To find f'+(4). we substitue value of 4 in the function. We get,

f'(+4) = lim $h \to 0+$ (f(4+h) - f(4))/ h [Here, if h>0, 4+h is greater than 4]

= lim $h \to 0+$ ((1/ (5- (4+h) -1)) / h [From definition, if x is > or equal to 4,we use 1/ (5-x)]

= lim $h \to 0+$ (1/(1-h)- (1-h)/(1-h)) / h

= lim $h \to 0+$ (h/(1-h)/h)

= 1

$f(x)$ = { 0 $if\ x \leq 0$

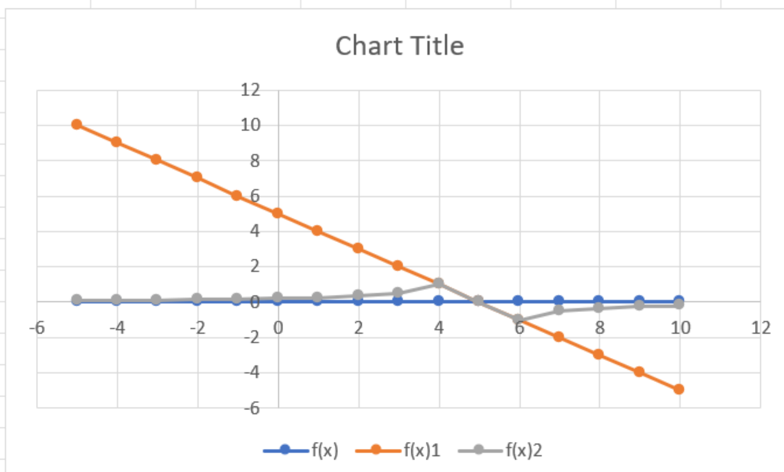
$5 - x\ if\ 0 < x < 4$

$1/(5 - x)\ if\ x \geq 4$}

(b) Sketch the graph of f.

Q No 3

x	f(x)	f(x)1	f(x)2
-5	0	10	0.1
-4	0	9	0.11111
-3	0	8	0.125
-2	0	7	0.14286
-1	0	6	0.16667
0	0	5	0.2
1	0	4	0.25
2	0	3	0.33333
3	0	2	0.5
4	0	1	1
5	0	0	#DIV/0!
6	0	-1	-1
7	0	-2	-0.5
8	0	-3	-0.33333
9	0	-4	-0.25
10	0	-5	-0.2



(c) Where is f discontinuous?

The graph of f is discontinuous at x=0 and x=5

(d) Where is f not differentiable?

The graph of f is not differentiable at x=0 and x=4 and x=5

4.If f is a differentiable function and $g(x) = xf(x)$, use the definition of a derivative

to show that $g'(x) = xf'(x) + f(x)$

To show that $g'(x) = xf'(x) + f(x)$, we'll use the definition of the derivative. The derivative of $g(x)$ with respect to x is defined as:

$g'(x) = \lim(x \to a)\ [g(x) - g(a)] / (x - a)$

Let's substitute the function $g(x) = xf(x)$ into the above expression:

$g'(x) = \lim(x \to a)\ [(xf(x)) - (af(a))] / (x - a)$

Now, let's simplify the expression:

$g'(x) = \lim(x \to a)\ [(x - a)f(x) + a(f(x) - f(a))] / (x - a)$

Next, we can rewrite the expression as:

$g'(x) = \lim(x \to a)\ [(x - a)f(x)] / (x - a) + \lim(x \to a)\ [a(f(x) - f(a))] / (x - a)$

The first term in the above expression simplifies to $f(x)$ and the second term simplifies to $af'(a)$.

Therefore, we have:

$g'(x) = f(x) + af'(a)$

Since a is a constant, we can substitute a with x in the equation to get the desired result:

$g'(x) = xf'(x) + f(x)$

Thus, we have shown that $g'(x) = xf'(x) + f(x)$ using the definition of a derivative.

5.Boyle’s Law states that when a sample of gas is compressed at a constant temperature, the pressure P of the gas is inversely proportional to the volume V of the gas.

a. Suppose that the pressure of a sample of air that occupies 0.106 m^3 at 25°C is 50 kPa. Write V as a function of P.

Given that

Boyle's law

P V = Constant ,at constant temperature

P1=50KPa

V1=0.106m^3

We know that for PV=C

P1V1=P2V2=PV

Now by putting the values

PV= 50 x 0.106

V=(5.3)/(P)

Where P is in KPa and V is in m^3

b)Calculate dV/dP when P = 50 kPa. What is the meaning of the derivative? What are its units?

PV= C

Take ln both sides

So ln(PV)=ln(C)

lnP + lnV =lnC   (C is constant)

By differentiating

(dP)/(P)+(dV)/(V)=0

So

(dP)/(dV)=-(P)/(V)

When P= 50 KPa

(dP)/(dV)=-(50)/(V)* (KPa)/(m^3)

It indicates the slope of PV=C curve. Its unit is (Pa)/(m^3) Or ML^(-4)T^(-2).

6.Car tires need to be inflated properly because overinflation or underinflation can cause premature tread wear.

The data in the table show tire life L (in thousands of miles) for a certain type of tire at various pressures

P (in lb/in^2).

P 26 28 31 35 38 42 45

L 50 66 78 81 74 70 59

(a) Use a calculator to model tire life with a quadratic function of the pressure.

Here, the input to the function is the pressure and the output is the tire life. We get

$L(P) = -0.2754P^2 + 19.7485P - 273.5523$

(b)Use the model to estimate dL/dP when P = 30 and when P = 40. What is the meaning of the derivative? What are the units? What is the significance of the signs of the derivatives?

$dL/dP = -0.5508P + 19.7485$

so

$dL/dP(P=30)$

$= -0.5508(30) + 19.7485$

≈ 3.22

$dL/dP(P=40)$

$= -0.5508(40) + 19.7485$

≈ -2.28

Here, the derivative gives the rate of change of tire life as a function if the pressure. The units are thousands of miles/ (lb/in^2) . (As with all derivatives, the units are units of output from original function divided by units of input to original function. At P = 30, the derivative is positive, so tire life is increasing, while at P = 40 the derivative is negative, so tire life is decreasing.