**1. Based on the experience of customs clearance inspections at the airport, the probability of luggage containing prohibited items is 10 —4. Given that the X-ray inspection machine has a probability of 1/10 of falsely identifying a regular luggage as containing prohibited items, and the probability of falsely identifying prohibited items as regular luggage is 10 —6, find the probability that it actually does contain prohibited items for a luggage belonging to someone determined by the X-ray inspection machine to contain prohibited items**

P(A): Probability that luggage contains prohibited items = 10^-4

P(B): Probability that X-ray identifies a luggage as containing prohibited items

P(B|A): Probability that X-ray identifies a luggage as containing prohibited items given that it does contain prohibited items = 1 - 10^-6 (because 10^-6 is the probability of falsely identifying prohibited items as regular)

P(B|~A): Probability that X-ray identifies a luggage as containing prohibited items given that it does not contain prohibited items = 1/10

We want to find P(A|B), which is the probability that luggage contains prohibited items given that X-ray identified it as such.

According to Bayes' theorem:

P(A|B) = P(B|A) * P(A) / P(B)

We can find P(B) using the law of total probability:

P(B) = P(B and A) + P(B and ~A) = P(B|A) * P(A) + P(B|~A) * P(~A)

Substituting the known values:

P(B) = (1 - 10^-6) * 10^-4 + 1/10 * (1 - 10^-4) ≈ 0.1

Now we can find P(A|B):

P(A|B) = (1 - 10^-6) * 10^-4 / 0.1 = 10^-6

So, the probability that a luggage actually contains prohibited items given that the X-ray machine identified it as such is approximately 10^-6. This is a very small probability, which means that even if the X-ray machine identifies a luggage as containing prohibited items, it's highly likely that this is a false alarm. This is due to the fact that the probability of a luggage containing prohibited items is very small (10^-4), and the X-ray machine has a relatively high false positive rate (1/10).

**2.There are two basketball teams A and B in the final for a 7-game series, where each team needs to win 4 out of 7 games. The probability of A winning in each game is 0.6 and B winning is 0.4 respectively. If defining the total game as N, N ∈ {4,5,6,7}, find the probability distribution of N and the expected value of N**

**Probability Distribution of N:**

The probability that team A wins the series in exactly N games can be calculated as follows:

- For N = 4, team A must win all 4 games, so the probability is (0.6)^4 = 0.1296.
- For N = 5, team A must win 3 of the first 4 games and the 5th game. This can be calculated as (4 choose 3) * (0.6)^4 * 0.4 = 0.3456.
- For N = 6, team A must win 3 of the first 5 games and the 6th game. This can be calculated as (5 choose 3) * (0.6)^4 * (0.4)^2 = 0.31104.
- For N = 7, team A must win 3 of the first 6 games and the 7th game. This can be calculated as (6 choose 3) * (0.6)^4 * (0.4)^3 = 0.186624.
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**Expected Value of N:**

The expected value of N can be calculated as the sum of the product of each value of N and its corresponding probability. In this case, it would be:
E[N] = 4 * P(N=4) + 5 * P(N=5) + 6 * P(N=6) + 7 * P(N=7) = 4 * 0.1296 + 5 * 0.3456 + 6 * 0.31104 + 7 * 0.186624 = 5.6

So, the expected number of games before team A wins the series is approximately 5.6 games.

**3. The values of the discrete random variable X are 0, 1, 2, 3 and the probability P(X) is as follows. Find the expected value of X.**

| **X** | **0** | **1** | **2** | **3** |
|---|---|---|---|---|
| **P(X)** | **0.2** | **0.1*(k+1)** | **0.3*(k+1)** | **0.2** |

The expected value of a discrete random variable X is calculated by summing the product of each outcome and its probability. In this case, the expected value E(X) is given by:

E(X)= $E(X) = \sum_{i=0}^{3} \quad x_i * P(x_i)$

where xi are the values of X and P(xi) are their corresponding probabilities.

Given the probabilities:

- P(X=0) = 0.2
- P(X=1) = 0.1*(k+1)
- P(X=2) = 0.3*(k+1)
- P(X=3) = 0.2

We can substitute these probabilities into the formula for E(X):

$E(X)=0*0.2+1*0.1*(k+1)+2*0.3*(k+1)+3*0.2$

Simplifying this gives:

$E(X)=0.1k+0.1+0.6k+0.6+0.6$

Combining like terms gives:

$E(X)=0.7k+1.3$

**4. Assuming that there are 30 male students and 20 female students in a class, the five students will be randomly selected to attend the speech contests organized by the student association. If the random variable X is the number of female students in the selected group, find the probability distribution of X**

Here:

- N is the total number of students (30 male + 20 female = 50 students),
- K is the total number of female students (20),
- n is the number of students selected (5), and
- k is the number of female students in the selected group.

The random variable X can take on values from 0 to 5 (the number of female students in the selected group). So, we can calculate the probability distribution of X as follows:

P(X = 0) = (20 choose 0) * (30 choose 5) / (50 choose 5) = 0.026

P(X = 1) = (20 choose 1) * (30 choose 4) / (50 choose 5) = 0.204

P(X = 2) = (20 choose 2) * (30 choose 3) / (50 choose 5) = 0.411

P(X = 3) = (20 choose 3) * (30 choose 2) / (50 choose 5) = 0.274

P(X = 4) = (20 choose 4) * (30 choose 1) / (50 choose 5) = 0.081

P(X = 5) = (20 choose 5) * (30 choose 0) / (50 choose 5) = 0.004

This gives the probability distribution of X, the number of female students in the selected group. Note that the “choose” notation, “n choose k”, represents the number of ways to choose k items from a group of n items without regard to the order of selection. The probabilities were calculated using a hypergeometric distribution. The total number of students is 50 (30 male + 20 female), the total number of female students is 20, and the number of students selected is 5. The random variable X can take on values from 0 to 5, which represent the number of female students in the selected group.

**5.In a batch of 12 TV sets, three of them are defective. Three TV sets will be randomly selected for inspection and the random variable X represents the number of good quality units in the inspection. If all three are good, the entire batch is accepted, otherwise, it is returned. Please answer the following questions**

**a. If the sampling is done without replacement, write the probability distribution**

**of X, the mean, and the variance. Also, fine the probability that the entire batch of TV sets can be accepted.**

**b. If the sampling is done with replacement, write the probability distribution of X, the mean, and the variance. fine the probability that the entire batch of TV sets can be accepted.**

**c. If the sampling is done without replacement, calculate the probability that the third one is one defective.**

a. The probability distribution of X if sampling is done without replacement is:

- P(X=0) = 0.025
- P(X=1) = 0.324
- P(X=2) = 0.540
- P(X=3) = 0.111

The mean of X is given by: E(X) = np = 3 * 0.7 = 2.1

The variance of X is given by: Var(X) = np(1-p)(N-n)/(N-1) = 3 * 0.7 * 9/11 = 1.8909

Here the calculations are based on the formulas for the mean and variance of a hypergeometric distribution, where:

- N is the total number of items,
- n is the number of items selected,
- p is the probability of success on a single trial, and
- K is the total number of successes.

b. When sampling is done with replacement, the problem describes a binomial distribution. The probability mass function of a binomial distribution is given by:

$P(X = k) = \binom{n}{k} * p^k * (1 - p)^{n-k}$where:

- n is the number of trials (3),

- p is the probability of success on a single trial (probability of selecting a good quality TV set = 9/12), and
- k is the number of successes.

The random variable X can take on values from 0 to 3. So, we can calculate the probability distribution of X as follows:

- P(X=0) =

  $\binom{3}{0} * (9/12)^0 * (1 - 9/12)^3$= 0.015625

- P(X=1) =

  $\binom{3}{1} * (9/12)^1 * (1 - 9/12)^2$= 0.140625

- P(X=2) =

  $\binom{3}{2} * (9/12)^2 * (1 - 9/12)^1$= 0.421875

- P(X=3
- =$\binom{3}{3} * (9/12)^3 * (1 - 9/12)^0$

  = 0.421875

The mean and variance of a binomial distribution are given by:

Mean =

n∗p

= 3 * 9/12 = 2.25

Variance = $n * p * (1 - p)$= 3 * 9/12 * 3/12 = 0.5625

The probability that the entire batch of TV sets can be accepted is P(X=3) = 0.421875. This is the probability that all three TV sets selected are good. If all three are good, the entire batch is accepted, otherwise, it is returned. So, the probability that the entire batch of TV sets can be accepted is 0.421875.

c. This is a conditional probability problem.

We have a total of 12 TV sets, with 9 good ones and 3 defective ones. If the first two TV sets selected are good, then there are 10 TV sets left, 8 of which are good and 2 are defective.

The probability that the third TV set is defective (given that the first two are good) is the ratio of the number of defective TV sets left to the total number of TV sets left, which is 2/10 = 0.2.