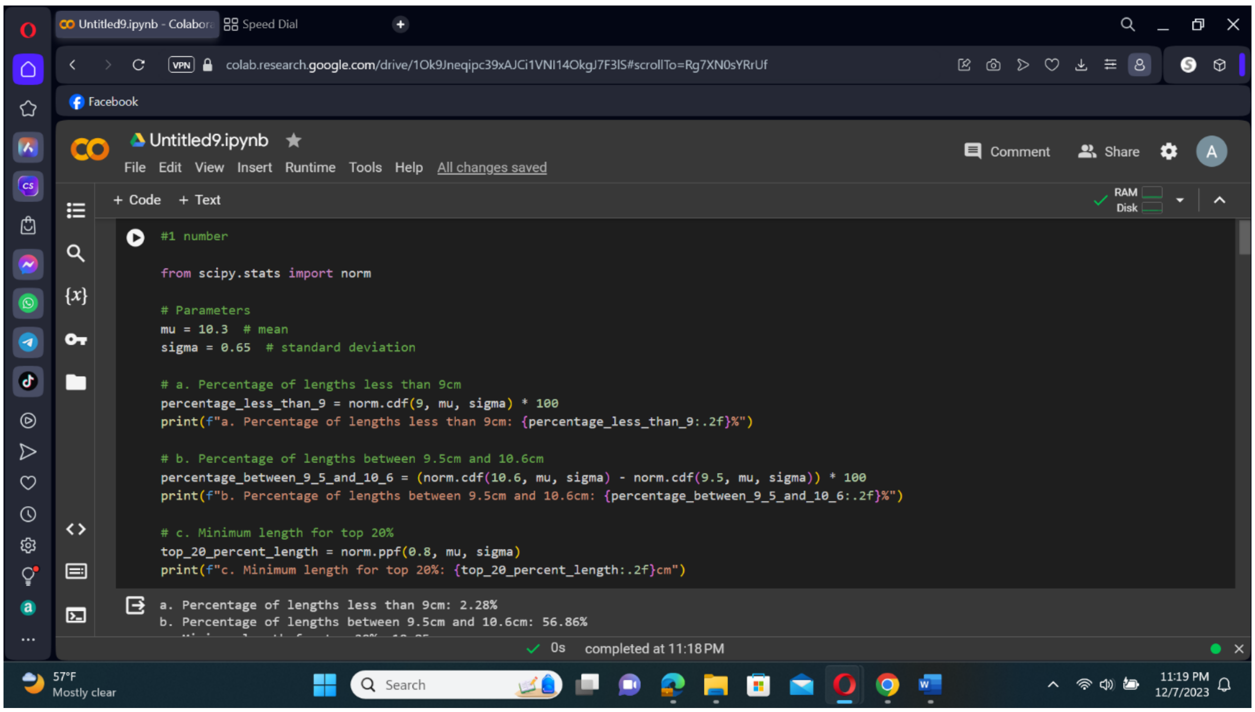
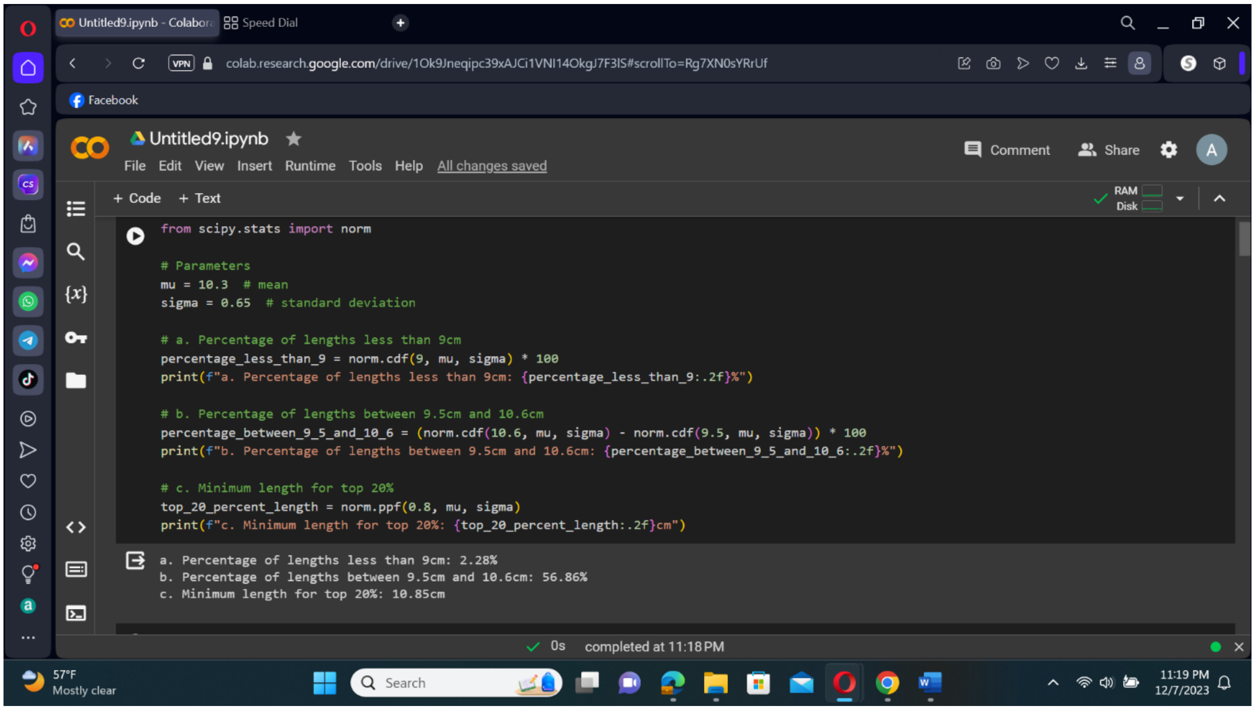
1. Assuming that the lengths of American anchovies appease the normal distribution with the mean μ = 10.3cm and standard deviation σ = 0.65cm, please find the percentages of the lengths in the population of American anchovies.

a. Less than 9cm.

b. Between 9.5cm and 10.6cm.

c. What is the minimum length if a restaurant claimed that the lengths of the sold anchovies are in the top of 20%?





2. If the random variables X and Y are normal distributions with μ = 10 & σ = 3 and μ = 15 & σ = 8, namely, $X \sim N(10, 3)$ and $Y \sim N(15, 8)$, and they are independent, what is the probability distribution and statistical parameters of (1) $X + Y$ (2) $X - Y$ (3) $3X$ (4) $4X + 5Y$.

(1) $X + Y$

The mean is obtained by summing the individual means: $\mu x + \mu y$ = 10 + 15, resulting in a mean of 25. The variance, calculated as the sum of squared standard deviations, is 3^2 + 8^2, equaling 73. Therefore, the probability distribution is denoted as N(25, √73).

(2) $X - Y$

The mean, derived from the difference of individual means ($\mu x - \mu y$ = 10 - 15), is -5. The variance remains consistent at 73. The corresponding probability distribution is represented as N (-5, √73).

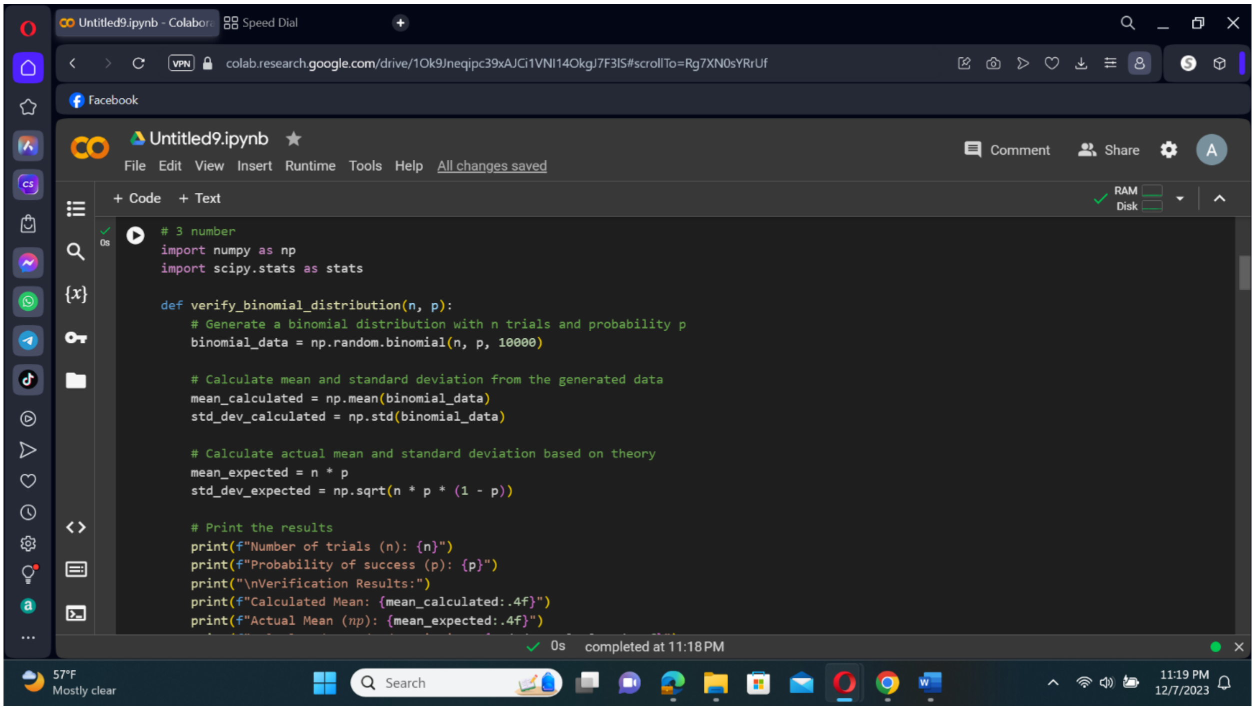
(3) $3X$

With the mean of 3 times μx (3* μx), the result is 3 * 10 = 30. The variance calculated as (3. σx) ^2, becomes (3*3) ^2, resulting in 81. Consequently, the probability distribution is N (30, 9).

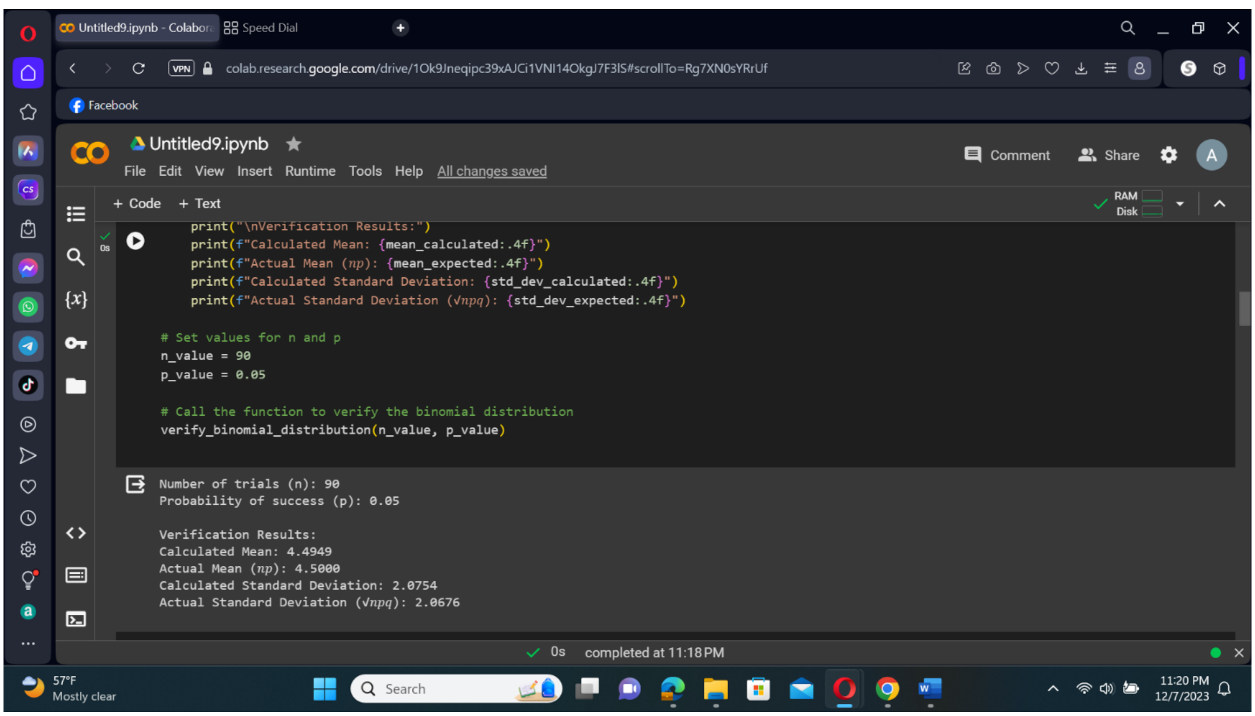
(4) $4X + 5Y$

The mean for this expression (4. μx + 5. μy) is calculated as 4 * 10 + 5 * 15 = 40 + 75, yielding a mean of 115. The variance, derived from (4. σx)^2 + (5. σy)^2, becomes (4*3)^2 + (5*8)^2, resulting in 344. The probability distribution is expressed as N(115, √344).

3. For the students in Engineering School, please write Python program to verify the mean $\mu = np$ and standard deviation σ = sqrt npq for p=0.05 and selecting any n greater than 50 in binomial distribution.



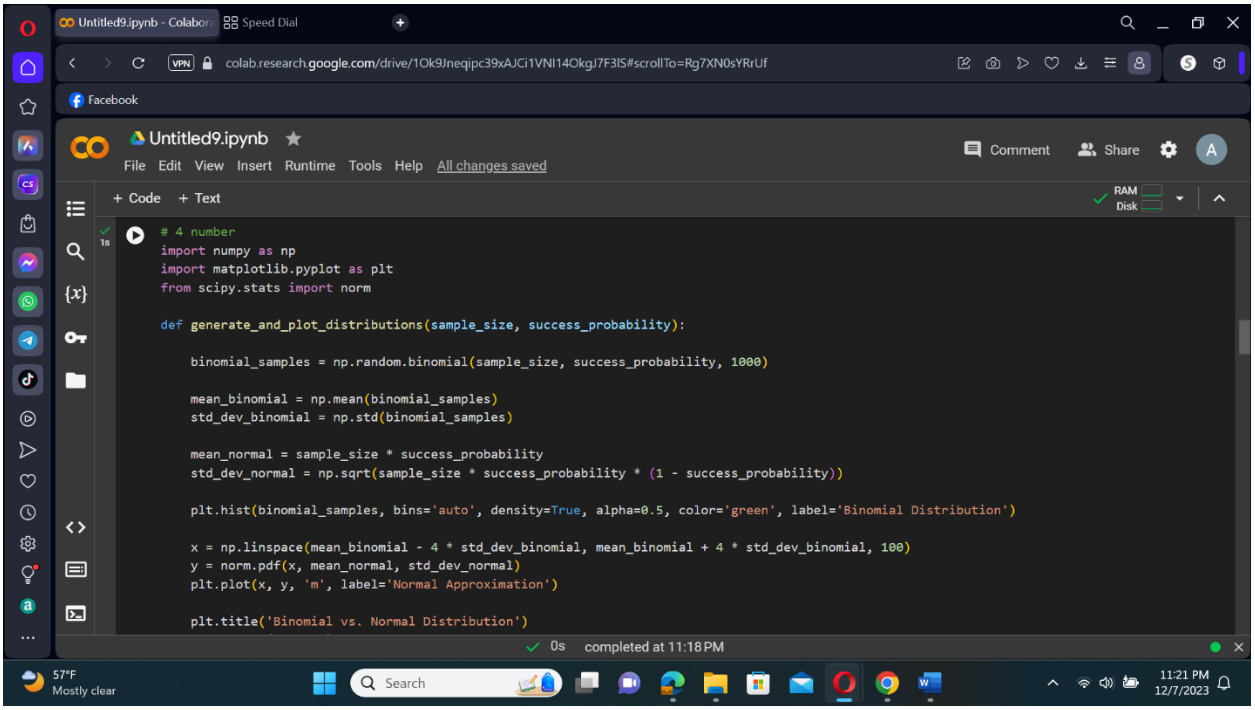
```python
# 3 number
import numpy as np
import scipy.stats as stats

def verify_binomial_distribution(n, p):
    # Generate a binomial distribution with n trials and probability p
    binomial_data = np.random.binomial(n, p, 10000)

    # Calculate mean and standard deviation from the generated data
    mean_calculated = np.mean(binomial_data)
    std_dev_calculated = np.std(binomial_data)

    # Calculate actual mean and standard deviation based on theory
    mean_expected = n * p
    std_dev_expected = np.sqrt(n * p * (1 - p))

    # Print the results
    print(f"Number of trials (n): {n}")
    print(f"Probability of success (p): {p}")
    print("\nVerification Results:")
    print(f"Calculated Mean: {mean_calculated:.4f}")
    print(f"Actual Mean (np): {mean_expected:.4f}")
```

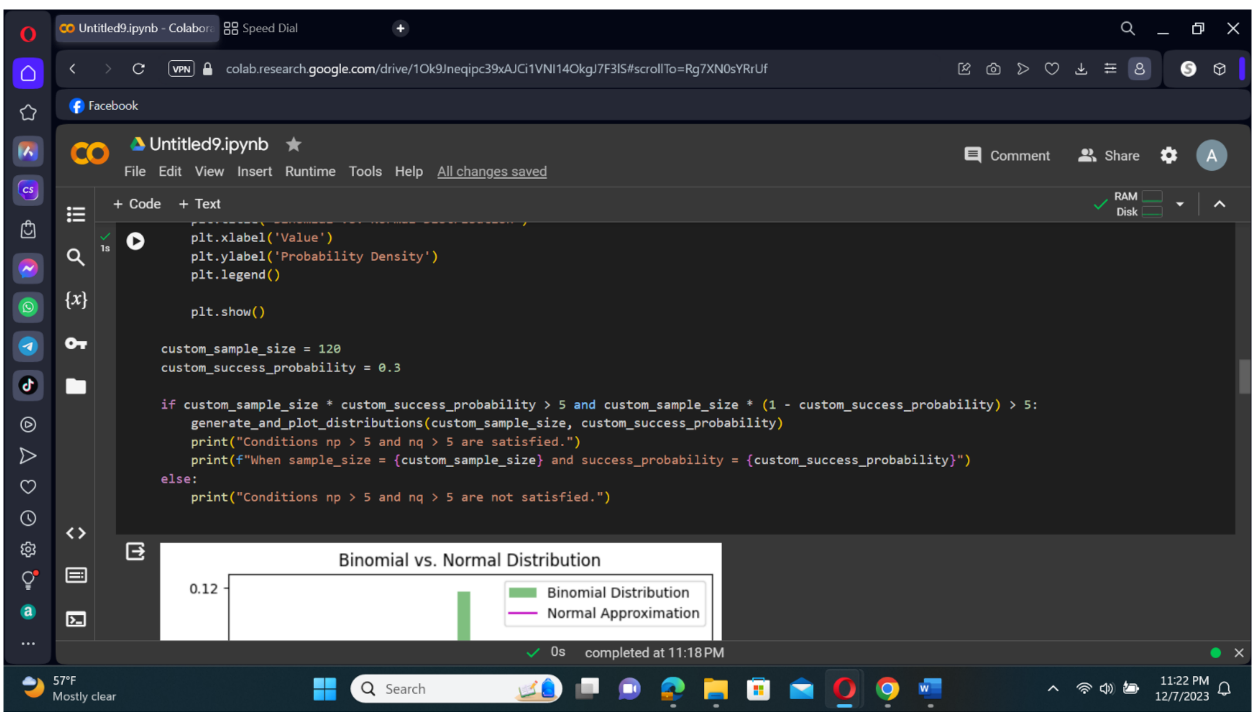


```python
    print("\nVerification Results:")
    print(f"Calculated Mean: {mean_calculated:.4f}")
    print(f"Actual Mean (np): {mean_expected:.4f}")
    print(f"Calculated Standard Deviation: {std_dev_calculated:.4f}")
    print(f"Actual Standard Deviation (√npq): {std_dev_expected:.4f}")

# Set values for n and p
n_value = 90
p_value = 0.05

# Call the function to verify the binomial distribution
verify_binomial_distribution(n_value, p_value)
```

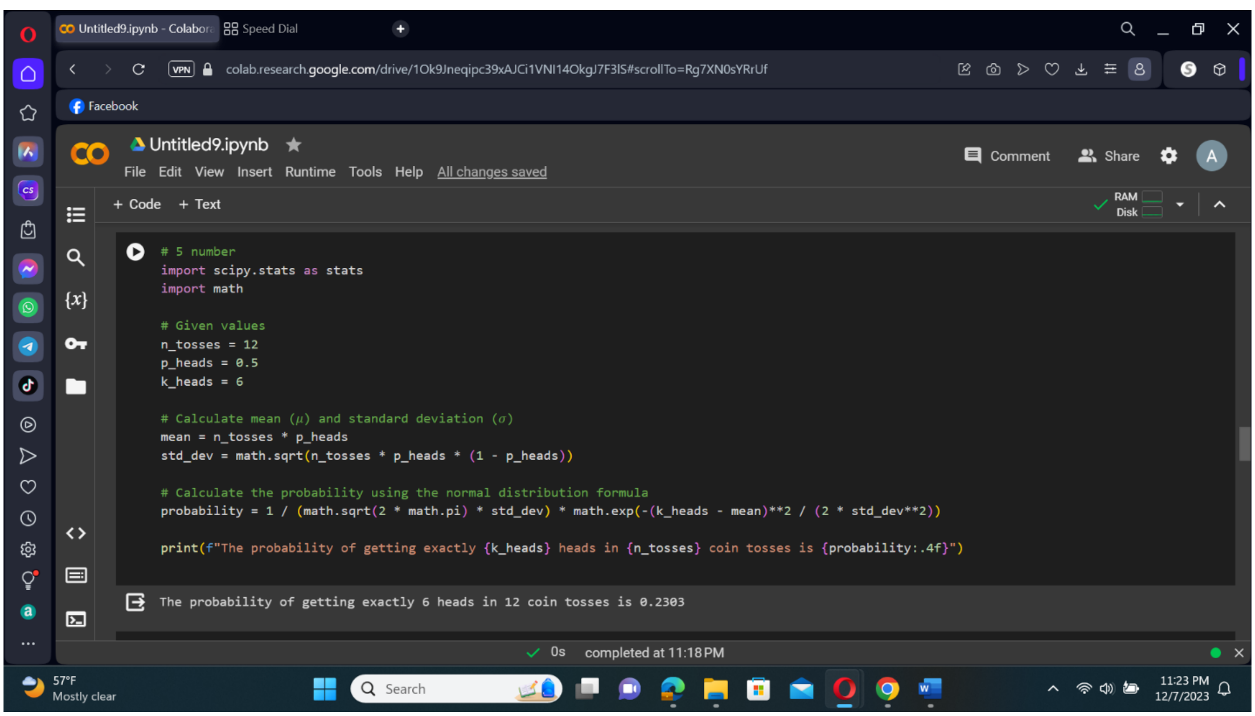
```
Number of trials (n): 90
Probability of success (p): 0.05

Verification Results:
Calculated Mean: 4.4949
Actual Mean (np): 4.5000
Calculated Standard Deviation: 2.0754
Actual Standard Deviation (√npq): 2.0676
```

4.In general, if $np > 5$ and $nq > 5$ in binomial distribution, binomial probabilities can be approximated using the normal distribution. Please select any big enough n and p's values to verify in Python program or Excel and plot the histogram.

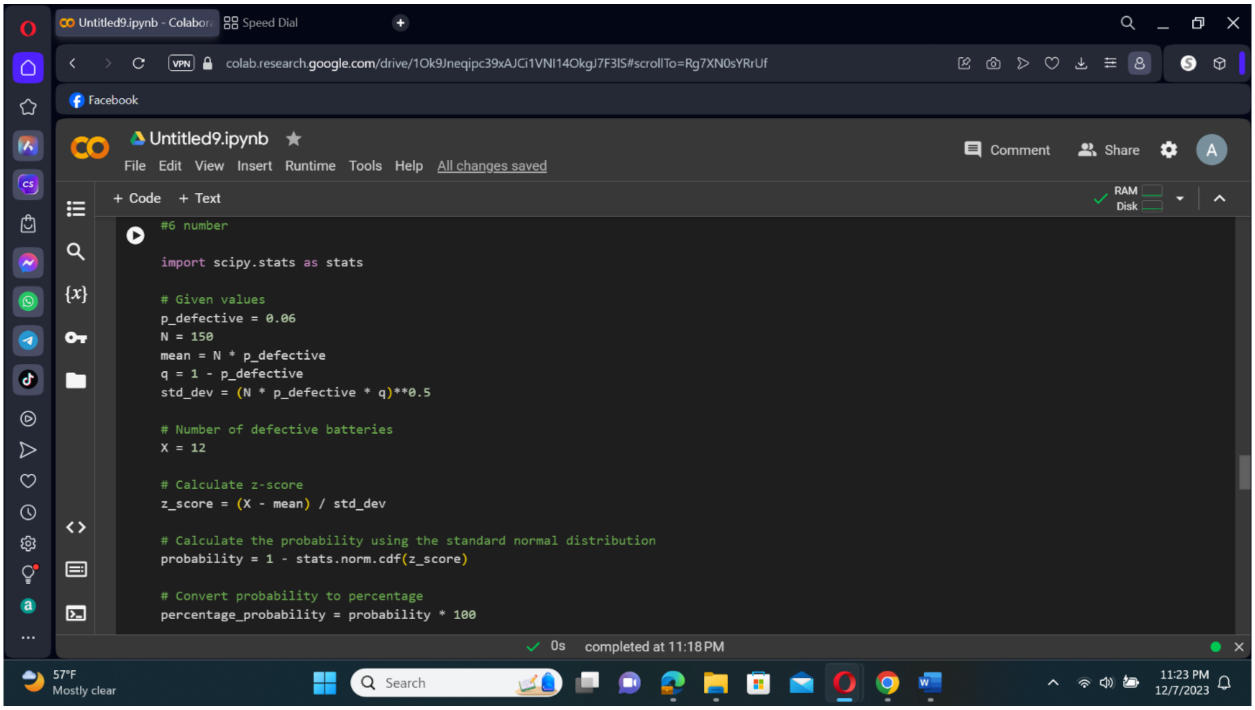


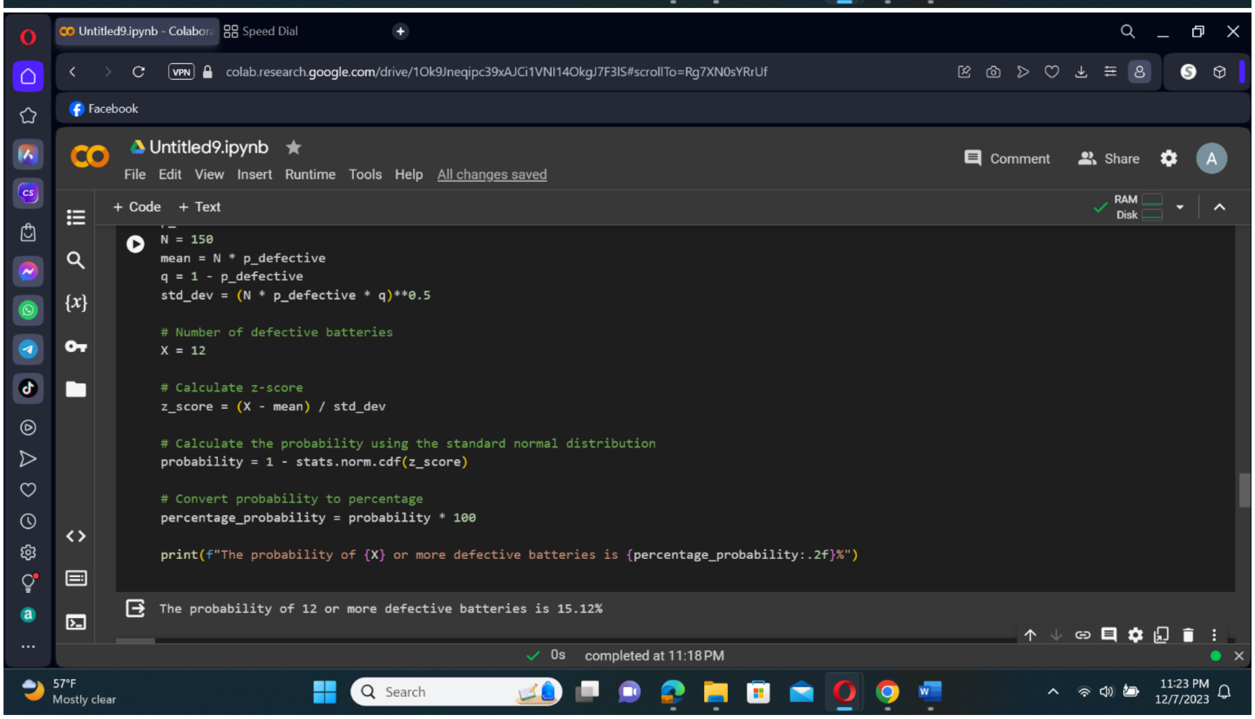


5. In coin tossing experiments, please find the probability of the exact 6 heads from 12 tossing by ONLY using the normal distribution method.

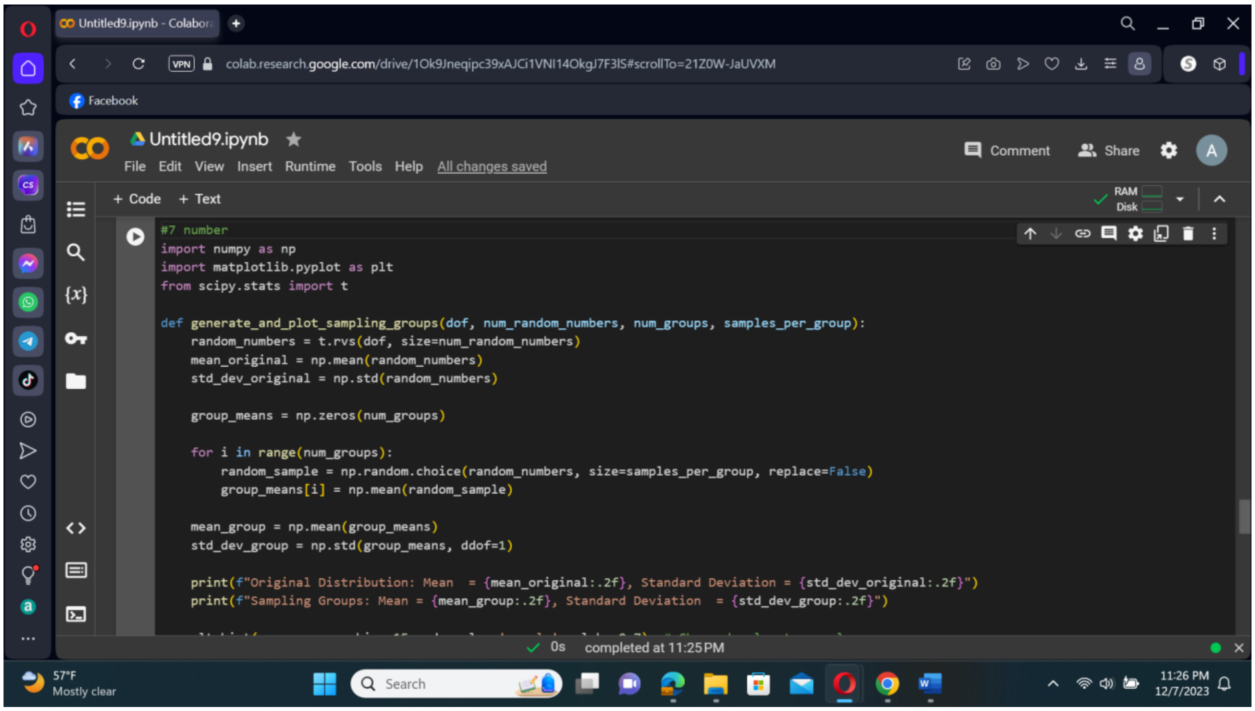


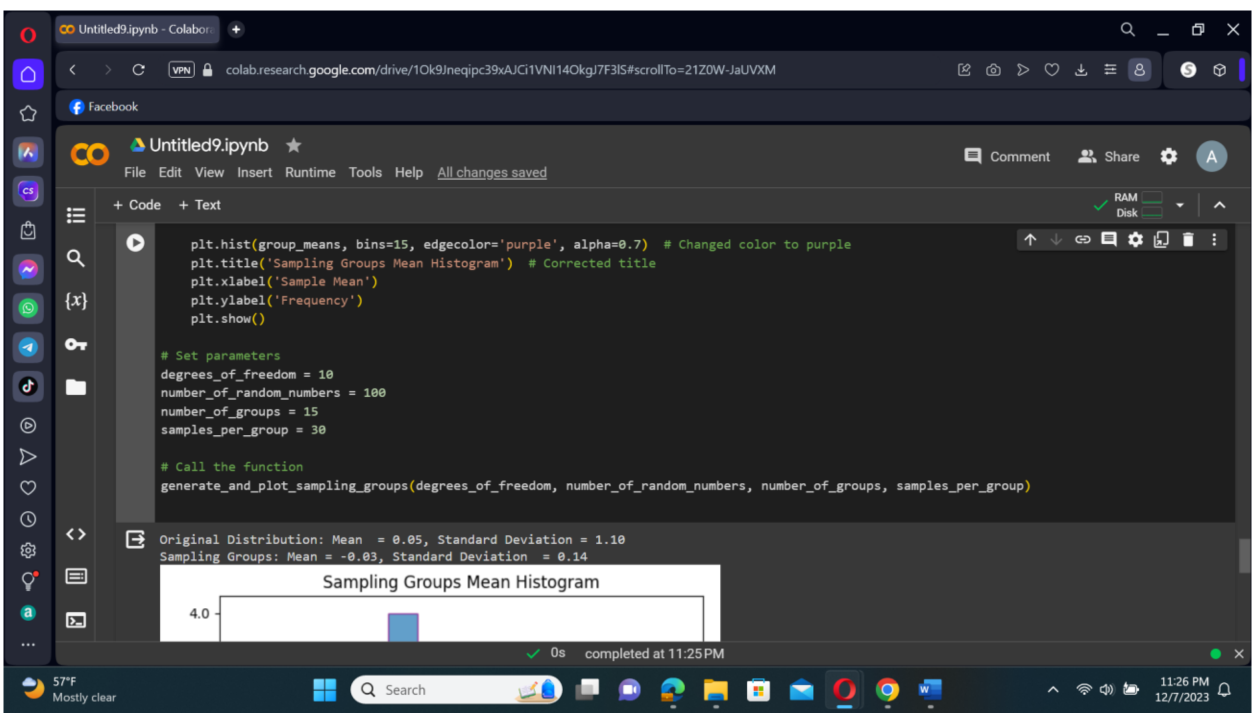
6.Given that the defective rate of a product of the batteries in a manufacturing company is 6%, 150 batteries are randomly selected from the population. Please find the probability of 12 or more defective ones in them by ONLY using the normal distribution method.

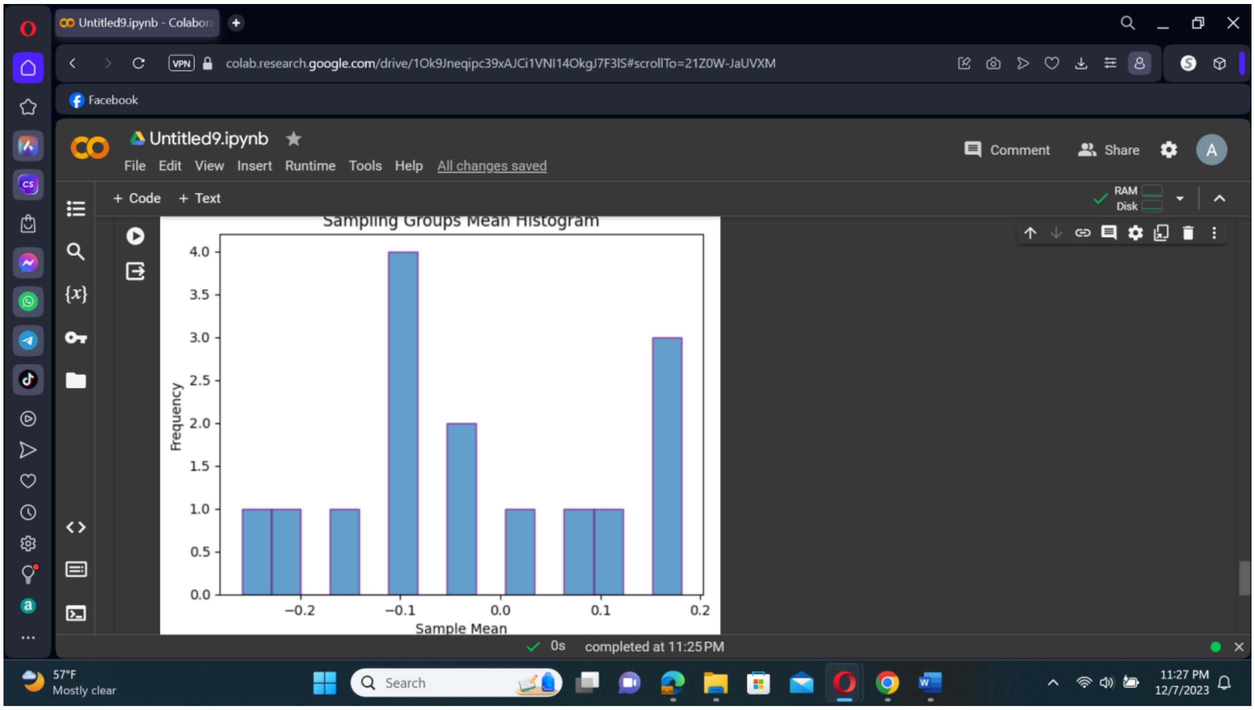




7. For the students in Engineering School, please write a Python program by calling functions in the following link to create 100 random numbers in T distribution with df =10 (degree of freedom) and calculate the mean μ and standard deviation σ. After that, the 30 samples will be randomly selected from these random numbers in each sampling group. A total of 15 sampling groups should be created. Based on Central Limit Theorem (CLT), the mean value x in total 15 sampling group is roughly the mean μ of 100 random numbers and $\sigma x = \sigma$/sqrt$n$. Please verify it and plot the histogram, which should be normal distribution. For Business school students, complete the above process in Excel.



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RAM
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Sample Mean
0s completed at 11:25PM
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