* + - 1. **Verify the following logic equivalence by the truth table in Excel.**
         1. ~ (𝑝 ⋃~𝑞) ⋃ (~𝑝⋂ ~𝑞) ≡~p

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **~(𝑝 ⋃~𝑞)⋃(~𝑝⋂ ~𝑞)≡~p** | | | | | | |
| p | q | ~p | ~q | p V ~q | ~p ∧ ~q | ~(p V ~q) V (~p ∧ ~q) |
| TRUE | TRUE | FALSE | FALSE | TRUE | FALSE | FALSE |
| TRUE | FALSE | FALSE | TRUE | TRUE | FALSE | FALSE |
| FALSE | TRUE | TRUE | FALSE | FALSE | FALSE | TRUE |
| FALSE | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE |

1. ~ ((~𝑝 ⋂𝑞) ⋃ (~𝑝⋂ ~𝑞)) ⋃(𝑝⋂𝑞) ≡𝑝

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **~ ((~𝑝 ⋂𝑞) ⋃ (~𝑝⋂ ~𝑞)) ⋃(𝑝⋂𝑞) ≡𝑝** | | | | | | | | | | |
| p | q | ~p | ~q | ~p ∧ q | ~p ∧ ~q | p ∧ q | (~p∧q)V(~p∧~q) | ~((~p∧q)V(~p∧~q)) | ~((~p∧q)V(~p∧~q)) V (p∧q) |  |
| TRUE | TRUE | FALSE | FALSE | FALSE | FALSE | TRUE | FALSE | TRUE | TRUE |  |
| TRUE | FALSE | FALSE | TRUE | FALSE | FALSE | FALSE | FALSE | TRUE | TRUE |  |
| FALSE | TRUE | TRUE | FALSE | TRUE | FALSE | FALSE | TRUE | FALSE | FALSE |  |
| FALSE | FALSE | TRUE | TRUE | FALSE | TRUE | FALSE | TRUE | FALSE | FALSE |  |

1. (𝑝 ⋂(~(~𝑝⋂𝑞))) ⋃ (𝑝⋂ 𝑞) ≡𝑝

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | **(𝑝⋂(~(~𝑝⋂𝑞)))⋃(𝑝⋂𝑞)≡𝑝** | |  |  |
| p | q | ~p | p ∧ q | ~p ∧ q | ~(~p ∧ q) | p ∧ (~(~p ∧ q)) | (p ∧ (~(~p ∧q))) V (p ∧ q) |
| TRUE | TRUE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE |
| TRUE | FALSE | FALSE | FALSE | FALSE | TRUE | TRUE | TRUE |
| FALSE | TRUE | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE |
| FALSE | FALSE | TRUE | FALSE | FALSE | TRUE | FALSE | FALSE |

***2.* Let the symbol ⊕ denote exclusive or; so 𝑝 ⊕ 𝑞 ≡ (𝑝⋃𝑞) ⋂∼(𝑝⋃𝑞). Hence the   
truth table for p ⊕ q is as follows:  
 \*Note: p and q have the same value, the result of exclusive or will be 0 or   
false, otherwise it is 1 or true.  
# Find the following values or verify the logic equivalence in Excel.**

a. (𝑝 ⊕𝑝) ⊕𝑝

|  |  |  |
| --- | --- | --- |
| **p** | **p ⊕p** | **(p ⊕ p) ⊕p** |
| FALSE | FALSE | FALSE |
| TRUE | TRUE | FALSE |

b. Is (𝑝 ⊕ 𝑞) ⊕ 𝑟≡𝑝 ⊕ (𝑞 ⊕ 𝑟)?

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p ⊕ q** | **q ⊕ r** | **(p⊕ q) ⊕ r** | **p ⊕ (q ⊕ r)** |
| 0 | 0 | 0 | FALSE | FALSE | FALSE | FALSE |
| 1 | 0 | 0 | TRUE | FALSE | TRUE | TRUE |
| 0 | 1 | 0 | TRUE | TRUE | TRUE | TRUE |
| 1 | 1 | 0 | FALSE | TRUE | FALSE | FALSE |
| 0 | 0 | 1 | FALSE | TRUE | TRUE | TRUE |
| 1 | 0 | 1 | TRUE | TRUE | FALSE | FALSE |
| 0 | 1 | 1 | TRUE | FALSE | FALSE | FALSE |
| 1 | 1 | 1 | FALSE | FALSE | TRUE | TRUE |

Therefore, (𝑝 ⊕ 𝑞) ⊕ 𝑟 ≡ 𝑝 ⊕ (𝑞 ⊕ 𝑟).

1. Is (𝑝 ⊕𝑞)⋂𝑟 ≡ (𝑝⋂𝑟) ⊕ (𝑞⋂𝑟) ?

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **p ⊕ q** | **(p ⊕ q) ∧ r** | **p ∧ r** | **q ∧ r** | **(p ∧ r) ⊕ (q ∧ r)** |
| 0 | 0 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE |
| 1 | 0 | 0 | TRUE | FALSE | FALSE | FALSE | FALSE |
| 0 | 1 | 0 | TRUE | FALSE | FALSE | FALSE | FALSE |
| 1 | 1 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE |
| 0 | 0 | 1 | FALSE | FALSE | FALSE | FALSE | FALSE |
| 1 | 0 | 1 | TRUE | TRUE | TRUE | FALSE | TRUE |
| 0 | 1 | 1 | TRUE | TRUE | FALSE | TRUE | TRUE |
| 1 | 1 | 1 | FALSE | FALSE | TRUE | TRUE | FALSE |

Therefore, (𝑝 ⊕𝑞)⋂𝑟 ≡ (𝑝⋂𝑟) ⊕ (𝑞⋂𝑟).

***3:* Suppose that p and q are statements so that p→q is false. Find the truth values of each of the following:**

Answer: p→q is false and that q is False and p is true.

1. ~p→q

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p | q | ~p | p→q | ~p→q |
| TRUE | FALSE | FALSE | FALSE | TRUE |

1. p V q

|  |  |  |
| --- | --- | --- |
| p | q | p V q |
| TRUE | FALSE | TRUE |

1. q→p

|  |  |  |
| --- | --- | --- |
| p | q | q→p |
| TRUE | FALSE | TRUE |

***4*: If statement forms P and Q are logically equivalent, then 𝑃↔𝑄 is a tautology. Conversely, if 𝑃↔𝑄 is a tautology, then P and Q are logically equivalent. Use ↔ to convert each of the logical equivalences to a tautology. Then use a truth table to verify each tautology in Excel.**

1. ***p→(𝑞V𝑟) ≡ (𝑝∧~𝑞) → 𝑟***

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | ~p | ~q | qVr | p ∧ ~q | ~(p ∧ ~q) | p→(qVr) | (p∧~q)→r | p→(𝑞V𝑟) ≡ (𝑝∧~𝑞) → 𝑟 |
| 0 | 0 | 0 | TRUE | TRUE | FALSE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 0 | 0 | FALSE | TRUE | FALSE | TRUE | FALSE | FALSE | FALSE | TRUE |
| 0 | 1 | 0 | TRUE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 1 | 0 | FALSE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 0 | 0 | 1 | TRUE | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 0 | 1 | FALSE | TRUE | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE |
| 0 | 1 | 1 | TRUE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 1 | 1 | FALSE | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |

1. ***p ∧ (𝑞V𝑟) ≡ (𝑝∧𝑞) V (𝑝∧𝑟)***

(𝑝∧𝑞) V (𝑝∧𝑟) ≡ p∧ (q V r)

Using Distributive law

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | p ∧ q | q V r | p ∧ r | p∧(q V r) | (p ∧ q) V (p ∧ r) | p ∧ (𝑞V𝑟) ≡ (𝑝∧𝑞) V (𝑝∧𝑟) |
| 0 | 0 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE |
| 1 | 0 | 0 | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE |
| 0 | 1 | 0 | FALSE | TRUE | FALSE | FALSE | FALSE | TRUE |
| 1 | 1 | 0 | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE |
| 0 | 0 | 1 | FALSE | TRUE | FALSE | FALSE | FALSE | TRUE |
| 1 | 0 | 1 | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 0 | 1 | 1 | FALSE | TRUE | FALSE | FALSE | FALSE | TRUE |
| 1 | 1 | 1 | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE |

1. ***p→(𝑞→𝑟)≡(𝑝∧𝑞)→𝑟***

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | p ∧q | ~(p∧q) | ~p | ~q | q→ r | p→(q→ r) | (𝑝∧𝑞)→𝑟 | p→(𝑞→𝑟)≡(𝑝∧𝑞)→𝑟 |
| 0 | 0 | 0 | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 1 | 0 | 0 | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 0 | 1 | 0 | FALSE | TRUE | TRUE | FALSE | FALSE | TRUE | TRUE | TRUE |
| 1 | 1 | 0 | TRUE | FALSE | FALSE | FALSE | FALSE | FALSE | FALSE | TRUE |
| 0 | 0 | 1 | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 1 | 0 | 1 | FALSE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE | TRUE |
| 0 | 1 | 1 | FALSE | TRUE | TRUE | FALSE | TRUE | TRUE | TRUE | TRUE |
| 1 | 1 | 1 | TRUE | FALSE | FALSE | FALSE | TRUE | TRUE | TRUE | TRUE |
|  |  |  |  |  |  |  |  |  |  |  |

5. The famous detective Percule Hoirot was called in to solve a baffling murder mystery.   
He determined the following facts:  
a. Lord Hazelton, the murdered man, was killed by a blow on the head with a   
brass candlestick.  
b. Either Lady Hazelton or a maid, Sara, was in the dining room at the time of   
the murder.  
c. If the cook was in the kitchen at the time of the murder, then the butler killed   
Lord Hazelton with a fatal dose of strychnine.  
d. If Lady Hazelton was in the dining room at the time of the murder, then the   
chauffeur killed Lord Hazelton.  
e. If the cook was not in the kitchen at the time of the murder, then Sara was not   
in the dining room when the murder was committed.  
f. If Sara was in the dining room at the time the murder was committed, then   
the wine steward killed Lord Hazelton.  
Is it possible for the detective to deduce the identity of the murderer from these facts? If   
so, who did murder Lord Hazelton? (Assume there was only one cause of death.)

# Answer

From the question, it can be inferred that **fact C** is **not true**because he did not die from a fatal dose of **strychnine**. So, it can be concluded that at the time of the murder in **cook**was **not** in the **kitchen**. If the **cook**was not in the **kitchen**at the time of the **murder**, that means **Sara**was not in the **dining room**at the time of the **murder**from fact E. So it can be concluded that it was **Lady Hazelton**who was in the **dining room**. If Lady Hazelton was in the **dining room**at the time of the **murder,** the one who killed **Lord Hazelton**was the **chauffeur**from fact D. Hence, According to the above, the one who killed **Lord Hazelton**was the **chauffeur.**