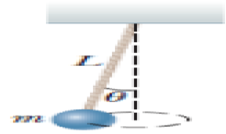
1. Consider a conical pendulum in the following figure with a bob of mass $m = 80.0$ kg on a string of length $L = 10.0$ m that makes an angle of $\theta = 5.00^{\circ}$ with the vertical. Determine (a) the horizontal and vertical components of the force exerted by the string on the pendulum and (b) the radial acceleration of the bob.



(a) Force Components

The tension (T) in the wire provides both the horizontal and vertical components that balance the forces on the bob.

Vertical component (T_v):

$$T_v = mg\cos(\theta)$$

Given:

$$m = 80\,\text{kg}, \quad g = 9.81\,\text{m/s}^2, \quad \theta = 5.00^\circ = 0.0873\,\text{radians}$$

Calculating:

$$T_v = 80\,\text{kg} \times 9.81\,\text{m/s}^2 \times \cos(0.0873)$$

$$T_v \approx 783.8\,\text{N (Newtons)}$$

This upward force balances the weight of the bob (mg).

Horizontal component (T_h):

$$T_h = T\sin(\theta)$$

Since $T_v = mg$, we can find T:

$$T = \frac{T_v}{\cos(\theta)} \approx \frac{783.8\,\text{N}}{\cos(0.0873)} \approx 787.3\,\text{N}$$

Then,

$$T_h = 787.3\,\text{N} \times \sin(0.0873) \approx 68.7\,\text{N}$$

(b) Radial Acceleration (a_r)

$$a_r = \frac{v^2}{r}$$

Where v is the tangential speed of the bob and r is the horizontal distance from the pivot to the bob.

$$r = L\sin(\theta) = 10.0\,\text{m} \times \sin(0.0873) \approx 0.872\,\text{m}$$

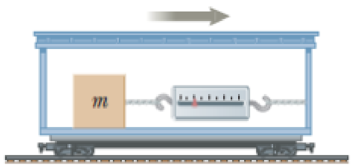
Using T_h as the centripetal force:

$$T_h = ma_r$$

Therefore:

$$a_r = \frac{T_h}{m} = \frac{68.7\,\text{N}}{80\,\text{kg}} \approx 0.86\,\text{m/s}^2$$

2. An object of mass $m = 5.00$ kg, attached to a spring scale, rests on a frictionless, horizontal surface as shown in the following figure. The spring scale, attached to the front end of a boxcar, reads zero when the car is at rest. (a) Determine the acceleration of the car if the spring scale has a constant reading of 18.0 N when the car is in motion. (b) What constant reading will the spring scale show if the car moves with constant velocity? Describe the forces on the object as observed (c) by someone in the car and (d) by someone at rest outside the car.



As the surface is frictionless, the only force acting on the object is the tension in the spring, which is read by the spring scale.

(a) Acceleration of the car

If the reading of the scale is 18.0 N, this means the tension in the spring, and hence the force on the object, is 18.0 N. The acceleration of the car and the object is given by the equation:

$$F = ma$$

Solving for acceleration a:

$$a = \frac{F}{m} = \frac{18.0\,\mathrm{N}}{5.00\,\mathrm{kg}} = 3.60\,\mathrm{m/s^2}$$

(b) Spring scale reading at constant velocity

If the car is moving with constant velocity, the acceleration of the car and the object is zero. Hence, the reading of the spring scale, or the tension in the spring, will be:

$$F = ma = 5.00\,\mathrm{kg} \times 0 = 0\,\mathrm{N}$$

(c) Observations from inside the car

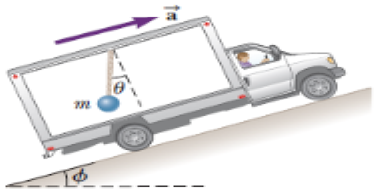
When the reading on the spring scale is 18.0 N (while the car is moving right), the person inside the car will observe that the object has no acceleration relative to them. However, since the scale reads 18.0 N, this indicates a force of 18.0 N is acting on the object to the left, which stretches the spring to this reading. Such forces are called pseudo-forces.

(d) Observations from outside the car

A person at rest outside the car will see that the object is accelerating to the right with an acceleration of $3.60\,\mathrm{m/s^2}$. Thus, the force acting on it is:

$$F = ma = 5.00\,\mathrm{kg} \times 3.60\,\mathrm{m/s^2} = 18.0\,\mathrm{N}$$

3. A truck is moving with constant acceleration α up a hill that makes an angle ϕ with the horizontal as in the following figure. A small sphere of mass m is suspended from the ceiling of the truck by a light cord. If the pendulum makes a constant angle θ with the perpendicular to the ceiling, what is α?



Forces Analysis

- Tension (T) acts along the cord. - Gravitational force (mg) acts vertically downward. - Inertial force (ma) due to truck's acceleration acts horizontally.

Equations of Motion

In the x-direction (along the incline):

$$\sum F_x = ma_x : +T\sin\theta - mg\sin\phi = ma \tag{1}$$

In the y-direction (perpendicular to the incline):

$$\sum F_y = ma_y : +T\cos\theta - mg\cos\phi = 0 \tag{2}$$

Solving for Tension T

From the y-direction equation:

$$T\cos\theta = mg\cos\phi \tag{3}$$

$$T = \frac{mg\cos\phi}{\cos\theta} \tag{4}$$

Substitute T into the x-direction equation

$$\frac{mg\cos\phi}{\cos\theta}\sin\theta - mg\sin\phi = ma$$
$$\frac{mg\cos\phi\sin\theta}{\cos\theta} - mg\sin\phi = ma$$
$$mg\left(\frac{\cos\phi\sin\theta}{\cos\theta} - \sin\phi\right) = ma$$
$$a = g\left(\frac{\cos\phi\sin\theta}{\cos\theta} - \sin\phi\right)$$
$$a = g(\cos\phi\tan\theta - \sin\phi)$$

Acceleration of the Truck α

$$\alpha = g(\cos\phi\tan\theta - \sin\phi) \tag{5}$$