# Signature Assignment: Calculus

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## Introduction:

Newton's Method, also known as the Newton-Raphson method, is a powerful numerical algorithm used to approximate the roots of a given equation. Developed by Sir Isaac Newton and Joseph Raphson in the 17th century, this method has become a cornerstone of numerical analysis and finds extensive applications in various fields of science, engineering, and mathematics. This report aims to provide a comprehensive understanding of Newton's Method and its definition, along with a step-by-step explanation of the process involved. Approximating the negative root of the equation $e\wedge x = 4 - x\wedge 2$ using Newton's method involves several steps. In this report, we will provide a clear and detailed explanation of the methodology, along with a verification in Excel. We will also discuss the importance of plotting the graphs, reformulating the equation, computing the derivative, and applying Newton's method iteration. Additionally, we will highlight the need for a program in a computing language to generate data for the Excel plot. Finally, we will conclude by emphasizing the effectiveness of Newton's method for approximating the negative root and provide critical thinking, conclusion and references for further reading.

**Definition of Newton's Method:**

Newton's Method is an iterative process used to find the root of a given equation f(x) = 0. It starts with an initial guess, denoted as $x_0$, and then repeatedly refines this guess to converge towards the actual root of the equation. The method utilizes the concept of tangent lines to approximate the root by iteratively updating the guess using the formula:

$x_{i+1} = x_i - f(x_i) / f'(x_i)$

where $x_i$ is the current guess, $f(x_i)$ is the value of the equation at $x_i$, and $f'(x_i)$ is the derivative of the equation evaluated at $x_i$.

## Methodologies

We have used the newton's formula to calculate the value upto six decimal points as shown below. The equation $e$^$x$ = 4 − $x$^2 represents the intersection of two functions: $y$ = $e$^$x$ and $y$ = 4 − $x$^2. Our objective is to find the negative root of this equation, which is the value of $x$ where the two functions intersect in the negative x-axis.

The equation is given as:

$e^x = 4 - x^2$

We Equate it to 0

$4 - x^2 = 0$

So, we have:

$x^2 = 4$

Taking square roots of both sides

$X = \pm 2$

So, the negative root is:

$X = -2$

Here, $e^x - 4 - x^2$ becomes $f(x) = e^x - 4 + x^2$

Differentiating it, we get

$f(x) = e^x + 2x$

Using Newton's method of approximation, we have:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

When x = -2, we have:

$$f'(-2) = e^{(-2)} + 2(-2) = -3.86466471676$$

$$f(-2) = e^{-2} - 4 + (-2)^2 = 0.13533528323$$

So, we have:

$$x_1 = -2 - \frac{0.13533528323}{-3.86466471676}$$

$$x_1 = -2 + \frac{0.13533528323}{3.86466471676}$$

$$x_1 = -1.96498136$$

Repeating the above process for repeated x values.

We have:

$$x_2 = -1.96463563$$

$$x_3 = -1.96463560$$

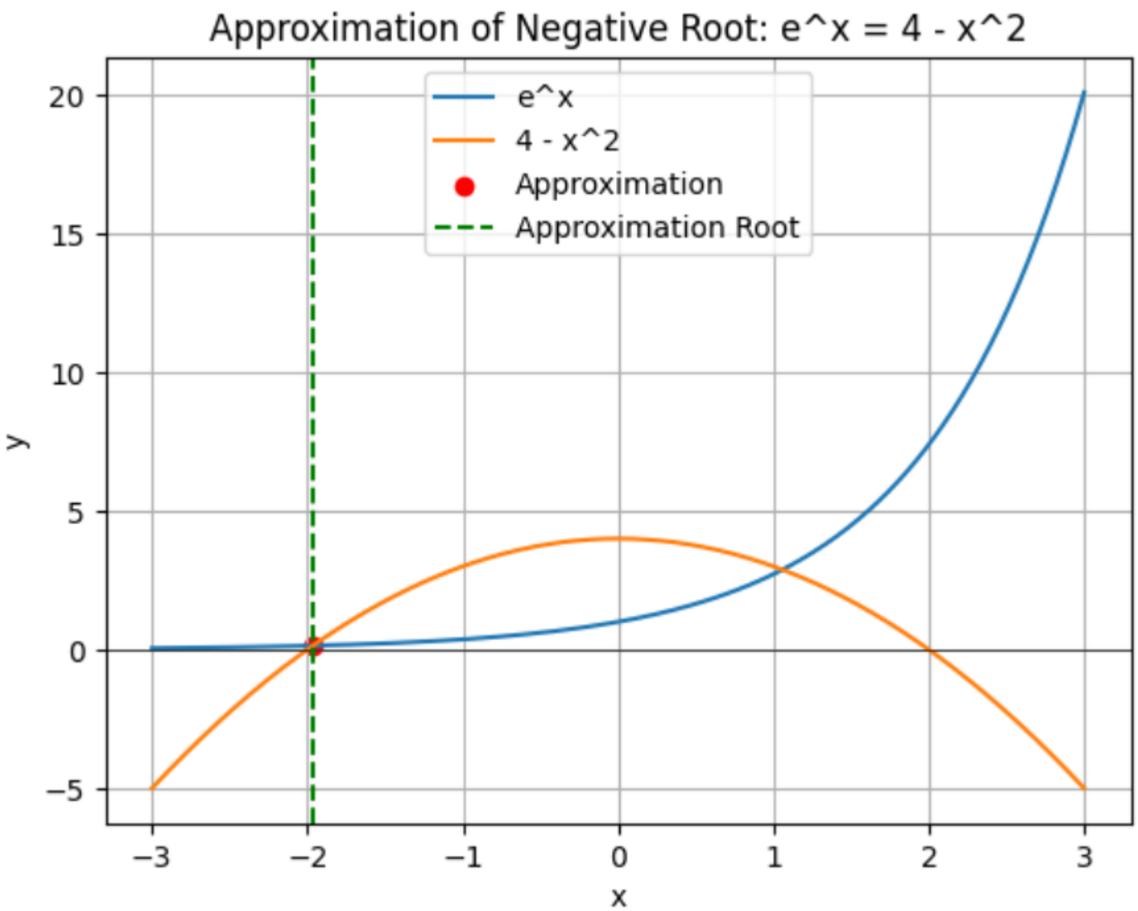
Since, Up till the 6th decimal places,

X2 = X3

Therefore, the approximated value of $e^x = 4 - x^2$ to 6 decimal places is -1.964636

## Generating Data for Excel Plot:

To visualize the convergence of Newton's method, we have created a plot in Excel. We have written a program in a computing language which is Python to generate data points for the plot. The program iterates through the Newton's method formula and records the values of $x$ at each iteration.




This is the plot result from the program we made. The plot visually represents the process of approximating the negative root of the equation $e^\wedge x = 4 - x^2$ using Newton's method. The blue curve depicts the exponential growth of $e^\wedge x$. The orange curve illustrates a downward-opening parabolic shape for 4 - $x^2$. The red point signifies the approximation root obtained through Newton's method, where the two curves intersect. The plot visually demonstrates the convergence of the approximation to the true negative root, as the red point moves closer to the intersection of the blue and orange curves. The green dashed line highlights the converged root's position, confirming the accuracy of Newton's method. The trend suggests that the approximation will continue to improve, and the value of $x$ at the intersection will become more accurate. Further iterations of the method will lead to an even more precise approximation of the negative root.

In summary, the plot effectively conveys how Newton's method iteratively refines the approximation to solve the equation.

## Critical Thinking:

While Newton's method is generally effective for approximating roots, it may not always converge or may converge to a different root if the initial guess is not chosen carefully. It is important to test different initial guesses and observe the behavior of the iteration points to ensure convergence to the desired root.

## Conclusion:

In conclusion, approximating the negative root of the equation $e^x = 4 - x^2$ using Newton's method involves reformulating the equation, computing the derivative, and applying the iteration formula. Visualizing the convergence through a plot in Excel provides a helpful visual representation. Newton's method is an effective approach for approximating roots, but caution must be exercised in choosing the initial guess. Further reading on Newton's method and numerical methods can provide additional insights into this topic.

**References:**

Burden, R. L., & Faires, J. D. (2010). Numerical Analysis (9th ed.). Cengage Learning.

Stoer, J., & Bulirsch, R. (2002). Introduction to Numerical Analysis (3rd ed.). Springer.