

# 550.400: Mathematical Modeling and Consulting

## Lecture Notes

**Instructor:**

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# Outline

Preliminaries

Principles

Tools

Arguments from Scale

Graphical Methods

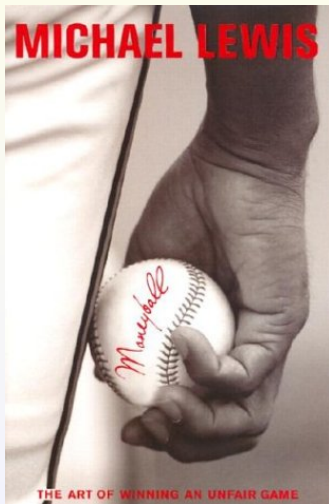
Basic Optimization

# Syllabus

- Grade Policy
- Attendance
- *Tentative* Schedule
- Blackboard
- Misc.

# What is Mathematical Modeling?

## Money Ball



# What is Mathematical Modeling?

## Trillion Dollar Bet

### [NOVA Online | Trillion Dollar Bet](#)

[www.pbs.org/wgbh/nova/stockmarket/](http://www.pbs.org/wgbh/nova/stockmarket/)

Welcome to the companion Web site to "**Trillion Dollar Bet**," originally broadcast on February 8, 2000. The film tells the fascinating story of the invention of the ...

[The Formula that Shook The ...](#) - [Transcripts](#) - [A Trader's Lexicon](#) - [Resources](#)

### [Videos for trillion dollar bet](#) - [Report videos](#)



#### [Trillion Dollar Bet 1 - YouTube](#)

[youtube.com](http://youtube.com)

Jan 8, 2009



#### [The Trillion Dollar Bet - YouTube](#)

[youtube.com](http://youtube.com)

Sep 15, 2007



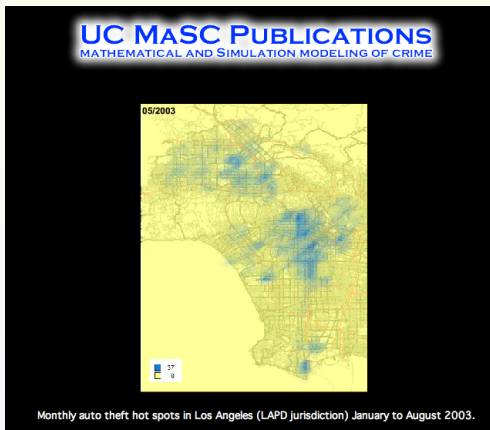
#### [The Midas Formula Stockmarket ...](#)

[youtube.com](http://youtube.com)

Aug 23, 2011

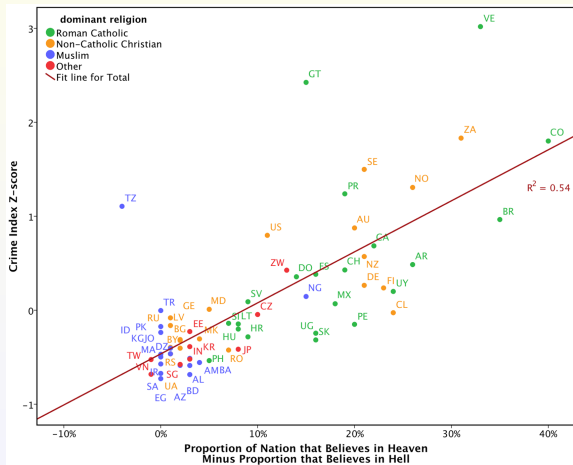
# What is Mathematical Modeling?

## LAPD Fighting Crime with Math



# What is Mathematical Modeling?

## Crime rates and religious beliefs



# More Project Ideas

<http://www.stat.berkeley.edu/>

<http://www.math.msu.edu/>

<http://www.mathgoespop.com/>



# Models and Reality

*The ultimate test of a model is how well it performs when it is applied to the problem it was designed to handle.*

*A model is used, it may lead to incorrect predictions. The model is often modified, frequently discarded, and sometimes used anyway because it is better than nothing. This is the way science develops.*

# Models and Reality

What makes Mathematical models useful? We must/have/have/have:

- formulate our ideas precisely and so are less likely to let implicit assumptions slip by,
- concise “language” which encourages manipulation,
- a large number of potential theorems available,
- high speed computers available for carrying out calculations.

# Properties of Models

As far as a model is concerned, the world can be divided into three parts:

- Things whose effects are neglected,
- Things that affect the model but whose behavior the model is not designed to study,
- Things the model is designed to study the behavior of.

## A recurring theme

Frequently Recurring Elements of doing a Project in Industry:

1. Work Statement,
2. Midterm Presentation,
3. Progress Report,
4. Final Presentation,
5. Final Report.



# What is Work Statement?

- The written proposal and definition of the project
- Your consulting team's "contract" with the sponsor
- It is ultimately given to the sponsor for review and signature

# What is Work Statement?

It sets forth:

- the nature of the project,
- the specific objectives of the project,
- the result expected,
- the “deliverable” for the project.

# What is Work Statement?

- The scope of the project must be within the time table for the course
- The deliverables are reasonable and appropriate

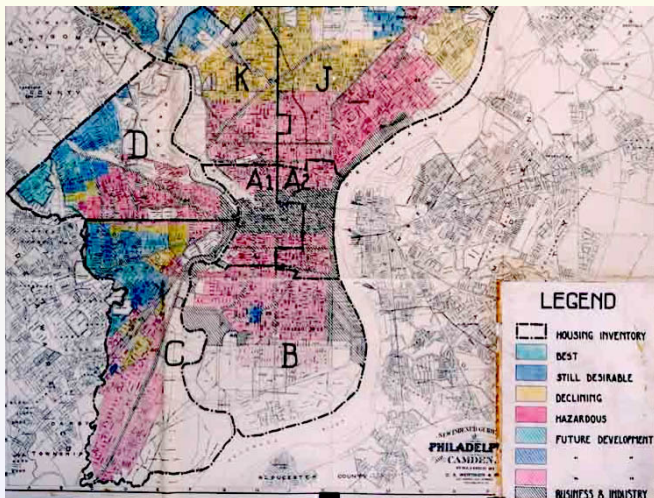
# What is Work Statement?

- Given the nature of research, it should not include promises that your consulting team cannot be certain to achieve
- It may be necessary after discussion and agreements among various parties to modify and renegotiate the work statement as the project progresses



# Example: Insurance Redlining

## Insurance Redlining



## Example: Insurance Redlining

### Insurance Redlining

*Insurance redlining* refers to the practice of refusing to issue insurance to certain types of people or within some geographic area.

### FAIR

The **FAIR** plan was offered by the city of Chicago as a default policy to homeowner who had been rejected by the voluntary market.

## Example: Insurance Redlining

### Sponsor

*The **U.S. Commission on Civil Rights** examined charges by several Chicago community organizations that insurance companies were redlining their neighborhoods.*

### Data

*The **number of FAIR plan policies** written and renewed in Chicago by zip code for the number of months of December 1977 through May 1978.*

## Example: Insurance Redlining

Variables to consider:

- `race` Racial composition in percentage of minority,
- `fire` Fire per 100 housing units,
- `theft` Theft per 100 housing units,
- `age` Theft per 1000 population,
- `involact` New FAIR plan policies and renewal per 100 housing units,
- `income` Median family income in thousands of dollars,
- `side` North or South side of Chicago.

# Work Statement: Introduction

Describe:

- the purpose of the project,
- a brief introduction of the sponsoring organization,
- a suitably condensed statement of the problem,
- some discussion of the relevance of the project to the sponsor.

## Work Statement: Introduction

The work statement should contain a short description of your sponsor.

For the insurance redlining example, *U.S. Commision on Civil Rights* would be the sponsor.

*Boilerplating* from the sponsor's webpage is often acceptable.

<http://www.usccr.gov>

## Work Statement: Problem Statement

*Can the insurance companies claim that the discrepancy is due to greater risks in some zip codes?*

*The insurance companies could claim that they were denying insurance in neighborhoods where they had sustained large fire-related losses and any discriminatory effect was a by-product of legitimate business practice.*

# Work Statement: Timeline & Deliverable

*“When I decide the time needed for the project, I first approximate the time that I might actually need, and then, request the sponsor the double of the approximated time.”*

**From Team to Sponsor** Presentations, Reports, Special Softwares.

**From Sponsor to Team** Regular meetings, Data & Contingences, Code & Code Documentation.



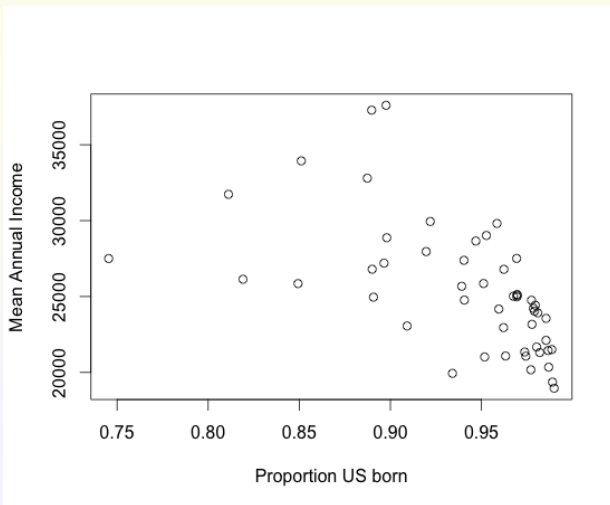
## Example: Ecological Fallacy

### Ecological Fallacy

*When data are collected at the group level, we may observe a correlation between two variables. The **ecological fallacy** is concluding that the same correlation holds at the individual level.*

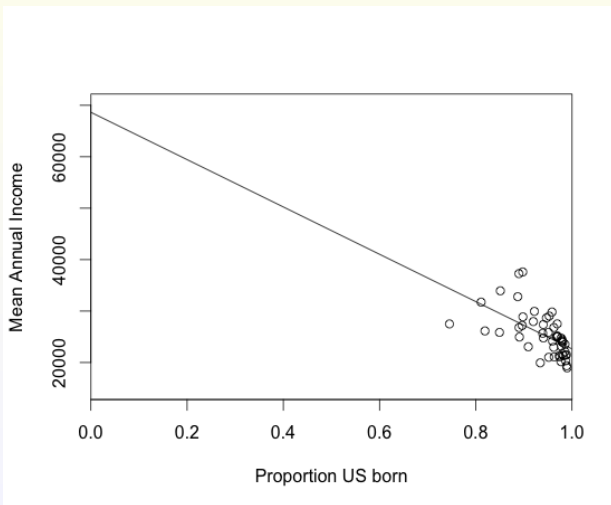
## Example: Ecological Fallacy

1998 annual per capita income and proportion U.S. born for 50 states plus D.C.



## Example: Ecological Fallacy

1998 annual per capita income and proportion U.S. born for 50 states plus D.C.



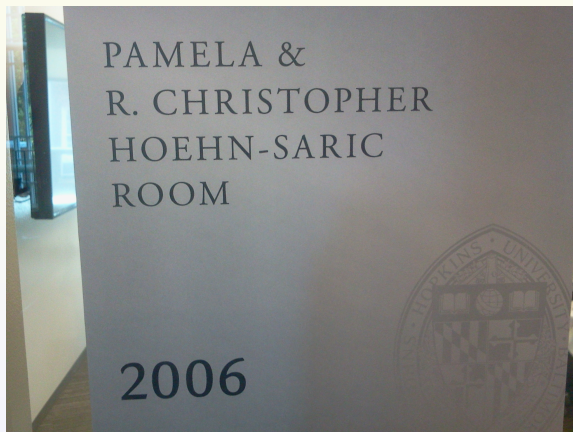
## Example: Insurance Redlining

*For the ecological fallacy example, the assumption would be that the incomes of the native born do not depend on the proportion of native born within the state (and similarly for naturalized citizens).*

*For the insurance redlining example, we only have aggregate data. We must inform the sponsor that unless more detailed data becomes available, the results for the aggregated data may not hold true at the individual level.*

# Presentations in this class

For your presentation recording needs



# Programmings in this class

- $\text{\LaTeX}$  :
  - moderncv
  - beamer
  - report
  - pgf/TikZ
- Git
  - git gui
- R:
  - lm
  - ggplot2
  - tikzDevice
  - R CMD build

# Where to get some help for $\text{\LaTeX}$

<http://en.wikibooks.org/wiki/LaTeX/>

# Tutorial: $\text{\LaTeX}$

$\text{\LaTeX}$  is a computer language for writing a scholarly paper:

Table: HTML vs  $\text{\LaTeX}$

	HTML	$\text{\LaTeX}$
Code	<pre>&lt;html&gt; . . . &lt;/html&gt;</pre> <hr/>	<pre>\begin{document} . . . \end{document}</pre> <hr/>
Compiler	Firefox and etc.	pdf $\text{\LaTeX}$ and etc.
Output	Web-page	PDF file



# Tutorial: $\text{\LaTeX}$

TeXworks is:

- an editing tool that is separate from  $\text{\LaTeX}$ ,
- available in Linux, OSX and Windows,
- available in:

<http://code.google.com/p/texworks/>

# Tutorial: $\text{\LaTeX}$

- Demo on preparing a resume using  $\text{\LaTeX}$  `moderncv` package:
  - Install  $\text{\LaTeX}$  (MikTeX in Windows and MacTeX in OSX),
  - Download `moderncv` package files from the course folder,
  - Change file names to reflect you,
  - Edit the TeX file,
  - Compile using your favorite  $\text{\LaTeX}$  editor,
  - Look at the resulting PDF file.

## Cautions: $\text{\LaTeX}$

There are numerous quirky  $\text{\LaTeX}$  rules:

- opening quotation is not the same as the closing quotation,
- period yields *two* blank spaces,
- for %, need to type `\%`,
- for `\`, need to type `\textbackslash`,
- for `/`, need to type `/`,
- for {, need to type `\{`,
- for \$, need to type `\$`,
- `~` yields a single blank space,
- and etc.

# Tutorial: Git

```
sudo apt-get install git
```

---

An alternative: `git gui`



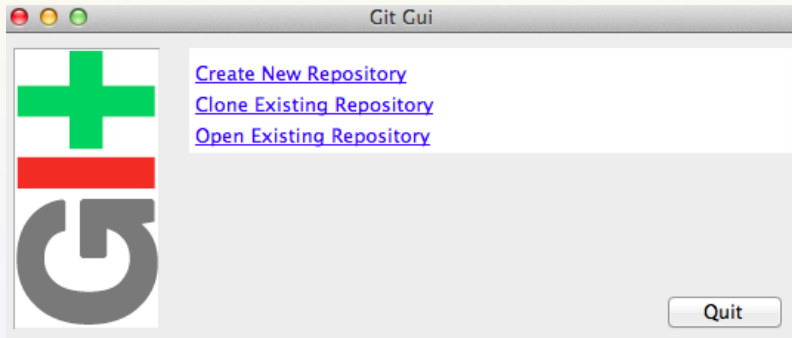
The screenshot shows the Git website. At the top, the Git logo (a red diamond with a white branching diagram) is followed by the word "git" in a large, bold, black font. To the right of the logo, the text "--distributed-is-the-new-centralized" is displayed in a smaller, grey font. Below the logo and text, there is a navigation menu with the following items: "About", "Documentation", "Downloads" (highlighted in red), "GUI Clients" (highlighted in red), "Logos", and "Community". To the right of the navigation menu, there is a large white box with the heading "GUI Clients" in a large, bold, black font. Below the heading, the text "Git comes with built-in GUI tools for com" and "third-party tools for users looking for plat" is visible. At the bottom of the white box, there is a button labeled "Show GUIs for all OSes" and a red text element that says "7 Mac GUIs a".

# Tutorial: Git

```
cd ~/
git clone http://cis.jhu.edu/~nhlee/550400.git
```

---

An alternative: `git gui`

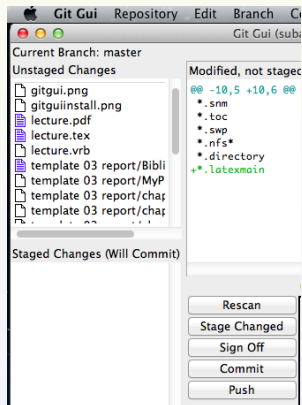


# Tutorial: Git

```
cd ~/550400.git  
git reset --hard HEAD  
git pull origin master
```

---

An alternative: git gui



# Tutorial: Git

- Demo I: build a *personal* Git folder
  - Create some files
  - Stage the files
  - Commit the files
- Demo II: build the *course* Git folder

# Unofficial Way to Access the Course Folder

`http://cis.jhu.edu/~nhlee/550400.html/`



# Demo: R

## R Studio

**>Welcome to RStudio**

RStudio™ is a free and open source integrated development environment (IDE) for R. You can run it on your desktop (Windows, Mac, or Linux) or even over the web using RStudio Server.

[Download RStudio for Windows, Mac or Linux](#)

**ScreenCast**  
RStudio in 2 minutes

```
library(ggplot2)
library(foreign)
library(dplyr)
library(plotly)

# Load data
diamonds <- read.csv("diamonds.csv")

# View data
view(diamonds)

# Summary
summary(diamonds)

# Scatter plot
ggplot(diamonds, aes(carat, price)) +
  geom_point()

# Faceted plot
ggplot(diamonds, aes(carat, price)) +
  facet_wrap(~ clarity)
```

**Diamond Pricing**

Price vs Carat

## R

**The R Project for Statistical Computing**  
www.r-project.org

**The R Project for Statistical Computing**

**PCA 3 view**  
(1-3) 60%

**Clustering: 4 groups**

**Factor 1 [41%]**

**Factor 3 [19%]**

**Getting Started:**

- R is a free software environment for statistical computing and graphics. It compiles and runs on a wide variety of UNIX platforms, Windows and MacOS. To [download R](#), please choose your preferred [CRAN mirror](#).
- If you have questions about R like how to download and install the software, or what the license terms are, please read our [answers to frequently asked questions](#) before you send an email.

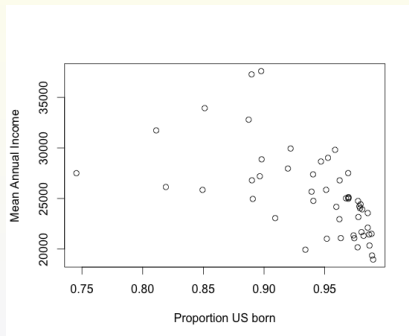
**News:**

- R version 2.15.1** (Roasted Marshmallows) has been released on 2012-06-22.
- The R Journal Vol.4/1** is available.
- useR! 2012**, took place at Vanderbilt University, Nashville Tennessee, USA, June 12-15, 2012.

# Demo: R

```
install.packages(faraway)
require(faraway)
data(eco)
plot(income ~ usborn,
     data=eco,
     xlab='Proportion US born',
     ylab='Mean Annual Income'
)
```

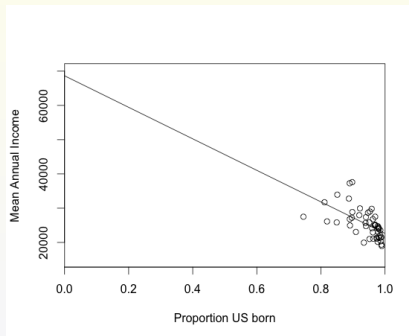
---



# Demo: R

```
g <- lm(income ~ usborn, eco)
summary(g)
plot(income ~ usborn,
     data = eco,
     xlab='Proportion US born',
     ylab='Mean Annual Income',
     xlim=c(0,1),
     ylim=c(15000,70000),
     xaxs='i')
abline(coef(g))
```

---



## Example: Insurance Redlining

```
data(chredlin);  
chredlin;  
head(chredlin);  
summary(chredlin);  
par(mfrow=c(1,1));  
pairs(chredlin);  
summary(lm(involact ~ race, chredlin));  
plot(involact ~ race, chredlin);  
abline(lm(involact ~ race, chredlin));  
plot(fire ~ race, chredlin);  
abline(lm(fire ~ race, chredlin));
```

---

# Seven Basic Principles

1. Set the context
2. Choose effective examples and analogies
3. Choose vocabulary to suit your readers
4. Decide whether to present  $\#$ s in text, tables, or figures
5. Report and interpret  $\#$ s in the text
6. Specify the direction *and* size of an association between variables
7. For many  $\#$ s, summarize overall pattern

# Creating Effective Tables

# Example: Cost of Packaging

# Example: The Nuclear Mission Arms Race



# Example: Maintaining Inventory