# Spatial Matching under Resource Competition


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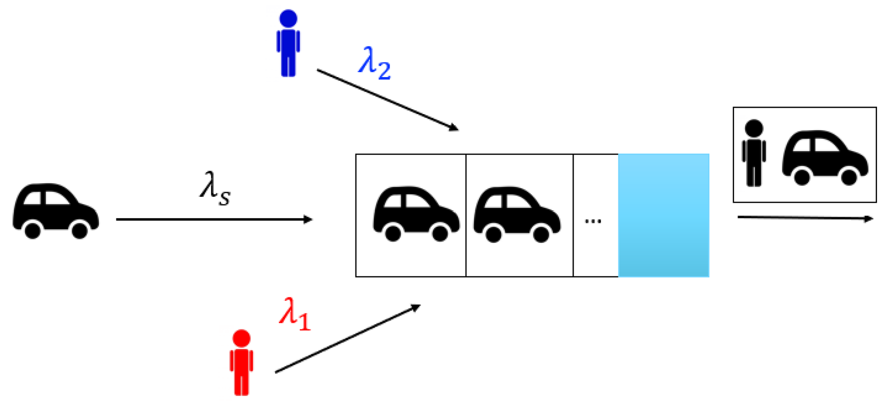
## Motivation

**Question:**
In the presence of spatial frictions (under which platforms require a large buffer of idle drivers to operate efficiently), can platform competition over multi-homing drivers lead to inefficient equilibria with high pick-up times?

## Model: Ride-hailing Platforms with Drivers Multi-homing

- **Matching duopoly**: customers arrive to platform j with rate $\lambda_j$ and drivers are replenished with rate $\lambda_s$ and multi-home.




- **Representative threshold policies**: Platforms choose thresholds $(n_1, n_2)$ on the minimum number of idle drivers to start accepting dispatches.

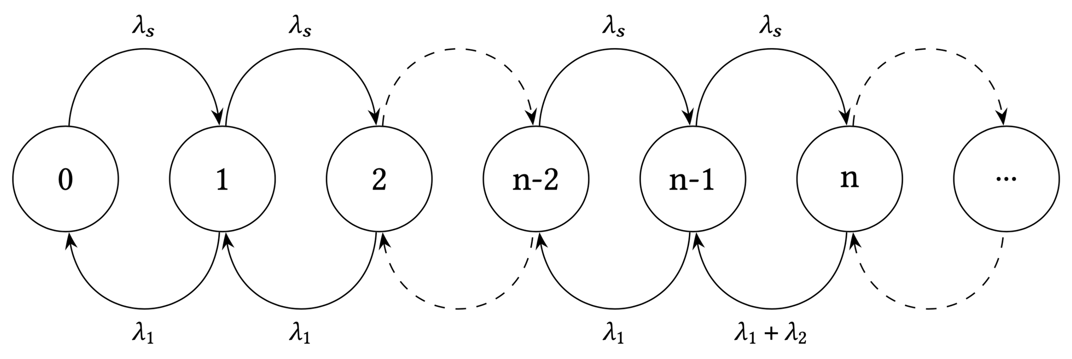



Figure 1. The threshold policies $(1, n)$ induce the birth-death process $N(1, n)$.

- **Cost function** $C_j(n_1, n_2)$: the cost for platform $j$ at thresholds $(n_1, n_2)$ is measured per demand request as the combination of three terms:
  1. Dispatch Cost (DC): $c_D \times \mathbb{E}$ [pick-up distance] $\times$ (rate of fulfilled requests).
  2. Idle Cost (IC): $c_I \times \mathbb{E}$ [number of idle drivers] $\times$ (market share).
  3. Unfulfillment Cost (UC): rate of rider requests that are not served.
- **Equilibrium concept:** $(n_1, n_2)$ is an $\varepsilon$-equilibrium iff for $j = 1, 2$, we have
$$C_j(n_j, n_{-j}) \le C_j(m, n_{-j}) + \varepsilon \quad \forall m \in \mathbb{N}\,.$$
- **Large-market limit:** the riders arrival rates are $\lambda_1\Lambda$ and $\lambda_2\Lambda$ and drivers arrival rate is $\Lambda$ with $\Lambda \to +\infty$ and $\lambda_1 + \lambda_2 > 1$.

## Monopolist

Informal proposition: the monopolist's optimal threshold $n^*$ is $\Theta(\sqrt{\Lambda})$ to balance the idle cost $O(\frac{n^*}{\Lambda})$ with dispatch cost $O(\frac{1}{n^*})$, since unfulfillment cost is invariant.

## Main Result: Equilibrium Classification

Let $c_D \in \mathbb{N}$ and assume $\lambda_1 \le \lambda_2$. Define $g \triangleq \lambda_2 - \lambda_1 \cdot \left(\sum_{i=1}^{+\infty} \frac{c_D}{c_D+i} \cdot \left(\frac{1}{\lambda_1+\lambda_2}\right)^i\right)$.

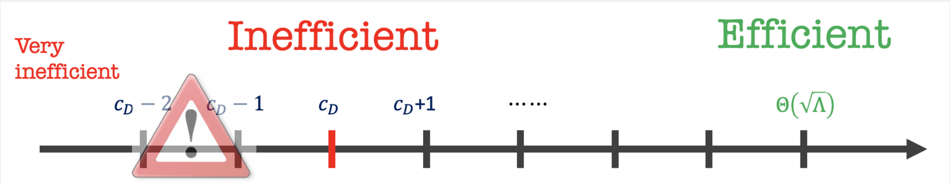
**Intuition**
For large enough $\Lambda$, any instance can be classified into two types of outcomes:

i. Inefficient equilibria of the form $(c_D, c_D)$, where there is no efficiency of scale, or
ii. Efficient $\varepsilon$-equilibria of the form $(c_D, \Theta(\sqrt{\Lambda}))$ where one platform generates efficiencies of scale.

More specifically, we distinguish three cases based on the sign of $g$:

**Theorem 1 (informal)**
For any large enough $\Lambda > 0$:

1. If $g > 0$,
   - $(c_D, c_D)$ is an equilibrium. ✔
   - $(c_D, \Theta(\sqrt{\Lambda}))$ is not an $\varepsilon$-equilibrium for any small $\varepsilon > 0$. ✖
2. If $g = 0$,
   - $(c_D, c_D)$ is an equilibrium. ✔
   - $(c_D, \Theta(\sqrt{\Lambda}))$ is an $\varepsilon$-equilibrium for any small $\varepsilon > 0$. ✔
3. If $g < 0$,
   - $(c_D, c_D)$ is not an equilibrium. ✖
   - $(c_D, \Theta(\sqrt{\Lambda}))$ is an $\varepsilon$-equilibrium for any small $\varepsilon > 0$. ✔
   - If $\lambda_1 < \frac{1}{c_D+1}$, $(c_D, n_2)$ is an equilibrium for some $n_2 = \Theta(\sqrt{\Lambda})$. ✔




## Price of Anarchy and Stability

A monopolist $M$ with demand arrival rate $(\lambda_1 + \lambda_2) \cdot \Lambda$ and optimal threshold $n^*$.

**Definitions:**

- Efficiency ratio:
$$R(n_1, n_2) = \frac{C_1(n_1, n_2) + C_2(n_1, n_2)}{(\lambda_1 + \lambda_2) C_M(n^*)}\,.$$
- Price of anarchy:
$$\text{PoA} = \limsup_{\Lambda \to +\infty} \sup_{\text{equilibrium } (n_1, n_2)} R(n_1, n_2).$$
- $\varepsilon$-price-of-stability:
$$\text{PoS}_\varepsilon = \limsup_{\Lambda \to +\infty} \inf_{\varepsilon\text{-equilibrium } (n_1, n_2)} R(n_1, n_2)$$

## Visual Theorem for PoA and PoS

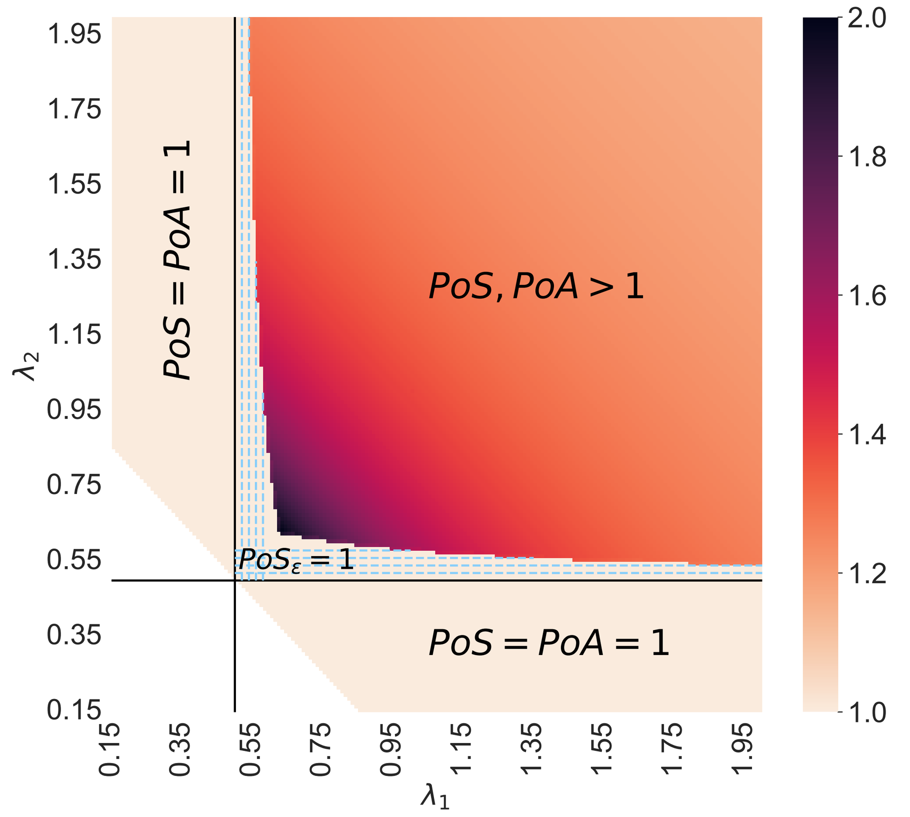



Figure 2. Large-market efficiency ratios with $c_D = 1$.

## Extension to Distance Thresholds

Platform $j$ adopts a distance threshold $\tau_j$ and accepts a ride request if and only if its distance to the nearest idle driver is less than $\tau_j$.

**Theorem 2 (informal)**
At least one of these statements holds:

1. For every small $\varepsilon > 0$ and large enough $\Lambda$, either $(1, \frac{1}{\sqrt{\Lambda}})$ is an $\varepsilon$-equilibrium or $(\frac{1}{\sqrt{\Lambda}}, 1)$ is an $\varepsilon$-equilibrium.
2. There exists $d$ such that for every large enough $\Lambda$ and every equilibrium $(\tau_1, \tau_2) \neq (0, 0)$, we have $\tau_1, \tau_2 \ge d$.

## Market Fragmentation: Boon or Bane?

A fragmented market is asymptotically as efficient as a monopolistic one. However, for a small $\Lambda$, we compare the overall market efficiency of (i) a fragmented market (with no multi-homing) with reduced spatial pooling but aligned incentives, and (ii) a competitive market with multi-homing, with potential spatial pooling but misaligned incentives.

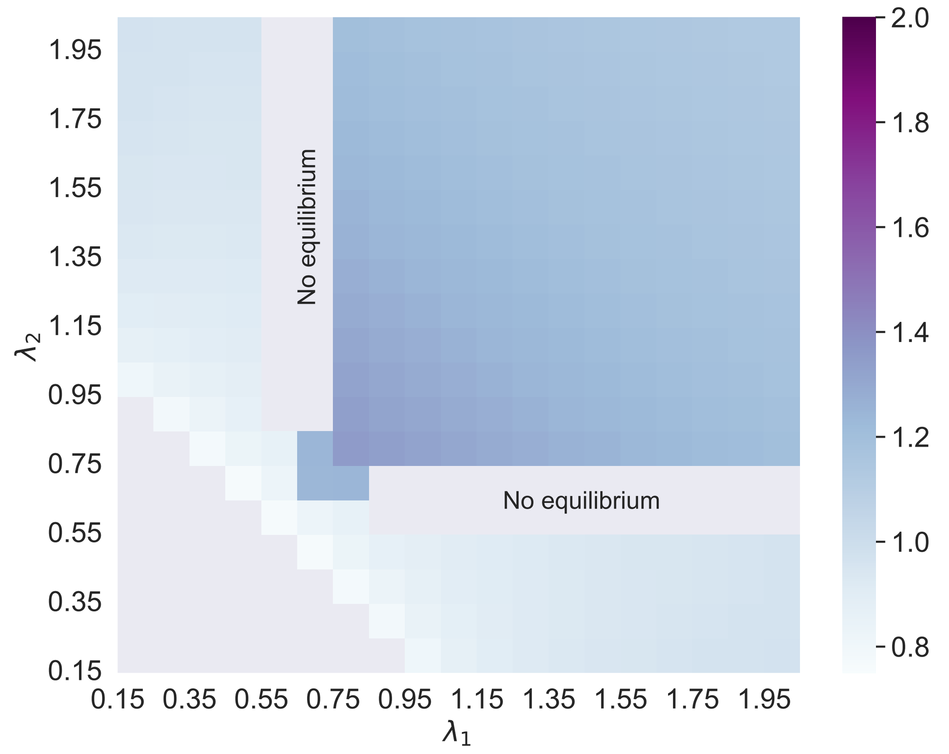



Figure 3. Equilibrium/fragmentation with $\Lambda = 5, c_D = 3, c_I = 0.02$.

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