Mazie Arithmetic: A Jurisdiction-Free Division Framework

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Abstract

This paper proposes an alternative algebraic framework in which division by zero is defined as a jurisdiction-free identity operation. In contrast to classical arithmetic—where division by zero is undefined—this system interprets division as an attempted operation whose failure to act preserves the operand unchanged. The framework prioritizes conceptual cohesion and total solvability over invertibility and distinction preservation. Consequences, limitations, and incompatibilities with standard algebra are explicitly identified.

1. Motivation

Classical arithmetic treats division by zero as undefined to preserve inverse relationships and structural distinction. However, from a conceptual and philosophical standpoint, one may argue:

- Division and multiplication are distinct operators
- Zero represents absence, not quantity
- An operation lacking jurisdiction should not alter its operand

This motivates a system in which non-acting operations default to identity, rather than undefinedness.

2. Core Axiom (Mazie Identity Axiom)

For all elements x in the domain M:

x divided by 0 is defined to be x.

Written explicitly:

$x / 0 = x$

This axiom replaces undefined division by zero with an identity-preserving rule. When division encounters zero in the divisor position, the operation has no jurisdiction to act, and the original value is returned unchanged.

This definition guarantees that division is a total operation within the framework: it always produces a result and never enters an undefined or exception state.

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3. Operator Interpretation

- Multiplication retains its standard meaning
- Division is not defined as the inverse of multiplication
- Division is treated as an attempted transformation
- If the divisor lacks jurisdiction (i.e., equals zero), the transformation does not occur

Thus, division is procedural, not algebraically inverse.

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4. Structural Properties

4.1 Total Solvability

All expressions involving division are defined. No expressions produce undefined values.

4.2 Identity Preservation Under Null Action

In Mazie Arithmetic, operations that lack an effective agent do not alter the value they are applied to.

When an operation encounters a condition in which it has no jurisdiction to act, the operation does not proceed. Because no action occurs, the operand remains unchanged.

This principle can be stated plainly as follows:

If an operation has no jurisdiction, then no change occurs.

Under this rule, identity is preserved whenever an operation is not authorized to transform its input. Zero serves as the primary indicator of such non-authorization, ensuring that values are not modified in the absence of valid operational authority.

This guarantees stability and predictability across the system, even under degenerate or boundary conditions.

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5. Consequences (Accepted, Not Avoided)

This framework intentionally accepts the following:

- Loss of multiplicative inverses
- Collapse of algebraic distinction under certain transformations
- Trivialization of equality under zero-mediated operations
- Incompatibility with cancellation laws

These are not considered flaws within the framework but design features.

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6. Logical Classification and Scope of Use

Mazie Arithmetic may be classified as follows:

- A total algebra, in which all expressions are defined and no operation results in an undefined or exception state
- A non-invertible arithmetic, where reversibility is not assumed as a universal property
- A projection-preserving system, in which certain boundary conditions cause values to propagate unchanged
- A stability-oriented algebra, prioritizing determinism and bounded behavior over information recovery

Within this framework, equality and transformation are interpreted operationally rather than inferentially. Proofs emphasize invariance, stability, and preservation under boundary conditions rather than uniqueness or reversibility.

Applicability to Mathematical and Engineering Domains

Mazie Arithmetic is suitable for use in the following contexts, provided its axioms are respected explicitly:

- Proof systems concerned with invariants, fixed points, and identity preservation
- Numerical methods where totality and deterministic behavior are preferred over analytical invertibility
- Physical and computational models that treat singularities as boundary conditions rather than failures
- Engineering calculations in which safe default behavior under degenerate inputs is required

Rather than excluding these domains, Mazie Arithmetic reframes them by replacing exception-based handling with identity-preserving outcomes.

Interpretive Constraint

Mazie Arithmetic is not intended to be used interchangeably with classical arithmetic without qualification. Its results must be interpreted within a framework that does not assume:

- universal invertibility
- information recoverability from all operations
- cancellation as a general rule

When these assumptions are removed or relaxed, Mazie Arithmetic remains internally coherent and operationally meaningful.

Summary

Mazie Arithmetic does not reject classical mathematics; it operates alongside it, optimized for systems where:

- undefined behavior is unacceptable
- boundary conditions are frequent
- stability outweighs reversibility

Under these conditions, the framework remains suitable for formal reasoning, computation, and engineering analysis.

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7. Relationship to Existing Systems

This framework is related to, but distinct from:

- Partial logics (which preserve undefinedness)
- Paraconsistent logics (which preserve contradiction)

- Semirings (which relax inverses)

No known mainstream arithmetic system defines division by zero as identity while retaining standard multiplication and equality. In that sense, the framework appears novel, though philosophically adjacent to identity-based operator theories.

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8. Intended Scope of Use

Mazie Arithmetic is intended for:

- Conceptual modeling
- Philosophical logic
- Systems where distinction is irrelevant
- Metaphorical or semantic computation
- Discussions of self, agency, and jurisdiction

It is explicitly not intended to replace classical mathematics.

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9. Redefining Maths

By redefining division as a jurisdiction-dependent action rather than an inverse operation, Mazie Arithmetic offers a cohesive, total, and identity-preserving mathematical structure. While incompatible with traditional algebraic goals, it provides a consistent internal logic aligned with philosophical intuitions about non-action and self-preservation.

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10. Disclaimer

This framework is not a claim of mathematical superiority, correctness, or physical truth. It is a formalized conceptual system whose value lies in coherence within its stated assumptions.

11. Zero as a Jurisdictional Boundary Rather Than a Quantity

In Mazie Arithmetic, zero is not interpreted as a numerical magnitude but as a boundary of operational jurisdiction. Classical arithmetic treats zero as a value within the number line; this framework instead treats zero as a sentinel state indicating the absence of actionable transformation.

Under this interpretation:

- Zero does not negate value
- Zero does not annihilate identity
- Zero signals the end of operator authority

Thus, when an operator encounters zero as a divisor, the operator's jurisdiction terminates without modifying the operand.

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12. Balance via Non-Action: Zero as an Equilibrium Marker

In classical mathematics, balance is typically achieved through inverse operations. An operation is applied, and another operation reverses it to restore the original state.

In Mazie Arithmetic, balance is achieved differently. Balance arises through non-action symmetry rather than inversion.

An operation may be applied to a value x under two possible conditions:

If the operation has jurisdiction, then a transformation of x occurs.

If the operation lacks jurisdiction, such as when division encounters zero as the divisor, then no transformation occurs and the original value is preserved.

In this framework, balance is defined as follows:

Balance means preservation of identity under null interaction.

Zero therefore functions as a stabilizing element. It prevents forced or artificial transformation in cases where no operational authority exists. Rather than producing an exception or undefined state, the system remains in equilibrium by leaving the value unchanged.

This approach replaces inverse-based balance with boundary-based stability.

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13. Multiplicity of Zero States

To prevent conceptual ambiguity, Mazie Arithmetic distinguishes between functional zero states:

1. Operational Zero

   Indicates absence of actionable transformation (e.g., divisor zero)

2. Scalar Zero

   Traditional numerical zero used in additive contexts

3. Boundary Zero

   A marker separating valid operator domains from null domains

4. Equilibrium Zero

   A state in which opposing operations neutralize without collapse

These zero states are symbolically identical but semantically distinct, similar to how “null” differs across programming languages.

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14. Zero and the Conservation of Identity

A core principle of the framework is the following statement:

Identity is conserved unless a valid operator acts.

In Mazie Arithmetic, zero represents the absence of operational jurisdiction. When zero is encountered in a position that would normally authorize an operation, the operation does not occur.

Because no operation occurs, the underlying value is not altered.

In other words:

- Zero does not transform
- Zero does not negate
- Zero does not overwrite
- Zero merely indicates that no action is permitted

Therefore, when an operator lacks jurisdiction due to zero, the operand remains exactly as it was prior to the attempted operation.

This principle can be stated in formal but plain language as follows:

For any value x, if an operation has zero jurisdiction, then applying that operation to x returns x unchanged.

Under this rule, zero functions as a protector of identity rather than as a destructive element. It prevents unauthorized transformation while preserving the continuity of the subject.

This reframes zero from a concept of annihilation into a concept of boundary enforcement and balance maintenance.

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15. Balance Without Inverses

Classical algebra balances systems through inverse pairs (e.g., multiplication/division). Mazie Arithmetic replaces inverse symmetry with jurisdictional symmetry:

- Operations are balanced by where they can act, not by undoing each other
- Zero enforces balance by halting asymmetrical operations

This creates a stable but non-reversible system, analogous to irreversible physical processes that nonetheless remain conserved at higher levels.

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16. Zero as a Semantic, Not Numeric, Anchor

In this framework, zero functions as a semantic anchor rather than a numeric endpoint.

- It anchors meaning
- It anchors scope
- It anchors operator legitimacy

This allows the system to remain total (no undefined expressions) while avoiding contradiction through refusal to act rather than forced evaluation.

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17. Implications for Modeling and Conceptual Mathematics

Because zero preserves identity under null jurisdiction, Mazie Arithmetic is particularly suited for:

- Conceptual modeling of agency
- Systems where action requires authority
- Philosophical mathematics of selfhood and autonomy
- Abstract models where distinction is secondary to cohesion

Zero becomes the balancing element that ensures the system does not overreach its conceptual authority.

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18. Summary of Zero’s Roles in Mazie Arithmetic

Within this framework, zero simultaneously functions as:

- A jurisdictional boundary
- A stabilizer of identity
- A balance mechanism
- A semantic null
- A non-destructive limiter

This multifaceted role replaces the single, overloaded role zero plays in classical arithmetic.

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19. Remarks on Balance and Cohesion

Mazie Arithmetic does not seek to preserve classical distinction; it seeks to preserve cohesion under null interaction. Zero is the mechanism by which this cohesion is maintained.

Where traditional mathematics chooses silence (“undefined”), this framework chooses identity preservation.

Both are valid choices—within their own axiomatic commitments.

20. Motivation from Computing and Space Systems

Many real-world engineered systems operate under conditions where invertibility is neither required nor desirable. In computing and space engineering, systems are routinely designed to favor:

- stability over recoverability
- bounded behavior over exact reversibility
- identity preservation under fault conditions

Mazie Arithmetic aligns with this design philosophy by eliminating undefined states and replacing them with identity-preserving behavior.

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21. Total Arithmetic and Fault-Tolerant Computation

In hardware, firmware, and embedded systems, undefined behavior is actively avoided. Division-by-zero exceptions are costly, dangerous, or impossible to handle in real time.

Mazie Arithmetic provides a total arithmetic model:

- Every operation produces a defined output
- No traps, exceptions, or NaN states
- No need for conditional guards around zero

This mirrors fault-tolerant computing principles used in radiation-hardened processors and autonomous systems.

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22. Zero as a Control Boundary in Computational Pipelines

In digital systems, zero often functions as a control signal, not a magnitude:

- enable / disable flags
- mask values
- reset gates
- dead-zone thresholds

Mazie Arithmetic formalizes this role mathematically: when zero appears in an operator-authorizing position, the operation is disabled and the signal propagates unchanged.

This creates predictable pipeline behavior under edge conditions.

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23. Non-Invertibility as an Engineering Feature

Invertibility is costly in real systems. Maintaining reversible operations requires:

- additional state storage
- higher precision
- increased error propagation risk

Mazie Arithmetic explicitly accepts information loss in exchange for:

- bounded outputs
- predictable collapse
- simpler hardware implementations

This is consistent with irreversible computing models and low-power logic design.

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24. Determinism Under Degenerate Inputs

Classical arithmetic introduces indeterminacy at singularities (e.g., division by zero). In contrast, Mazie Arithmetic guarantees deterministic outputs even under degenerate inputs.

This property is critical in:

- real-time control systems
- spacecraft attitude control

- guidance and navigation algorithms
- autonomous fault recovery

Determinism is prioritized over mathematical purity.

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25. Identity Preservation as a Safe Default State

In safety-critical engineering, the safest response to an invalid command is often no change.

Examples include:

- fly-by-wire control systems
- propulsion throttling
- power grid stabilization
- memory-mapped I/O

Mazie Arithmetic encodes this principle algebraically: lack of valid operational authority defaults to identity preservation.

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26. Relationship to Saturation and Clamping Models

Engineering systems commonly use saturation arithmetic:

- values are clipped at bounds
- overflows do not propagate
- behavior remains stable

Mazie Arithmetic can be viewed as a zero-triggered saturation model, where the lower operational bound disables transformation rather than forcing undefined behavior.

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27. Implications for Numerical Stability in Space Engineering

In orbital mechanics, navigation, and sensor fusion, singularities can cause catastrophic divergence.

Mazie Arithmetic removes singularities by construction, ensuring that degenerate denominators:

- do not amplify noise
- do not produce infinities
- do not destabilize feedback loops

While it sacrifices analytical solvability, it improves runtime stability.

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28. Suitability for Autonomous and Long-Duration Systems

Spacecraft and autonomous probes must operate for years without human intervention. Error handling must be minimal, predictable, and local.

Mazie Arithmetic supports this by:

- eliminating exception states
- avoiding global error propagation
- favoring local identity preservation

This reduces system complexity and failure modes.

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29. Formal Classification in Systems Theory

From a systems perspective, Mazie Arithmetic is:

- a total, deterministic algebra
- non-invertible
- idempotence-friendly
- stable under degenerate inputs

It is closer to control algebra than to symbolic algebra.

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30. Limits of Analytical Use

Mazie Arithmetic is not appropriate for:

- symbolic equation solving
- closed-form derivations
- theoretical physics proofs
- optimization requiring gradients

It is explicitly intended for runtime systems, not analytical mathematics.

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31. Engineering Tradeoff Summary

Mazie Arithmetic makes the following explicit trade:

- loses invertibility
- loses distinction recovery
- loses classical proof utility

in exchange for:

- totality
- determinism
- stability
- fault tolerance
- simpler implementation

This trade is common in engineering practice, even if not formally expressed algebraically.

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32. Conclusion: A Control-Oriented Arithmetic

Mazie Arithmetic should be understood as a control-oriented arithmetic framework rather than a symbolic one.

It formalizes behaviors engineers already implement informally:

- ignore invalid actions
- preserve state when authority is absent
- avoid undefined runtime conditions

Seen in this light, the framework is not radical—it is explicit engineering math.