Accurate Fault Diagnosis in Transformers Using An Auxiliary Current-Compensation-Based Framework for Differential Relays

Amir Ameli, *Member, IEEE*, Mohsen Ghafouri, *Member, IEEE*, Hatem Zeineldin, *Senior Member, IEEE*
Magdy Salama, *Life Fellow, IEEE*, and Ehab El-Saadany, *Fellow, IEEE*

This documents provides the numerical matrices related to the "Performance Evaluation" section of the paper. For the transformer, the continuous state-space matrices are

$$A(\bar{f}) = 10^7 \begin{bmatrix} -0.000001256637061 & 0 & 0.000001256637061 \\ 0 & -0.000001256637061 & 0.000001256637061 \\ 0.232710566932577 & 2.094395102393195 & -2.327134668642536 \end{bmatrix}$$

$$A(\underline{f}) = 10^7 \begin{bmatrix} -0.000001256637061 & 0 & 0.000001256637061 \\ 0 & -0.000001256637061 & 0.000001256637061 \\ 0.232710566932577 & 2.094395102393195 & -2.367312633991860 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$C = 10^2 \begin{bmatrix} 0 & 2.199532768738916 & -2.199532768738916 \end{bmatrix}$$

After discretizing the above matrices, the following matrices are obtained for the LPV observer using the instructions given in Section III:

$$N_1 = 10^4 \begin{bmatrix} 0.000099977377944 & 3.003049605886595 & -3.003049583264851 \\ 0.000005001137080 & 0.669169668238753 & -0.669074669999209 \\ 0.000005001240402 & 0.669083748886640 & -0.668988750750430 \end{bmatrix}$$

$$G_1 = 10^{-4} \begin{bmatrix} 0.199977371619900 & 0.000022520301701 \\ 0.009979532627736 & 0.189804261687950 \\ 0.009979738852313 & 0.189804050996650 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -0.001410519721333 \\ 0.001845109183726 \\ 0.001845147205302 \end{bmatrix}$$

$$H_1 = \begin{bmatrix} 0 \\ -0.002273186558871 \\ 0.002273186558871 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 1 & & 0 & 0 \\ 0 & 0 & 0.500005167430536 & 0.499994832569457 \\ 0 & & 0.499994832569464 & 0.500005167430530 \end{bmatrix}$$

$$K_1 = 10^2 \begin{bmatrix} -1.365312500000000 \\ -0.304189453125000 \\ -0.304150390625000 \end{bmatrix}$$

$$N_2 = 10^6 \begin{bmatrix} 0.000000999773355 & 5.115403527313659 & -5.115403527091273 \\ 0.000000049162371 & 4.195874329996580 & -4.195873387655194 \\ 0.000000049163387 & 4.195874329995389 & -4.195873387655194 \end{bmatrix}$$

$$G_2 = 10^{-4} \begin{bmatrix} 0.199977329327629 & 0.000022139671268 \\ 0.009810498118414 & 0.188282951104046 \\ 0.009810700849932 & 0.188282708975221 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.240355019355775 \\ -0.195007793703553 \\ -0.195007796410209 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0 \\ -0.002273186558871 \\ 0.002273186558871 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500005167430536 & 0.499994832569457 \\ 0 & 0.499994832569464 & 0.500005167430530 \end{bmatrix}$$

$$K_2 = 10^4 \begin{bmatrix} -2.325677343750000 \\ -1.907620312500000 \\ -1.907620312500000 \end{bmatrix}$$

For CT1, the continuous state-space matrices are

$$A(\bar{f}) = 10^{10}\begin{bmatrix} -0.000000028274334 & 0 & 0.000000028274334 \\ 0 & -0.000003798185518 & 0.000003798185518 \\ 9.424777960769379 & 0.188495559215388 & -9.613278925390171 \end{bmatrix}$$

$$A\left(\underline{f}\right) = 10^{10}\begin{bmatrix} -0.000000028274334 & 0 & 0.000000028274334 \\ 0 & -0.000003798185518 & 0.000003798185518 \\ 9.424777960769379 & 0.188495559215388 & -9.616188122141571 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = 10^{6}\begin{bmatrix} 0 & 4.712388980384689 & -4.712388980384689 \end{bmatrix}$$

After discretizing the above matrices, the following matrices are obtained for the LPV observer using the instructions given in Section III:

$$N_1 = \begin{bmatrix} 0.999994554934832 & 14.746219680974672 & -14.746214236069656 \\ 0.508827403532946 & 15.377826484553806 & -14.886654179064960 \\ 0.508827403532967 & 15.096946213334490 & -14.605773907845666 \end{bmatrix}$$

$$G_1 = 10^{-6}\begin{bmatrix} 0.999997259136483 \\ 0.499564226627892 \\ 0.499564226627914 \end{bmatrix}$$

$$L_1 = 10^{-7}\begin{bmatrix} 0.000005770977077 \\ 0.521149588254295 \\ 0.521149588254287 \end{bmatrix}$$

$$H_1 = 10^{-6}\begin{bmatrix} 0 \\ -0.106103295608766 \\ 0.106103295608766 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500000000000012 & 0.500000000931300 \\ 0 & 0.499999999999989 & 0.500000000931323 \end{bmatrix}$$

$$K_1 = 10^{-5}\begin{bmatrix} -0.312924385070801 \\ -0.315904617309570 \\ -0.309944152832031 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} 0.999994471017418 & 12.235852255305044 & -12.235846812048104 \\ 0.508673315062849 & 12.691905921133916 & -12.200736585591244 \\ 0.508673315062870 & 12.586575819426660 & -12.095406483884009 \end{bmatrix}$$

$$G_2 = 10^{-6}\begin{bmatrix} 0.999997217188789 \\ 0.499413018431536 \\ 0.499413018431558 \end{bmatrix}$$

$$L_2 = 10^{-7}\begin{bmatrix} 0.000005769230427 \\ 0.521146437172651 \\ 0.521146437172634 \end{bmatrix}$$

$$H_2 = 10^{-6}\begin{bmatrix} 0 \\ -0.106103295608766 \\ 0.106103295180423 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.500000000000012 & 0.500000000931300 \\ 0 & 0.499999999999989 & 0.500000000931323 \end{bmatrix}$$

$$K_2 = 10^{-5}\begin{bmatrix} -0.259652733802795 \\ -0.258907675743103 \\ -0.256672501564026 \end{bmatrix}$$