Numerical Example for Developing a UIO

This file explains the development procedure for a UIO through a numerical example for a single-generator single-area power system. The block digram for this system is shown in Fig. R1. Since this system has only one area, tie-line power deviations are not present in the states and outputs. The parameters of this system are $H_1 = 6.5$ s, $T_{g_{1,1}} = 0.08$ s, $T_{ch_{1,1}} = 0.4$ s, $R_{1,1} = 2.4$ Hz/MW, $D_{e_1} = 0.014$ MW/Hz, $k_1 = 0.7$, and $\beta_1 = D_{e_1} + \frac{1}{R_{1,1}} = 0.43$ MW/Hz. Therefore, matrices A_c, $B_{c,u}$, and C are

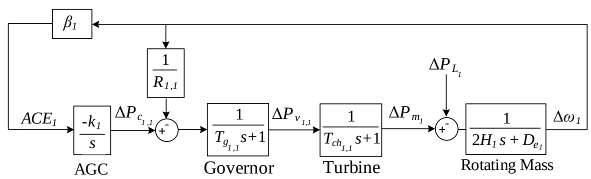

Figure R1: Block diagram of a single-area power system.

$$A_c = \begin{bmatrix} -0.0011 & 0.0769 & 0 & 0 \\ 0 & -2.5 & 2.5 & 0 \\ -5.2083 & 0 & -12.5 & 12.5 \\ -0.301 & 0 & 0 & 0 \end{bmatrix} \tag{R1a}$$

$$B_{c,u} = \begin{bmatrix} -0.0769 & 0 & 0 & 0 \end{bmatrix}^T \tag{R1b}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{R1c}$$

As a result, $n = 4$, $m = 1$, and $p = 2$. For $T_s = 2$ s, matrices A and B_u after discretization are

$$A = \begin{bmatrix} 0.9217 & 0.0288 & 0.0058 & 0.1148 \\ -0.8410 & -0.0155 & -0.0026 & 0.9419 \\ -0.9487 & -0.0259 & -0.0050 & 0.9289 \\ -0.5848 & -0.0145 & -0.0028 & 0.9717 \end{bmatrix} \tag{R2a}$$

$$B_u = \begin{bmatrix} -0.15 & 0.07 & 0.10 & 0.05 \end{bmatrix}^T \tag{R2b}$$

By selecting $\alpha = 2$ and forming $M_{u,2}$ and $M_{u,1}$ using (46e) and (52), (51) is satisfied. Hence, the required delay for the UIO can be chosen as $\alpha = 2$.

To design the UIO's gain, L, the general form of (47) is selected. L_2 must be equal to B_u, i.e.,

$$L_2 = \begin{bmatrix} -0.15 & 0.07 & 0.10 & 0.05 \end{bmatrix}^T \tag{R3}$$

Additionally, matrix Q should be obtained by solving (54) as follows:

$$Q \times \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.15 & 0 & 0 \\ 0.05 & 0 & 0 \\ -0.13 & -0.15 & 0 \\ 0.13 & 0.05 & 0 \end{bmatrix} = \left[\begin{array}{c|cc} 0 & 0 & 0 \\ \hline 1 & 0 & 0 \end{array}\right] \tag{R4}$$

As a result,

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.58 & 1.40 & 0.85 & 2.78 \end{bmatrix} \tag{R5}$$

After identifying L_2 and Q, L_1 must be designed to make the UIO stable. Since matrices A, B_u, and C introduced in (R1c) and (R2) satisfy (57), there is an L_1 that meets the stability condition. To find L_1, matrices S_1 and S_2 are obtained using (55) and Θ_α in (46b):

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = Q \times \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.92 & 0.03 & 0.01 & 0.12 \\ -0.59 & -0.02 & 0 & 0.97 \\ 0.75 & 0.02 & 0.01 & 0.25 \\ -1.09 & -0.03 & -0.01 & 0.86 \end{bmatrix}}_{\Theta_\alpha} \tag{R6}$$

where Q is shown in (R5). Therefore, S_1 and S_2 are

$$S_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{R7a}$$

$$S_2 = \begin{bmatrix} -7.44 & -0.22 & -0.04 & 3.44 \end{bmatrix} \tag{R7b}$$

Selecting $[-0.1, -0.05, 0, 0.1]$ as four stable and sufficiently small poles for A' in (55), L_1 must be designed such that the poles of (55) are located at these stable locations. Using pole placement techniques, the following L_1 is obtained:

$$L_1 = \begin{bmatrix} 0.68 & -137.80 & 808.83 & 0.90 \end{bmatrix}^T \tag{R8}$$

Using Q, L_2, and L_1 in (R5), (R3), and (R8), the numerical value of the UIO's gain given by (53) is:

$$L = \begin{bmatrix} 0.68 & 0 & 0.68 & -0.21 & -0.13 & -0.42 \\ -137.80 & 0 & -0.35 & 0.11 & 0.06 & 0.21 \\ 808.83 & 0 & -0.47 & 0.14 & 0.09 & 0.28 \\ 0.90 & 0 & -0.21 & 0.06 & 0.04 & 0.13 \end{bmatrix} \tag{R9}$$

By finding L, the UIO is appropriately designed to estimate the states of the system using (47) and system output measurements, without requiring its inputs, ΔP_{L_1}.