**COMPSCI/SFWRENG 2FA3**
**Discrete Mathematics with Applications II**
**Winter 2021**

# Assignment 10

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Assignment 10 consists of two problems. You must write your solutions to the problems using LaTeX.

Please submit Assignment 10 as two files, `Assignment_10_`*`YourMacID`*`.tex` and `Assignment_10_`*`YourMacID`*`.pdf`, to the Assignment 10 folder on Avenue under Assessments/Assignments. *`YourMacID`* must be your personal MacID (written without capitalization). The `Assignment_10_`*`YourMacID`*`.tex` file is a copy of the LaTeX source file for this assignment (`Assignment_10.tex` found on Avenue under Contents/Assignments) with your solution entered after each problem. The `Assignment_10_`*`YourMacID`*`.pdf` is the PDF output produced by executing

```
pdflatex Assignment_10_YourMacID
```

This assignment is due **Sunday, April 11, 2021 before midnight.** You are allow to submit the assignment multiple times, but only the last submission will be marked. **Late submissions and files that are not named exactly as specified above will not be accepted!** It is suggested that you submit your preliminary `Assignment_10_`*`YourMacID`*`.tex` and `Assignment_10_`*`YourMacID`*`.pdf` files well before the deadline so that your mark is not zero if, e.g., your computer fails at 11:50 PM on April 11.

**Although you are allowed to receive help from the instructional staff and other students, your submission must be your own work. Copying will be treated as academic dishonesty! If any of the ideas used in your submission were obtained from other students or sources outside of the lectures and tutorials, you must acknowledge where or from whom these ideas were obtained.**

## Problems

1. [**4 points**] Recall the diagonalization argument used in the Week 12 Exercises to show that the set of real numbers in the interval $[0, 1]$ is uncountable. If we tried to use this same argument to show that the set of rational numbers in the interval $[0, 1]$ is uncountable, where and why would the argument fail?

   **Aamina Hussain, hussaa54, April 16, 2021**

   The diagonalization argument to show that the set of real numbers in the interval $[0, 1]$ is uncountable is proven by showing that the new number formed by the diagonal represents a real number in $[0, 1]$. However, this argument would fail for the set of rational numbers in the interval $[0, 1]$ since there is no way to prove that the number formed by the diagonal is actually a rational number (meaning there is no way to prove that the number formed by the diagonal actually represents a rational number in $[0, 1]$).

2. Let $\Sigma$ be a finite alphabet and $A, B \subseteq \Sigma^*$ be r.e. sets.

   a. [**8 points**] Prove that $A \cup B$ is r.e.

   **Aamina Hussain, hussaa54, April 16, 2021**
   Since $A$ and $B$ are both r.e., that means they both have Turing machines that enumerate them. Let $M_A$ be the Turing machine that enumerates $A$ and $M_B$ be the Turing machine that enumerates $B$. $A \cup B$ is r.e. if there exists a Turing machine that enumerates its members. They can be enumerated if the tape looks like this: $A_0 \textvisiblespace A_1 \textvisiblespace A_2 \textvisiblespace ...$ Given $x \in \Sigma^*$ as input, we can create a Turing machine $M$ that enumerates the members of $A \cup B$. Let $C = A \cup B$. For any $x$ as input, run $M_A$ and $M_B$ simultaneously in order to get all the members in $A$ and all the members in $B$. (If x exists in either $A$ or $B$, then it will exist in $C$.) Let $C_0 = A_0, C_1 = B_0, C_2 = A_1, C_3 = B_1, ...$ etc. ($M = A_0 \textvisiblespace B_0 \textvisiblespace A_1 \textvisiblespace B_1 \textvisiblespace A_2 \textvisiblespace B_2 \textvisiblespace ...$) Since $M$ can enumerate $C$, this means that $C$, or $A \cup B$, is r.e.

   b. [**8 points**] Prove that $A \cap B$ is r.e.

   **Aamina Hussain, hussaa54, April 16, 2021**
   Since $A$ and $B$ are both r.e., that means they both have Turing machines that enumerates them. Let $M_A$ be the Turing machine that enumerates $A$ and $M_B$ be the Turing machine that enumerates $B$. $A \cap B$ is r.e. if there exists a Turing machine that enumerates its members. They can be enumerated if the tape looks like this: $A_0 \textvisiblespace A_1 \textvisiblespace A_2 \textvisiblespace ...$ Given $x \in \Sigma^*$ as input, we can create a Turing machine $M$ that enumerates the members of $A \cap B$.

Let $C = A \cap B$. For any $x$ as input, run $M_A$ and $M_B$ simultaneously in order to get all the members in $A$ and all the members in $B$. (If x exists in BOTH $A$ and $B$, then it will exist in $C$.) Since $A$ and $B$ are both r.e., and the number of members in $C$ is less than or equal to the number of members in $A \cap B$, then $C$ can also obviously be enumerated. Therefore, $C$, or $A \cup B$, is r.e.