COMPSCI/SFWRENG 2FA3

Discrete Mathematics with Applications II

Winter 2021

Assignment 9

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Assignment 9 consists of two problems. You must write your solutions to the problems using LaTeX.

Please submit Assignment 9 as two files, `Assignment_9_YourMacID.tex` and `Assignment_9_YourMacID.pdf`, to the Assignment 9 folder on Avenue under Assessments/Assignments. *YourMacID* must be your personal MacID (written without capitalization). The `Assignment_9_YourMacID.tex` file is a copy of the LaTeX source file for this assignment (`Assignment_9.tex` found on Avenue under Contents/Assignments) with your solution entered after each problem. The `Assignment_9_YourMacID.pdf` is the PDF output produced by executing

```
pdflatex Assignment_9_YourMacID
```

This assignment is due **Sunday, April 4, 2021 before midnight.** You are allow to submit the assignment multiple times, but only the last submission will be marked. **Late submissions and files that are not named exactly as specified above will not be accepted!** It is suggested that you submit your preliminary `Assignment_9_YourMacID.tex` and `Assignment_9_YourMacID.pdf` files well before the deadline so that your mark is not zero if, e.g., your computer fails at 11:50 PM on April 4.

Although you are allowed to receive help from the instructional staff and other students, your submission must be your own work. Copying will be treated as academic dishonesty! If any of the ideas used in your submission were obtained from other students or sources outside of the lectures and tutorials, you must acknowledge where or from whom these ideas were obtained.

Problems

1. **[10 points]** Let $\Sigma = \{a, b\}$ and

$$L = \{x \in \Sigma^* \mid \#a(x) \text{ and } \#b(x) \text{ are both even}\}.$$

Construct a total Turing machine that accepts L. Present the TM using a transition table or diagram formally, and describe how it works informally.

Aamina Hussain, hussaa54, April 4, 2021

Let $M = (Q, \Sigma, \Gamma, \vdash, \_, \delta, s, t, r)$ be the TM where:

$Q = \{s, q_1, q_2, q_3, t, r\}$
$\Sigma = \{a, b\}$
$\Gamma = \Sigma \cup \{\vdash, \_\}$
δ is defined by the following table:

	$\vdash$	a	b	$\_$
s	$(s, \vdash, R)$	(q_3, a, R)	(q_1, b, R)	$(t, -, -)$
q_1	$(r, \vdash, R)$	(q_2, a, R)	(s, b, R)	$(r, -, -)$
q_2	$(r, \vdash, R)$	(q_1, a, R)	(q_3, b, R)	$(r, -, -)$
q_3	$(r, \vdash, R)$	(s, a, R)	(q_2, b, R)	$(r, -, -)$
t	$(t, \vdash, R)$	$(t, -, -)$	$(t, -, -)$	$(t, -, -)$
r	$(r, \vdash, R)$	$(r, -, -)$	$(r, -, -)$	$(r, -, -)$

This TM only moves right, starting from the left endmarker, and every time it reads from the input tape, it goes to the corresponding state. State s is when there are an even number of a's and b's. State q_1 is when there are an odd number of b's but an even number of a's. State q_2 is when there are an odd number of a's and an odd number of b's. State q_3 is when there are an odd number of a's and an even number of b's. You can only transition to the acceptance state t if you are in state s when you read the first blank symbol $\_$.

2. [**10 points**] Let $\Sigma = \{a, b\}$ and

$$L = \{x \in \Sigma^* \mid \#a(x) = \#b(x)\}.$$

Construct a total Turing machine that accepts L. Present the TM using a transition table or diagram formally, and describe how it works informally.

Aamina Hussain, hussaa54, April 4, 2021

Let $M = (Q, \Sigma, \Gamma, \vdash, \lrcorner, \delta, s, t, r)$ be the TM where:

$Q = \{s, q_1, q_2, q_3, t, r\}$
$\Sigma = \{a, b\}$
$\Gamma = \Sigma \cup \{\vdash, \lrcorner, x\}$
δ is defined by the following table:

	$\vdash$	a	b	x	$\lrcorner$
s	$(s, \vdash, R)$	(q_1, x, R)	(q_2, x, R)	(s, x, R)	$(t, -, -)$
q_1	$(r, \vdash, R)$	(q_1, a, R)	(q_3, x, L)	(q_1, x, R)	$(r, -, -)$
q_2	$(r, \vdash, R)$	(q_3, x, L)	(q_2, b, R)	(q_2, x, R)	$(r, -, -)$
q_3	$(s, \vdash, R)$	(q_3, a, L)	(q_3, b, L)	(q_3, x, L)	$(r, -, -)$
t	$(t, \vdash, R)$	$(t, -, -)$	$(t, -, -)$	$(t, -, -)$	$(t, -, -)$
r	$(r, \vdash, R)$	$(r, -, -)$	$(r, -, -)$	$(r, -, -)$	$(r, -, -)$

a. Begin in the start state s.

b. If there is an a available before a b on the input tape: Move right until you see the first a, replace it with an x, and transition into state q_1. Then continue moving right until you see the first b, replace it with an x, and transition into state q_3. Then move left until you reach the left endmarker and transition into state s.

c. If there is a b available before an a on the input tape: Move right until you see the first b, replace it with an x, and transition into state q_2. Then continue moving right until you see the first a, replace it with an x, and transition into state q_3. Then move left until you reach the left endmarker and transition into state s.

d. Repeat step b)/step c) until all the a's and b's are replaced with x's.

e. You can only transition to the acceptance state t if you are in state s when you read the first blank symbol $\lrcorner$.