COMPSCI/SFWRENG 2FA3
Discrete Mathematics with Applications II
Winter 2021

Assignment 6

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Revised: March 10, 2021

Assignment 6 consists of two problems. You must write your solutions to the problems using LaTeX.

Please submit Assignment 6 as two files, `Assignment_6_`*`YourMacID`*`.tex` and `Assignment_6_`*`YourMacID`*`.pdf`, to the Assignment 6 folder on Avenue under Assessments/Assignments. *`YourMacID`* must be your personal MacID (written without capitalization). The `Assignment_6_`*`YourMacID`*`.tex` file is a copy of the LaTeX source file for this assignment (`Assignment_6.tex` found on Avenue under Contents/Assignments) with your solution entered after each problem. The `Assignment_6_`*`YourMacID`*`.pdf` is the PDF output produced by executing

```
pdflatex Assignment_6_YourMacID
```

This assignment is due **Sunday, March 14, 2021 before midnight.** You are allow to submit the assignment multiple times, but only the last submission will be marked. **Late submissions and files that are not named exactly as specified above will not be accepted!** It is suggested that you submit your preliminary `Assignment_6_`*`YourMacID`*`.tex` and `Assignment_6_`*`YourMacID`*`.pdf` files well before the deadline so that your mark is not zero if, e.g., your computer fails at 11:50 PM on March 14.

Although you are allowed to receive help from the instructional staff and other students, your submission must be your own work. Copying will be treated as academic dishonesty! If any of the ideas used in your submission were obtained from other students or sources outside of the lectures and tutorials, you must acknowledge where or from whom these ideas were obtained.

Background

For any three regular expressions α, β, and γ, the following properties hold:

1. Commutativity of union: $\alpha + \beta = \beta + \alpha$
2. Associativity of union: $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$
3. Associativity of concatenation: $(\alpha\beta)\gamma = \alpha(\beta\gamma)$
4. Distribution of union: $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$ and $(\beta + \gamma)\alpha = \beta\alpha + \gamma\alpha$.
5. $\emptyset$ is the identity element for union: $\alpha + \emptyset = \alpha$.
6. $\emptyset$ is the annihilator element for concatenation: $\alpha\emptyset = \emptyset\alpha = \emptyset$.
7. ϵ is the identity element for concatenation: $\alpha\epsilon = \alpha$.
8. Idempotence of Kleene star: $\alpha^{**} = \alpha^*$.
9. Some additional properties of Kleene star:
   a. $\alpha^* + \epsilon = \alpha^*$
   b. $\alpha^* + \alpha^* = \alpha^*$
   c. $\alpha^*\alpha^* = \alpha^*$
   d. $\alpha\alpha^* + \epsilon = \alpha^*$
   e. $\alpha^*\alpha + \epsilon = \alpha^*$

Problems

1. The regular expression $\alpha = (0110 + 01)(10)^*$ can be written in a more simplified way.
   a. **[10 points]** By reasoning based on $L(\alpha)$, define a shorter regular expression equal to α. Explain your reasoning.
   b. **[10 points]** Using the properties explained above, prove that the expression you came up with in (a), is indeed equal to α.

Aamina Hussain, hussaa54, March 14, 2021

part a)

A shorter regular expression β that is equal to α is $\beta = 01(10)^*$. The language that the regular expression α describes says that the string can begin with:

1. 01 followed by an arbitrary amount of 10s, or
2. 0110 followed by an arbitrary number of 10s

However, in #2, 0110 ends with 10, meaning it follows strings described in #1. 0110 is 01 followed by an arbitrary number of 10s, in this case one 10. 0110 is $01(10)^1$. We can therefore remove the 0110 from α in order to shorten it.

$$
\begin{aligned}
& L(\alpha) & \\
&= L((0110+01)(10)^*) & \langle \text{sub. for } \alpha \rangle \\
&= L(0110+01)L((10)^*) & \langle \text{definition of L} \rangle \\
&= (L(0110) \cup L(01))(L(10))^* & \langle \text{definition of L} \rangle \\
&= (\{0110\} \cup \{01\})\{10\}^* & \langle \text{definition of L} \rangle \\
&= \{0110\}\{10\}^* \cup \{01\}\{10\}^* & \langle \text{concat. distributes over union} \rangle \\
&= \{0110\}\{\epsilon, 10, 1010, ...\} \cup \{01\}\{\epsilon, 10, 1010, ...\} & \langle \text{definition of asterate} \rangle \\
&= \{0110, 011010, 01101010, ...\} \cup \{01, 0110, 011010, ...\} & \langle \text{definition of concatenation} \rangle
\end{aligned}
$$

As you can see, the union of the two sets above will result in a set that is equal to the set on the right above. This means that everything in the set on the left is in the set on the right. Therefore, we don't need the set on the left of the union. More specifically, we do not need 0110. Continuing the above calculation below:

$$
\begin{aligned}
&= \{01, 0110, 011010, ...\} & \langle \text{definition of union} \rangle \\
&= \{01\}\{\epsilon, 10, 1010, ...\} & \langle \text{definition of concatenation} \rangle \\
&= \{01\}\{10\}^* & \langle \text{definiton of asterate} \rangle \\
&= L(01)(L(10))^* & \langle \text{definition of L} \rangle \\
&= L(01)L((10)^*) & \langle \text{definition of L} \rangle \\
&= L(01(10)^*) & \langle \text{definition of L} \rangle \\
&= L(\beta) & \langle \text{sub. for } \beta \rangle
\end{aligned}
$$

part b)

$$
\begin{aligned}
& (0110+01)(10)^* & \langle \alpha \rangle \\
&= (0110+01\epsilon)(10)^* & \langle \epsilon \text{ is the identity element for concatenation} \rangle \\
&= (01(10+\epsilon))(10)^* & \langle \text{Distribution of union} \rangle \\
&= 01((10+\epsilon)(10)^*) & \langle \text{Associativity of concatenation} \rangle \\
&= 01(10(10)^* + \epsilon(10)^*) & \langle \text{Distribution of union} \rangle \\
&= 01(10(10)^* + (10)^*) & \langle \epsilon \text{ is the identity element for concatenation} \rangle \\
&= 01(10(10)^* + (10)^* + \epsilon) & \langle \text{Additional property of Kleene star (9.a)} \rangle \\
&= 01(10(10)^* + \epsilon + (10)^*) & \langle \text{Commutativity of union} \rangle \\
&= 01((10)^* + (10)^*) & \langle \text{Additional property of Kleene star (9.d)} \rangle \\
&= 01(10)^* & \langle \text{Additional property of Kleene star (9.b); } \beta = \alpha \text{ simplified} \rangle
\end{aligned}
$$