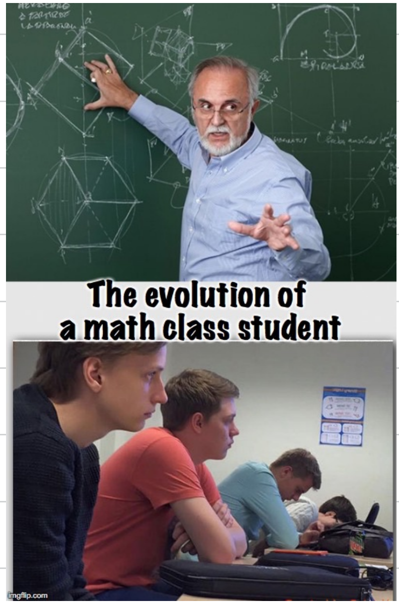
Session -3 # LINEAR ALGEBRA - 3 Feb 01, 2024

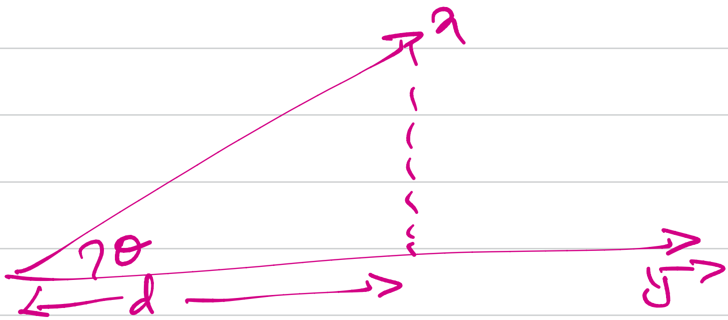


AGENDA

1. How to shift Lines ??
2. Why weight vector is $\perp$r to Line.
3. Shortest distance b/w origin & a Line.
4. " " " a point & " ".

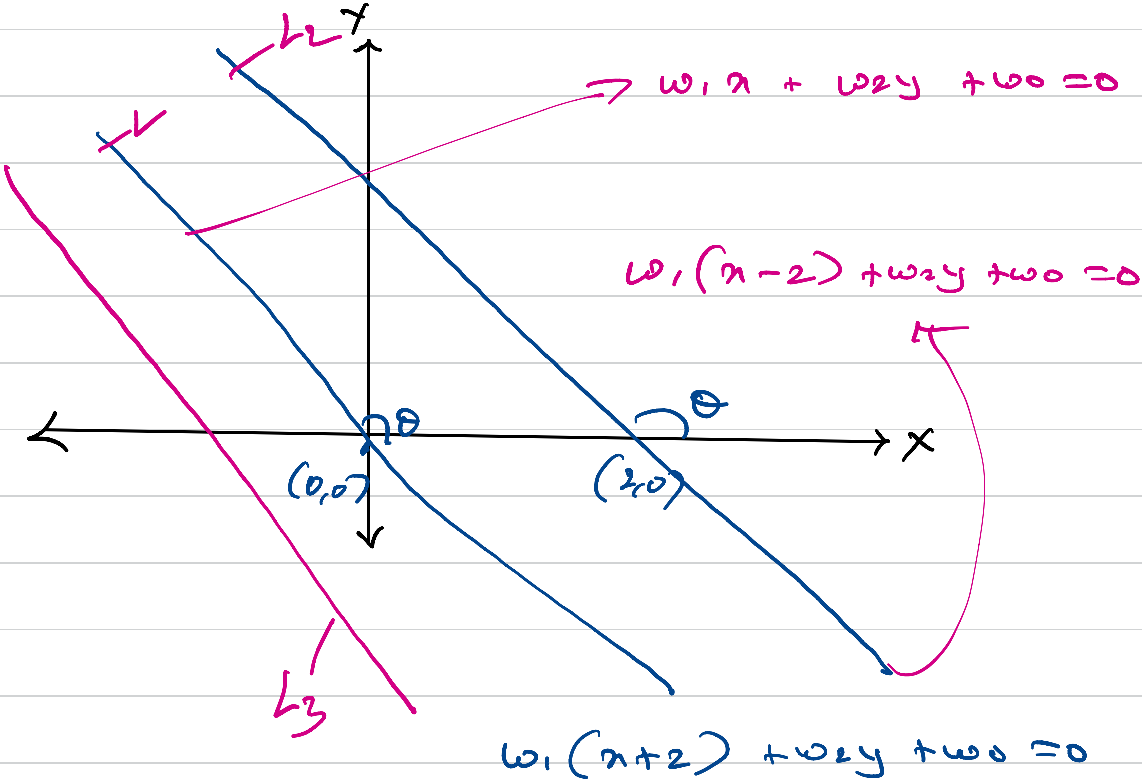
① Slope → $\tan(\theta) = \frac{P}{b} = \frac{y_2 - y_1}{x_2 - x_1}$

② ($\angle$) Angle b/w 2 Ve = $\cos(\theta)$ → Cosine similarity

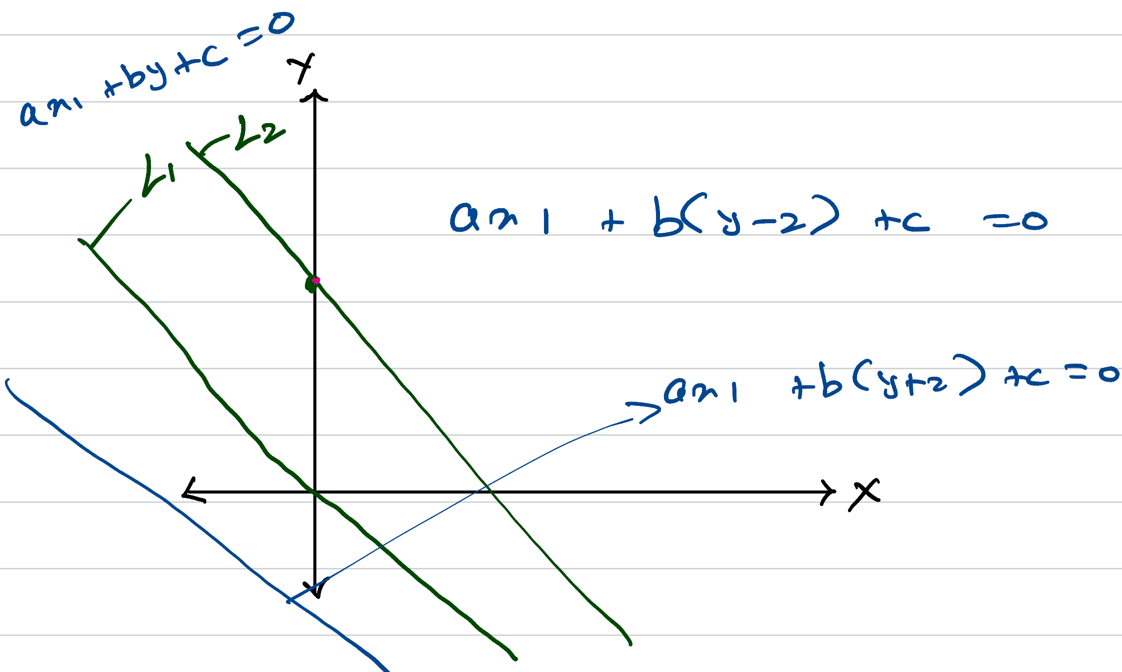


$d = x \cos\theta$

How to shift line



Y
X
L2
L
L3
θ
θ
(0,0)
(2,0)
$w_1 x + w_2 y + w_0 = 0$
$w_1(x-2) + w_2 y + w_0 = 0$
$w_1(x+2) + w_2 y + w_0 = 0$



$a x_1 + by + c = 0$
Y
X
L1
L2
$a x_1 + b(y-2) + c = 0$
$a x_1 + b(y+2) + c = 0$

$$w_1 x + w_2 y + w_0 = 0$$

$\hookrightarrow y = mx + c$

$$y = -\frac{w_1 x}{w_2} - \frac{w_0}{w_2}$$ $\rightarrow$ If this intercept

and if pass through origin $\rightarrow$ w_0 has to be zero

$$w_1(x-2) + w_2(y-2) + w_0 = 0$$

$$\Rightarrow w_1 x - 2w_1 + w_2 y - 2w_2 + w_0 = 0$$

$$w_1 x + w_2 y + w_0 - 2w_1 - 2w_2$$

$$y = \left(\frac{2w_1 + 2w_2 - w_0}{w_2}\right) - \frac{w_1 x}{w_2}$$

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

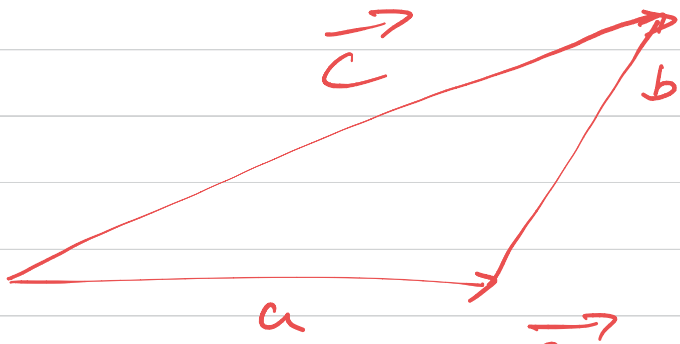
$$w_1(x_1 - a) + w_2 x_2 + w_0 = 0$$

x axis (→) → right

y axis (→) → up

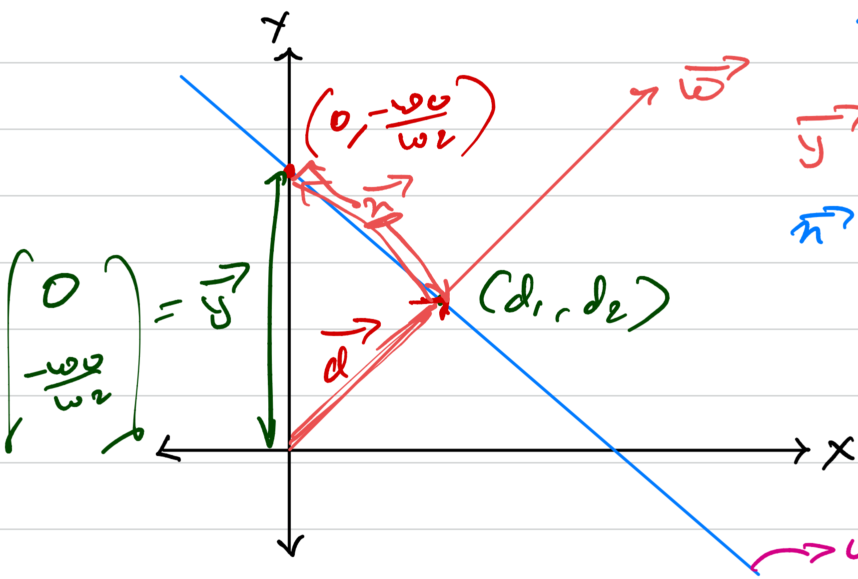
(against your logic disclaimer)

→ Vector



$\vec{c} = \vec{a} + \vec{b}$

Law of vector addition



$\vec{y} = \vec{x} + \vec{a}$

$\vec{x} = \vec{y} - \vec{d}$

→ $w_1 x + w_2 y + w_0 = 0$

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad \vec{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$w_1 d_1 + w_2 d_2 + w_0 = 0 \quad - \; 1$$

$$\vec{d} = K \times \vec{w} \quad - \; 2$$

$$\Rightarrow K \times \hat{w} \Longleftrightarrow \hat{w} = \frac{\vec{w}}{\|w\|}$$

$$\Updownarrow$$

$$= \begin{bmatrix} \dfrac{w_1}{\sqrt{w_1^2 + w_2^2}} \\ \dfrac{w_2}{\sqrt{w_1^2 + w_2^2}} \end{bmatrix}$$

$$\vec{d} = \begin{bmatrix} \dfrac{K \cdot w_1}{\sqrt{w_1^2 + w_2^2}} \\ \dfrac{K \, w_2}{\sqrt{w_1^2 + w_2^2}} \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$d_1 = \frac{K \cdot \omega_1}{\sqrt{\omega_1^2 + \omega_2^2}} \; ; \quad d_2 = \frac{K \cdot \omega_2}{\underline{\sqrt{\omega_1^2 + \omega_2^2}}}$$

3

$$\omega_1 d_1 + \omega_2 d_2 + \omega_0 = 0$$

$$\omega_1 \left(\frac{K\omega_1}{\sqrt{\omega_1^2 + \omega_2^2}} \right) + \omega_2 \left(\frac{K\omega_2}{\sqrt{\omega_1^2 + \omega_2^2}} \right) + \omega_0 = 0$$

$$K \left(\frac{\omega_1^2}{\sqrt{\omega_1^2 + \omega_2^2}} + \frac{\omega_2^2}{\sqrt{\omega_1^2 + \omega_2^2}} \right) + \omega_0 = 0$$

$$\sqrt{\omega_1^2 + \omega_2^2} \rightarrow \|\omega\|$$

$$\Rightarrow K = \frac{-\omega_0 \times \|\omega\|}{\underline{\omega_1^2 + \omega_2^2}} \quad (4)$$

$$d_1 = \frac{K \cdot \omega_1}{\sqrt{\omega_1^2 + \omega_2^2}} \; ; \quad d_2 = \frac{K \cdot \omega_2}{\underline{\sqrt{\omega_1^2 + \omega_2^2}}}$$

From 3

3

$$d_1 = \frac{-w_0 \times \cancel{\|w\|}}{w_1^2 + w_2^2} \times \frac{w_1}{\cancel{\sqrt{w_1^2 + w_2^2}}}$$

$\cancel{\|w\|}$

$$= \frac{-w_0 \, w_1}{w_1^2 + w_2^2}$$

$$d_2 = \frac{-w_0 \, w_2}{w_1^2 + w_2^2} \Rightarrow \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-w_1 w_0}{w_1^2 + w_2^2} \\[2ex] \dfrac{-w_0 \, w_2}{w_1^2 + w_2^2} \end{bmatrix}$$

$$y = \begin{bmatrix} 0 \\ \dfrac{-w_0}{w_2} \end{bmatrix}$$

$$\vec{x} = \vec{y} - \vec{d}$$

$$\vec{x} = \begin{bmatrix} 0 - \left(\dfrac{-w_1 w_0}{w_1^2 + w_2^2} \right) \\[2ex] \dfrac{-w_0}{w_2} - \left(\dfrac{-w_0 \, w_2}{w_1^2 + w_2^2} \right) \end{bmatrix}$$

$$= \quad \vec{x} \begin{bmatrix} \dfrac{w_1 w_0}{w_1^2 + w_2^2} \\[2ex] \dfrac{-w_0(w_1^2 + w_2^2) + w_0 w_2^2}{w_2(w_1^2 + w_2^2)} \end{bmatrix}$$

$$d^T \cdot \vec{x} \qquad \begin{bmatrix} \dfrac{-w_1 w_0}{w_1^2 + w_2^2} \\[2ex] \dfrac{-w_0 w_2}{w_1^2 + w_2^2} \end{bmatrix}$$

$$\underbrace{d^T}_{\begin{bmatrix} \dfrac{-w_1 w_0}{w_1^2 + w_2^2} & -\dfrac{w_0 w_2}{w_1^2 + w_2^2} \end{bmatrix}} \times \begin{bmatrix} \dfrac{w_1 w_0}{w_1^2 + w_2^2} \\[2ex] \dfrac{-w_0(w_1^2 + w_2^2) + w_0 w_2^2}{w_2(w_1^2 + w_2^2)} \end{bmatrix}$$

$$\boxed{1 \times 2 \qquad 2 \times 1}$$

$$= 1$$

$$\Rightarrow \quad \underbrace{\frac{-w_1^2 w_0^2}{(w_1^2 + w_2^2)}} - \underbrace{\frac{w_2 w_0}{w_1^2 + w_2^2} \left(\frac{w_2^2 w_0 - w_0(w_1^2 + w_2^2)}{w_2(w_1^2 + w_2^2)} \right)}$$

$$\downarrow$$

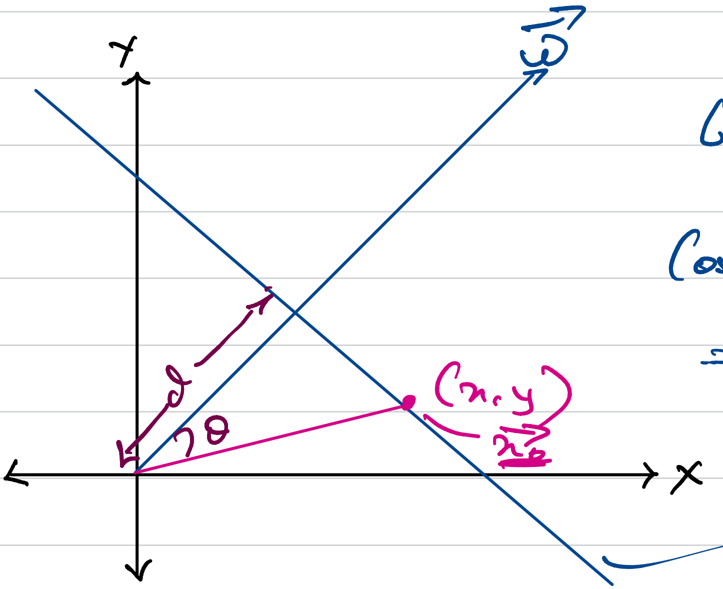
$$-\frac{\omega_1^2\omega_0^2}{(\omega_1^2+\omega_2^2)^2} + \frac{\omega_2^3\omega_0^2 - \omega_0^2\omega_2(\omega_1^2+\omega_2^2)}{\omega_2(\omega_1^2+\omega_2^2)^2}$$

$$\Rightarrow \frac{\cancel{\omega_1^2\omega_0^2\omega_2} + \cancel{\omega_2^3\omega_0^2} - \cancel{\omega_0^2\omega_2\omega_1^2} - \cancel{\omega_0^2\omega_2^3}}{\omega_2(\omega_1^2+\omega_2^2)^2}$$

$$\Rightarrow \underline{0}$$

$$d^T \cdot \vec{x} = 0$$

② Shortest distance to the line from Origin



$$\cos\theta = \frac{w^T \cdot x_0}{\|w\| \|x_0\|}$$

$$\cos\theta = \frac{d}{\|x_0\|}$$

$w^T x + w_0 = 0$

I

$$w^T x_0 + w_0 = 0$$

$$w^T x_0 = -w_0 \quad — (1)$$

$$d = \|x_0\| \cos\theta$$

$$\Rightarrow \cancel{\|x_0\|} \times \frac{w^T \cdot x_0}{\|w\| \cancel{\|x_0\|}}$$

$$d = \frac{w^T \cdot x_0}{\|w\|}$$

$$d = \frac{w^T \cdot x_0}{\|w\|} \Rightarrow \frac{-w_0}{\|w\|}$$

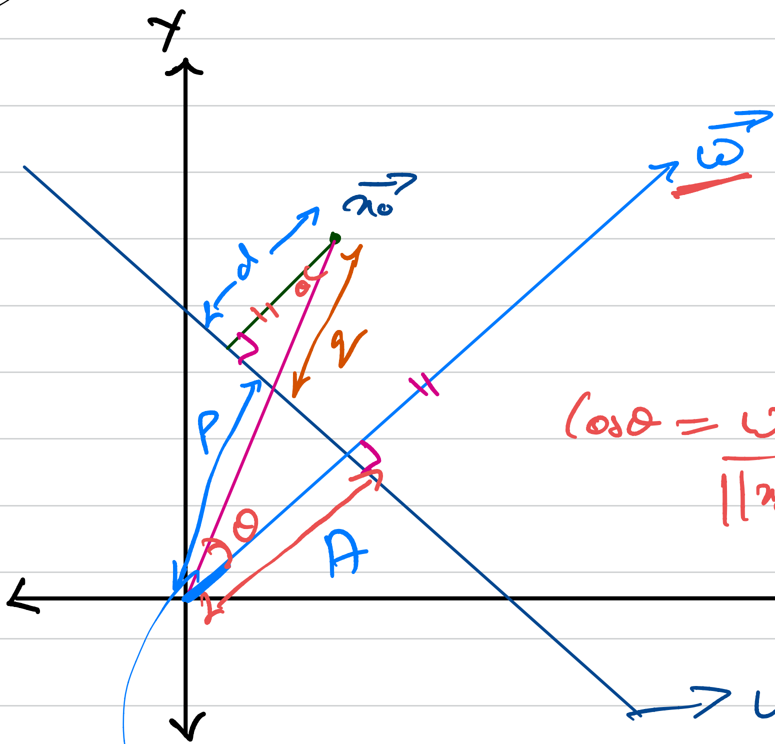
$$\boxed{d = \frac{-w_0}{\|w\|}}$$

II

$$d = \begin{bmatrix} \underbrace{\frac{-w_1 w_0}{w_1^2 + w_2^2}}_{a} \\ \underbrace{\frac{-w_0 w_2}{w_1^2 + w_2^2}}_{b} \end{bmatrix}$$

$$\|d\| = \sqrt{a^2 + b^2}$$

③ Dist b/w any pt & any line.



Alternate interior angle

$$\cos\theta = \frac{\omega^T \cdot \vec{x_0}}{||x_0||\,||\omega||}$$

$\rightarrow \omega^T x + \omega_0 = 0$

$$\cos\theta = \frac{A}{p} \Rightarrow p = \frac{A}{\cos\theta} \quad \text{— ①}$$

$$\cos\theta = \frac{d}{q}$$

$$d = q\cos\theta \rightarrow 2$$

$$p + q = ||\vec{x_0}|| \Rightarrow q = ||\vec{x}|| - p$$

$$r = \left|\,|\vec{x}|\,\right| - \frac{A}{\cos\theta}$$

$$d = r\cos\theta \Rightarrow d = \left(\|x\| - \frac{A}{\cos\theta}\right)\cos\theta$$

$$\Rightarrow d = \|x\|\underline{\cos\theta} - A$$

$$\cancel{\|x_0\|} \times \frac{w^T \cdot \vec{x_0}}{\|w\|\,\cancel{\|x_0\|}} - A$$

$$\frac{w^T \cdot \vec{x_0}}{\|w\|} - A$$

$$\Rightarrow \frac{w^T \cdot \vec{x_0}}{\|w\|} - \left(-\frac{w_0}{\|w\|}\right)$$

$$\Rightarrow \frac{w^T \cdot \vec{x_0} + w_0}{\|w\|}$$

$$\boxed{d = \frac{w^T \cdot \vec{x_0} + w_0}{\|w\|}}$$

Q. $x + y + z = 0$, Point $(30, 45, 0)$

$$w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \vec{x_0} = \begin{bmatrix} 30 \\ 45 \\ 0 \end{bmatrix}$$

$$\sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} = \|w\|$$

$$w^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 45 \\ 0 \end{bmatrix}$$

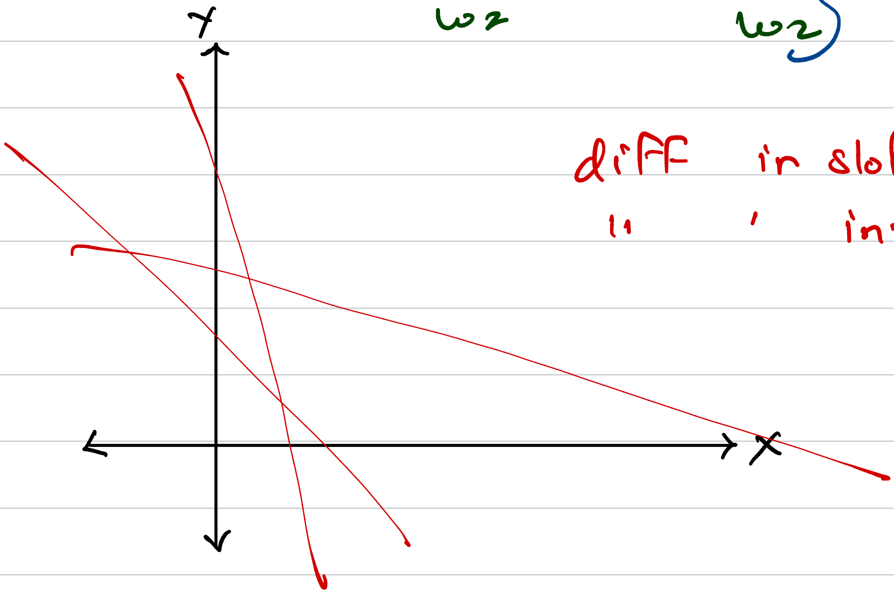
$$\frac{30 + 45 + 1 \times 0 + 0}{\sqrt{3}} \Rightarrow \frac{75}{\sqrt{3}}$$

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + w_0$$

$$y = mx + c$$

$$y = -\frac{w_1}{w_2} x - \frac{w_0}{w_2}$$ only on y axis



diff in slop

" " intercept