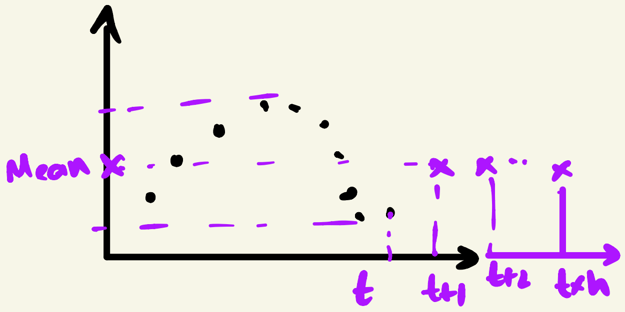
Lec-3 Time Series-2

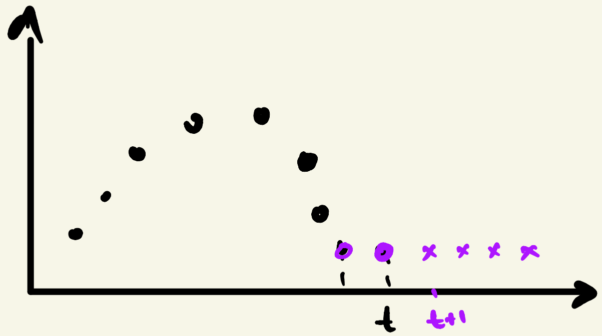
- Smoothing methods for forecasting
    - Moving average forecasting
    - Simple exponential smoothing
    - Double exponential smoothing
    - Triple exponential smoothing

- Concept of Stationarity

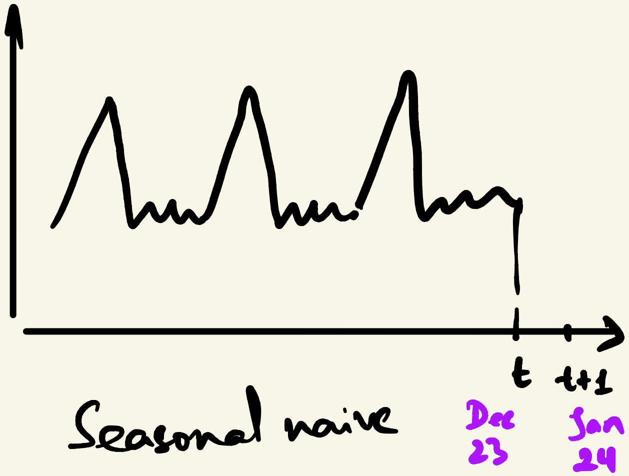
Recap



Mean

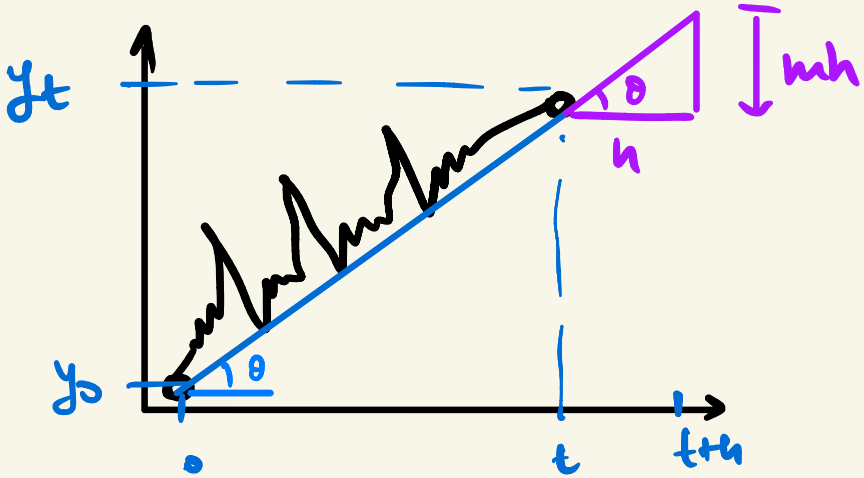


Naive $\hat{y}_{t+h} = y_t$



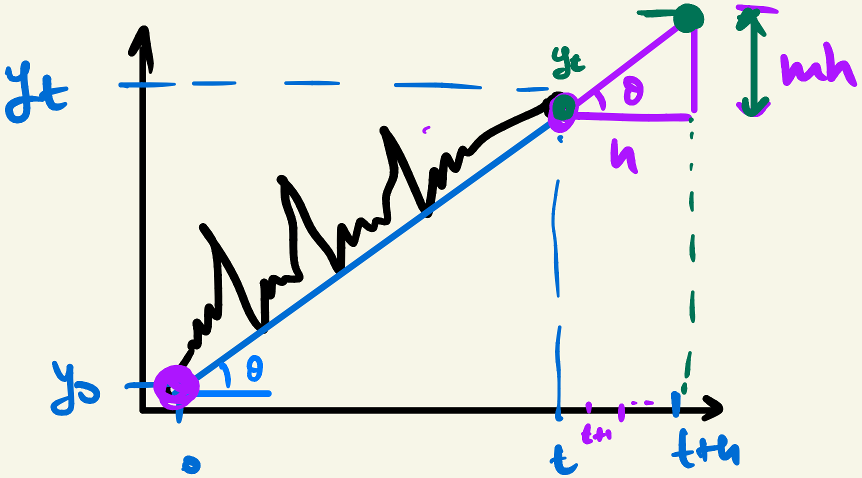
Seasonal naive

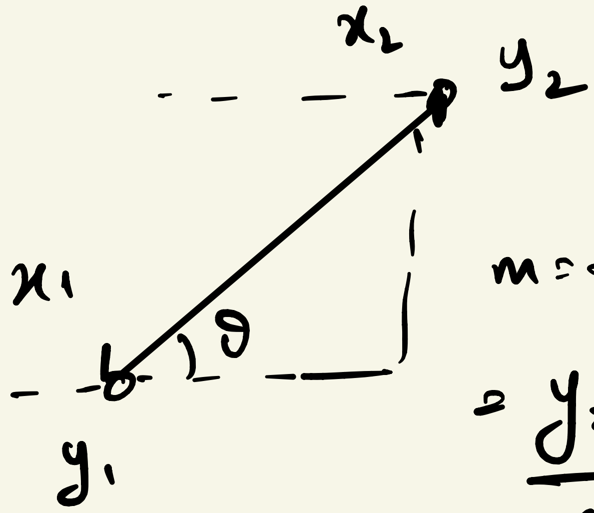
T = time period $\Rightarrow \hat{y}_{t+h} = y_{t+h-T}$



Drift

$$\text{Slope} = m = \frac{y_t - y_0}{t}$$





$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

Drift

$$\text{Slope} = m = \frac{y_t - y_0}{t}$$

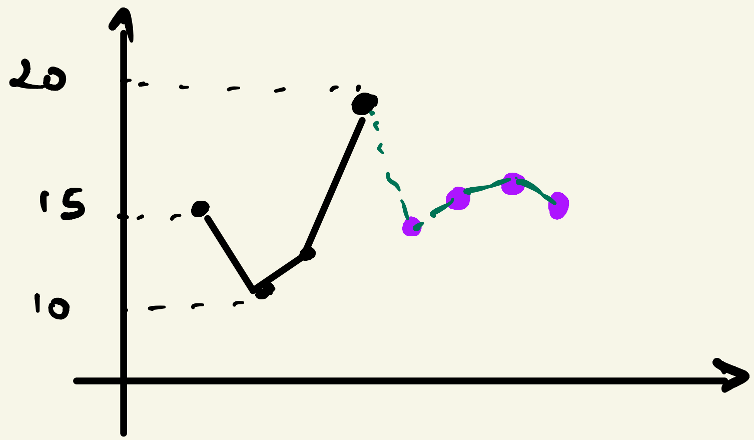
$$\boxed{\hat{y}_{t+h} = y_t + m \times h}$$

Smoothing methods

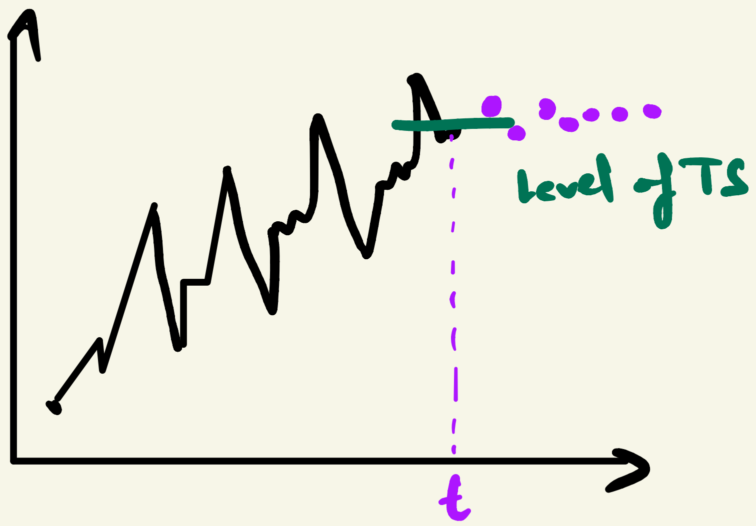
① Moving average forecasting

window = 3

t-3	t-2	t-1	t	t+1	t+2	t+3	t+4
15	10	12	20	14	15.33	16.44	15.25



$\hat{y}_{t+1}$ = MA of last 3 values

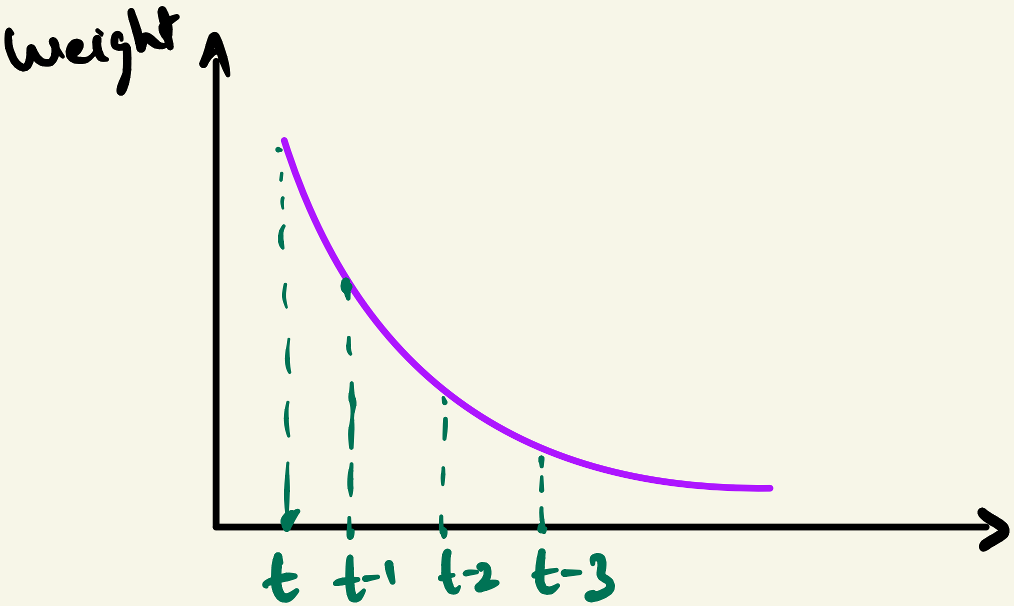


② Simple exponential smoothing

Date	Sales

Aug	Sep	Oct	Nov23	Dec23	Jan 24
	t-3	t-2	t-1	t	t+1
	15	10	12	20	

* Idea: As we go further into the past the weightage of older months should decrease



Eg. $$\hat{y}_{t+1} = 0.5\, y_t + 0.25\, y_{t-2} + 0.125\, y_{t-3} + \cdots$$

discussion
- → generate forecasts for known months
- → generate forecasts for unknown months

$t \to$ Dec 23

SES : $\rightarrow \quad \hat{y}_t = \alpha y_t + (1-\alpha)\hat{y}_{t-1}$

$\Rightarrow \hat{y}_{t-1} = \alpha y_{t-1} + (1-\alpha)\hat{y}_{t-2}$

$\Rightarrow \hat{y}_t = \alpha y_t + (1-\alpha)\left[\alpha y_{t-1} + (1-\alpha)\hat{y}_{t-2}\right]$

$= \alpha y_t + \alpha(1-\alpha) y_{t-1} + (1-\alpha)^2 \hat{y}_{t-2}$

$\Rightarrow \hat{y}_{t-2} = \alpha y_{t-2} + (1-\alpha)\hat{y}_{t-3}$

$\Rightarrow \hat{y}_t = \alpha y_t + \alpha(1-\alpha)y_{t-1} + (1-\alpha)^2\left[\alpha y_{t-2} + (1-\alpha)\hat{y}_{t-3}\right]$

$= \alpha y_t + \alpha(1-\alpha)y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + (1-\alpha)^3 \hat{y}_{t-3}$

$$\Rightarrow \hat{y}_t = \alpha y_t + \alpha(1-\alpha) y_{t-1} + \alpha(1-\alpha)^2 y_{t-2} + \alpha(1-\alpha)^3 y_{t-3} + \cdots$$

Eg. $\alpha = 0.1 \Rightarrow 1-\alpha = 0.9$

$$\hat{y}_t = 0.1 \times y_t + 0.1 \times 0.9\, y_{t-1} + 0.1 \times (0.9)^2 y_{t-2} + 0.1 \times (0.9)^3 y_{t-3} + \cdots$$

$$= 0.1 \times y_t + 0.09\, y_{t-1} + 0.081\, y_{t-2} + 0.072\, y_{t-3} + \cdots$$

Eg. $\alpha = 0.9 \Rightarrow (1-\alpha) = 0.1$

$$\hat{y}_t = 0.9\, y_t + 0.9 \times 0.1\, y_{t-1} + 0.9 \times (0.1)^2 y_{t-2} + 0.9 \times 0.1^3 y_{t-3} \cdots$$

$$= 0.9\, y_t + 0.09\, y_{t-1} + 0.009\, y_{t-2} + 0.009\, y_{t-3}$$

Eg. $\alpha = 1$

$$\hat{y}_t = y_t + 0 + 0 + 0 + 0$$

⇒ weight only to the most recent data pt.

Eg. $\alpha = 0.001 \Rightarrow 1-\alpha \simeq 1$

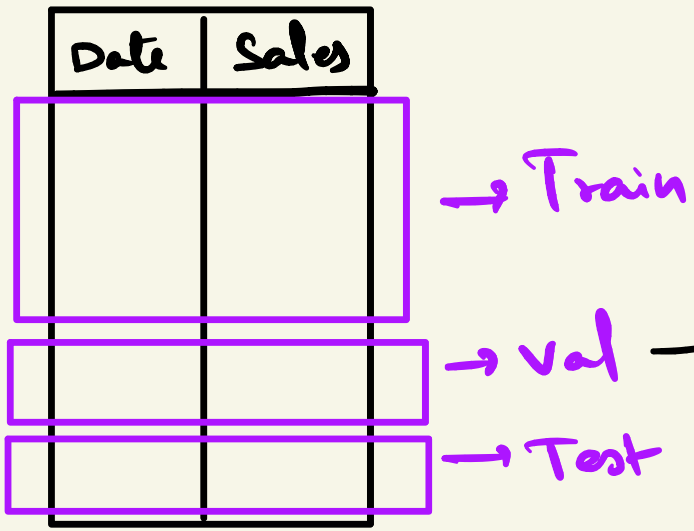
$$\hat{y}_t \simeq \alpha y_t + \alpha y_{t-1} + \alpha y_{t-2} + \alpha y_{t-3} + \dots$$

→ All data pts are getting equal weight

⇒ forecast is going to be close to global mean

Value of α

*



→ find the best value of α

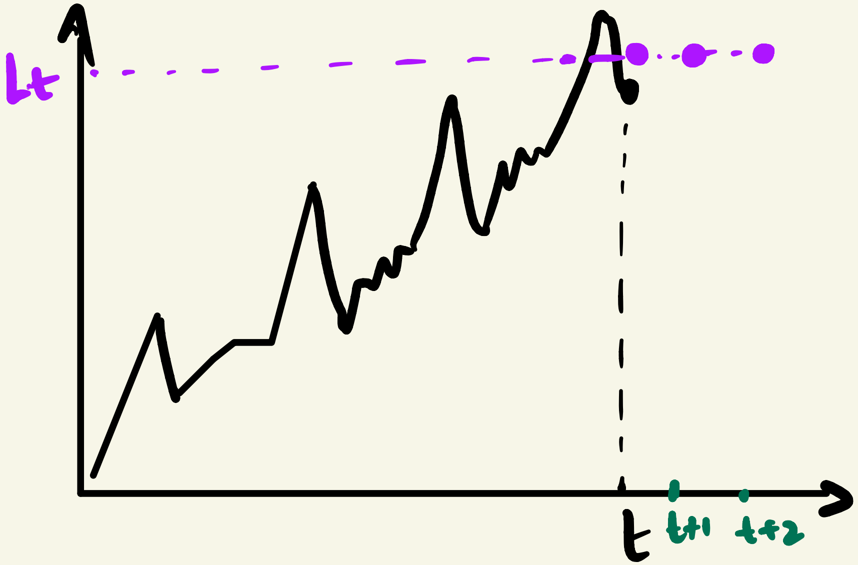
* $\alpha = \dfrac{1}{2 \times \text{seasonality}}$

for eg. seasonality = 12 months

$\alpha = \dfrac{1}{2 \times 12}$

$$\hat{y}_t = \alpha y_t + (1-\alpha)\hat{y}_{t-1} = \text{Level at } t = L_t$$

$$\hat{y}_{t+h} = \hat{y}_t = L_t$$



Final mathematical model:

$$\hat{y}_{t+h} = L_t = \alpha y_t + (1-\alpha)\hat{y}_{t-1}$$

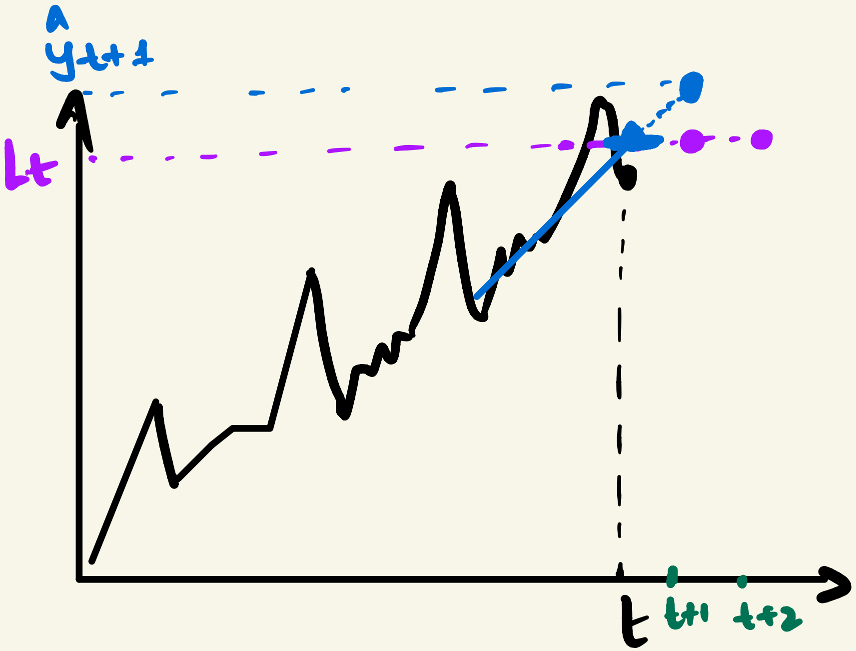
For eg. $h = 1$

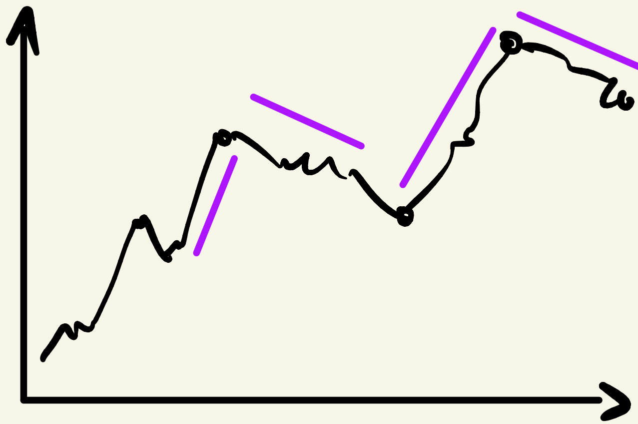
$$\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\hat{y}_{t-1}$$

③ Double exponential smoothing (Holt's method)

↳ * Level + Trend

* incorporate past trends as well
* more recent trend should have more weightage.





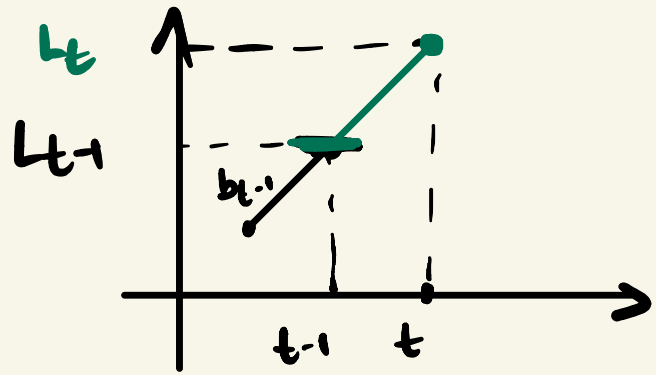
$$\hat{y}_{t+h} = L_t + h \times b_t$$

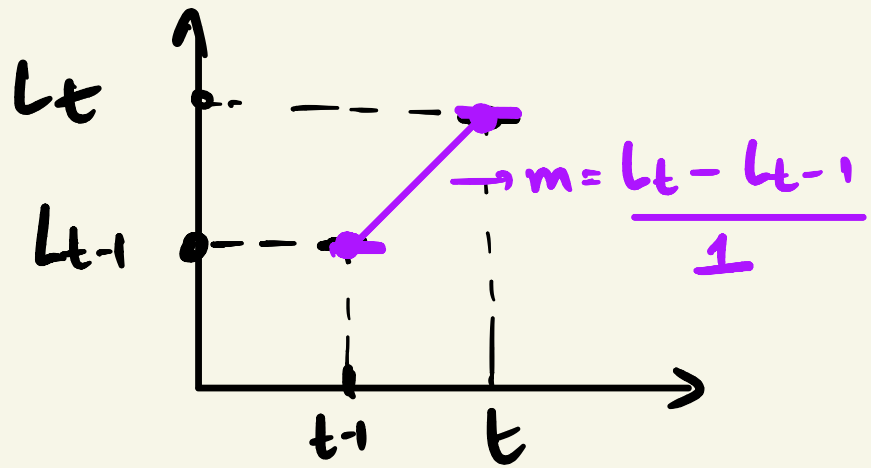
forecasting for known months

forecasting for unknown months

$$L_t = \alpha\, y_t + (1-\alpha)\left[L_{t-1} + b_{t-1}\right]$$

$$b_t = \beta \underbrace{\left(\frac{L_t - L_{t-1}}{1}\right)}_{\text{Current slope}} + (1-\beta)\underbrace{b_{t-1}}_{\text{all prev. slo}}$$





⑤ Triple exponential smoothing (Holt Winter's method)

↳ * level + trend + seasonality

* all past seasonality should be taken into account
* more recent seasonal value should get more weightage

$$\hat{y}_{t+h} = L_t + h b_t + S_{t+h-T}$$

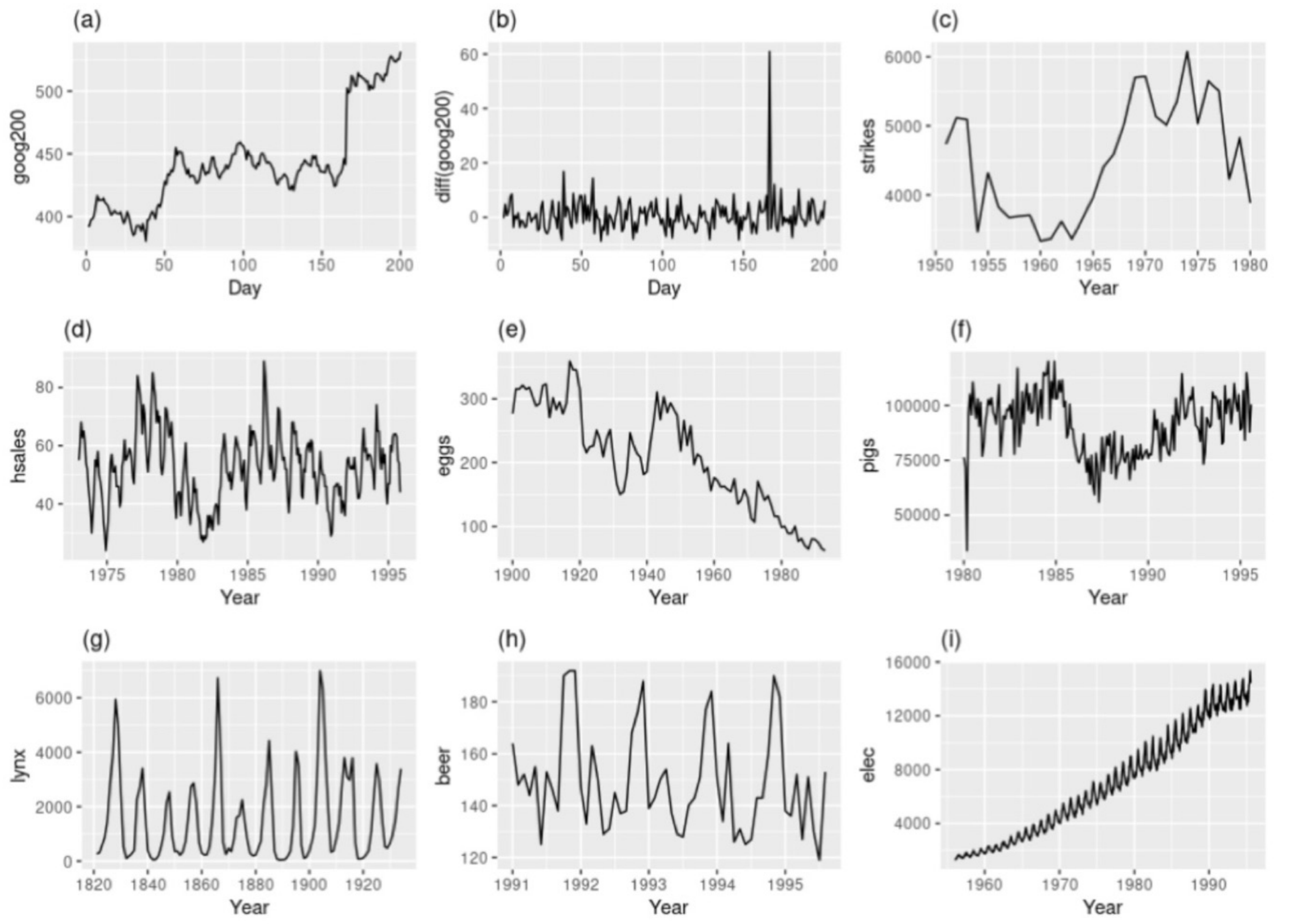
$$L_t = \alpha(y_t - \underbrace{S_{t-T}}) + (1-\alpha)(L_{t-1} + b_{t-1})$$

Subtract the effect of seasonal component from level

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta) b_{t-1}$$

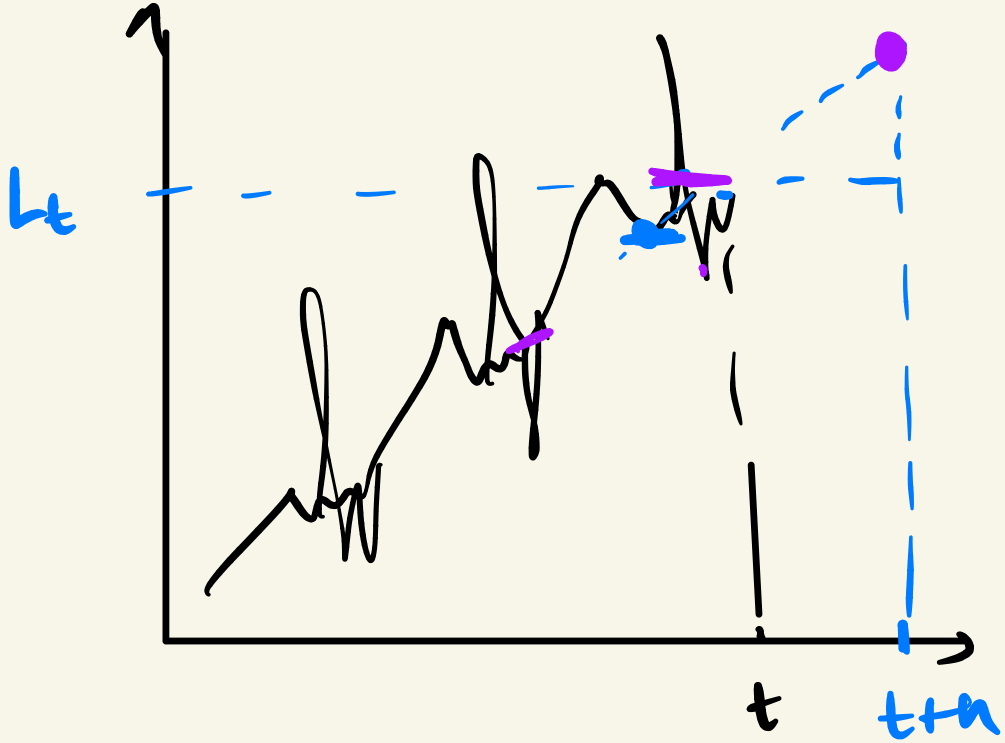
$$s_t = \gamma \underbrace{(y_t - l_{t-1} - b_{t-1})}_{\text{current seasonal component}} + (1-\gamma) \underbrace{s_{t-\tau}}_{\text{past seasonal components}}$$

Stationarity



(a)
goog200
Day
(b)
diff(goog200)
Day
(c)
strikes
Year
(d)
hsales
Year
(e)
eggs
Year
(f)
pigs
Year
(g)
lynx
Year
(h)
beer
Year
(i)
elec
Year

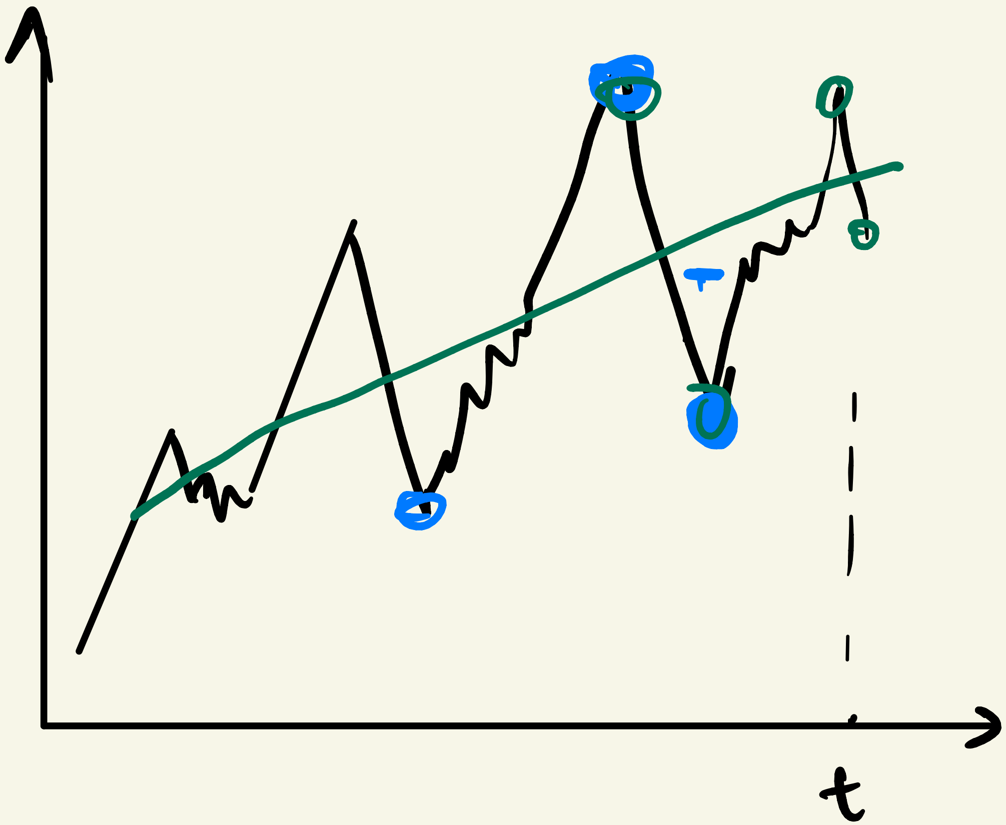
Doubt



$$L_t = \alpha y_t + (1-\alpha)\hat{y}_{t-1}$$

$$\hat{y}_{t+h} = L_t + h \times b_t$$

Level info → known datapoint

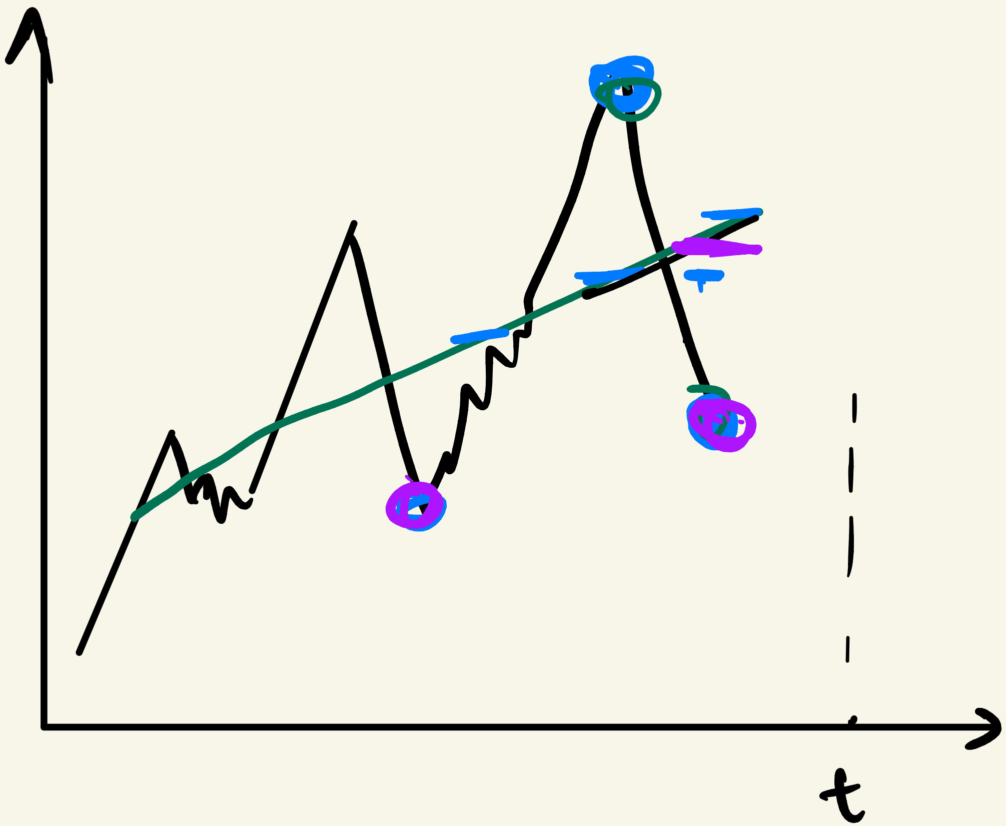


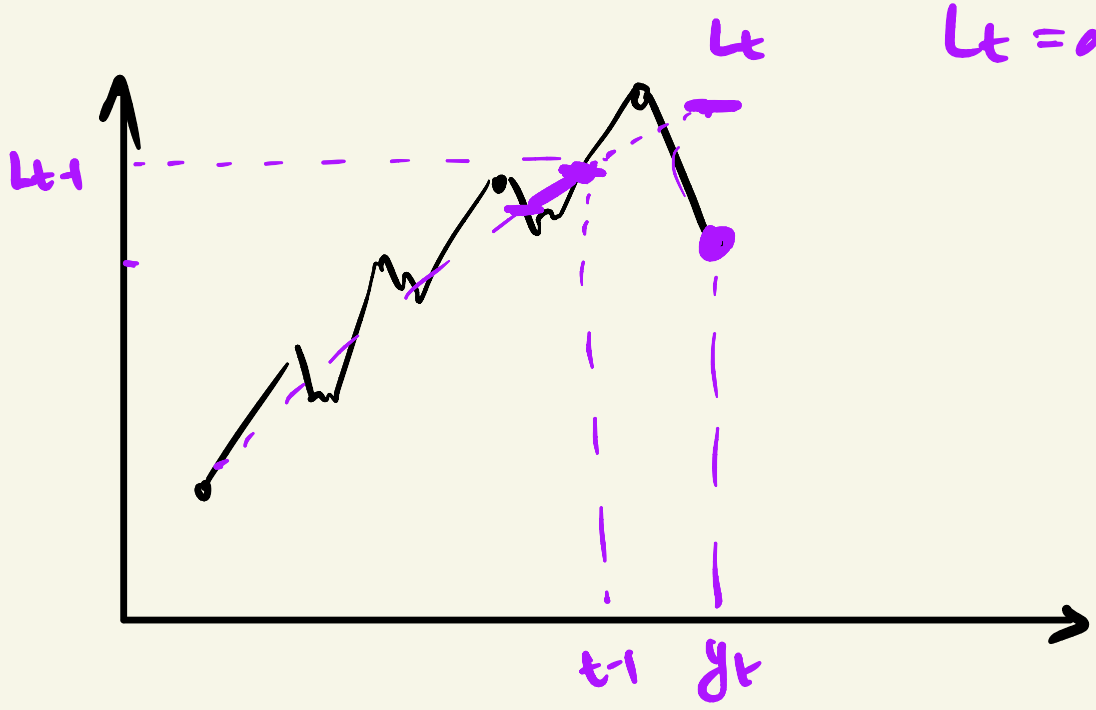
$$L_t = \alpha \underline{y_t} + (1-\alpha) L_{t-1}$$

Smoothing

$$10 \quad \underbrace{12 \quad 13 \quad 30}_{18} \quad 14 \quad 10$$

MA → level





$$L_t = \alpha y_t + (1-\alpha)(L_{t-1} + b_{t-1})$$