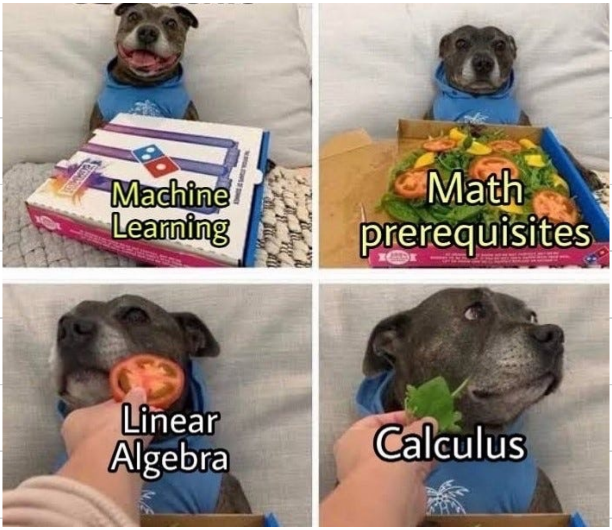
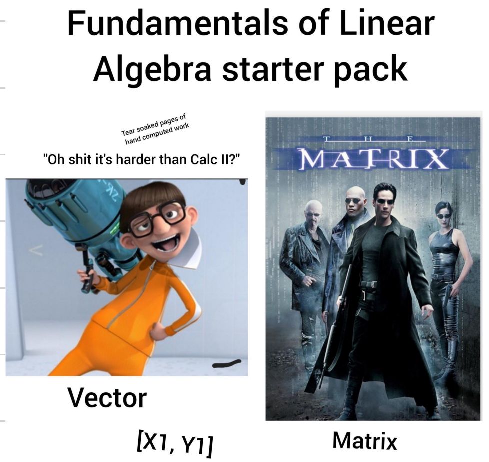
Session-2 — Linear Algebra-2





Agenda

1. Manually optimising a Classifier
2. Vector
   - → Intro
   - → Representation of Vector
   - → Magnitude of Vectors
   - → Norm of Vector
   - → Types of distances → Euclidion
     - → Manhattan
3. Matrix multiplication
4. Angle b/w two vectors
5. Connection b/w Linear algebra & Coord.geom
6. Unit Vector, Trigonometry & Projection.

$98x + 25y + \frac{26}{14} = 14$

$y = mx + c$

$-25y = 98x -$

$y = \frac{98}{25}$

Manual optimization of Classifier

x_1	y_1	x_3
33	27	0
13	6.5	0
43	39	0
22.6	16	0
13	35	
33.8	7.3	1
46	13	
10	19	1
22	30	
24	43	1

x_2	y_2	y_3
63	77	0
67	97	0
85.7	60	
93.8	90	1
91.4	105	1
73	114.8	1
41.7	66.7	
50.3	89.5	1
79	88	
56	104.5	1

1. Will 1 line be able to separate the two clusters — No
2. What do I use?? → Plane
3. Can of 3d plane??

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0$$

$$ax + by + c = 0$$ } 2d - hyperplane → generate 3d - hyperplane

VECTOR

Physics

1. Vector → magnitude & direction

ML

1. Vector → 1d collection of nos

1d → Vector
2d → matrix
3d → Tensor

denotion : $\vec{x}$ / $\bar{x}$ ≠ mean

[1, 2, 3, 4, 5, 6] → Representation

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix}_{6\times 1}$$

→ default Format of writing a vector

→ column vector

$x \in \mathbb{R}^6$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

row vector

column vector

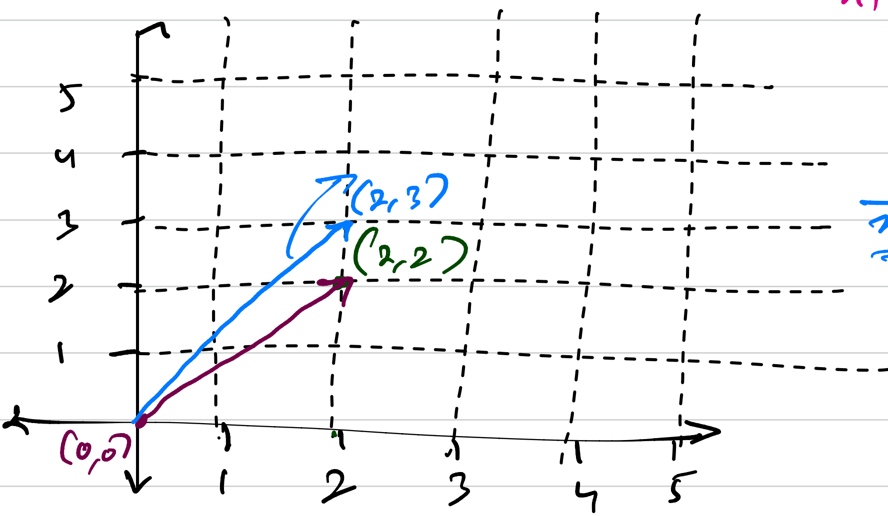
$$\boxed{\mathbb{R}}$$

$$\underset{\text{vector}}{x} \underset{\text{belongs to}}{\in} \underset{\mathbb{R} \text{ represents Real nos.}}{\mathbb{R}}^{\overset{\text{dimension}}{d}}$$

$$\underbrace{\text{* Did an alkaline test for salt water}}_{\text{went for swimming}}$$

Pictorial representation

$$\vec{x} = \begin{bmatrix} a \\ b \end{bmatrix}$$



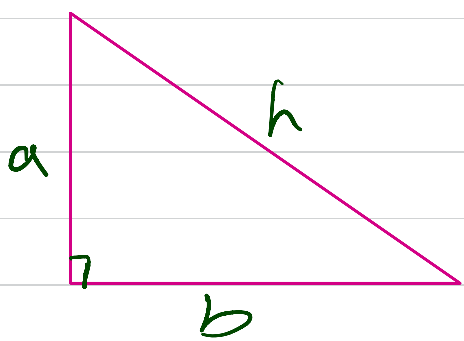
x axis

$$\vec{x_1} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

y axis

$$\vec{x_2} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$a = 2, \; b = 3$



$$h = \sqrt{a^2 + b^2}$$

only applicab

↓

angle

↓

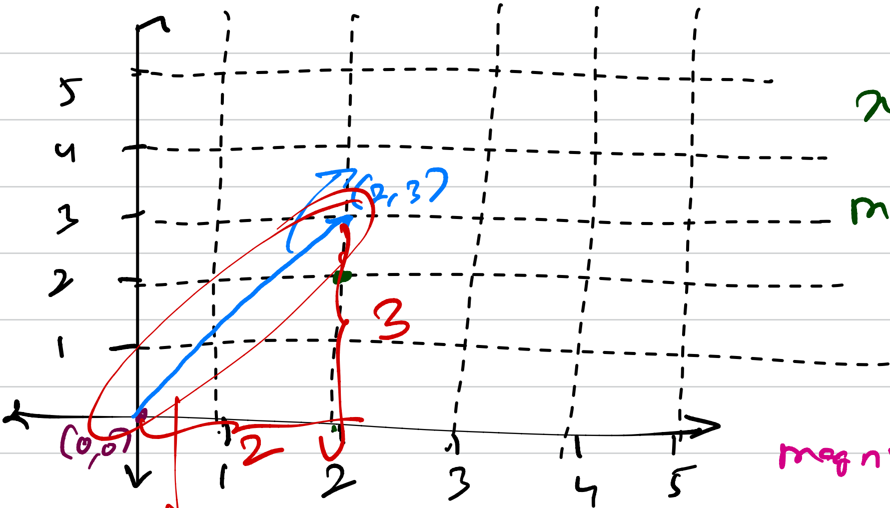
90°

$$= \sqrt{4+9}$$

$$= \sqrt{13}$$

$x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

magnitude =



magnitude

magnitude = $\sqrt{2^2 + 3^2}$

$= \sqrt{13}$

Vector $\vec{x} = \begin{bmatrix} a \\ b \end{bmatrix}$

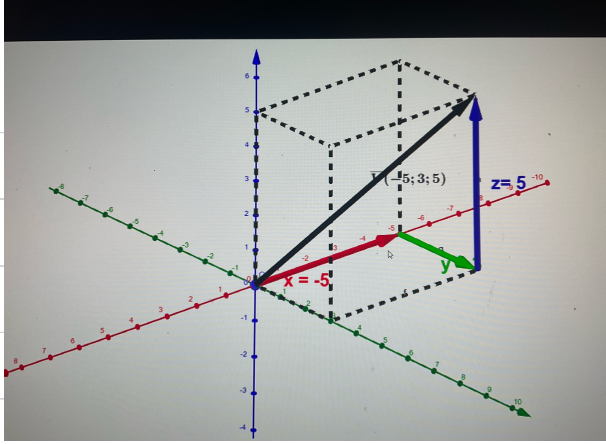
magnitude $= \sqrt{a^2 + b^2}$

$\bar{y} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

magnitude $= \sqrt{a^2 + b^2 + c^2}$

$\vec{p} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$

magnitude $= \sqrt{a_1^2 + a_2^2 - \cdots a_n^2}$



$\rightarrow$ **NORM** $\rightarrow$ nothing but magnitude

$\vec{x} \rightarrow$ Norm $\Rightarrow \|\vec{x}\|$ notation for norm

$$\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \text{Norm} = \sqrt{2^2 + 3^2} = \|x\|$$

Euclidean norm

$$\Updownarrow$$

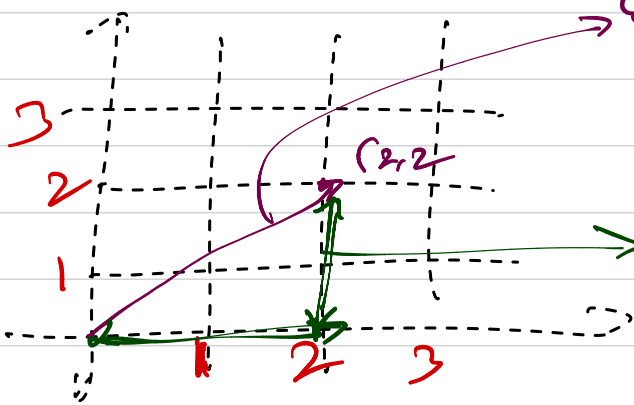
$$\|x\|_2$$

→ manhattan distance

displacement

Euclidian distance

$\sqrt{2^2+2^2}$



→ manhattan distance

$a+b$

$\Rightarrow 4$

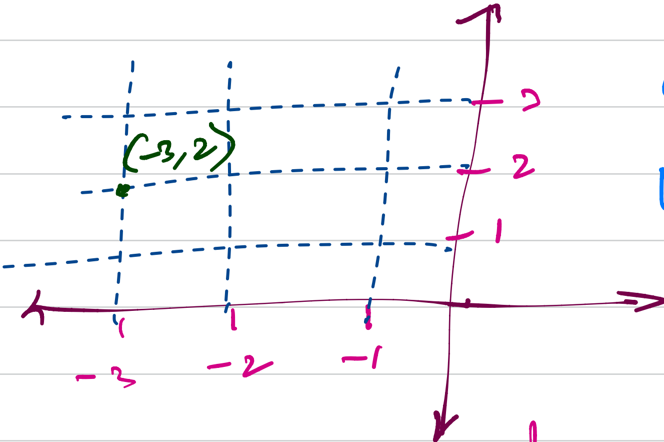
distance

$||x||_2$ → Euclidian distance → L2 norm

$||x||_1$ → Manhattan distance → L1 norm

$||x||$ → Euclidian distance

Euclidian distance = Euclidian norm



Euclidean dist = $\sqrt{13}$

Manhattan = $\cancel{-3 + 2}$

can dist be negative??

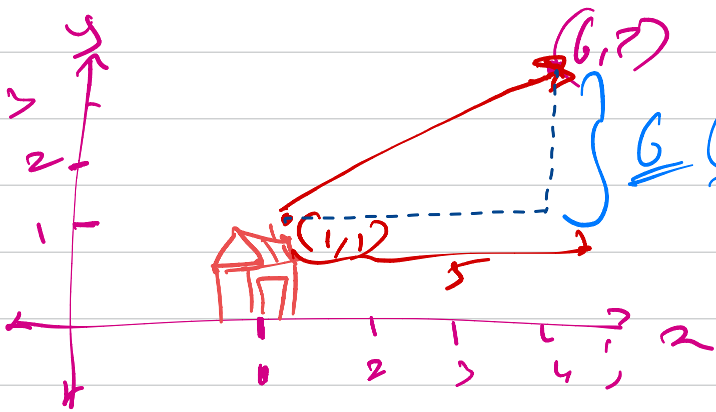
No

$|a| + |b|$

$|-3| + |2| = \underline{5}$

$$\left.\begin{matrix} \sqrt{(-3)^2 + 2^2} \\ \sqrt{(|-3|)^2 + 2^2} \end{matrix}\right\} \text{doesn't matter}$$

→ So far, I've taken origin as reference point



$\underline{6}$ $\underline{(7-1)}$

$$\text{Euclidean} = \sqrt{6^2 + 5^2}$$

$$\text{Manhattan} = |5| + |6| = \underline{\underline{11}}$$

Matrix multiplication

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \quad \begin{array}{cc} 1 & 2 \\ \hline \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \end{array}_{2\times 2}$$

$A_{m\times n}$, $B_{p\times q}$

$n = p$ should be equal

$$\begin{bmatrix} 1\times 5 + 2\times 7 & 1\times 6 + 2\times 8 \\ 3\times 5 + 4\times 7 & 3\times 6 + 4\times 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$A_{m\times n} \,/\, B_{p\times q} \Rightarrow \underline{C_{m\times q}}$$

$$m\times \boxed{n \cdot p}\times q$$

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times 1} \qquad \vec{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2\times 1}$$

$$\vec{x}^T = \begin{bmatrix} 1, & 2 \end{bmatrix}_{\underline{1\times 2}} \qquad \vec{y} \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{\underline{2}\times 1}$$

$$\vec{x}^T \cdot \vec{y} = \begin{bmatrix} 1\times 3 + 2\times 4 \end{bmatrix}$$

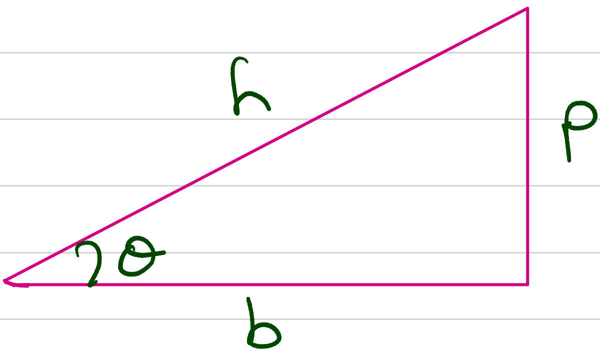
$$= \underline{\begin{bmatrix} 11 \end{bmatrix}}$$

$$\vec{x}^T \cdot y \;\cancel{=}\; \vec{y}^T \cdot x$$

np.dot(x, y)

dot Product

Trigonometry



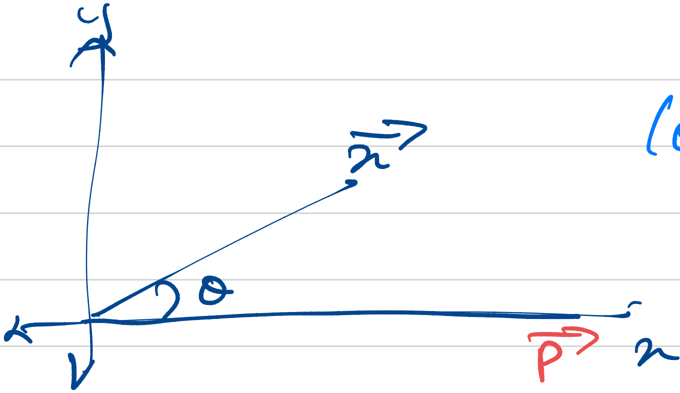
$$\sin(\theta) = \frac{p}{h}$$

$$\cos(\theta) = \frac{b}{h}$$

$$\tan(\theta) = \frac{p}{b}$$

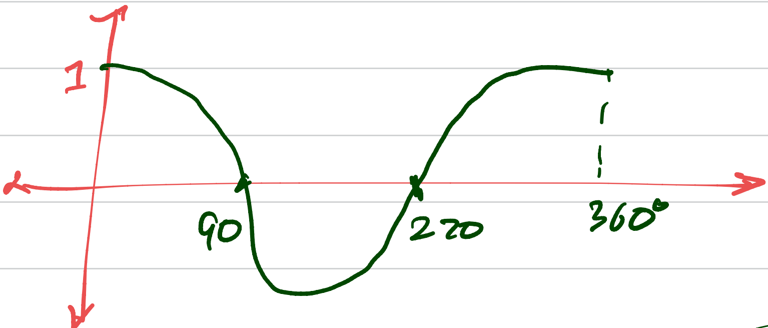
Pandit	bhadri	Prasad
hari	hari	bol
$\sin\theta$	$\cos\theta$	$\tan\theta$

Deg/Rad	$\sin(x)$	$\cos(x)$	$\tan(x)$
0° or 0 360° or 2π	0	1	0
30° or $\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45° or $\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60° or $\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90° or $\frac{\pi}{2}$	1	0	undefined
120° or $\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
135° or $\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
150° or $\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
180° or π	0	-1	0



$$\cos(\theta) = \frac{\vec{x}^T \cdot \vec{p}}{\boxed{|\vec{x}| \times |\vec{p}|}}$$

Proof in next class



$\cos(\theta) > 0$

what range

$(0-90)$ & $(270-360)$

How Coord. geom meets Linear algebra

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} \vec{x_1} \\ \vec{x_2} \end{bmatrix}$$

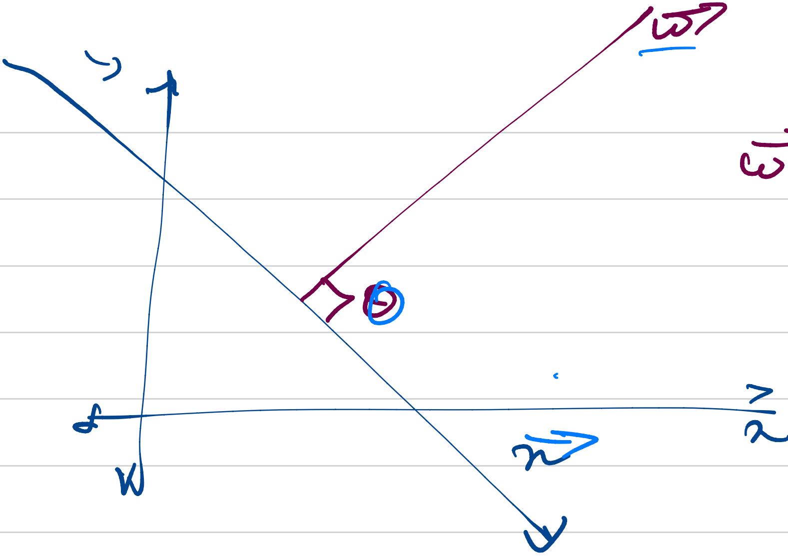
$$\vec{w}^T \cdot \vec{x} = \begin{bmatrix} w_1 & w_2 \end{bmatrix}_{1\times 2} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{2\times 1}$$

$$= \underbrace{w_1 x_1 + w_2 x_2 + w_0 = 0}_{\text{2d - hyperplane / Line}}$$

For n dim - Vector

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_d \end{bmatrix} \qquad \vec{x} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_c \end{bmatrix}$$

$$\underbrace{\vec{w}^T \cdot \vec{x} + w_0 = 0}_{\text{d - dim - hyperplane}}$$



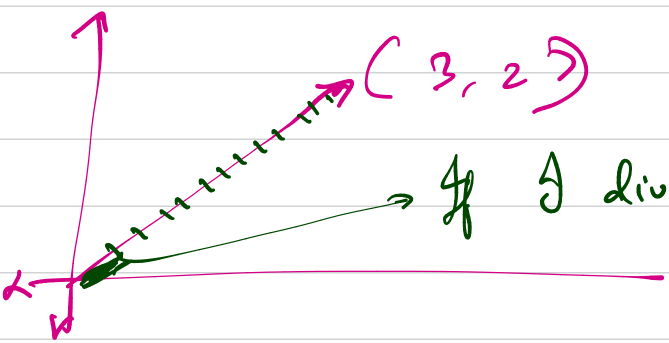
$\vec{w}$ always $\perp$r to $\vec{x}$ vecto

$$\cos(\theta) = \frac{\vec{w}^T \cdot \vec{x}}{\|\vec{w}\| \, \|\vec{x}\|}$$

$$\theta = \vec{w}^T \cdot \vec{x}$$

If $= 0$, $\theta = 90$??

Unit Vector



$(3, 2)$ $\sqrt{13}$

If I divide into $\sqrt{13}$ Parts
length of each Part $= 1$

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad \hat{w} = \frac{\vec{w}}{||w||}$$

→ vecto

→ modu

unit vector

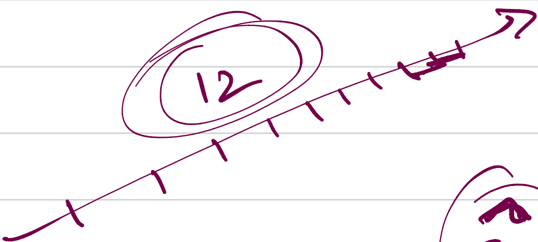
$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \sqrt{1+1} = \sqrt{2}$$

$$\hat{w} = \begin{bmatrix} \dfrac{w_1}{\sqrt{w_1^2 + w_2^2}} \\ \dfrac{w_2}{\sqrt{w_1^2 + w_2^2}} \end{bmatrix}$$

$$a^T = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \qquad b = [8, 0]$$

2×1 $\qquad$ 1×2

2×2



→ divide into 12

↳ length of each part = 1

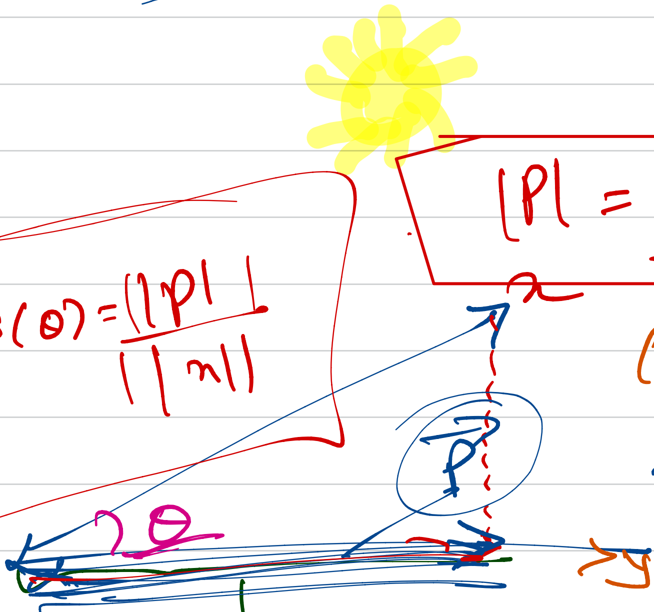
$$\hat{x} = \frac{\vec{x}}{||x||}$$

Vector Projection

$$\cos(\theta) = \frac{||p||}{||x||}$$

$$\boxed{|P| = ||x|| \cos(\theta)}$$

$$\cos\theta = \frac{\vec{x}^T \cdot y}{||x|| \, ||y||}$$



Projection of x (shadow)

$\theta_1 \rightarrow$ Angle b/w $\vec{P}$ & $\vec{y}$

$$\cos(\theta_1) = \frac{\vec{p}^T \cdot \vec{y}}{||p|| \, ||y||}$$

$\cos(0) = 1$

$$1 = \frac{\vec{p}^T \cdot \vec{y}}{||p|| \, ||y||} \Rightarrow \boxed{||p|| = \frac{\vec{p}^T \cdot \vec{y}}{||y||}}$$

$$||P|| = \underline{\vec{P}^T \cdot \vec{y}}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 2 & 3 & 5 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{bmatrix}$$

$3 \times \boxed{2 \quad 2} \times 3 = \underline{2} \times 3$

$$\begin{bmatrix} \underline{1\times2 + 2\times6 + 3\times9} & 1\times3 + 2\times7 + 3\times10 & 1\times5 + 2\times8 + 3\times4 \\ 4\times2 + 5\times6 + 6\times9 & 4\times3 + 5\times7 + 6\times10 & \sim . \end{bmatrix}$$