

Lec-7 Recommender Systems - 1

- Walmart Case Study
- Apriori Algorithm
- Market Basket Analysis
- Association Rules
- Association Metrics

Dataset link:

- - -

Dataset

<u>Invoice no.</u>	<u>Item-name</u>	<u>Quantity</u>
1	A	2
1	B	6
1	C	5
2	X	.
2	Y	.

↓ Transform

<u>Invoice no.</u>	A Item 1	B Item 2	C Item 3	X Item 4	Y Item 5	- - -
1	1	1	1	0	0	
2	0	0	0	1	1	
3						
4						
5						

D = set of all the items

$$= \{ I_1, I_2, I_3, \dots, I_n \}$$

$$\|D\| = n$$

$$T_1 = \{ I_1, I_2, I_3 \}$$

$$T_2 = \{ I_4, I_5 \}$$

⋮

$$T_m =$$

$$T = \{ T_1, T_2, T_3, \dots, T_m \}$$

↳ set of all transaction.

$$\|T\| = m$$

* Usually $\|T\| > \|D\|$

* $T_i \subseteq D$

$$\text{Eg. } T_1 = \{1, 3, 4\}$$

$$T_2 = \{1, 2, 4, 5\}$$

$$T_3 = \{1, 3, 5, 6\}$$

$$T_4 = \{1, 3, 4, 5\}$$

$$T_5 = \{1, 3, 6\}$$

$$\text{Q1) frequency of } \{1, 3\} = 4$$

$$\text{Q2) freq. of } \{1, 4\} = 3$$

$$\text{Q3) freq. of } \{1, 3, 4\} = 2$$

$T_1 = \{1, 3, 4\} \rightarrow$ list all the subsets.

Subsets = $\{\emptyset, \{1\}, \{3\}, \{4\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1, 3, 4\}\}$

} all of the item combination that can be thought of from the T_1

of subsets = 8

$|S| = n \Rightarrow$ how many subsets of S ?

Ans. 2^n

$T_1 = \{1, 3, 4\}$ → every item has Yes/No option

Y/N Y/N Y/N
 $2 \times 2 \times 2 = 2^3$ to be in the subset

Eg. YYN ⇒ {1, 3}

YN N = {1}

$$T_1 = \{1, 3, 4\}$$

$$T_2 = \{1, 2, 4, 5\}$$

$$T_3 = \{1, 3, 5, 6\}$$

$$T_4 = \{1, 3, 4, 5\}$$

$$T_5 = \{1, 3, 6\}$$

NOTE: To recommend items using a frequency based approach, we need to count the frequency of all item combination.

frequency based approach

$$\{1, 2\} \quad 1$$

$$\{1, 2, 4\} = 1$$

$$\{1, 3\} \quad 4$$

$$\{1, 2, 3\} = 0$$

$$\{1, 4\} \quad 3$$

$$\{1, 3, 4\} = 2$$

$$\{1, 5\} \quad 3$$

$$\{1, 3, 5\} = 2$$

$$\{1, 6\} \quad 2$$

Freq threshold ≥ 3

Q) if we have m transaction and n times ?

How many computations will be needed to implement the frequent based approach in the worst scenario ?

Ans. $m \times 2^n \rightarrow$ very very large

Apriori Algorithm

$$T_1 = \{1, 3, 4\}$$

$$T_2 = \{1, 2, 4, 5\}$$

$$T_3 = \{1, 3, 5, 6\}$$

$$T_4 = \{1, 3, 4, 5\}$$

$$T_5 = \{1, 3, 6\}$$

frequency based approach

$$\{1\} \quad 5$$

$$\{2\} \quad 1$$

$$\{3\} \quad 4$$

$$\{4\} \quad 3$$

$$\{5\} \quad 3$$

$$\{6\} \quad 2$$

$$\{1, 2\} \quad 1$$

$$\{1, 3\} \quad 4$$

$$\{1, 4\} \quad 3$$

$$\{1, 5\} \quad 3$$

$$\{1, 6\} \quad 2$$

$$\{1, 2, 4\} = 1$$

$$\{1, 2, 3\} = 0$$

$$\{1, 3, 4\} = 2$$

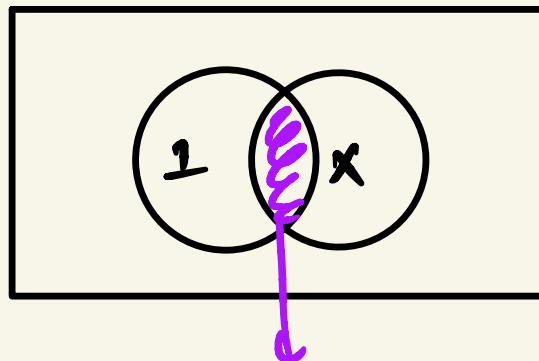
$$\{1, 3, 5\} = 2$$

Q) If frequency of $\{1\}$ is 5, what is the frequency of $\{1, X\}$, where X can be element, going to be?

A) > 5

B) < 5 ✓

C) Can't say



$$\{1, X\}$$

$\{1\}$ is a subset of $\{1, X\}$

$\{1, X\}$ is a superset of $\{1\}$

} frequency of subset

\geq freq. of superset

* In apriori, we will set a frequency threshold at the beginning \rightarrow Eg. freq. threshold = 3

Apriori Algorithm

$$T_1 = \{1, 3, 4\}$$

$$T_2 = \{1, 2, 4, 5\}$$

$$T_3 = \{1, 3, 5, 6\}$$

$$T_4 = \{1, 3, 4, 5\}$$

$$T_5 = \{1, 3, 6\}$$

frequency based approach

{1} 5

{2} 1

{3} 4

{4} 3

{5} 3

{6} 2

↳ eliminated

\Rightarrow Any combination which contains 2 or 6
will have lesser frequency than this,
So they are automatically

$\{1, 3\}$ 4

$\{1, 4\}$ 3

$\{1, 5\}$ 3

$\{3, 4\}$ 2

↳ we don't need to count any combination
which has $\{3, 4\}$ in it.

frequency based approach + frequency threshold
= Apriori Algorithm

Support

Support = $\frac{\text{frequency of the item combination}}{\text{total \# of transactions}}$

$$T_1 = \{1, 3, 4\}$$

$$T_2 = \{1, 2, 4, 5\}$$

$$T_3 = \{1, 3, 5, 6\}$$

$$T_4 = \{1, 3, 4, 5\}$$

$$T_5 = \{1, 3, 6\}$$

$$\text{Support } \{1, 3\} = 4/5 = 0.8$$

$$\text{Support } \{1, 2\} = 1/5 = 0.2$$

$$\text{Support } \{4, 6\} = 0/5 = 0$$

$$\text{Support} \in \{0, 1\}$$

Association rules

- ① Bread \dashrightarrow Milk
- ② Pizza \dashrightarrow Coke
- ③ Pizza \dashrightarrow Garlic Bread

Pizza \dashrightarrow Coke

Antecedent Consequent

if Pizza then Coke

if Antecedent then consequent

Q) Does the following association rule make sense?

If coke then pizza \rightarrow No

\Rightarrow The Association Rules are not symmetric

Association rules \rightarrow Association metrics

⑧ If Bread then Butter
↓
Ante. Conse.

Which is false \rightarrow Butter is antecedent

Association Metrics

$X \rightarrow 1000$ $Y \rightarrow 10000$
 $X, Y \rightarrow 900$ $X, Y \rightarrow 900$

① Confidence

$$\text{Confidence}(X \rightarrow Y) = \frac{\text{Support}(X, Y)}{\text{Support}(X)}$$

$$= \frac{\# \text{ of transactions containing } X \text{ and } Y \text{ both}}{\# \text{ of transactions containing } X}$$

Q) Is $\text{Confidence}(X \rightarrow Y) = \text{Confidence}(Y \rightarrow X)$

⇒ No. Confidence is asymmetric

$$\text{Confidence}(Y \rightarrow X) = \frac{\text{Support}(X, Y)}{\text{Support}(Y)}$$

Eg. If $X \rightarrow Y$ works but not $Y \rightarrow X$

Confidence($X \rightarrow Y$) high but Confidence($Y \rightarrow X$)
low

Q) high support, low confidence

low support, high confidence



eg. Special cornflakes \rightarrow Milk

iff

- * if A and B are independent
 $P(A \cap B) = P(A) * P(B)$
- * if A and B have some ^{tive} association, they are not independent.

$$\begin{aligned} P(A \cap B) &= P(A) * P(B) \\ &< P(A) * P(B) \\ &> P(A) * P(B) \end{aligned}$$

$$\text{lift}(X, Y) = \frac{\text{Support}(X, Y)}{\text{Support}(X) \times \text{Support}(Y)}$$

e) $\text{lift}(X, Y) > 1 \Rightarrow$ +ive correlation or association

$= 1 \Rightarrow$ no correlation

$< 1 \Rightarrow$ -ive correlation

$\text{lift} \in [0, \infty)$

Leverage \rightarrow Same as lift conceptually

$$\text{leverage} = \frac{\text{Support}(X, Y) - \text{Support}(X) * \text{Support}(Y)}{\text{Support}(X, Y)}$$

$0 < \text{leverage}(X, Y) \leq 1 \Rightarrow +\text{ve corr.}$

$\text{lev}(X, Y) = 0 \Rightarrow \text{no corr.}$

$-1 \leq \text{lev}(X, Y) < 0 \Rightarrow -\text{ve correlation}$