Session-4 LINEAR ALGEBRA -4 Feb 03, 2024

AGENDA

1. Half spaces
2. Distance b/w two planes
3. How to find the halfspace a point belongs to
4. Mathematical representation → Gain / Loss function

Halfspaces

Q. $x + y = 0$, Point $(1, 2)$

$ax + by + c = 0$

$$\boxed{d = \frac{w^T \cdot \vec{x_0} + w_0}{||w||}}$$

$\rightarrow$ If my eqn.

$\underbrace{ax + by + c = 0}$

dist b/w $\downarrow$ & $\underline{(x_1, y_1)}$

$(\vec{x_0})$

$\rightarrow \dfrac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \Longleftrightarrow \dfrac{w^T \cdot \vec{x_0} + w_0}{||w||}$

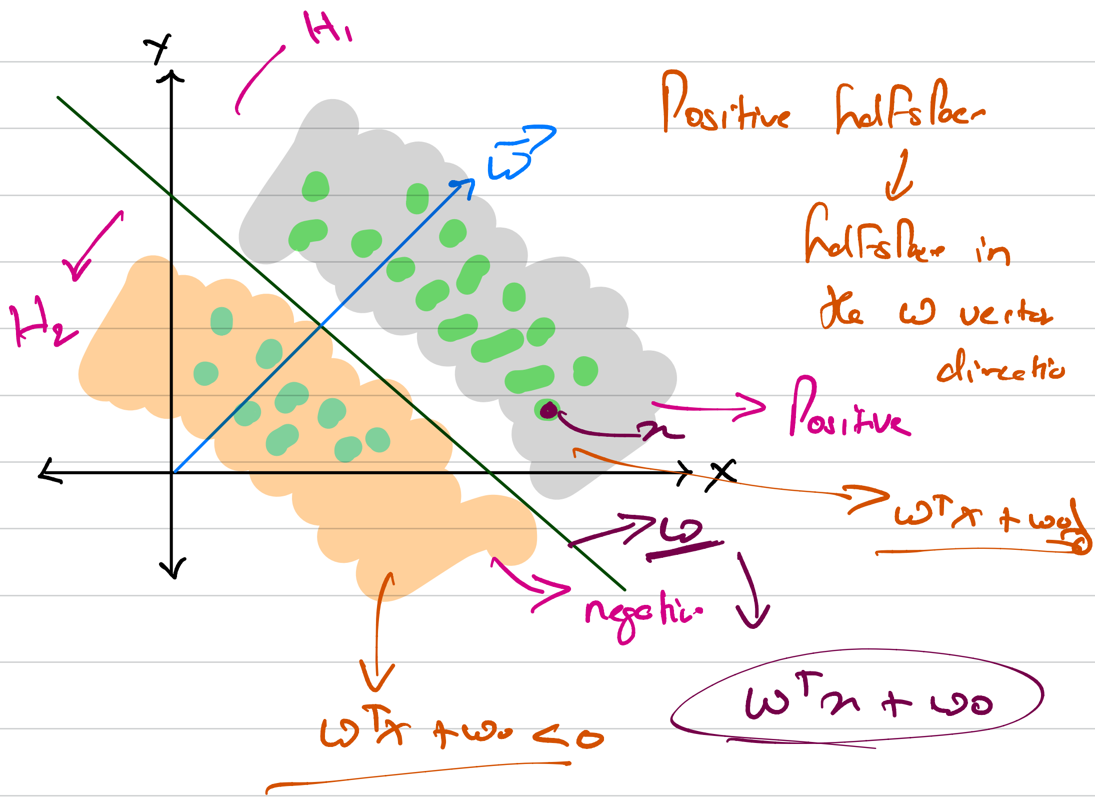
$a = 1$
$b = 1$
$c = 0$

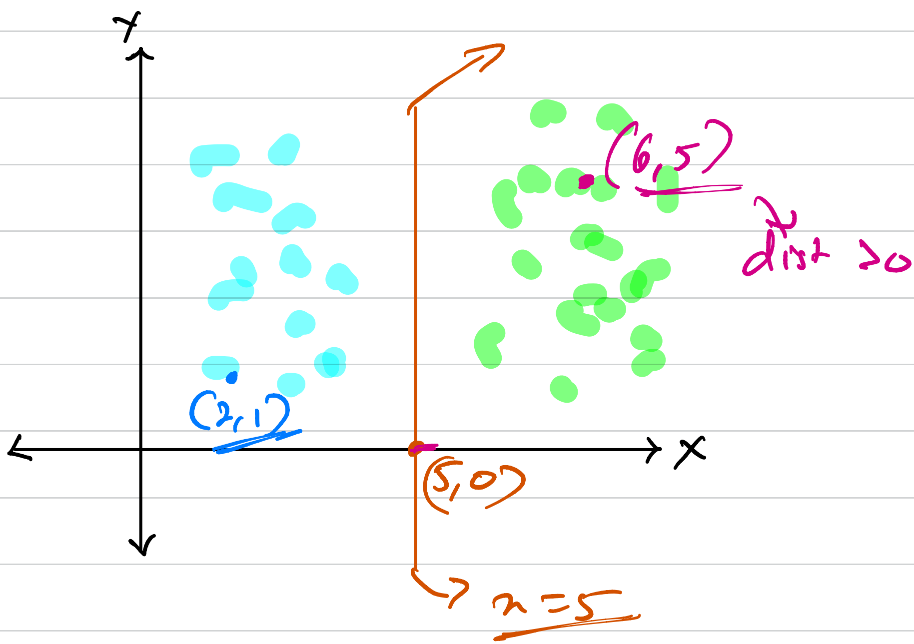
$\begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\Downarrow$

$w_1 x_1 + w_2 x_2$

$$\frac{1 \cdot 1 + 1 \cdot 2}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$


H1
H2
Y
X
w
Positive halfspace
↓
halfspace in the w vector direction
x
Positive
$w^T x + w_0$
w
negative
$w^T x + w_0 < 0$
$w^T x + w_0$


Y
X
(6,5)
dist > 0
(2,1)
(5,0)
x=5

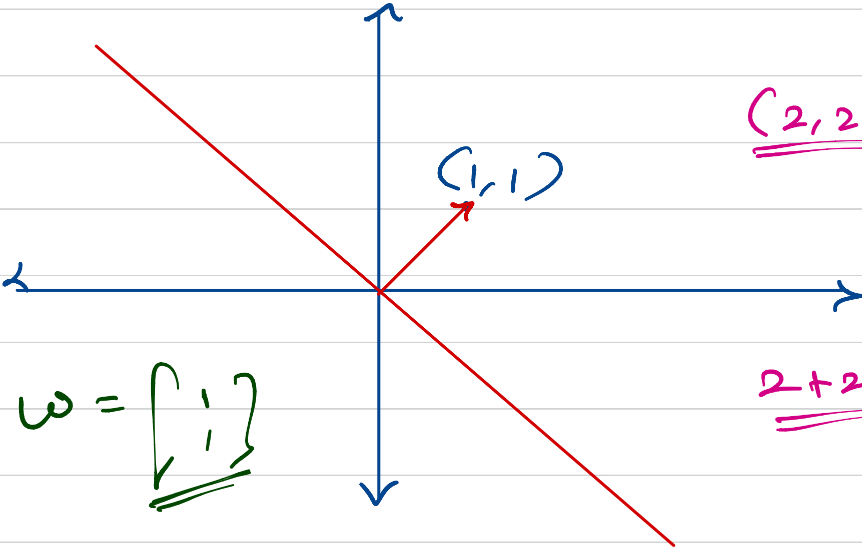
$ax + by + c = 0$

$a=1 \quad b=1 \quad c=0$

$1\cdot x + 1\cdot y + 0 = 0$

$x + y = 0$

$x + y = 0$ $<$



$(2, 2)$

$w = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$2 + 2 > 0$

$\rightarrow$ +ve

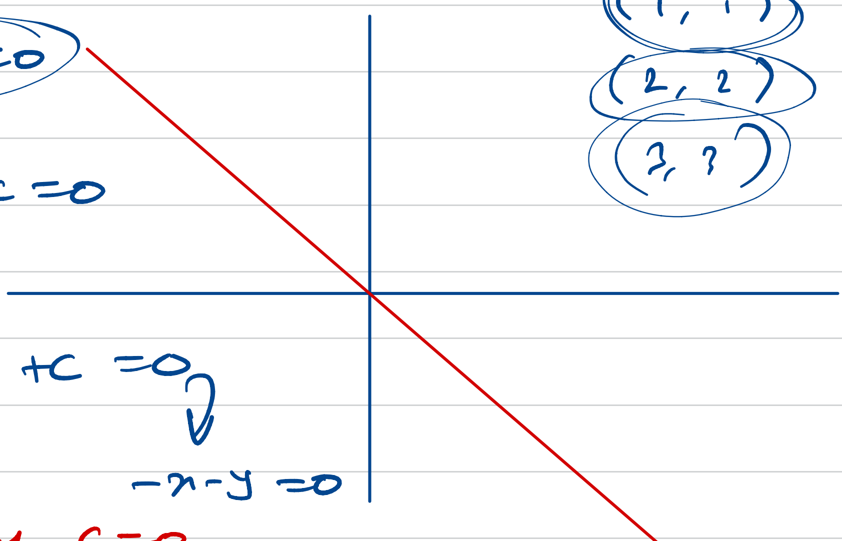
$-x - y = 0$

$(1, 1)$

$(2, 2)$

$(?, ?)$

$ax + by + c = 0$



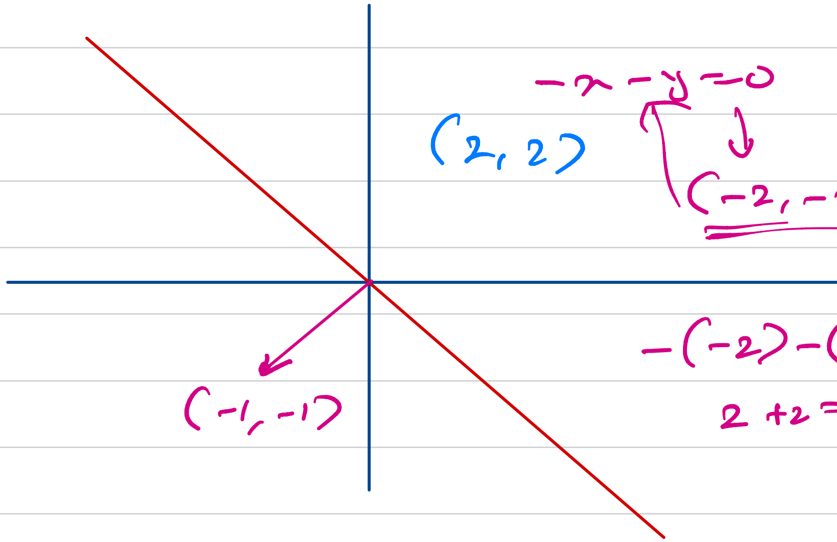
$a \cdot x + b \cdot y + c = 0$

$-x - y = 0$

$a = -1, \; b = -1, \; c = 0$

$a, b = -1$

$$w = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$



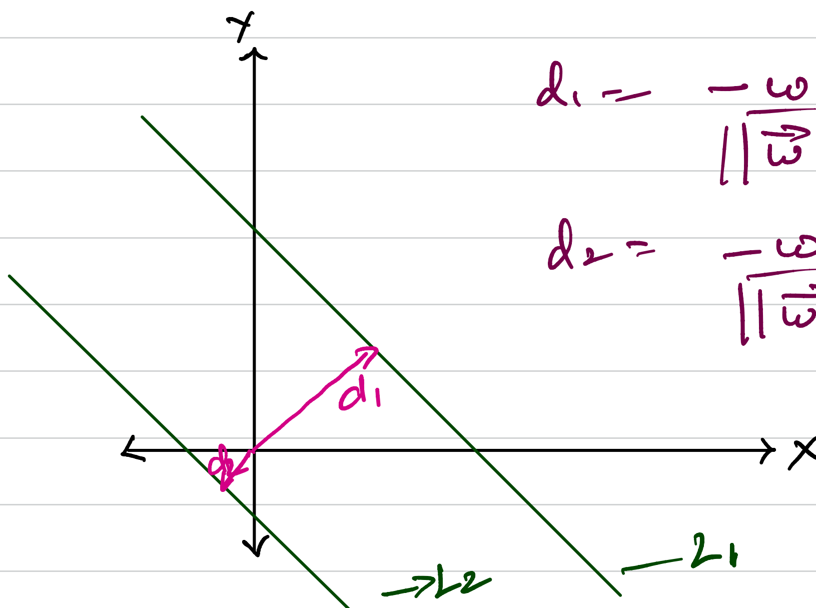
$-x - y = 0$

$(-2, -2)$

$-(-2) - (-2) = 0$

$2 + 2 = 4 > 0$

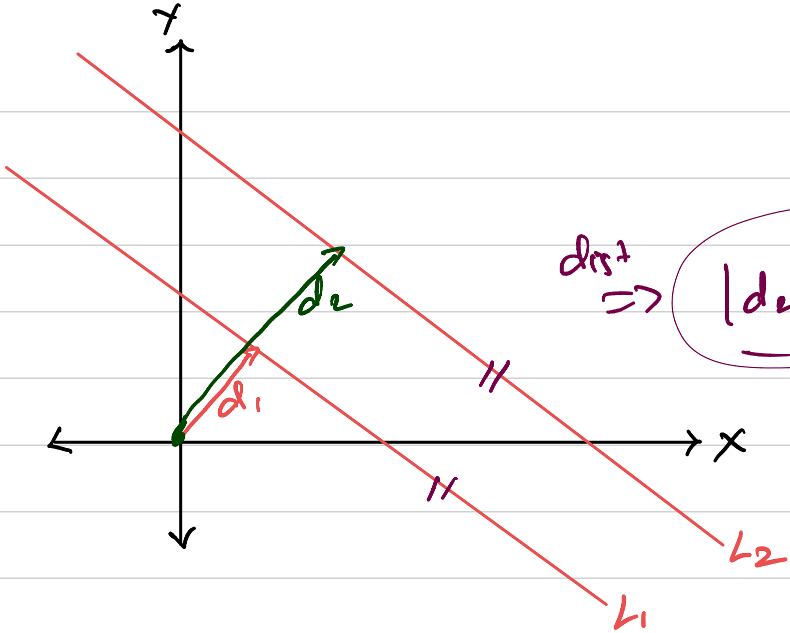
★ Dist b/w 2 Parallel Lines



$$d_1 = \frac{-w_0}{\|\vec{w}\|}$$

$$d_2 = \frac{-w_0'}{\|\vec{w}\|}$$

$|d_1| + |d_2|$



dist $\Rightarrow$ $|d_2| - |d_1|$

$$4x_1 + 3x_2 + 3 = 0$$
$$16x_1 + 12x_2 + 7 = 0$$

$w_1 = [4, 3]$

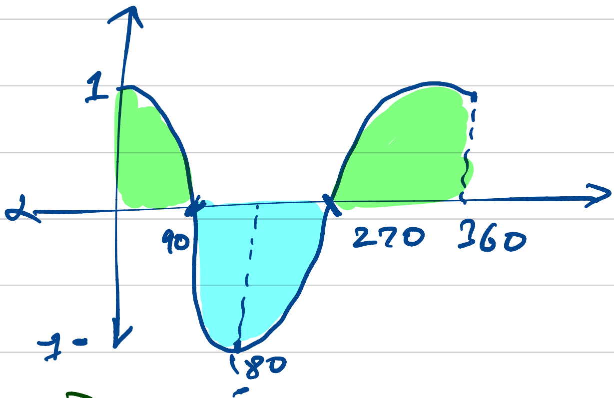
$w_2 = [16, 12]$

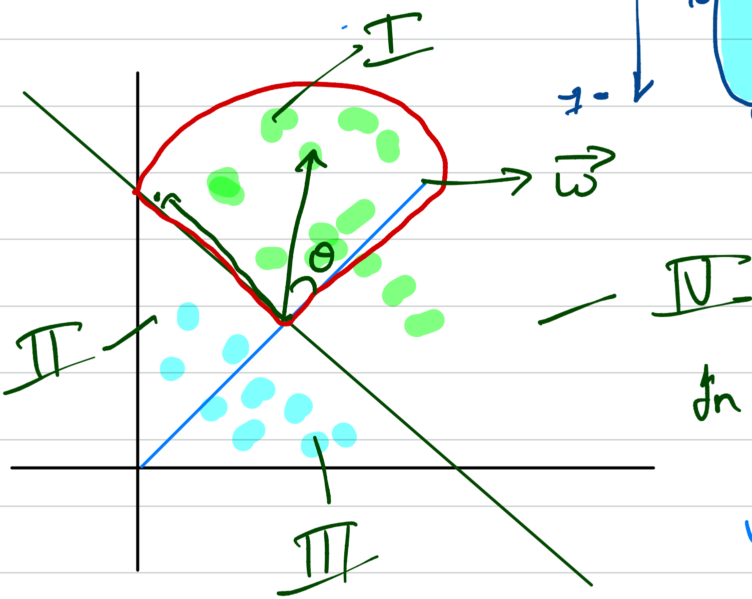
$$L_1 = \frac{-w_0}{||w||} \Rightarrow \frac{-3}{5}$$

$$L_2 = \frac{-w_0}{||w||} = \frac{-7}{20}$$

$$\left|-\frac{3}{5}\right| - \left|\frac{-7}{20}\right| = \frac{1}{4}$$

→ Another way of finding out if point is in +/- halfspace





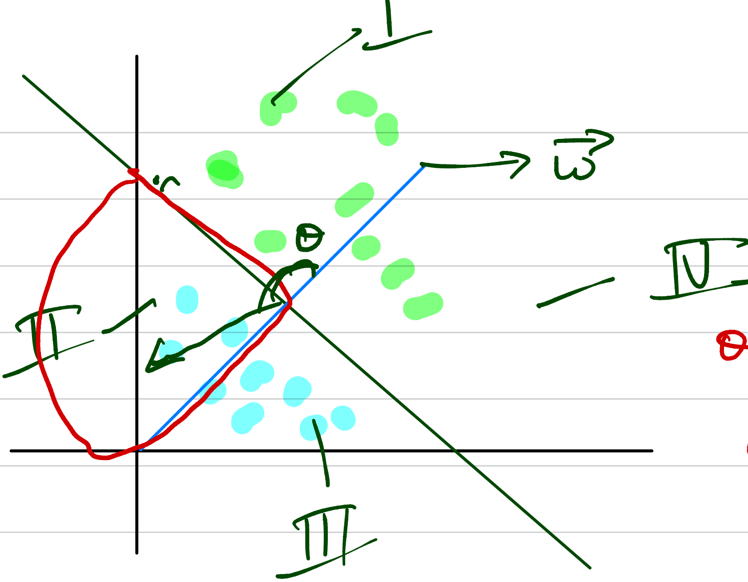
In Quad I

$0 < \theta < 90$

$\cos\theta$

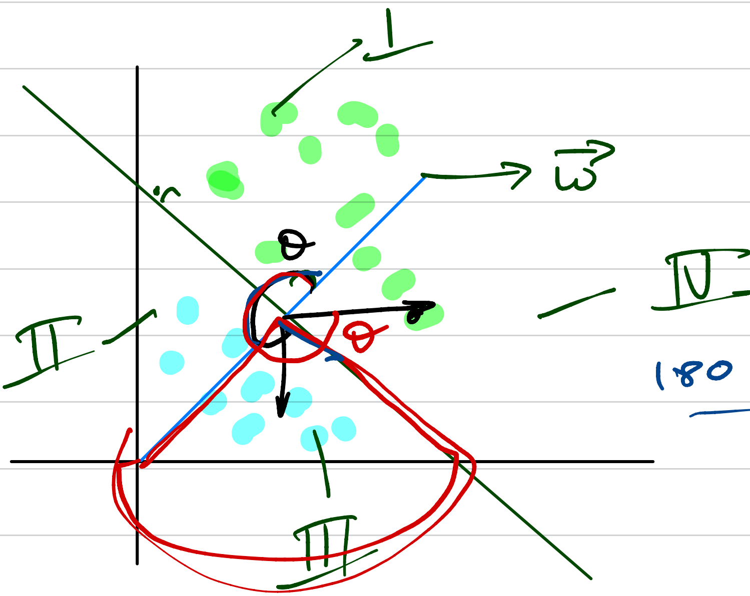
↓

0 to 1



$\theta \rightarrow 90$ to 180

$\cos\theta \rightarrow \underline{(-)ve}$



$\underline{180 < \theta < 270}$

$\cos\theta \Rightarrow \underline{(-)ve}$

IV $\rightarrow$ $\theta \rightarrow 270$ to 360

$\theta \rightarrow \underline{(+)\omega}$

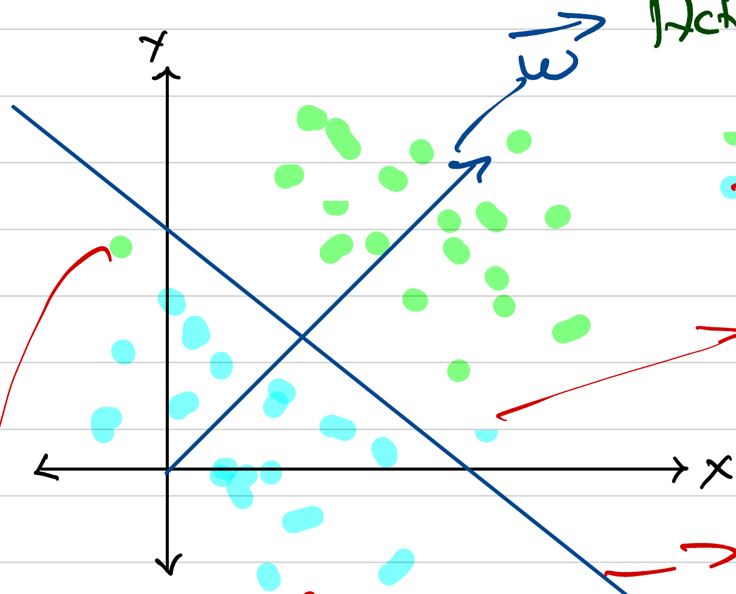
$(3,3)$

$3x + 2y + 24 = 0$

$\text{Loss} \Rightarrow \dfrac{w^T \cdot \vec{x}}{||w||\,||x||}$

LOSS FUNCTION

→ Gain function



Actual → +1

1

actual = -1

Predicted ⇒ +1

→ $w^T x + w_0 = 0$

Actual → (+)ve

Alloted → (-)ve

mis-classified point

All ML models are inaccurate
but some of them are
useful

Misclassified Case

Actual → (+)ve
Predicted → (−)ve

Actual → (−)ve
Predicted → (+)ve

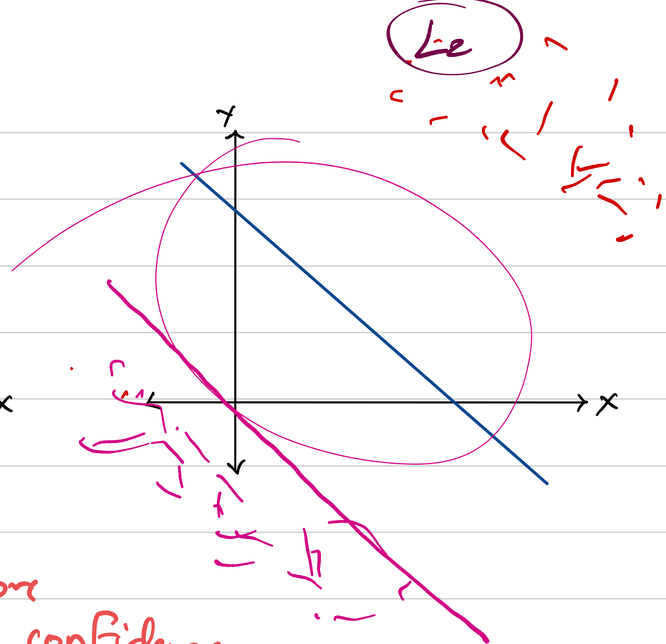
misclassif

→ How do we assign labels

$$\vec{d} = \frac{\vec{w}^T \cdot \vec{x} + w_0}{\|\vec{w}\|}$$

if $d > 0 \rightarrow \underline{+1}$

$d < 0 \rightarrow \underline{\underline{-1}}$



more dist → more confidence

→ Mathematical notation

$$X = \{\vec{x_i}, y_i\}_{i=1}^{n}$$

x_i → feature Vector

y_i → Actual label

+1

-1

$$\frac{ax_1 + by_2 + c}{\sqrt{a^2+b^2}}$$

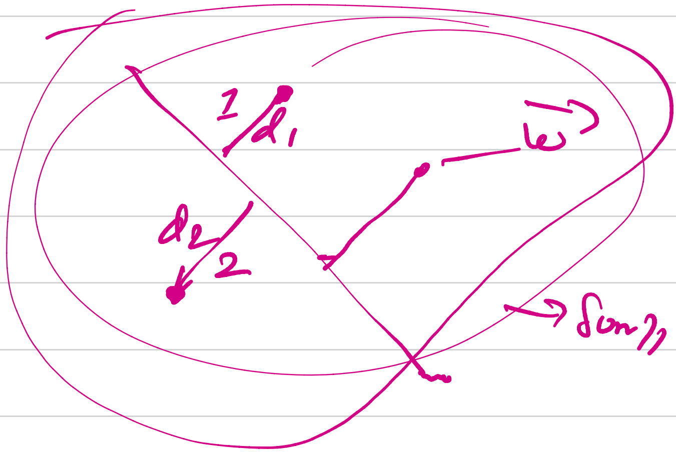
$$\frac{w^T\vec{x} + w_0}{||w||}$$

$$\text{Gain}(X, \vec{w}, w_0) \Rightarrow \max \sum_{i=1}^{n} \left(\frac{w^T \cdot x_i + w_b}{\|\vec{w}\|} \right)$$

max

What problems do I have??

$d_1 + d_2 = -1$



Possible Solution?:

① $|x| \rightarrow$ Not differentiable

$\downarrow$

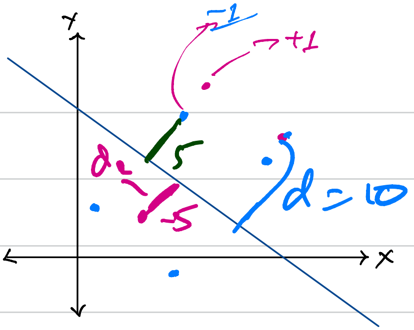
Calculus

② $x^2 \rightarrow$ Possible

$\downarrow$

because $\Rightarrow$ differentiable

$\rightarrow$ lots of calculation



① Actual point → (−)ve

$d_1 \rightarrow \underline{\underline{+ve}} = +5$

$d_1 \times (\text{sign}(\text{Actual point}))$

$d_1 \times (-1)$

$5 \times (-1) \Rightarrow \underline{\underline{-5}}$

② Actual point → $\underline{\underline{(+)ve}}$

$d_2 \Rightarrow (-)ve \quad \underline{\underline{-5}}$

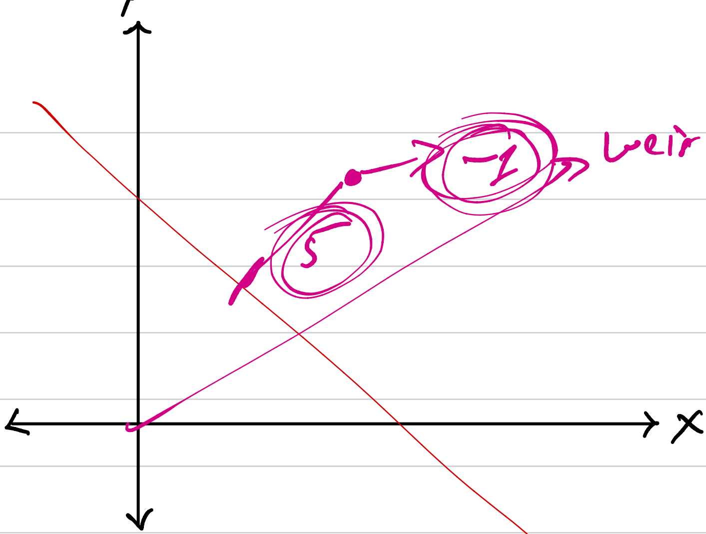
$d_2 \times \text{Sign}(\text{actu pt})$

$d_2 \times 1 = -5$

③ Actual pt → (+)ve

$d_3 \rightarrow (+)ve = 10$

$d_3 \times 1 = 10$



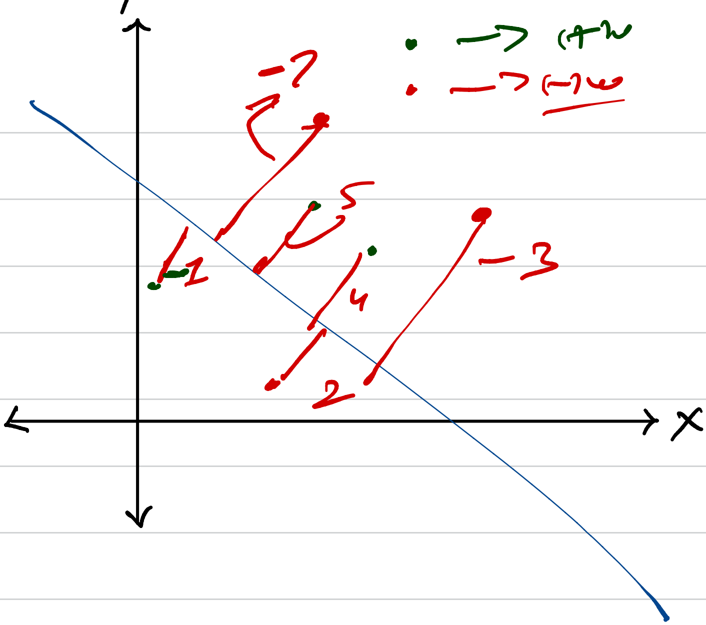
$5x - 1$

$\Rightarrow -5$

If misclassif $\longrightarrow$ penalizin gain

$$\text{Gain}(x, \vec{w}, w_0) \Rightarrow \underbrace{\sum_{i=1}^{n}}_{\text{max}} \left(\frac{w^T \cdot x_i + w_0}{\|\vec{w}\|} \right) \times \underbrace{y_i}_{\text{actual label}}$$

Whi



→ Correct → (+)ve

mis-cls → (-)ve

G → max $G(\vec{x}, \vec{w}, w_0)$

loss → minimize

↳ $-G$ → loss function

	①	②
Gain :	5	7
Loss =	-5	-7

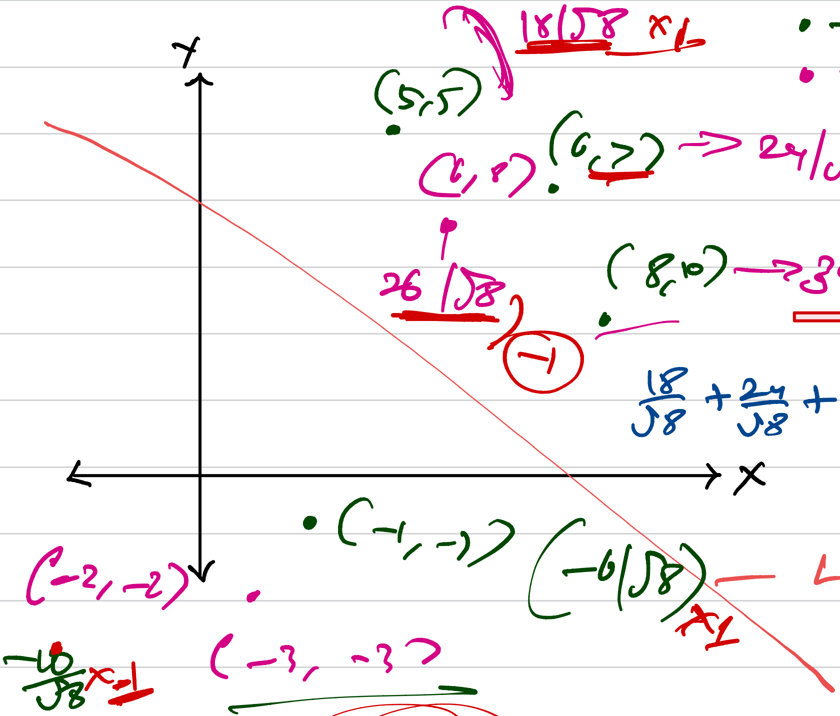
$L(x, \vec{w}, w_0) = -G(x, \vec{w}, w_0)$

(1) Convexity

(2) Global minima

• → (+)ve

• → (−)ve

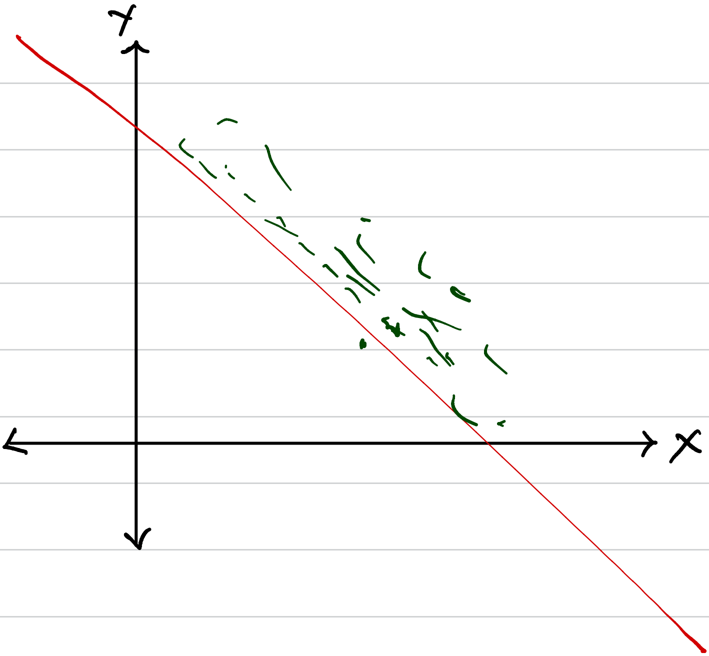


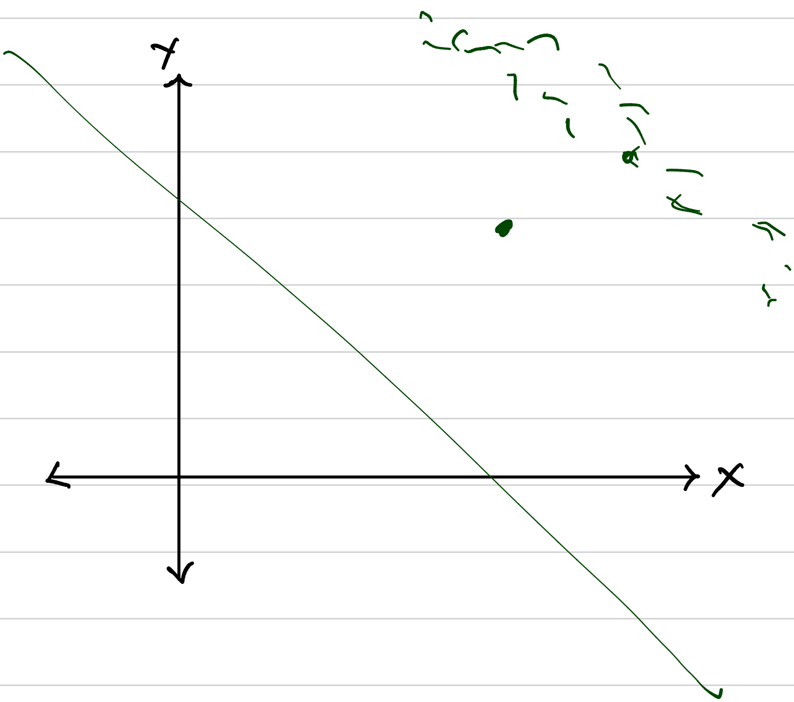
$\frac{18}{\sqrt{8}} + \frac{24}{\sqrt{8}} + \frac{34}{\sqrt{8}} - \frac{26}{\sqrt{8}}$

$\frac{-10}{\sqrt{8}} + \frac{14}{\sqrt{8}} - \frac{6}{\sqrt{8}}$

$2x + 2y - 2 = 0$

$200x + 200y - 200 = 0$


Y
X


Y
X