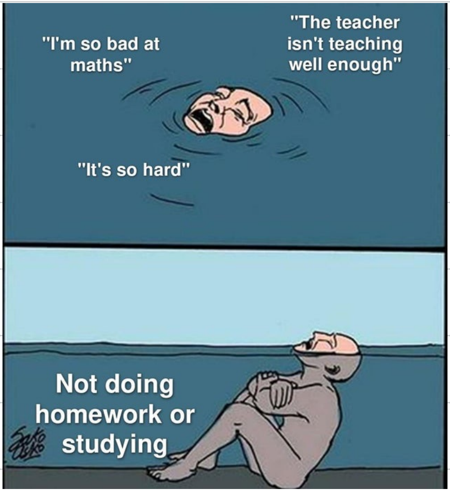
Session 6 # Optimization - 1 Feb 08, 2024

Need For Calculus

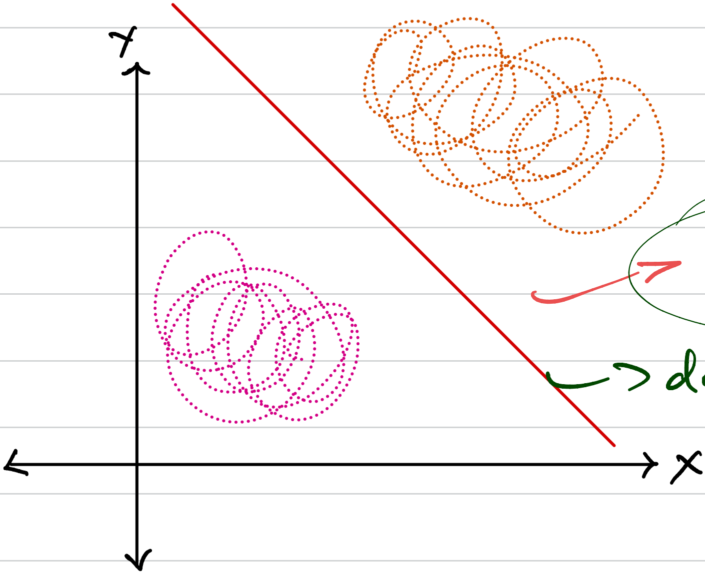
Calculus in Mathematics*

Agenda

1. Basic Intuition For Classifier
2. Simple Searching algorithm
3. How to solve optimization problem
4. Relationship b/w distance and gain functions
5. Functions
6. Limits - Continuity
   Domain & Range



Recap



$y = mx + c$

→ decision boundary

$\tan(90) = \infty$

$y = mx + c$

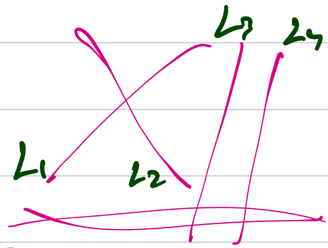
→ not possible in an eqn $y = mx + c$

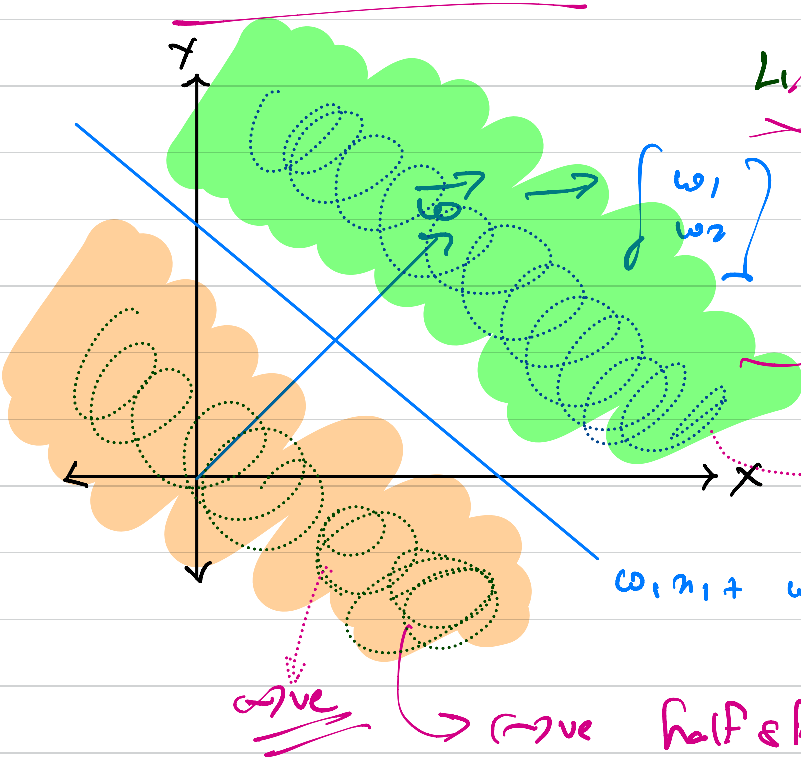
$w_1 x + w_2 y + w_0 = 0$ → 2d - hyperplane

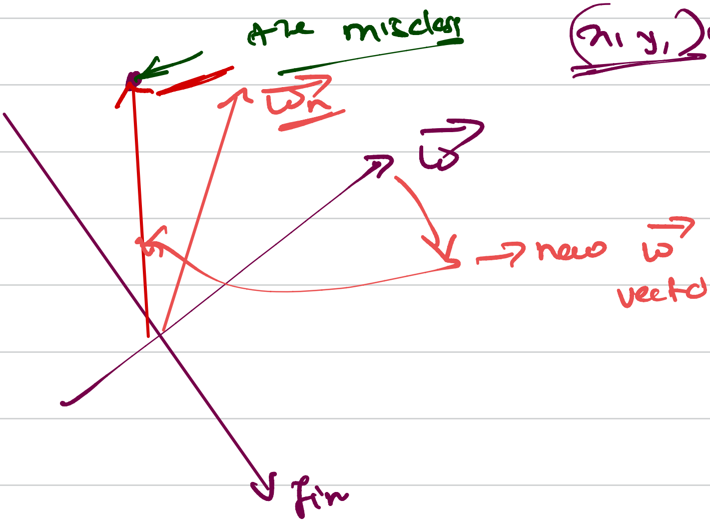
extend this ⇒ $y = \frac{-w_1 x}{w_2} - \frac{w_0}{w_2}$

$w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$

learnable Parameters



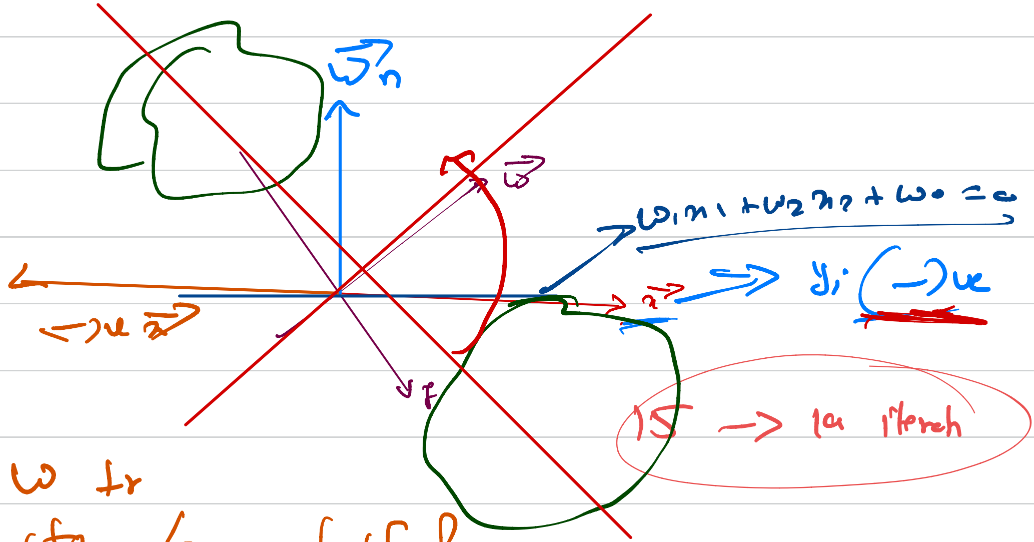




$(x_1, y_1) \rightarrow w^T x + w_0$

If $> 0 \rightarrow$ (+)ve

$< 0 \rightarrow$ (-)ve

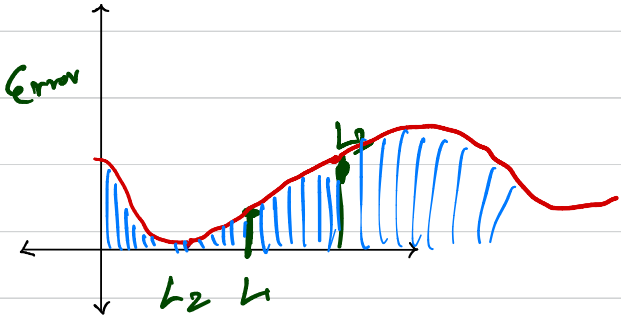


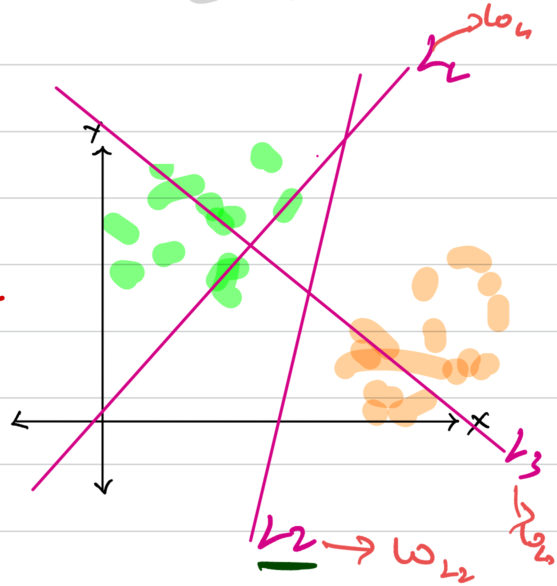
→ w tr

→ (+)ve / (-)ve half planes

→ Vector addn

Basic Intuition For Classification





Simplest Searching Algorithm

$w_3x_3 + w_1x_1 + w_2x_2 + w_0 = 0$

→ I want to find best set of w_0, w_1 & w_2

→ Points are in Range → -10 to 10

$$\underbrace{100}_{}\quad 1$$

→ $w_1 \rightarrow [-10, 10], 0.1 \rightarrow [-10, -9.9 \ldots -0.1]\ [0]\ [0.1 \ldots 10]$

$201 \qquad 200$

$w_2 \rightarrow [-10, 10], 0.1 \rightarrow \underline{201}$

$w_0 \rightarrow [-10, 10], 0.1 \rightarrow \underline{201}$

$$\underset{w_1}{\boxed{201}} \times \underset{w_2}{\boxed{201}} \times \underset{w_0}{\boxed{201}} \rightarrow \underline{201^3}$$

→ $\underline{1\ \mu s}$ → arithmetic operation

↳ 10^{-6} seconds

How To Solve Optimization Problem

Gradient descent

1. maxima & minima
2. Calculus in multivariable
3. Calculus in single variable system
4. Derivative slope & tangent
5. Limits, continuity & differentiability

Defining The Classification Problem Mathematically

$$D = \{(\underbrace{x_i}_{\text{Features}},\ \overset{\text{labels}}{y_i}) : x_i \in R^d,\ y \in \underbrace{1, -1}_{\text{only in our use-case for now}}\}_{i=1}^{n}$$

Goal: $\text{Input} \rightarrow \boxed{f(x)} \rightarrow \text{output}$

$$w^T x + w_0 > 0 \rightarrow +1$$
$$< 0 \rightarrow -1$$

2 years	→	20 lakhs
3 year	→	30 lakhs
5 years	→	50 lakhs

6 years → 60 lakhs

$y = 10 \times x$

$\Rightarrow$ Predicted $= 10 \times 2$
$= 20$

actual

$25 - 20 \Rightarrow 5$ error

10 m

$2 \rightarrow 25$ L

$f(\bar{x}) = \boxed{\hat{y}_i} \rightarrow$ Predicted value

Not a unit vector

$y_i \rightarrow$ actual

Ideally $y_i = \hat{y}_i$

will measure $\rightarrow$ how good of a job am I doing

$G(D, \bar{w}, \bar{w_0}) \rightarrow$ We want to optimize as much as possible

$\bar{w} \downarrow w_1 \,\&\, w_2$

Loss function $= -$ Gain function

$$g(\bar{x}_i, y_i, \vec{w}, w_0) = \left(\frac{w^T x_i + w_0}{\|w\|}\right) \times y_i$$

① V. high Gain value

or

① Parameter, $\vec{w}$ & w_0

Train a Fish detector → 100% for one Fish

Not for other Fish

→ We want $\underline{w^*}$ → most optimal set of Parameters

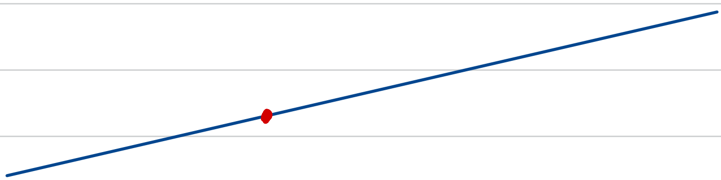
$$\vec{w}^*, w_0^* = \underset{\vec{w}, w_0}{\operatorname{argmax}} G(D, \vec{w}, w_0)$$

Relationship b/w distance & Gain funct

1. $G(D, \vec{w}, w_0) = \sum_{i=1}^{n} g(\vec{x_i}, y_i, \vec{w}, w_0)$

2. $G(D, \vec{w}, w_0) = \prod_{i=1}^{n} g(\vec{x_i}, y_i, \vec{w}, w_0)$

Q ntile thing can be $\underline{0}$

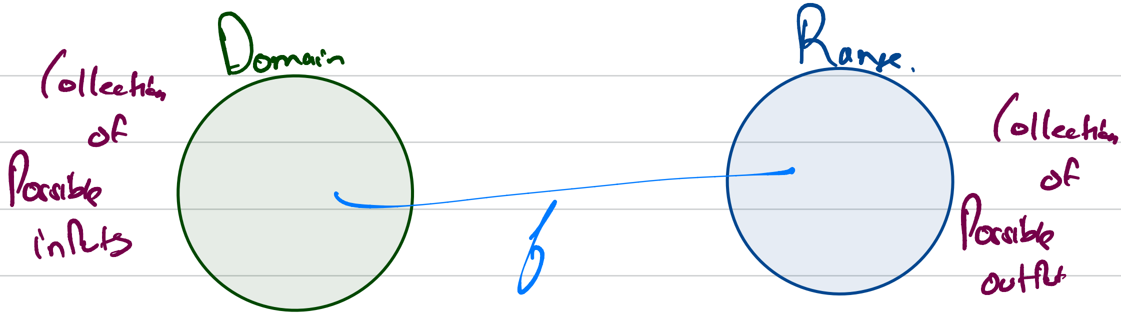


FUNCTIONS

$f(x) = y$

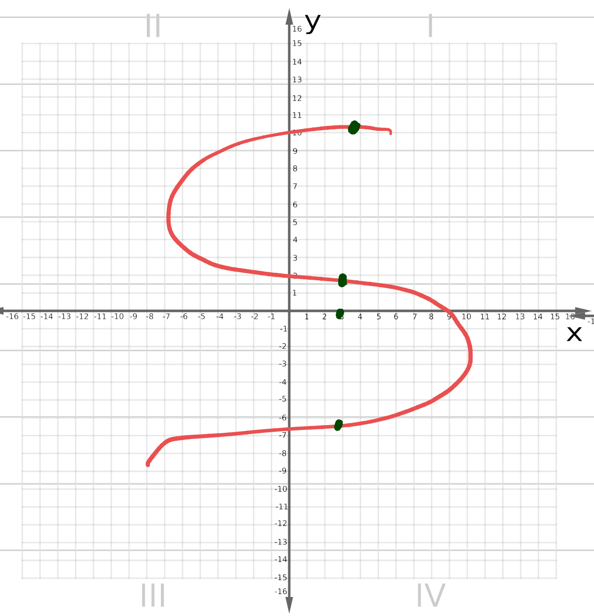
Input → Domain

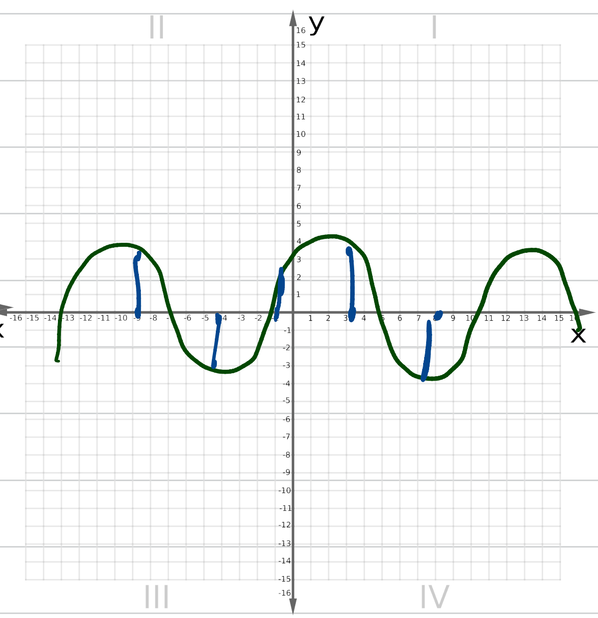
output → Range

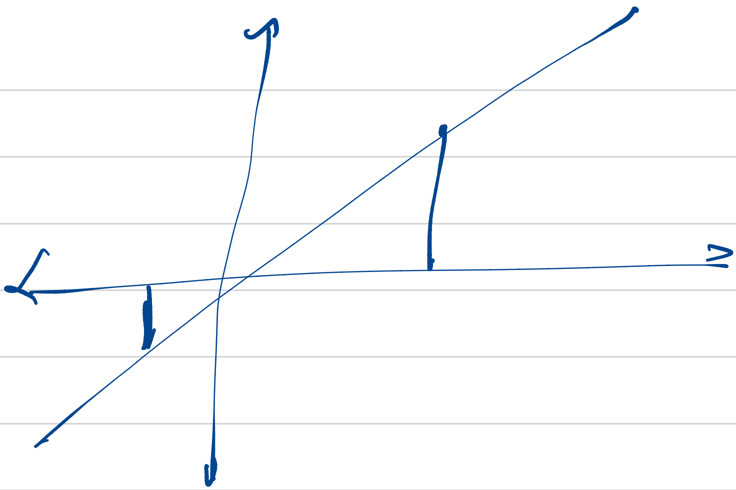


For any x → can have only 1 output
input

		Domain	Range
x^2	→	$(-\infty, \infty)$	$[0, \infty)$
$\lvert x \rvert$	→	" "	" "







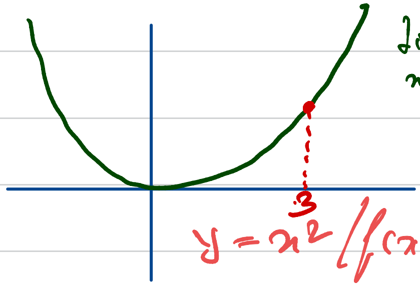
Continuous Vs Non-Continuous Functions

-> Continuous ->

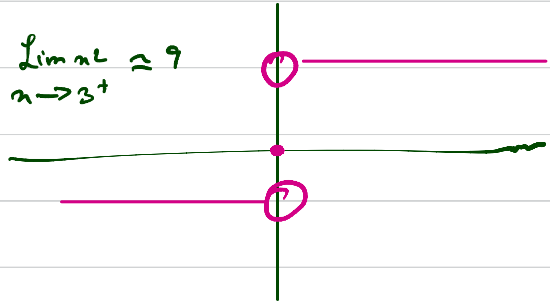
You can draw without lifting your Pen

Discontinuous functi

have to lift Pen



$\lim_{x \to 3^-} x^2 = \lim_{x \to 3^+} x^2 \approx 9$



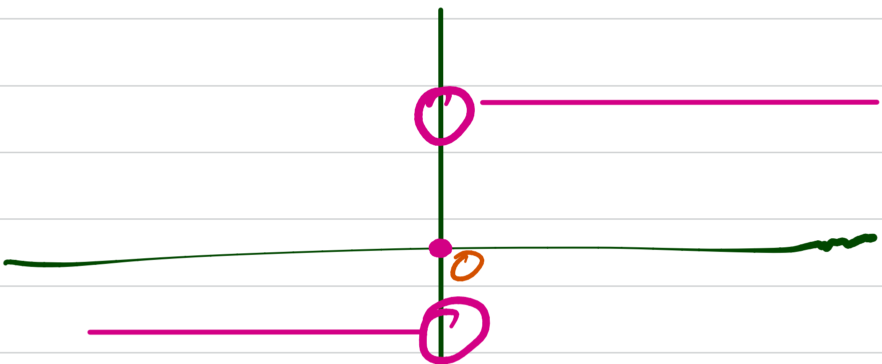
$y = x^2$ // $f(x) = x2$

Limit

$$y = f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

$x > 0 \Rightarrow 0.0000000001$

$x < 0 \Rightarrow -0.000000001$



Left hand Limit

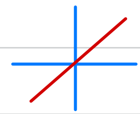
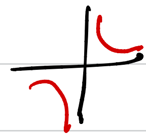
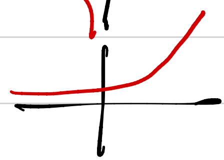
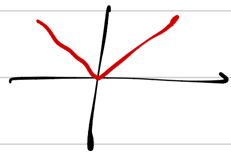
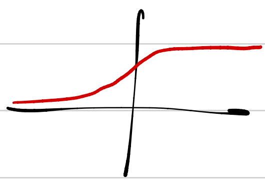
Right hand Limit

(LHL) $0^- \Longleftrightarrow 0^+$ (RHL)

$\lim_{x \to 0^+} f(x) = 1$ / $\lim_{x \to 0^-} f(x) = -1$

If LHL $\neq$ RHL $\Rightarrow$ func is discontinuous

Most Important functions

Function	Domain	Range	Con/Dis	Plot
① $y=x$	$(-\infty, \infty)$	$(-\infty, \infty)$	Continuous	
② $y=\frac{1}{x}$	$(-\infty, \infty)-\{0\}$	$(-\infty, \infty)$	Discontinuous	
③ $y=e^x$	(∞, ∞)	$(0, \infty)$	Continuous	
④ $y=\lvert x\rvert$	(∞, ∞)	$(0, \infty)$	Continuous	
⑤ $y=\text{Log}(x)$	$(0, \infty)$	$(-\infty, \infty)$	Continuous	
⑥ $y=\frac{1}{1+e^{-x}}$	$(-\infty, \infty)$	$(0, 1)$	Continuous	
⑦ $y=\sin\theta$	$(-\infty, \infty)$	$[-1, 1]$	Continuous