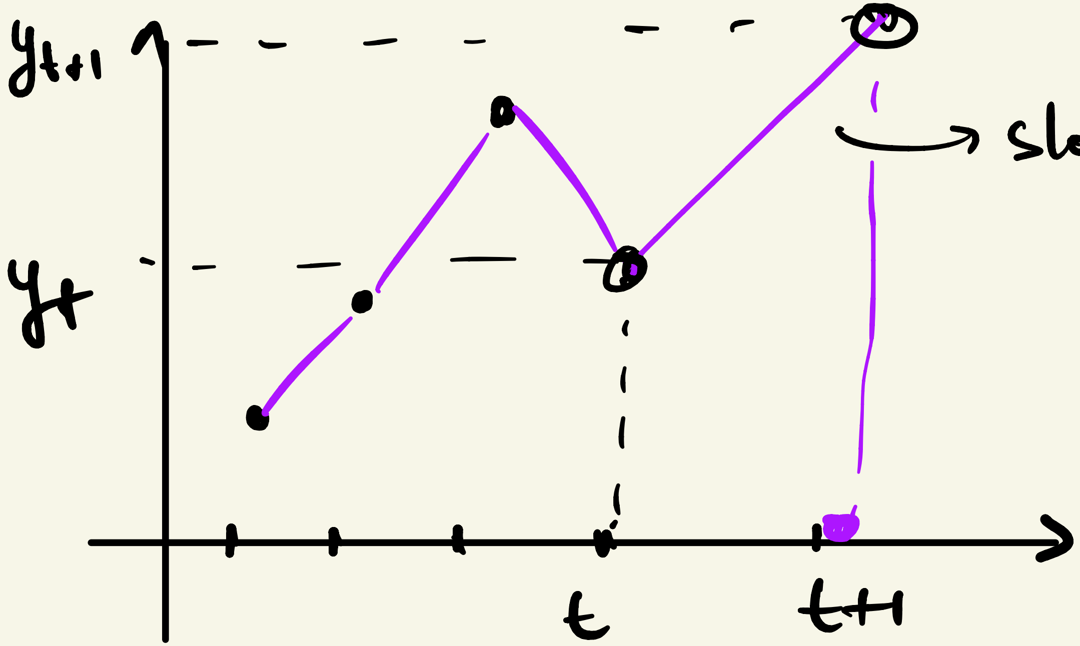
Lec 4 : Time Series - 3

- Stationarity
- ACF and PACF
- AR model
- MA model
  - ARMA
  - ARIMA
  - SARIMA
  - SARIMAX

t	y_t
	10,000
	12,000
	13,500
	12500

Sales → continuous numerical variable



→ slope $= \dfrac{y_{t+1} - y_t}{(t+1) - t}$

$$= \frac{y_{t+1} - y_t}{1}$$

→ in continuous functions → $\dfrac{dy}{dt}$ = slope of the curve

t Date	y_t Sales	y'_t derivative of sales
	a_1	a_1
	a_2	$a_2 - a_1$
	a_3	$a_3 - a_2$
	a_4	$a_4 - a_3$
	a_5	$a_5 - a_4$

differencing

$\xrightarrow{\text{Int.}}$

y sales

$a_1 = a_1$

$(a_2 - a_1) + a_1 = a_2$

$(a_3 - a_2) + (a_2 - a_1) + a_1 = a_3$

cumulative sum

(cumsum)

↓

Integration

y Bread

↓

$\boxed{\text{Derivative}}$ Toaster

↓

y' Toast

↓

$\boxed{\text{Integration}}$ anti toaster

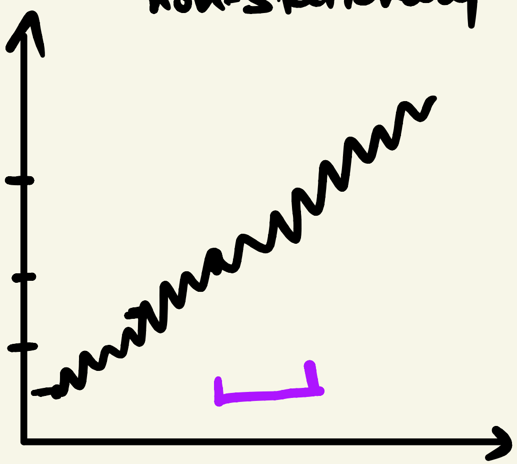
↓

y Bread

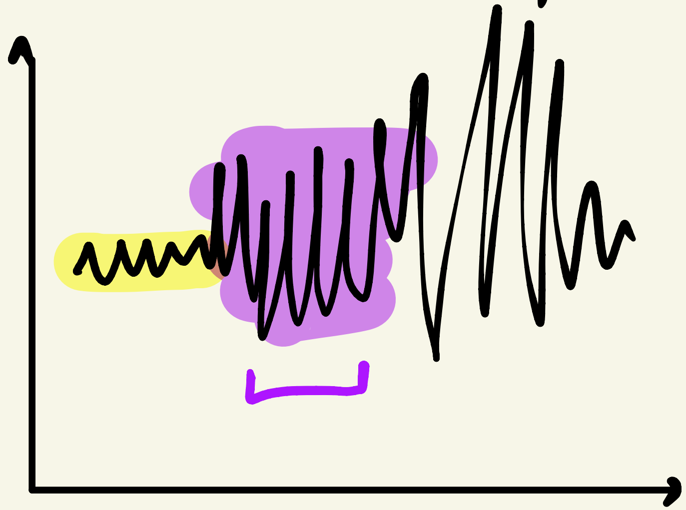
Stationarity

* A time series is said to stationary if its statistical parameters (mean, variance etc) they are not dependent on time.

non-stationary



non-stationary



How to make TS stationary

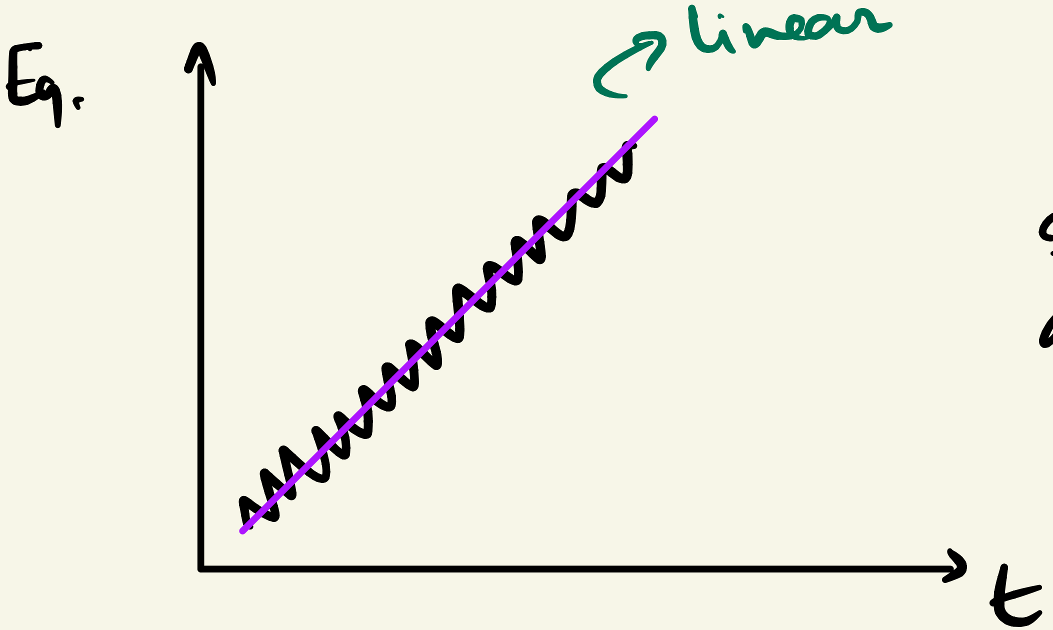
(i) De-trending

(a) TS decomposition

$$y(t) = \underbrace{b(t)}_{\text{Trend}} + \overbrace{S(t)}^{\text{Seasonality}} + \underbrace{\varepsilon(t)}_{\text{residuals}}$$

$$y(t) - b(t) = \text{de-trended series}$$

b) Differencing



$b(t) = mt + c$ (linear)

$$\frac{db}{dt} = m$$

$y(t) = b(t) + S + \varepsilon$

↓ Differencing

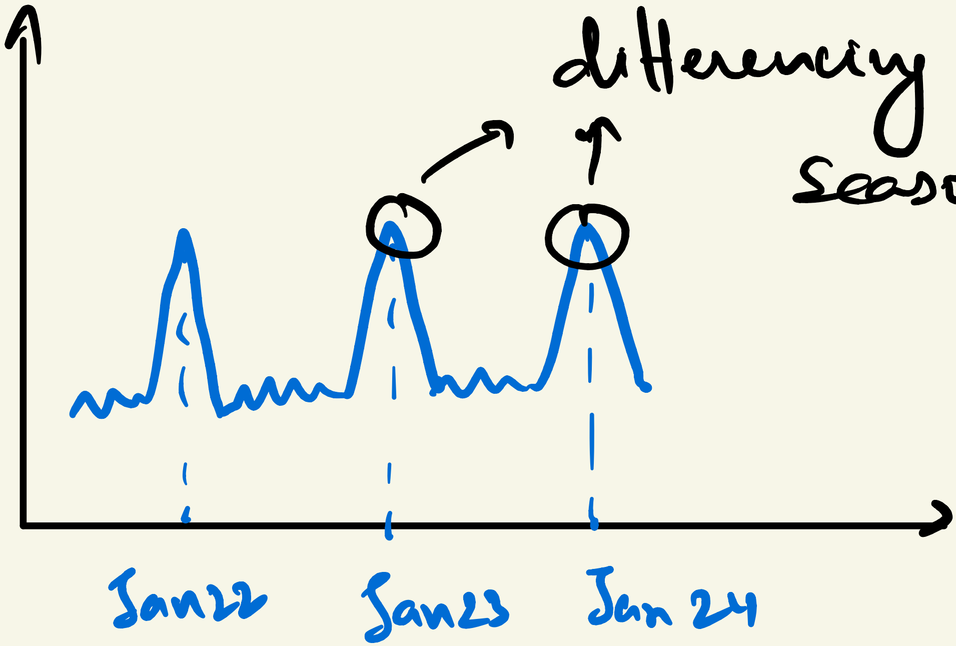
$y'(t) = b'(t)$ (will this remove the trend)

Ans. Yes

t	y	$y\text{-lag}(1)$	$y' = y - y.\text{lag}1$
1	a_1		a_1
2	a_2	a_1	$a_2 - a_1$
3	a_3	a_2	$a_3 - a_2$
4	a_4	a_3	$\vdots$
5	a_5	a_4	
		a_5	

② Deseasonalizing

$T = \text{period} = m$



differencing to remove these seasonal peaks

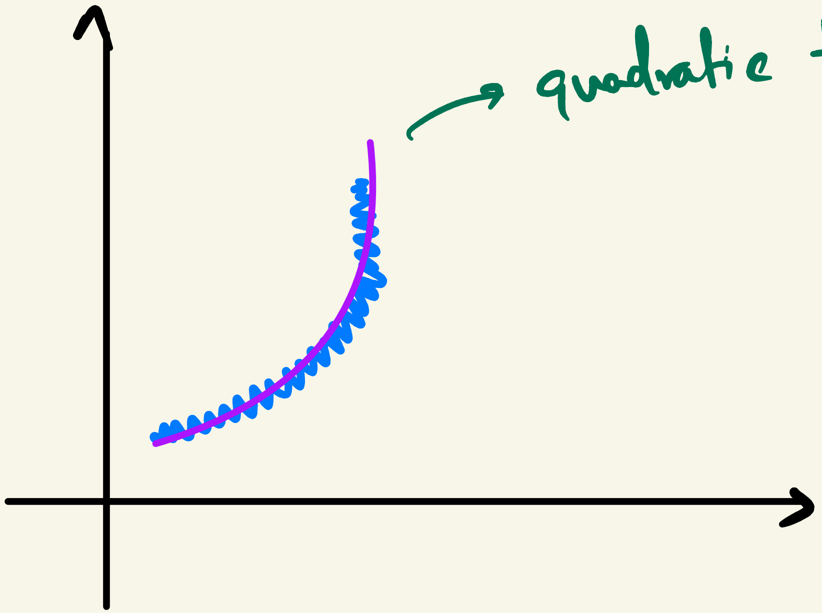
$\rightarrow$ $\underbrace{\text{diff}(12)}$

m-differencing

m = period of seasonality

TS.diff(m) $\rightarrow$ de-seasonalize

Ques



quadratic trend $\Rightarrow$ $\underbrace{b(t) = a_1 t^2 + a_2 t + a_3}_{\text{double differencing to remove trend}}$

$\underbrace{\texttt{mobile\_sales.Sales.diff(1).diff(1)}}_{y''}$

Autocorrelation

Hypothesis : Current month sales are correlated to prev. month sales.

	t	y
Jan	1	a_1
Feb	2	a_2
Mar	3	a_3
Apr	4	a_4

$a_3 \xleftrightarrow{\text{correlation}} a_2$

$a_4 \longleftrightarrow a_3$

verify this mathematically

↓

Pearson Correlation Coeff (y , y_lag1) = PCC

if PCC = 1 ⇒ strong +ive correlation

PCC = -1 ⇒ strong -ive correlation

} Strong dependance

PCC = 0 ⇒ No correlation } no dependance lets say PCC $(y, y\_lag1) = 0.9 \Rightarrow$ y_lag1 to predict y

Hypothesis 2: Current month sales is correlated with 2 months back sales

$\downarrow$

verify $\Rightarrow$ PCC (y, y_lag2)

if PCC $(y, y\_lag2) = 0.8$

$\Rightarrow$ can we use y_lag2 to predict y

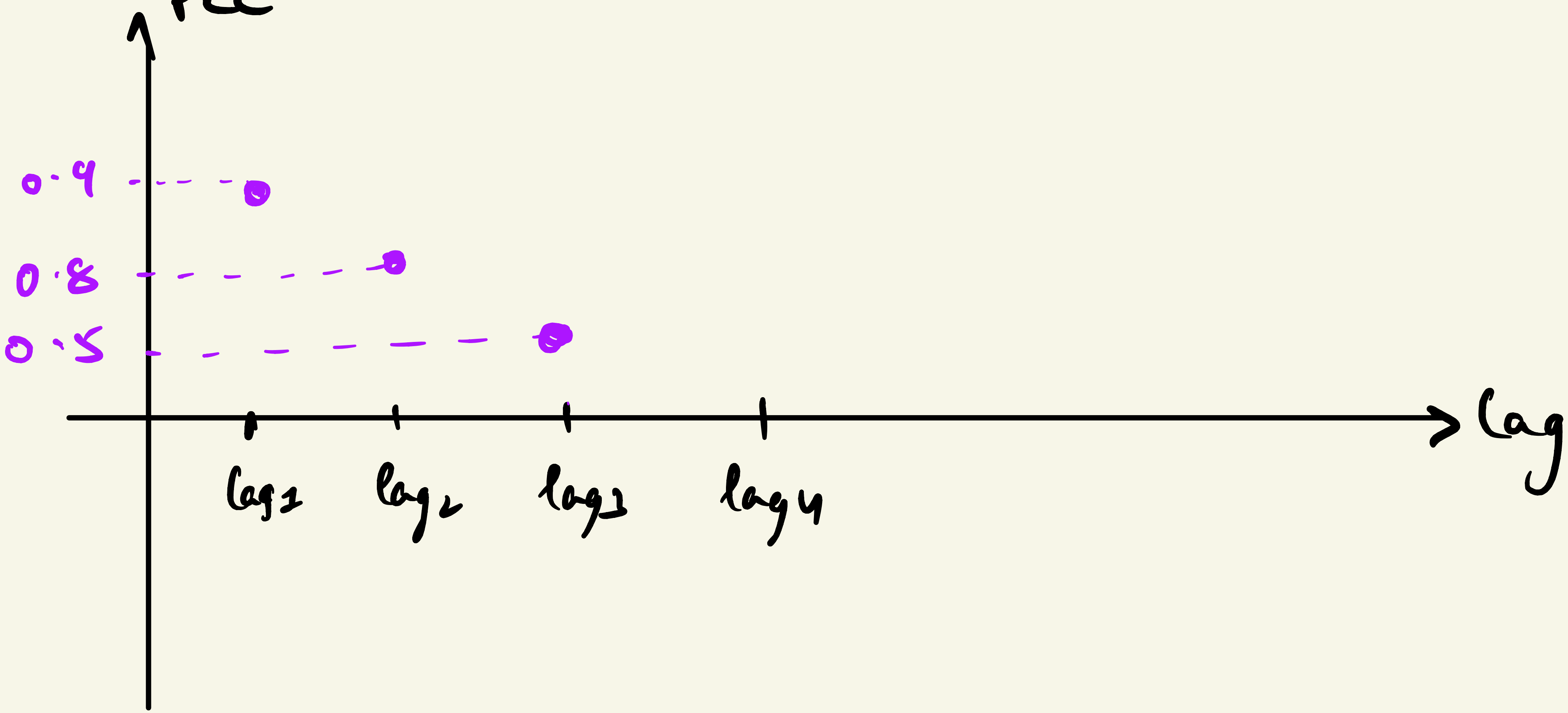
Ans Yes

t	y	y_lag1	y_lag2	y_lag3 - - - - -
1	a_1			
2	a_2	a_1		
3	a_3	a_2	a_1	
4	a_4	a_3	a_2	
5	a_5	a_4	a_3	

PCC_1

PCC_2

Autocorrelation function Plot (ACF plot)



Partial autocorrelation

	t	y
Jan	1	a_1
Feb	2	a_2
Mar	3	a_3
Apr	4	a_4

a_1 is correlated with a_2

a_2 is correlated with a_3

⇒ This can cause correlation between a_1 and a_3

+ This correlation is not direct, it is manifesting because of intermediate value a_2

⇒ This is an example of indirect correlation corr (y, y_lag2) = Direct_corr (y, y_lag2)

+ indirect_corr (y, y_lag1, y_lag2)

* Partial autocorrelation only measures direct correlations

	t	y
Jan	1	a_1
Feb	2	a_2
Mar	3	a_3
Apr	4	a_4

PACF → direct correlation

Correlation vs causation

Eg. Ice cream sales and # of people drowning at beach.

observation: Ice cream sales ↑ and # of people drowning ↑ at beach

⇒ These two are correlated but one is not the cause of the other.

Eg. Ice cream sales ↑ temperature ↑, # of people drowning ↑

Correlation as well as causation

Correlation as well as causation

Can we use # of people drowning to predict Ice cream sales?

Ans. Yes it can be used. But the forecast might not be good.

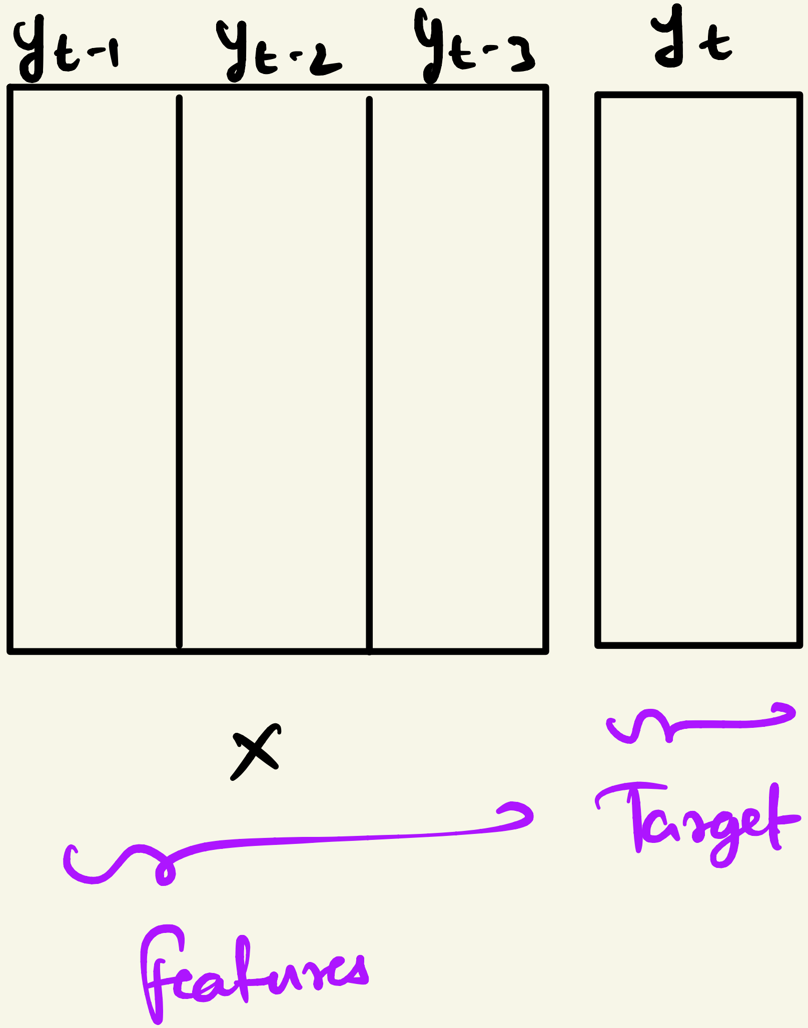
If possible use temperature instead of Ice-cream sales.

Auto Regression (AR model)

t	Sal y	y_lag1	y_lag2	y_lag3 - - -
1	a_1			
2	a_2	a_1		
3	a_3	a_2	a_1	
4	a_4	a_3	a_2	
5	a_5	a_4	a_3	

$$\left.\begin{aligned} PCC_1 &= 0.8 \\ PCC_2 &= 0.5 \\ PCC_3 &= 0.4 \end{aligned}\right\} \text{Statistically significant}$$

we can use y_lag1, y_lag2, y_lag3 to predict y



→ Train a linear regression model

$$\rightarrow \hat{y}_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \alpha_3 y_{t-3} + \alpha_0$$

→ learn $\alpha_1, \alpha_2, \alpha_3, \alpha_0$ using training data

$\Rightarrow$ $\alpha_1, \alpha_2, \alpha_3 \rightarrow$ are weights for features $y_{t-1}, y_{t-2}, y_{t-3}$

order of AR model (p)

p = order = how many past/lag values to take for prediction.

$p = 3 \Rightarrow$ use lag 1, lag 2, lag 3

$p = 1 \Rightarrow$ use lag 1

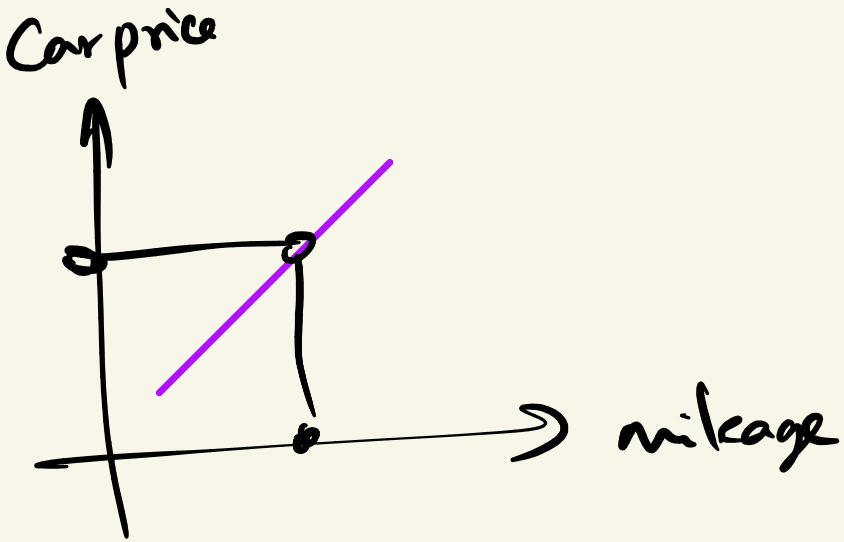
8) How to determine p?

a) use the ACF plot

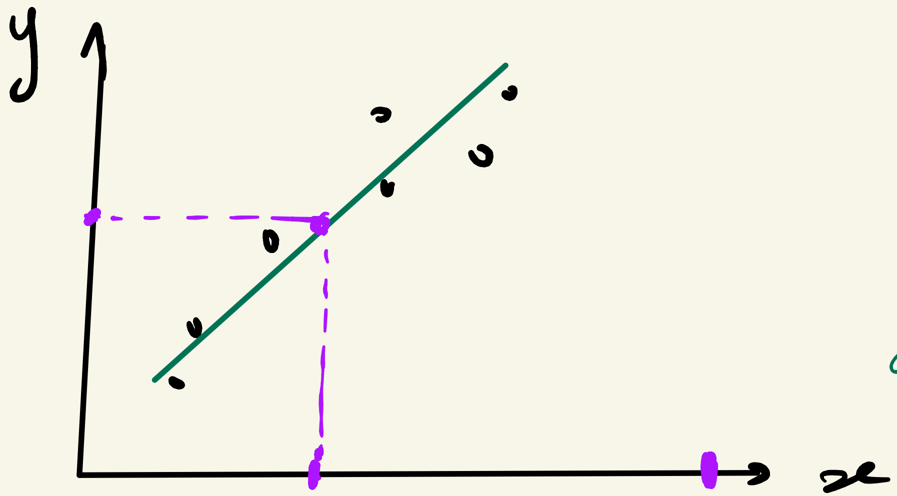
b) Hyperparameter Tuning → Train, val, test

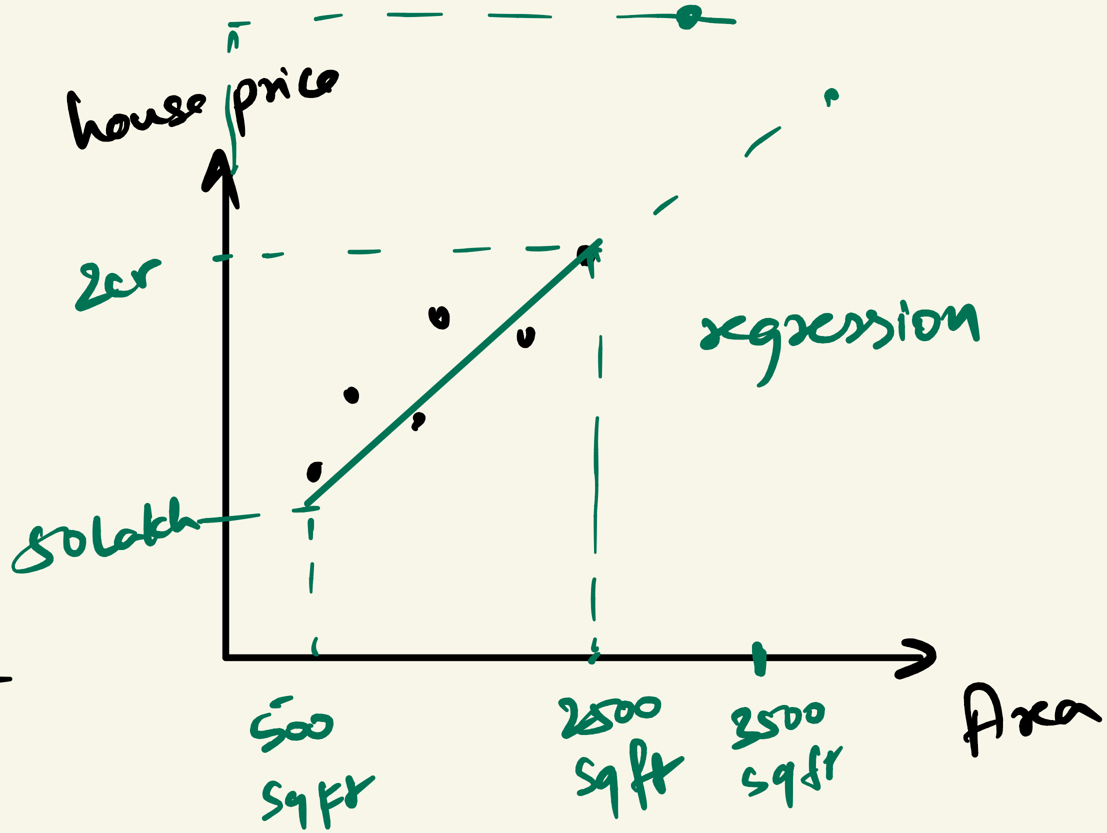
Doubts

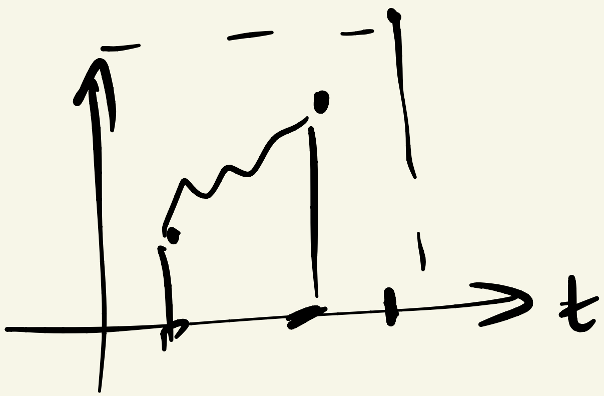
Regression vs forecasting



forecasting


y
x


house price
2cr
50lakh
regression
500 sqft
2500 sqft
2500 sqft
Area


t