Differential Equations

(MATH-108)

Dr. Aamir Alaud Din

Institute of Geographical Information Systems (IGIS)
School of Civil and Environmental Engineering (SCEE)
National University of Sciences and Technology (NUST), Islamabad

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What is Mathematics?

I am not asking the formal definition. Rather, I am asking your perception about mathematics as an engineer.

Looking at Mathematics

$$s = ut + \frac{1}{2}at^2$$

1. What is ut?
2. What is at^2?

Mathematics is a language

Urdu

Urdu is a language of communication in Pakistan.

English

English is a language of communication in England, United States of America, Australia, and many other countries.

Mathematics

Mathematics is a language of communication in Physics and Engineering.

Engineering Mathematics and Way of Studying Engineering Mathematics

1. Models, Modeling and Simulation
2. Our mathematical background and tutorials
3. A promise of daily study
4. No use of mobile phones & laptops (only for modeling and graphing etc.)
5. Registers of practice

$$\begin{cases} x - 2y + z = -1 \\ 2x + 7y \quad\quad = 8 \\ 3x + 5y + z = 7 \end{cases}$$

- Infinite solutions.

$$x^2 - 1 = 0$$

- One equation, one solution, one curve.

$$\frac{d^2y}{dx^2} + y^2\frac{dy}{dx} = 0$$

- One equation, one solution, infinite curves.

Outline

1. Objectives
2. Definitions and terminologies
3. Classification of Differential Equations

Outline

Objectives

After taking this lecture and studying, you should be able to

1. Define and explain different terminologies related to differential equations.
2. Describe the classification of differential equations.

Outline

- The derivative dy/dx of a function $y = f(x)$ is itself another function $g(x) = f'(x)$.

Definition

Differential Equation

An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables, is said to be a differential equation (DE). ▶

Example: $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 0$

Example: $\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 u}{\partial y^2} = 0$

Notation

- Leibniz notation: dy/dx, d^2y/dx^2, d^3y/dx^3, d^ny/dx^n
- Prime Notation: y', y'', y''', $y^{(n)}$
- Newton's Dot or Flyspek Notation: Used in derivatives with respect to time is $\ddot{s} = -32$ which is $d^2s/dt^2 = -32$
- Subscript Notation: u_{xx} which is d^2u/dx^2

Order

- The order of a differential equation is the order of the highest derivative in the equation.

$$\frac{d^2y}{dx^2} + 5\left(\frac{dy}{dx}\right)^3 - 4y = e^x \tag{1}$$

$$2\frac{\partial^4 u}{\partial x^4} + \frac{\partial^2 u}{\partial t^2} = 0 \tag{2}$$

- Equations (1) and (2) are second-orde and fourth-order differential equations.

- Consider the differential equation

$$6xy\frac{dy}{dx} + x^2 + y^2 = 0 \tag{3}$$

- If we multiply this equation by dx, we get

$$(x^2 + y^2)dx + 6xydy = 0 \tag{4}$$

- Equation (4) is called the differential form of the differential equation (3) and generally written as $M(x, y)dx + N(x, y)dy = 0$. ▶

- A differential equation in one dependent variable can also be written as $F(x,\ y,\ y',\ y'', ...,\ y^{(n)}) = 0$

Linear and Nonlinear DE

- Consider the nth-order differential equation in y

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + ... + a_1(x)\frac{dy}{dx} + a_0(x)y - g(x) = 0 \quad (5)$$

- Equation (5) is said to be linear in y if it has the following two properties

1. The dependent variable y and all its derivatives $y', y'', ..., y^{(n)}$ are of the first degree; that is, the power of each term involving y is 1.
2. The coefficients $a_0, a_1, ..., a_n$ of $y, y', y'', ..., y^{(n)}$ depend at most on the independent variable x.

Nonlinear Differential Equation

- If a differential equation does not possesses the above two properties, it is said to be a nonlinear differential equation. ▶

Homogeneous and Nonhomogeneous DEs

- The nth-order DE (5) can also be written as

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x) \qquad (6)$$

- Equation (6) is said to be homogeneous if $g(x) = 0$, otherwise, nonhomogeneous.

Definitions and Terminologies

Solution

- The solution of an nth-order differential equation is a function of the independent variable x and is sometimes denoted by $y(x)$.
- A solution $y(x)$ can be verified by substituting $y(x)$ and its required derivatives into the DE.

Interval of Definition

- The solution of a DE is not always valid for the interval $(-\infty, \infty)$.
- The domain in which the solution is valid is called the interval of definition, interval of validity, or the domain of the solution.

Solution Curves

- The solution of a DE involves arbitrary constants.
- If the constants are not evaluated and assumed values are used, multiple solution curves are obtained.
- The constants are evaluated by initial or boundary conditions.
- The evaluation of constants is called initial value problem (IVP) or boundary value problem (BVP). ▶

Outline

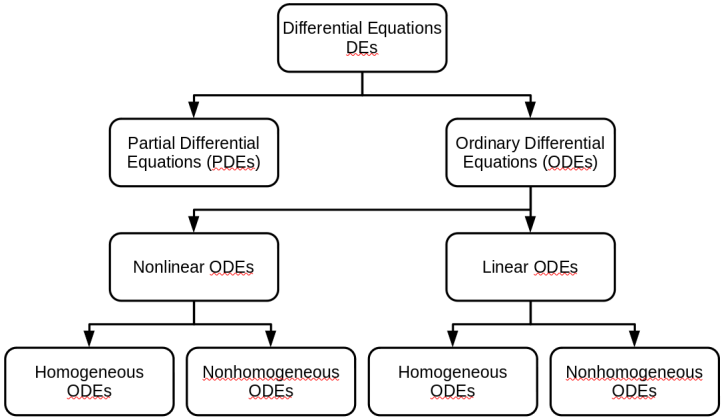

Figure: Classification of Differential Equations

THANK YOU