Linear Algebra and Ordinary Differential Equations
(MATH-121)

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Lecture 1
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What is Mathematics?

I am not asking the formal definition. Rather, I am asking your perception about mathematics as an engineer.

Looking at Mathematics

$$s = ut + \frac{1}{2}at^2$$

1. What is ut?
2. What is at^2?

Mathematics is a language

Urdu

Urdu is a language of communication in Pakistan.

English

English is a language of communication in England, United States of America, Australia, and many other countries.

Mathematics

Mathematics is a language of communication in Physics and Engineering.

Engineering Mathematics and Way of Studying Engineering Mathematics

1. Models, Modeling and Simulation
2. Our mathematical background and tutorials
3. A promise of daily study
4. No use of mobile phones & laptops (only for modeling and graphing etc.)
5. Registers of practice

Course and Grading

Course Outline

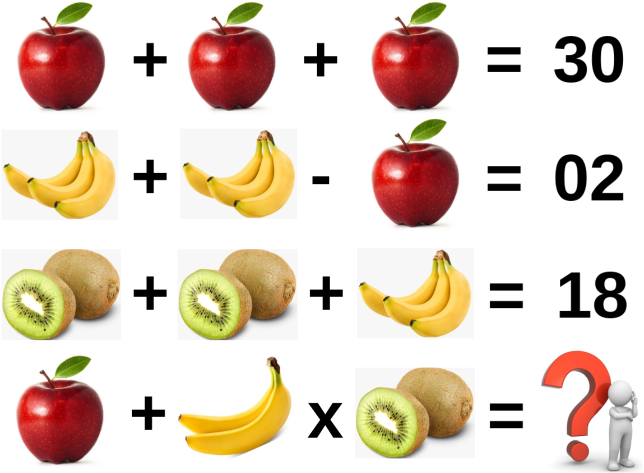
Definitions and terminologies of linear algebra, introduction to ordinary differential equations (ODEs), mathematical techniques to solve linear, nonlinear, homogeneous, nonhomogeneous ODEs of first, second and higher order. Applications of first and second order ODEs, Laplace and inverse Laplace transforms, partial differentiation and ODEs.

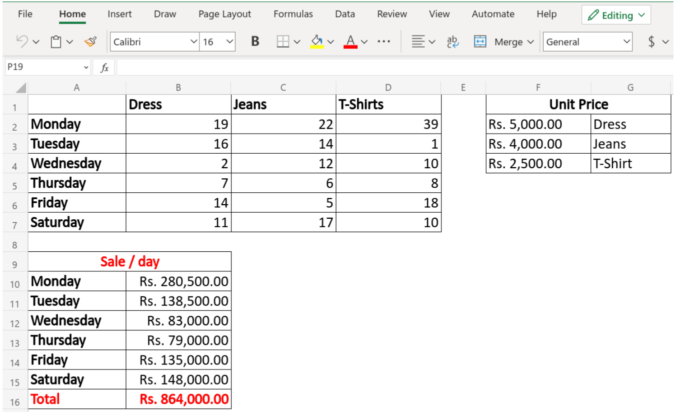
Grading Policy

As per NUST policy.

Linear Algebra

Opener





	Dress	Jeans	T-Shirts
Monday	19	22	39
Tuesday	16	14	1
Wednesday	2	12	10
Thursday	7	6	8
Friday	14	5	18
Saturday	11	17	10

Unit Price	
Rs. 5,000.00	Dress
Rs. 4,000.00	Jeans
Rs. 2,500.00	T-Shirt

Sale / day	
Monday	Rs. 280,500.00
Tuesday	Rs. 138,500.00
Wednesday	Rs. 83,000.00
Thursday	Rs. 79,000.00
Friday	Rs. 135,000.00
Saturday	Rs. 148,000.00
Total	Rs. 864,000.00

Figure: Real world example of matrix multiplication in excel

Opener

```python
#!/usr/bin/env pytyhon3
# -*- coding: <encoding-name> -*-i

# Program: sale_per_day.py
# Programmer: Aamir Alaud Din
# Date: 2022.03.22

# Objective(s):
#   To teach application of linear algebra in real life.

import numpy as np

units_sold = np.array([19, 22, 39, 16, 14, 1, 2, 12, 10, 7, 6, 8, 14, 5, 18,
    11, 17, 10]).reshape(6, 3)
unit_price = np.array([5000, 4000, 2500]).reshape(3, 1)

sale_per_day = np.dot(units_sold, unit_price)

days = np.array(['Monday', 'Tuesday', 'Wednesday', 'Thursday', 'Friday',
    'Saturday'])

for i in range(len(days)):
    print("%-12sRs. %-.2f" % (days[i], sale_per_day[i]))
print()
print("Total = Rs. %-.2f" % np.sum(sale_per_day))
```

Figure: Real world example of matrix multiplication in python

Opener


```
Monday      Rs. 280500.00
Tuesday     Rs. 138500.00
Wednesday   Rs. 83000.00
Thursday    Rs. 79000.00
Friday      Rs. 135000.00
Saturday    Rs. 148000.00

Total = Rs. 864000.00
```

Figure: Result of matrix multiplication from a python code

Outline

1. Objectives
2. Definitions
3. Basic Operations
4. Properties

Outline

Objectives

After taking this lecture and studying, you should be able to

1. Define in your own words and elaborate basic definitions related to matrices.
2. Carry out basic operations on matrices like, addition, multiplication, and taking transpose of matrices etc.
3. Describe the properties on matrix operations like commutative, associative, and distributive properties etc.

Outline

Definitions

- A matrix is a collection of numbers or functions arranged in rows and columns and is represented by an uppercase letter.

$$A = \begin{bmatrix} a_{11} & a_{12} \cdots\cdots & a_{1n} \\ a_{21} & a_{22} \cdots\cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} \cdots\cdots & a_{mn} \end{bmatrix}$$

- The numbers or functions in the matrix are called the entries or elements of the matrix.

- The horizontal arrangement of elements is called a row whereas the verticial arrangement is called a column of the matrix.

Objectives
Definitions
Basic Operations
Properties

Order

If a matrix has m rows and n columns, it is said to be of order $m \times n$. For example, matrix A, shown below, is of order 3×2.

$$A = \begin{bmatrix} 7 & 3 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$$

Row Vector

If a matrix has only one row, it is called a row vector. For example, the matrix R, shown below, is a row vector.

$$R = \begin{bmatrix} 1 & -8 & 4 \end{bmatrix}$$

Column Vector

If a vector has only one columns, it is called a column vector. For example, the matrix C, shown below, is a column vector.

$$C = \begin{bmatrix} 9 \\ 0 \\ 1 \end{bmatrix}$$

Square Matrix

If a matrix has equal number of rows and columns, it is called a square matrix. The matrix S, shown below, is a square matrix.

Definitions

$$S = \begin{bmatrix} 3 & 5 & 0 \\ 1 & -5 & 1 \\ 8 & 2 & -7 \end{bmatrix}$$

Rectangular Matrix

If a matrix has unequal number of rows and columns, it is called a rectangular matrix.

$$R = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 9 & 11 & 13 & 15 \\ 17 & 19 & 21 & 23 \end{bmatrix}$$

Definitions

- An element in i-th row and j-th column of a matrix is represented by the symbol a_{ij}.
- In the matrix R, shown above

$$a_{23} = 13$$

Diagonal Matrix

A square matrix A is called a diagonal matrix if $a_{ij} = 0$ for all $i \neq j$.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{7}{3} & 0 \\ 0 & 0 & -8 \end{bmatrix}$$

Transpose

A matrix A^T is called the transpose of matrix $A = [a_{ij}]$ if

$$A^T = [a_{ji}]$$

If $A = \begin{bmatrix} 1 & 5 & -8 \\ 2 & -1 & 9 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & 2 \\ 5 & -1 \\ -8 & 9 \end{bmatrix}$

Null Matrix

A matrix O is called a null or zero matrix if all $a_{ij} = 0$.

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O_{2\times 3}$$

Definitions

- A null or zero matrix is equivalent to the integer 0.

Identity Matrix

A square matrix I is called an identity matrix if $a_{ij} = 1$ for all $i = j$ and $a_{ij} = 0$ for all $i \neq j$.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- An identity matrix is equivalent to integer 1 in the operation of matrix multiplication.

Definitions

Triangular Matrix

A square matrix $A = [a_{ij}]$ is called triangular matrix if $a_{ij} = 0$ for all $i < j$ or $i > j$.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ -1 & 4 & 0 \\ 7 & 6 & -2 \end{bmatrix} \text{ and } A = \begin{bmatrix} 7 & 6 & -2 \\ 0 & 4 & -1 \\ 0 & 0 & 3 \end{bmatrix} \text{ are triangular matrices.}$$

Upper Triangular Matrix

A triangular matrix $A = [a_{ij}]$ is upper triangular if $a_{ij} = 0$ for all $i > j$.

Lower Triangular Matrix

A triangular matrix $A = [a_{ij}]$ is lower triangular if $a_{ij} = 0$ for all $i < j$.

Definitions

The matrix $A = \begin{bmatrix} 7 & 6 & -2 \\ 0 & 4 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ is an upper triangular matrix.

The matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ -1 & 4 & 0 \\ 7 & 6 & -2 \end{bmatrix}$ is a lower triangular matrix.

Scalar Matrix

A square matrix $A = [a_{ij}]$ is called a scalar matrix if all a_{ij} are equal for all $i = j$.

The matrix $A = \begin{bmatrix} -6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -6 \end{bmatrix}$ is a scalar matrix.

Symmetric Matrix

A square matrix A is called a symmetric matrix if $A^T = A$.

The matrix $A = \begin{bmatrix} 3 & -1 & 6 \\ -1 & 5 & 2 \\ 6 & 2 & 1 \end{bmatrix}$ is symmetric because

$$A^T = \begin{bmatrix} 3 & -1 & 6 \\ -1 & 5 & 2 \\ 6 & 2 & 1 \end{bmatrix} = A.$$

Outline

Basic Operations

- Two matrices A and B are said to be conformable for addition/subtraction if they have equal sizes.
- For two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ of sizes $m \times n$, their addition is defined as
$$C = A + B = [a_{ij}] + [b_{ij}] = [(a_{ij} + b_{ij})]$$
- If k is a real number then the scalar multiple kA of matrix $A = [a_{ij}]$ is $B = kA = k[a_{ij}] = [ka_{ij}]$
- Two matrices A and B are said to be conformable for multiplication if number of columns of the first matrix are equal to the number of rows of the second matrix.

Basic Operations

- If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two matrices of sizes $m \times p$ and $p \times n$, respectively, then their product is given by

$$AB = \left(\sum_{k=1}^{p} a_{ik} b_{kj} \right)_{m \times n}$$

- Rows are multiplied by columns as shown below.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} (1)(5) + (2)(7) & (1)(6) + (2)(8) \\ (3)(5) + (4)(7) & (3)(6) + (4)(8) \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 29 & 50 \end{bmatrix}$$

Outline

Properties

- $A + B = B + A$
- $A + (B + C) = (A + B) + C$
- $(k_1 k_2)A = k_1(k_2 A)$
- $k_1(A + B) = k_1 A + k_1 B$
- $(k_1 + k_2)A = k_1 A + k_2 A$
- $(A^T)^T = A$ and $(kA)^T = kA^T$
- $(A + B)^T = A^T + B^T$
- $(AB)^T = B^T A^T$

- Compute the weekly and daily sales for the six days of the week in the opener.
- Try in MS Excel or MATLAB (Not mandatory).

THANK YOU