IMPLEMENTATION OF TRIGONOMETRIC FUNCTIONS USING CORDIC ALGORITHM ON FPGA

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OUTLINE

- First Section
 - Circular-CORDIC Algorithm (BASIC)

- Second Section
 - Implementation in MATLAB
 - Implementation of Circular CORDIC in RTL
 - References

MOTIVATION

If a unit vector with co-ordinates $(x_1, y_1) = (1, 0)$ is rotated by an angle θ , its new co-ordinate will be $(x_2, y_2) = (\cos \theta, \sin \theta)$. Thus, by finding the (x_2, y_2) , $\cos \theta$, $\sin \theta$ can easily be computed.

Circular-CORDIC Algorithm (BASIC)

Pseudorotations

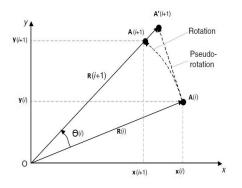


Figure: pseudorotation step in CORDIC[3]

$$R(i+1) = R(i)(1 + \tan^2 \theta(i))^{1/2}$$

Domain of Convergence

• In rotation mode, convergence of $\alpha(n)$ to 0 is possible because each CORDIC angle is more than half the previous angle.

$$\tan^{-1}(2^{-(i+1)}) \geq 0.5 * \tan^{-1}(2^{-i})$$

- Thus, domain of convergence is $-99.7^o \le \alpha \le 99.7^o$ where $\pm 99.7^o$ is the sum of the CORDIC angle.
- Beyond the above range, we can use Trigonometric identities to convert the problem within the domain of convergence.

Implementation in MATLAB

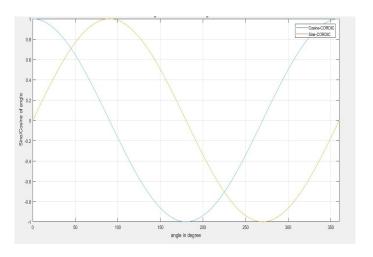


Figure: Trigonometric Functions using CORDIC [3]

Implementation of Circular CORDIC in RTL

- The digital design should be able to compute Sine and Cosine of the input angles $\in [0, 2\pi]$ including the floating point angles like 89.45°, 29.7° etc.
- A 32-bit binary scaling system has been used to represent angles[5]. The resolution of the design is $360^{\circ}/2^{32} = 8.381903172 \times 10^{-8}$.
- To convert the angle from degrees to 32-bit value, multiply the angle^o by 2³², then divide it 360°. Flnally, convert the decimal value to binary.
- The upper two bits represent the quadrant[6].
 - 2b'00 = represents I quadrant i.e (0 π /2) range
 - 2b'01 = represents II quadrant i.e $(\pi/2 \pi)$ range
 - 2b'10 = represents III quadrant i.e $(\pi$ $3\pi/2)$ range
 - $2b'11 = represents IV quadrant i.e (3\pi/2 2\pi) range$

Testbench tested for different values of input angle using Xilinx-ISim software. After 16-clock cycles, Output Data was displayed on the simulator screen. Ouput value is scaled 25000 times (i.e Xout = 25000 * $\cos \alpha$ and Yout = 25000 * $\sin \alpha$).

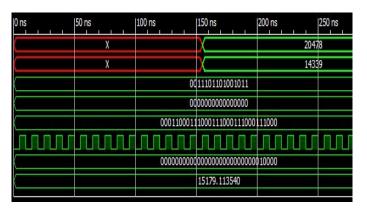


Figure: $\alpha = 35^{\circ}$

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