

# Graph Coloring.

m - no. of colors.

Algorithm: coloring( $k$ )

{

while (true)

1 NextValue( $x$ ); — which color to // assign  $x[k]$  to a legal color

if ( $x[k] = 0$ ) then // no such color possible.

return;

if ( $k = n$ ) then

write( $x[1:n]$ );

else

coloring( $k+1$ );

coloring( $x$ )

coloring( $x$ )

}

The ~~below~~ <sup>above</sup> algo finds legal color for  $k^{th}$  vertex of graph. A value for  $x[k]$  is determined in the range  $(1..m)$ .  $x[k]$  is assigned the next highest numbered color while maintaining distinct from the adjacent vertices of vertex  $x$ .

If no such color exists then  $x[k]$  is assigned 0.  $x[1]..x[k-1]$  have been assigned integer values in the range  $[1, m]$

Such that adjacent vertices have distinct integers  
Graph is represented by its boolean adjacency matrix.

# Algorithm Next Value (k)

{ while (true)

{

$x[k] = (x[k] + 1) \bmod (m+1)$ ; // next highest color.  
if ( $x[k] = 0$ ) then

return; // all colors have been used.

for  $j = 1$  to  $n$  do // check if this color is distinct vertex  
// from adjacent vertices colors.

{ if ( $(G[k][j] \neq 0)$  and  $x[k] = x[j]$ ) then

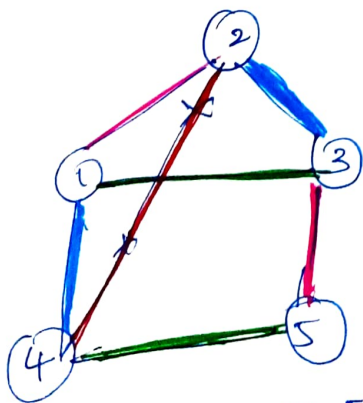
break; // if  $(k, j)$  is an edge of adjacent  
// vertices have same color.

if ( $j = n+1$ ) then

return

} // otherwise try to find another color.

Ex:



$x[1]$	$x[2]$	$x[3]$	$x[4]$	$x[5]$
0	0	0	0	0
1	2			

$k=1$

$$G = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

$m=3$

~~B~~, B, G  
(1, 2, 3)

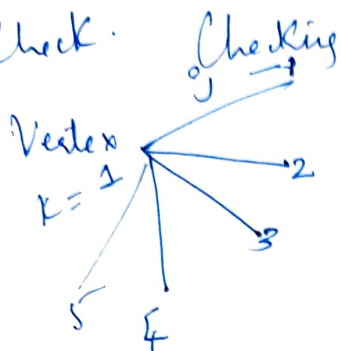
finding color for vertex 1  
mcoloring (1) →

$$\text{Next value } [1] \rightarrow x[1] = (0+1) \bmod (3+1)$$

$$= 1 \bmod 4$$

$$x[k] \rightarrow x[1] = 1$$

Check.



Checking adjacency. (refer matrix).

1-1 (Edge b/w 1 → 1 → no → value = 0)

1-2 → 1-0 (Check if both have same color?)

1-3 → 1-0

1-4 → 1-0

0-value of color.

$$(x[k] = 1.$$

$$x[1] = 0$$

no color

Cond: false

∴ algorithm will send  $x[k] = 1$ .

Coz none of the color matches as initials

they are set at value 0.

$$\begin{array}{c} x[1] \quad x[2] \quad x[3] \quad x[4] \quad x[5] \\ \hline 1 \quad 0 \quad 0 \quad 0 \quad 0 \\ \hline \downarrow \\ 1 \quad 2 \end{array}$$

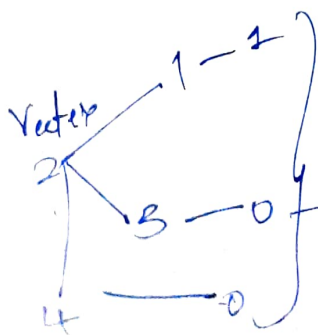
$k=2$

Sending value  $k=2$  to mcoloring(2) algorithm

mcoloring (2) → Next value (2)

$$x[2] = (0+1) \bmod 4 = 1$$

but 1 is already assigned to vertex 1.



Color assigned to 3 will be 2. as 2 is not assigned to it or adjacent ones & III as way algorithm will run.