

TSP. Example. NP. Hard Problem

TSP. Travelling Salesperson Problem.

Here, the problem starts ^{with one starting} from one City visiting all Cities only once and coming back to the same City.

Here we need to find the optimal solution with smallest cost chosen for the travel.

(minimization problem).

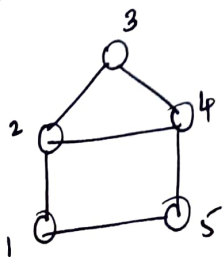
which is similar to the Hamiltonian Cycle (Circuit).

So to prove TSP is NP Complete we first need to prove that. ~~it is is~~

So we will take a problem which already is NP. Hard and is NP. Complete (which is Hamiltonian Circuit).

Thus if TSP also has its own ^{non} deterministic algorithm we say it is NP-Complete problem.

First ^{step} \rightarrow If Hamiltonian Cycle is reducible to TSP.

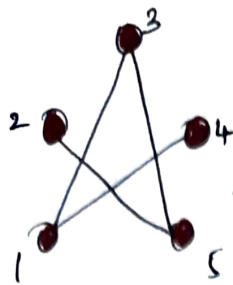


$G = (V, E)$

~~Step~~ ^{Step}

Construct a Complete Graph $G' = (V, E')$ (New Edges.)

2a)

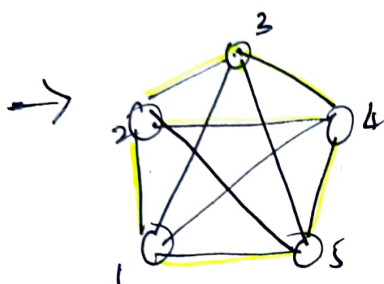


$G(V, E)$

Observe here the new Edges are drawn with which are not present in Original Graph.



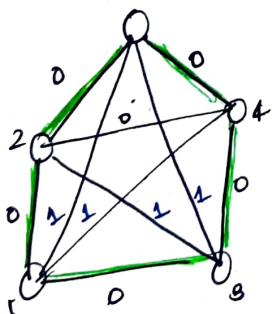
Combine this new Graph with Original one.



Complete Graph.

(2b) Define the cost function.

$$C(i, j) = \begin{cases} 0, & i, j \in E \\ 1, & i, j \in E' \end{cases} \quad \begin{aligned} &\text{Cost is 0 if } i, j \text{ belongs to Original} \\ &\text{Edge Set (E).} \\ &\text{Cost = 1 if } i, j \text{ belongs to new} \\ &\text{Edge Set (E')} \end{aligned}$$



Now construct the minimum cost Path. i.e. TSP.

$\min_{\text{cost}} = 0$. reflecting original graph

→ We now showed that Graph G has a hamiltonian cycle if and only if Graph G' has a tour of cost atleast 0 (zero).

Thus we have proved that TSP is NP-Complete problem.