Example 3: Out of 800 families with 5 children each, how many would you expect to have (a) 3 boys (b) 5 girls (c) either 2 or 3 boys. Assume equal probabilities for boys and girls.

Solution: Probability of boy $= P(B) = p = \frac{1}{2}$, and probability of girl $= P(G) = q = \frac{1}{2}$. $n =$ number of trials $= 5$, $X =$ no. of boys in a family

a. Probability of a family having 3 boys

$$= P(X = 3) = 5_{C_3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-2}$$

$$= \frac{5!}{2!3!}\left(\frac{1}{2}\right)^5 = \frac{10}{32} = \frac{5}{16}$$

Expected number of families having 3 boys out of 5 children $= 800\left(\frac{5}{16}\right) = 250$, i.e., 250 families have 3 boys out of 5 children.

b. $P(X = 0) = P$ (all girls) $= \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

Expectation $= 800 \times \frac{1}{32} = 25$.

c. $P(X = 2) = P(2 \text{ boys}) = 5_{C_2}\left(\frac{1}{2}\right)^5 = \frac{5}{8}$

Expectation $= 800 \times \frac{5}{8} = 500$.

Example 4: Determine the probability distribution of the number of bad eggs in a box of 6 chosen at random if 10% of eggs are bad, in a large consignment.

Solution: Probability of a bad egg $= p = \frac{10}{100} = 0.1$. Let $X =$ number of bad eggs, $n = 6$. The required B.D. $= b(x; 6, 0.1) = 6_{C_x}(.1)^x(.9)^{6-x}$, for $x = 0, 1, 2, 3, 4, 5, 6$.

X :	0	1	2	3	4	5	6
$P(X)$:	.5311	.35429	0.098	0.015	0.001215	0.000054	0

Example 5: Assume that 50% of all engineering students are good in mathematics. Determine the probabilities that among 18 engineering students (a)

exactly 10 (b) at least 10 (c) at most 8 (d) at least 2 and at most 9, are good in maths.

Solution: Let X = number of engineering students who are good in maths:

p = prob of good in maths $= \frac{50}{100} = \frac{1}{2}$, $n = 18$

$b(x; n, p) = b(x; 18, \frac{1}{2}) = 18_{C_x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{18-x}$

a. Exactly 10 students out of 18 are good in maths

$$P(X = 10) = 18_{C_{10}} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{8} = .1670$$

From tables (A2 to A7)

$$P(X = 10) = \sum_{x=0}^{10} b\left(x; 18, \frac{1}{2}\right) - \sum_{x=0}^{9} b\left(x; 18, \frac{1}{2}\right)$$

$$= .7597 - .5927 = .1670$$

b. $$P(X \geq 10) = \sum_{x=10}^{18} 18_{C_x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{18-x}$$

$$= \sum_{x=0}^{18} b\left(x; 18, \frac{1}{2}\right) - \sum_{x=0}^{9} b\left(x; 18, \frac{1}{2}\right)$$

$$= 1 - .5927 = .4073$$

c. $$P(X \leq 8) = \sum_{x=0}^{8} 18_{C_x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{18-x} = .4073$$

from table with $n = 18$, $x = 8$, $p = \frac{1}{2}$.

d. $$P(2 \leq x \leq 9) = \sum_{x=2}^{9} 18_{C_x} \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{18-x}$$

$$= \sum_{x=0}^{9} b\left(x; 18, \frac{1}{2}\right) - \sum_{x=0}^{1} b\left(x; 18, \frac{1}{2}\right).$$

$$= .5927 - .0007 = .5920.$$

Example 6: The probability of a man hitting a target is $\frac{1}{3}$. (a) If he fires 5 times, what is the probability of his hitting the target at least twice? (b) How many times must he fire so that the probability of his hitting the target at least once is more than 90%?

Solution: Probability of hitting $= p = \frac{1}{3}$

probability of no hit (or failure) $= q = \frac{2}{3}$

a. X = number of hits (successes), $n = 5$

$$P(X \geq 2) = \sum_{x=2}^{5} 5_{C_x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}$$

$$= 1 - \sum_{x=0}^{1} 5_{C_x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{5-x}$$

$$= 1 - \left(\frac{2}{3}\right)^5 - 5_{C_1} \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^4 = \frac{131}{243}.$$

b. The probability of not hitting the target is q^n in n trials (fires). Thus to find the smallest n for which the probability of hitting at least once $1 - q^n$ is more than 90%.

i.e., $\quad 1 - q^n > 0.9$

or $\quad 1 - \left(\frac{2}{3}\right)^n > 0.9 \quad$ i.e., $\left(\frac{2}{3}\right)^n < 0.1$

For $n = 6$, $2^6 = 64 < (0.1)3^6 = 72.9$ this is true. In other words, he must fire 6 times.