APTITUDE MASTERY SERIES

MODULE 9 – PROBABILITY

1. What is the probability that a number selected from 1 to 30 is a prime number, when each of the given numbers is equally likely to be selected?

(a) 9/30 (b) 8/30 **(c) 10/30** (d) 11/30

Solution:

X = {2,3,5,7,11,13,17,19,23,29}

n(X) = 10

n(S) = 30

Hence required probability,

$= \frac{n(\mathrm{X})}{n(S)}$

$= \frac{10}{30}$

2. There are four hotels in a town. If 3 men check into the hotels in a day then what is the probability that each checks into a different hotel?

(a) 6/7 (b) 1/8 **(c) 3/8** (d) 5/9

Solution:

Total cases of checking in the hotels = 4^3 ways

Cases, when 3 men are checking in different hotels = 4 x 3 x 2 = 24 ways

Required probability:

= 24 / 4^3

= 3 / 8

3. In a drawer there are 4 white socks, 3 blue socks and 5 grey socks. Two socks are picked randomly. What is the possibility that both the socks are of same colour?

(a) 4/11 (b) 1 (c) 2/33 **(d) 19/66**

Solution:

Probability = $\frac{What\ we\ want}{Total}$

OR = Add AND = Multiply

Total socks = 4 + 3 + 5 = 12

We want same color socks

So we want 2 white OR 2 blue OR 2 grey socks

For white:

Probability of 1st sock being white = $\frac{4}{12}$

Probability of 2nd sock being white = $\frac{3}{11}$

White Probability = $\frac{4}{12}$ x $\frac{3}{11}$ = $\frac{1}{11}$

Similarly,

Blue Probability = $\frac{3}{12}$ x $\frac{2}{11}$ = $\frac{1}{22}$

Grey Probability = $\frac{5}{12}$ x $\frac{4}{11}$ = $\frac{5}{33}$

Therefore, Total Probability = $\frac{1}{11} + \frac{1}{22} + \frac{5}{33} = \mathbf{\frac{19}{66}}$

4. When two dice are thrown simultaneously, what is the probability that the sum of the two numbers that turn up is less than 11?

(a) 5/6 **(b) 11/12** (c) 1/6 (d) 1/12

Solution:

Instead of finding the probability of this event directly, we will find the probability of the non-occurrence of this event and subtract it from 1 to get the required probability.

Combination whose sum of 12 is (6,6)

Combinations whose sum of 11 is (5,6), (6,5).

Therefore, there are totally 3 occurrences out of 36 occurrences that satisfy the given condition.

Probability whose sum of two numbers is greater than or equal to 11 = 3 / 36 = 1 / 12.

Hence probability whose sum of two numbers is lesser than 11 = 1 - 1 / 12 = 11 / 12.

5. Three unbiased coins are tossed. What is the probability of getting at least 2 tails?

(a) 0.75 **(b) 0.5** (c) 0.25 (d) 0.2

Solution:

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

E = {HTT, THT, TTH, TTT}

n(S) = 8

n(E) = 4

P(E) =n(E) / n(S) = 4/8 = 0.5

6. What is the possibility of having 53 Thursdays in a non-leap year?

(a) 1/7 (b) 6/7 (c) 1/365 (d) 53/365

Solution:

A non-leap year has 365 days, which has 52 weeks (364 days) means 52 Thursdays.

Thus there is just 1 day extra.

We want it to be Thursday.

Total possibilities are 7 (Sunday to Saturday means 7 days)

Therefore, Probability of 53 Thursdays = $\frac{1}{7}$

7. On rolling a dice 2 times, the sum of 2 numbers that appear on the uppermost face is 8. What is the probability that the first throw of dice yields 4?

(a) 2/36 **(b) 1/36** (c) 1/6 (d) 1/5

Solution:

A dice has 6 faces

So there are 6 possible outcomes

Dice is rolled once AND then again

So, total possibilities = 6 x 6 = 36

The sum should be 8 of the three throws.

So which combination of numbers from 1 to 6 will yield us a sum of 8?

They are – (2,6); (6,2); (3,5); (5,3); (4,4)

So there are total 5 possibilities where addition is 8

But only 1 possibility where first throw of dice is 4.

So, probability for first throw to be 4 and sum to be 8 = $\frac{1}{36}$

8. Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart, is:

(a) 47/100 **(b) 13/102** (c) 29/34 (d) 3/20

Solution:

n(S) = $^{52}C_2$ = (52 * 51)/(2 * 1) = 1326

n(E) = number of ways of choosing 1 spade out of 13 and 1 heart out of 13

n(E) = $^{13}C_1$ * $^{13}C_1$ = 13 * 13 = 169

P(E) = n(E)/n(S) = 169/1326 = 13/102

9. A box contains 50 balls, numbered from 1 to 50. If three balls are drawn at random with replacement, what is the probability that sum of the numbers are odd?

(a) 1/2 (b) 1/3 (c) 2/7 (d) 1/5

Solution:

There are 25 odd and 25 even numbers from 1 to 50.

Sum will be odd if = odd + odd + odd, odd + even + even, even + odd + even, even+ even + odd

P= (1/2)*(1/2)*(1/2) + (1/2)*(1/2)*(1/2) + (1/2)*(1/2)*(1/2) + (1/2)*(1/2)*(1/2) = 4/8 → 1/2

10. In a class, 40% of the students study math and science. 60% of the students study math. What is the probability of a student studying science given he/she is already studying math?

(a) 0.54 (b) 0.72 **(c) 0.67** (d) 0.84

Solution:

P(M and S) = 0.40

P(M) = 0.60

P(S|M) = P(M and S)/P(S) = 0.40/0.60 = 2/3 = 0.67

11. A speaks truth in 55% cases and B speaks truth in 75% cases. Find the percentage of cases they are likely to contradict each other in stating the fact?

(a) 36.4% (b) 56.8% (c) 63.2% **(d) 47.5%**

Solution:

From the given problem we get,

Probability of A speaking the truth = P(A) = 55/100

Probability of A lying = P(AC) = 1 – P(A) = 45/100

Probability of B speaking the truth = P(B) = 75/100

Probability of B lying = P(BC) = 1 – P(B) = 25/100

There exist two cases of contradictions

i. A speaks truth and B lies.

ii. A lies and B speaks the truth.

<u>Case i</u>

Probability = P(X1) = P(A) . P(BC)

= 55/100 x 25/100

= 11/80

<u>Case ii</u>

Probability = P(X2) = P(AC) . P(B)

= 45/100 x 75/100

= 27/80

So the total probability that A and B will contradict each other is

= 11/80 + 27/80

= 19/40

Therefore the required percentage = 47.5%

12. Doctors have devised a test for leptospirosis that has the following property: For any person suffering from lepto, there is a 90% chance of the test returning positive. For a person not suffering from lepto, there is an 80% chance of the test returning negative. It is known that 10% of people who go for testing have lepto. If a person who gets tested gets a +ve result for lepto (as in, the test result says they have got lepto), what is the probability that they actually have lepto?

(a) 7/10 (b) 8/11 **(c) 1/3** (d) 1/2

Solution:

Let us draw the possibilities in this scenario.

Prob (patient having lepto) = 0.9

Prob (patient not having lepto) = 0.1

Given that patient has lepto, Prob (test being positive) = 0.9

Given that patient has lepto, Prob (test being negative) = 0.1

Given that patient does not have lepto, Prob (test being negative) = 0.8

Given that patient does not have lepto, Prob (test being positive) = 0.2

Now, we are told that the test turns positive. This could happen under two scenarios – the patient has lepto and the test turns positive and patient does not have lepto and the test turns positive.

Probability of test turning positive = $0.9 \times 0.1 + 0.9 \times 0.2 = 0.27$.

Now, we have not been asked for the probability of test turning positive. We are asked for the probability of patient having lepto given that he/she tests positive. So, the patient has already tested positive. So, this 0.27 includes the set of universal outcomes. Or, this 0.27 sits in the denominator.

Within this 0.27, which subset was the scenario that the patient does indeed have lepto?

This is the key question. This probability is $0.1 \times 0.9 = 0.09$. So, the required probability = 0.09/0.27 = 1/3

So, if a patient tests positive, there is a 1 in 3 chance of him/her having lepto. This is the key reason that we need to be careful with medical test results.

Correct Answer : **1/3**

13. If all the rearrangements of the word AMAZON are considered, what is the probability that M will feature between the 2As?

(a) 1/3 (b) 1/6 (c) 2/5 (d) 3/8

Solution:

For this type of question, we need to consider only the internal arrangement within the M and 2As.

M and 2As can be rearranged as AMA, AAM, or MAA.

So, the probability that M will feature between the 2As is 1/3.

Now, let us think why we need to consider only the M and 2As.

Let us start by considering a set of words where the M and 2 As are placed at positions 2, 3 and 5.

The other three letters have to be in slots 1, 4 and 6

Three letters can be placed in three different slots in 3! = 6 ways.

Now with __ M A __ A __ there are 6 different words.

With __ A M __ A __ there are 6 different words.

With __ A A __ M __ there are 6 different words.

For each selection of the positions for A,A and M, exactly one-third of words will have M between the two A's.

This is why only the internal arrangement between A, A and M matters.

So, probability of M being between 2 As is 1/3.

14. A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is (1/7) and the probability of wife's selection is (1/5). What is the probability that only one of them is selected?

(a) 2/7 (b) 1/7 (c) 3/4 (d) 4/5

Solution:

Let $\quad A$ = Event that the husband is selected

and $\quad B$ = Event that the wife is selected.

Then, $P(A) = \frac{1}{7}$ and $P(B) = \frac{1}{5}$.

$\therefore \quad P(\bar{A}) = \left(1 - \frac{1}{7}\right) = \frac{6}{7}$ and $P(\bar{B}) = \left(1 - \frac{1}{5}\right) = \frac{4}{5}$.

$\therefore$ Required probability = P [(A and not B) or (B and not A)]

$= P[(A \text{ and } \bar{B}) \text{ or } (B \text{ and } \bar{A})]$

$= P(A \text{ and } \bar{B}) + P(B \text{ and } \bar{A})$

$= P(A) \cdot P(\bar{B}) + P(B) \cdot P(\bar{A}) = \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{1}{5} \times \frac{6}{7}\right) = \frac{10}{35} = \frac{2}{7}$.

15. Amit throws three dice in a special game of Ludo. If it is known that he needs 15 or higher in this throw to win, then find the chance of his winning the game.

(a) 5/54 (b) 17/216 (c) 13/216 (d) 15/216

Solution:

Event definition is: 15 or 16 or 17 or 18.

15 can be got as: 5 and 5 and 5 (one way)

Or

6 and 5 and 4 (Six ways)

Or

6 and 6 and 3 (Three ways)

Total 10 ways

16 can be got as: 6 and 6 and 4 (3 ways)

Or

6 and 5 and 5 (3 ways)

Total 6 ways

17 has 3 ways and 18 has 1 way of appearing.

Thus, the required probability is: (10 + 6 + 3 + 1) / 216

=20/216

=5/54

HOME WORK

16. Two small squares on a chess board are chosen at random. Find the probability that they have a common side?

(a) 1/12 **(b) 1/18** (c) 2/15 (d) 3/14

Solution:

The common side could be horizontal or vertical. Accordingly, the number of ways that event can occur is:

n (E) = 8 x 7 + 8 x 7 = 112

n (S) = $^{64}C_2$

$= \frac{2 \times 8 \times 7 \times 2}{64 \times 63} = 1/18$

17. There are two bags, one of them contains 5 red and 12 white balls, neither bag being empty. How should the balls be divided so as to give a person who draws one ball from either bag;

(i) the least chance of drawing a red ball.

(a) 3/35 **(b) 5/32** (c) 7/32 (d) 1/16

(ii) the greatest chance of drawing a red ball.

(a) 3/4 (b) 2/3 **(c) 5/8** (d) 5/7

Solution:

(i) For the least chance of drawing a red ball the distribution has to be 5 Red + 11 white in one bag and 1 white in the second bag. This gives us

1/2 x 5/16 + 1/2 x 0 = 5/32

(ii) For the greatest chance of drawing a red ball the distribution has to be 1 Red in the first bag and 4 red + 12 white balls in the second bag. This gives us

1/2 x 1 + 1/2 x 4/16 = 5/8.

18. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 4 or 6?

(a) 1/12 (b) 1/3 (c) 7/15 **(d) 7/18**

Solution:

Clearly, n(S) = 6 x 6 = 36.
Let E be the event that the sum of the numbers on the two faces is divisible by 4 or 6. Then
E = {(1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (6, 2), (6, 6)}
∴ n(E) = 14. Hence, P(E) =

$$\frac{n(E)}{n(S)} = \frac{14}{36} = \frac{7}{18}$$

19. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are queens and the third card drawn is an ace?

(a) 2/5530 (b) 3/5525 **(c) 2/5525** (d) 4/5525

Solution:

Let Q denote the event that the card drawn is queen and A be the event that the card drawn is an ace. Clearly, we have to find P (QQA)

Now P(Q) = 4/52

Also, P (Q|Q) is the probability of second queen with the condition that one queen has already been drawn. Now there are three queen in (52 - 1) = 51 cards.

Therefore P(Q|Q) = 3/51

P(A|QQ) is the probability of third drawn card to be an ace, with the condition that two queens have already been drawn. Now there are four aces in left 50 cards.

Therefore P(A|QQ) = 4/50

By multiplication law of probability, we have

P(QQA) = P(Q) P(Q|Q) P(A|QQ)

= 4/52 × 3/51 × 4/50

= 2/5525.

20. A family has two children. Find the probability that both the children are girls given that at least one of them is a girl?

(a) 1/4 (b) 2/3 **(c) 1/3** (d) 2/4

Solution:

Let b stand for boy and g for girl. The sample space of the experiment is

S = {(g, g), (g, b), (b, g), (b, b)}

Let E and F denote the following events:

E : ‘both the children are girls’

F : ‘at least one of the child is a girl’

Then E = {(g, g)} and F = {(g,g), (g,b), (b,g)}

Now E n F = {(g,g)}

Thus P(F) = 3/4

and P (E n F)= 1/4

Therefore P(E|F) = P(E ∩ F)/P(F) = (1/4)/(3/4) = 1/3