

*APTITUDE MASTERY SERIES*

**MODULE 13 – ALLIGATIONS AND MIXTURES**

1. A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?

(a) 26 litres

(b) **29.16 litres**

(c) 28 litres

(d) 28.2 litres

***Solution:***

Suppose a container contains  $x$  units of a liquid from which  $y$  units are taken out and replaced by water. After  $n$  operations, quantity of pure liquid

$$= x \left(1 - \frac{y}{x}\right)^n \text{ units.}$$

milk contained by the container now

$$= 40 \left(1 - \frac{4}{40}\right)^3$$

$$= 40 \left(1 - \frac{1}{10}\right)^3$$

$$= 40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$$

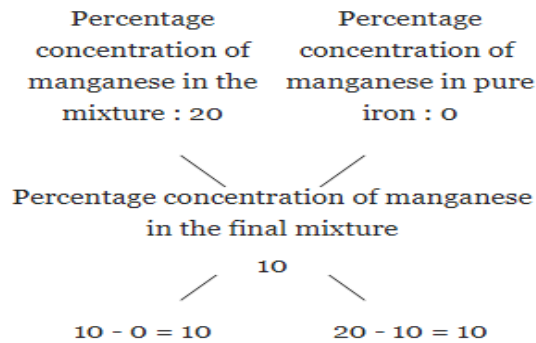
$$= \frac{4 \times 9 \times 9 \times 9}{100}$$

$$= 29.16$$

2. In 1 kg mixture of iron and manganese, 20% is manganese. How much iron should be added so that the proportion of manganese becomes 10%?
- (a) 1.5 kg                      (b) 2 kg                      (c) 5 kg                      (d) 1 kg

**Solution:**

By rule of alligation,



=> Quantity of the mixture : Quantity of iron = 10 : 10 = 1 : 1

Given that quantity of the mixture = 1 kg

Hence quantity of iron to be added = 1 kg

3. Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed with a third variety of tea in the ratio 1 : 1 : 2. If the mixture is worth Rs. 153 per kg, what is the price of the third variety per kg?
- (a) Rs.182.50                      (b) Rs.170.5                      (c) **Rs.175.50**                      (d) Rs.180

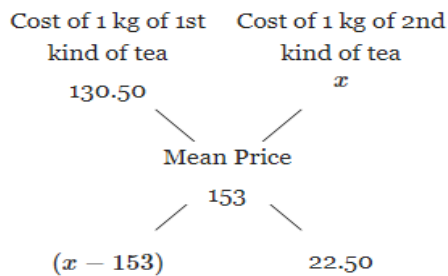
**Solution:**

Tea worth Rs. 126 per kg and Rs. 135 per kg are mixed in the ratio 1 : 1

So their average price =  $\frac{(126 + 135)}{2} = 130.5$

Hence let's consider that the mixture is formed by mixing two varieties of tea, one at Rs. 130.50 per kg and the other at Rs.  $x$  per kg in the ratio 2 : 2, i.e., 1 : 1. Now let's find out  $x$ .

By rule of alligation,



$$(x - 153) : 22.5 = 1 : 1$$

$$\Rightarrow x - 153 = 22.50$$

$$\Rightarrow x = 153 + 22.5 = 175.5$$

4. The cost of Type 1 material is Rs. 15 per kg and Type 2 material is Rs.20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then what is the price per kg of the mixed variety of material?
- (a) Rs. 19                      (b) Rs. 16                      **(c) Rs. 18**                      (d) Rs. 17

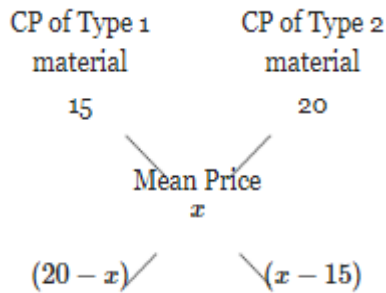
***Solution:***

Cost Price(CP) of Type 1 material is Rs. 15 per kg

Cost Price(CP) of Type 2 material is Rs. 20 per kg

Let Cost Price(CP) of resultant mixture be Rs. $x$  per kg

By rule of alligation,



$$\Rightarrow \text{Type 1 material : Type 2 material} = (20 - x) : (x - 15)$$

Given that Type 1 material : Type 2 material = 2 : 3

$$\Rightarrow (20 - x) : (x - 15) = 2 : 3$$

$$\Rightarrow \frac{(20 - x)}{(x - 15)} = \frac{2}{3}$$

$$\Rightarrow 3(20 - x) = 2(x - 15)$$

$$\Rightarrow 60 - 3x = 2x - 30$$

$$\Rightarrow 90 = 5x$$

$$\Rightarrow x = \frac{90}{5} = 18$$

$\Rightarrow$  price per kg of the mixed variety of material = Rs.18

5. Find the ratio in which rice at Rs. 7.20 a kg be mixed with rice at Rs. 5.70 a kg to produce a mixture worth Rs. 6.30 a kg.

(a) 4 : 3

(b) 2 : 3

(c) 3 : 4

(d) 3 : 2

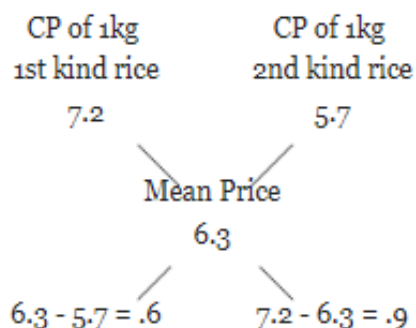
**Solution:**

CP of 1kg 1st kind rice = Rs.7.20

CP of 1kg 2nd kind rice = Rs.5.70

CP of 1kg mixed rice = Rs.6.30

By rule of alligation,



Required Ratio = .6 : .9 = 6:9 = 2:3

6. A dishonest milkman sells his milk at cost price, but he mixes it with water and thereby gains 25%. What is the percentage of water in the mixture?

(a) 25%

(b) 20%

(c) 22%

(d) 24%

**Solution:**

Let CP of 1 litre milk = Rs.1

SP of 1 litre mixture = CP of 1 litre milk = Rs.1

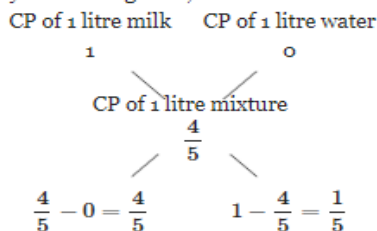
Gain = 25%

Hence CP of 1 litre mixture

$$= \frac{100}{(100 + \text{Gain}\%)} \times \text{SP}$$

$$= \frac{100}{(100 + 25)} \times 1 = \frac{100}{125} = \frac{4}{5}$$

By rule of alligation,



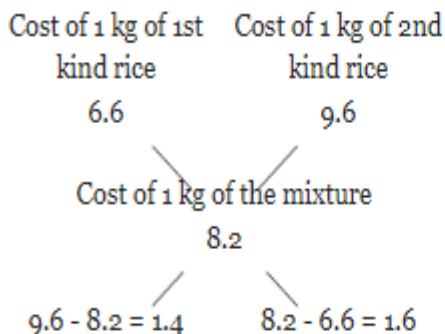
$$\Rightarrow \text{Quantity of milk : Quantity of water} = \frac{4}{5} : \frac{1}{5} = 4 : 1$$

$$\text{Hence percentage of water in the mixture} = \frac{1}{5} \times 100 = 20\%$$

7. How many kg of rice at Rs. 6.60 per kg be mixed with 56 kg of rice at Rs. 9.60 per kg to get a mixture worth Rs. 8.20 per kg?
- (a) 56 kg                      (b) 52 kg                      (c) 44 kg                      **(d) 49 kg**

**Solution:**

By rule of alligation,



Quantity of 1st kind rice : Quantity of 2nd kind rice = 1.4 : 1.6 = 7 : 8

=> Quantity of 1st kind rice : 56 = 7 : 8

=> Quantity of 1st kind rice =  $56 \times \frac{7}{8} = 49$

8. 3 litre of water is added to 11 litre of a solution containing 42% of alcohol in the water. The percentage of alcohol in the new mixture is?
- (a) 25%                      (b) 20%                      (c) 30%                      **(d) 33%**

**Solution:**

We have a 11 litre solution containing 42% of alcohol in the water.

=> Quantity of alcohol in the solution =  $\frac{11 \times 42}{100}$

Now 3 litre of water is added to the solution.

=> Total quantity of the new solution = 11 + 3 = 14

$$\begin{aligned} \text{Percentage of alcohol in the new solution} &= \frac{\frac{11 \times 42}{100}}{14} \times 100 \\ &= \frac{11 \times 3}{100} = 33\% \end{aligned}$$

9. In 40 litres of a mixture, the ratio of milk to water is 7:1. In order to make the ratio of milk to water as 3:1, the quantity of water that should be added to the mixture will be:
- (a)  $6\frac{2}{3}$  litre                      (b)  $5\frac{2}{3}$  litre                      (c)  $4\frac{1}{3}$  litre                      (d) 6 litre

**Solution:**

By rule of alligation,

Concentration of water in pure water : 1	Concentration of water in mixture : $\frac{1}{8}$
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$$\begin{array}{c}
 \diagdown \quad \diagup \\
 \text{Concentration of water in} \\
 \text{the final mixture : } \frac{1}{4} \\
 \diagup \quad \diagdown \\
 \frac{1}{4} - \frac{1}{8} = \frac{1}{8} \qquad 1 - \frac{1}{4} = \frac{3}{4}
 \end{array}$$

$$\text{Quantity of water : Quantity of mixture} = \frac{1}{8} : \frac{3}{4} = 1 : 6$$

Given that quantity of mixture = 40 litre

$$\Rightarrow \text{Quantity of water : 40} = 1 : 6$$

$$\Rightarrow \text{Quantity of water} = 40 \times \frac{1}{6} = 6\frac{2}{3} \text{ litre}$$

10. A trader has 1600 kg of sugar. He sells a part at 8% profit and the rest at 12% profit. If he gains 11% on the whole, find the quantity sold at 12%.

- (a) 1200 kg                      (b) 1400 kg                      (c) 1600 kg                      (d) 800 kg

**Solution:**

By rule of alligation,

$$\begin{array}{c}
 \% \text{ Profit by selling} \quad \% \text{ Profit by selling} \\
 \text{part1} \qquad \qquad \text{part2} \\
 8 \qquad \qquad \qquad 12 \\
 \diagdown \quad \diagup \\
 \text{Net \% Profit} \\
 11 \\
 \diagup \quad \diagdown \\
 12 - 11 = 1 \qquad 11 - 8 = 3
 \end{array}$$

$$\Rightarrow \text{Quantity of part1 : Quantity of part2} = 1 : 3$$

Given that total quantity = 1600 kg

Hence, quantity of part2 (quantity sold at 12% profit)

$$= 1600 \times \frac{3}{4} = 1200$$

11. Rs. 460 was divided among 41 boys and girls such that each boy got Rs. 12 and each girl got Rs. 8. What is the number of boys?
- (a) 33                                      (b) 30                                      (c) 36                                      (d) 28

**Solution:**

Assume that the number of boys =  $b$  and number of girls =  $g$

number of boys + number of girls = 41

$$\Rightarrow b + g = 41 \quad \dots(1)$$

Given that each boy got Rs.12 and each girl got Rs.8. Then the total amount is Rs.460

$$\Rightarrow 12b + 8g = 460 \quad \dots(2)$$

Now we need to solve these equations to get  $b$  and  $g$ .

$$(1) \times 8 \Rightarrow 8b + 8g = 8 \times 41 = 328 \quad \dots(3)$$

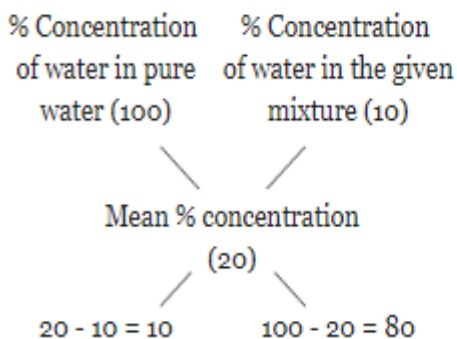
$$(2) - (3) \Rightarrow 4b = 460 - 328 = 132$$

$$\Rightarrow b = \frac{132}{4} = 33$$

12. How many litres of water must be added to 16 liters of milk and water containing 10% water to make it 20% water?
- (a) 4 litre                                      (b) 2 litre                                      (c) 1 litre                                      (d) 3 litre

**Solution:**

By rule of alligation,



$$\Rightarrow \text{Quantity of water : Quantity of the mixture} = 10 : 80 = 1 : 8$$

Here quantity of the mixture = 16 litres

$$\Rightarrow \text{Quantity of water : } 16 = 1 : 8$$

$$\Rightarrow \text{Quantity of water} = 16 \times \frac{1}{8} = 2 \text{ litre}$$

13. In what ratio must water be mixed with milk to gain  $16\frac{2}{3}\%$  on selling the mixture at cost price?

(a) 6 : 1

(b) 1 : 6

(c) 1 : 4

(d) 4 : 1

**Solution:**

Let CP of 1 litre milk = Rs.1

SP of 1 litre mixture = CP of 1 litre milk = Rs.1

$$\text{Gain} = 16\frac{2}{3}\% = \frac{50}{3}\%$$

$$\begin{aligned}\text{CP of 1 litre mixture} &= \frac{100}{(100 + \text{Gain}\%)} \times \text{SP} \\ &= \frac{100}{\left(100 + \frac{50}{3}\right)} \times 1 = \frac{100}{\left(\frac{350}{3}\right)} \\ &= \frac{300}{350} = \frac{6}{7}\end{aligned}$$

By rule of alligation,

CP of 1 litre water	CP of 1 litre milk
0	1
$\swarrow \quad \searrow$ CP of 1 litre mixture $\frac{6}{7}$ $\swarrow \quad \searrow$	
$1 - \frac{6}{7} = \frac{1}{7}$	$\frac{6}{7} - 0 = \frac{6}{7}$

$$\text{Quantity of water : Quantity of milk} = \frac{1}{7} : \frac{6}{7} = 1 : 6$$

14. A merchant has 1000 kg of sugar part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is?

(a) 300

(b) 400

(c) 600

(d) 500

**Solution:**

By rule of alligation,

Profit% by selling	Profit% by selling
1st part	2nd part
8	18
$\swarrow \quad \searrow$ Net % profit 14 $\swarrow \quad \searrow$	
$18 - 14 = 4$	$14 - 8 = 6$

$$\Rightarrow \text{Quantity of part}_1 : \text{Quantity of part}_2 = 4 : 6 = 2 : 3$$

Total quantity is given as 1000 kg. So quantity of part<sub>2</sub> (quantity sold at 18% profit)

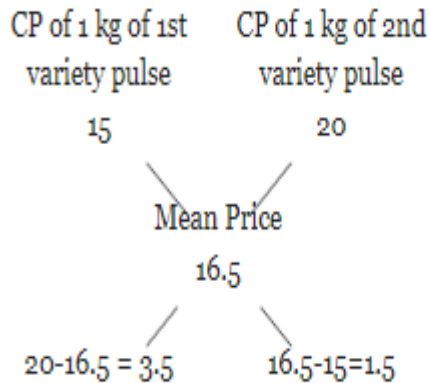
$$= 1000 \times \frac{3}{5} = 600 \text{ kg}$$



15. In what ratio must a grocer mix two varieties of pulses costing Rs.15 and Rs. 20 per kg respectively to obtain a mixture worth Rs.16.50 per kg?
- (a) 1 : 2                                      (b) 2 : 1                                      (c) 3 : 7                                      (d) 7 : 3

**Solution:**

By rule of alligation,



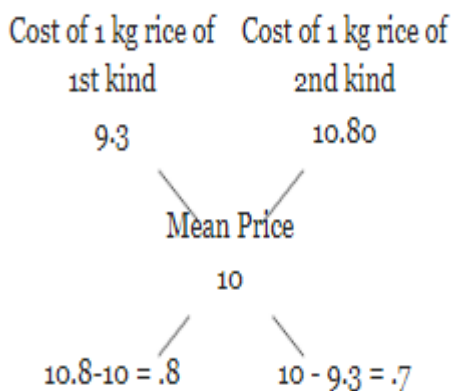
Required Ratio =  $3.5 : 1.5 = 35 : 15 = 7 : 3$

## HOME WORK

16. In what ratio should rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs.10 per kg?
- (a) 7 : 8                                      (b) 8 : 7                                      (c) 6 : 7                                      (d) 7 : 6

**Solution:**

By rule of alligation,



Required ratio =  $.8 : .7 = 8 : 7$ .

17. 8 litres are drawn from a cask full of wine and is then filled with water. This operation is performed three more times. The ratio of the quantity of wine now left in cask to that of the water is 16 : 65. How much wine did the cask originally hold?

- (a) 30 litres                      (b) 26 litres                      **(c) 24 litres**                      (d) 32 litres

**Solution:**

Let initial quantity of wine =  $x$  litre

After a total of 4 operations, quantity of wine

$$= x \left(1 - \frac{y}{x}\right)^n = x \left(1 - \frac{8}{x}\right)^4$$

Given that after a total of 4 operations, the ratio of the quantity of wine left in cask to that of water = 16 : 65

$$\Rightarrow \frac{x \left(1 - \frac{8}{x}\right)^4}{x} = \frac{16}{81}$$

$$\Rightarrow \left(1 - \frac{8}{x}\right)^4 = \left(\frac{2}{3}\right)^4$$

$$\Rightarrow \left(1 - \frac{8}{x}\right) = \frac{2}{3}$$

$$\Rightarrow \left(\frac{x-8}{x}\right) = \frac{2}{3}$$

$$\Rightarrow 3x - 24 = 2x$$

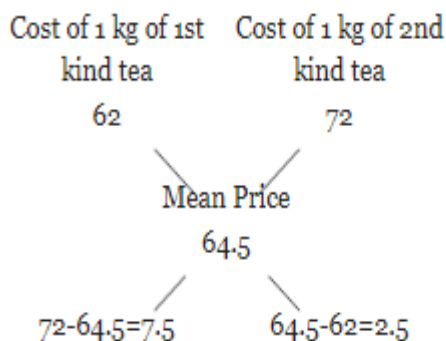
$$\Rightarrow x = 24$$

18. In what ratio must tea at Rs. 62 per kg be mixed with tea at Rs. 72 per kg so that the mixture must be worth Rs. 64.50 per kg?

- (a) 1 : 2                      (b) 2 : 1                      **(c) 3 : 1**                      (d) 1 : 3

**Solution:**

By rule of alligation,



Required Ratio = 7.5 : 2.5 = 3 : 1

19. John bought 20 kg of wheat at the rate of Rs.8.50 per kg and 35 kg at the rate of Rs.8.75 per kg. He mixed the two. Approximately at what price per kg should he sell the mixture to make 40% profit at the cost price?

(a) Rs.12                                      (b) Rs.8                                      (c) Rs.16                                      (d) Rs.20

**Solution:**

$$\begin{aligned} \text{CP} &= 20 \times 8.5 + 35 \times 8.75 \\ &= 170 + 306.25 = 476.25 \end{aligned}$$

$$\text{Profit} = 40\%$$

$$\begin{aligned} \text{SP} &= \frac{(100 + \text{Profit}\%)}{100} \times \text{CP} \\ &= \frac{(100 + 40)}{100} \times 476.25 \\ &= \frac{140}{100} \times 476.25 \\ &= \frac{140}{4} \times 19.05 = 35 \times 19.05 \end{aligned}$$

$$\text{Total quantity} = 20 + 35 = 55 \text{ kg}$$

$$\begin{aligned} \text{SP per kg} &= \frac{35 \times 19.05}{55} = \frac{7 \times 19.05}{11} \\ &\approx \frac{7 \times 19}{11} \approx \frac{133}{11} \approx 12 \end{aligned}$$

20. Some amount out of Rs.7000 was lent at 6% per annum and the remaining was lent at 4% per annum. If the total simple interest from both the fractions in 5 years was Rs. 1600, the sum lent at 6% per annum was

(a) Rs. 2000                                      (b) Rs. 2200                                      (c) Rs. 2400                                      (d) Rs. 1800

**Solution:**

Total simple interest received , I = Rs.1600

Principal , p = 7000

period, n = 5 years

Rate of interest, r = ?

$$\text{Simple Interest, } I = \frac{pnr}{100}$$

$$\Rightarrow 1600 = \frac{7000 \times 5 \times r}{100}$$

$$\Rightarrow r = \frac{1600 \times 100}{7000 \times 5} = \frac{160}{35} = \frac{32}{7} \%$$

By rule of alligation,

Rate of interest %      Rate of interest %  
from part1                  from part2

6

4

Net rate of interest %

$$\frac{32}{7}$$

$$\frac{32}{7} - 4 = \frac{4}{7}$$

$$6 - \frac{32}{7} = \frac{10}{7}$$

$$\Rightarrow \text{Part1 : part2} = \frac{4}{7} : \frac{10}{7} = 4 : 10 = 2 : 5$$

Given that total amount is Rs.7000. Therefore, the amount lent at 6% per annum (part1 amount)

$$= 7000 \times \frac{2}{7} = \text{Rs. 2000}$$