APTITUDE MASTERY SERIES

MODULE 8 – PERMUTATION AND COMBINATION

1. In how many ways five chocolates can be chosen from an unlimited number of Dairy Milk, Five-Star and Perk chocolates?

(a) 65 **(b) 243** (c) 21 (d) 3125

Solution:

For each selection there are 3 ways of doing it. Thus, there are a total of 3 x 3 x 3 x 3 x 3 = 243. Hence, option (b) is correct.

2. In how many ways is it possible to choose a white square and a black square on a chess board so that the squares must not lie in the same row or column?

(a) 56 (b) 896 (c) 60 **(d) 768**

Solution:

The white square can be selected in 32 ways and once the white square is selected 8 black squares become ineligible for selection. Hence, the black square can be selected in 24 ways.

32 x 24 = 768. Option (d) is correct.

Alternatively

The white square can be chosen in 32 ways.

If we remove the row and the column which contains the chosen white square, we will be left with 7 Rows & 7 Columns containing a total of 49 squares (24 black and 25 white). We would have removed 15 squares (7 white and 8 black)

*Required ways = **32*24 = 768***

3. Out of 8 consonants and 5 vowels, how many words can be made, each containing 4 consonants and 3 vowels?

(a) 700 (b) 504000 **(c) 3528000** (d) 7056000

Solution:

3 vowels can be selected in = $^5C_3 = 10$ ways

4 consonants can be selected from 8 in $^8C_4 = 70$ ways

Required number of words = $^5C_3 \times {}^8C_4 \times 7! = 70 \times 10 \times 7! = 3528000$

4. In the board meeting of a FMCG Company, everybody present in the meeting shakes hand with everybody else. If the total number of handshakes is 78, the number of members who attended the board meeting is:

(a) 7 (b) 9 (c) 11 **(d) 13**

Solution:

Let 'n' people attended the board meeting, then total number of handshakes = $^nC_2 = 78$

By solving or by checking the options we get n = 13.

5. How many integers, greater than 999 but not greater than 4000, can be formed with the digits 0, 1, 2, 3 and 4 if repetition of digits is allowed?

(a) 376 (b) 375 (c) 500 (d) 499

Solution:

The integers can be identified as follows:

4-digit numbers starting with 1: 5 x 5 x 5 = 125

4-digit numbers starting with 2: 5 x 5 x 5 = 125

4-digit numbers starting with 3: 5 x 5 x 5 = 125 + the number 4000.

Hence, the answer would be 125 + 125 + 125 + 1 = 376. Option (a) is correct.

6. A new flag is to be designed with six vertical stripes using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent stripes have the same colour is:

(a) 12 x 81 (b) 16 x 192

(c) 20 x 125 (d) 24 x 216

Solution:

For the first vertical stripe we can use any of 4 colours, for the second we would have only 3 options, same for the third to the sixth stripe. Hence, the required answer would be 4 x 3 x 3 x 3 x 3 x 3 = 12 x 81. Option (a) is correct.

7. All six letters of the name SACHIN are arranged to form different words without repeating any letter in any one word. The words so formed are then arranged as in a dictionary. What will be the position of the word SACHIN in that sequence?

(a) 436 (b) 590 **(c) 601** (d) 751

Solution:

All words starting with A, C, H, I and N would be before words starting with S. So we would have 5! Words (=120 words) each

starting with A, C, H, I and N. Thus, a total of 600 words would get completed before we start off with S. SACHIN would be the first word starting with S, because A, C, H, I, N in that order is the correct alphabetical sequence. Hence, SACHIN would be the 601^{st} word. Hence, option (c) is correct.

8. In a question paper, there are four multiple-choice questions. Each question has five choices with only one choice as the correct answer. What is the total number of ways in which a candidate will not get all the four answers correct?

(a) 19 **(b) 624** (c) 1024 (d) 120

Solution:

5^4 would be the total number of ways in which the questions can be answered. Out of these there would be only 1 way of getting all 4 correct. Thus, there would be 624 ways of not getting all answers correct.

9. If we have to make 7 boys sit with 7 girls around a round table, then the number of different relative arrangements of boys and girls that we can make so that there are no two boys nor two girls sitting next to each other is:

(a) $2 \times (7!)^2$ (b) $7! \times 7!$

(c) 6! x 7! (d) None of these

Solution:

First step – arrange 7 boys around the table according to the circular permutations rule. i.e. in 6! Ways.

Second step – now we have 7 places and have to arrange 7 girls on these places. This can be done in ${}^7P_7$ ways. Hence, the total number of ways = $6! \times 7!$

10. In how many ways a cricketer can score 200 runs with fours and sixes only?

(a) 13 **(b) 17** (c) 19 (d) 16

Solution:

200 runs can be scored by scoring only fours or through a combination of fours and sixes. Possibilities are 50 x 4, 47 x 4 + 2 x 6, 44 x 4 + 4 x 6, … A total of 17 ways.

11. Find the sum of all 5 digit numbers formed by the digits 1, 3, 5, 7, 9 when no digit is being repeated.

(a) 4444400 (b) 8888800

(c) 13333200 **(d) 6666600**

Solution:

Five numbers can be arranged in 5! = 120 ways

In which, the last digit can be 1, 3, 5, 7, 9

_ _ _ _ 1

_ _ _ _ 3

_ _ _ _ 5

_ _ _ _ 7

_ _ _ _9

Sum of all these 1 + 3 + 5 + 7 + 9 = 25

In 120 ways we have 24 sets which gives the sum as 25. Hence, 24 x 25 = 600.

So the unit place will be ‘0’ and 60 is carried to the ten’s place.

In ten’s place again we get a sum of 600 + 60 (carried from unit’s place) = 660. Hence, in ten’s place we will have ‘0’ and 66 is carried to the hundred’s place.

In hundred’s place again we get a sum of 600 + 66 (carried from ten’s place) = 666. Hence, in ten’s place we will have ‘6’ and 66 is carried to the thousand’s place and it continues.

Hence, the answer is option (d).

12. A team of 8 students go on an excursion in two cars of which one can seat 5 and the other only 4. In how many ways can they travel?

(a) 9 (b) 26 **(c) 126** (d) 3920

Solution:

There are 8 students and the maximum capacity of the cars together is 9. We may divide the 8 students as follows:

(i) 5 students in the first car and 3 in the second

(ii) 4 students in the first car and 4 in the second

Hence, in (i), 8 students are divided into groups of 5 and 3 in ${}^8C_3$ ways. Similarly, in case (ii), 8 students are divided into two groups of 4 and 4 in ${}^8C_4$ ways.

Therefore, the total number of ways in which 8 students can travel is ${}^8C_3 + {}^8C_4 = 56 + 70 = 126$.

13. How many numbers using all digits can be formed from 0, 1, 2, 5, 6, 7, 8 but not starting with 0?

(a) 7^7 (b) 7! (c) 6×7^6 **(d) 6 x 6!**

Solution:

At 1st position only 6 digits can come as 0 is not included.

At 2nd position 6 digits can come as repetition is not allowed.

At 3rd position 5 digits can come. This means for last 6 digits the possible number that can be formed is 6!

So, required number of possible numbers that can be formed will be 6 x 6!

14. The number of ways in which 8 persons can be seated at a round table if 2 particular persons must always sit together is?

(a) 1440 (b) 720 (c) 288 (d) 2880

Solution:

Method 1

8 persons can be seated in 7! Ways. But, when 2 persons are sitting together they will be considered as a single unit and number of ways of their arrangement will be 6! And their independent arrangement will also be taken into account.

Hence, the number of ways are 2 x 6! = 1440

Method 2

(n-1)! is the number of ways n people can be made to sit at a round table. Here 2 people need to sit together. So, we consider these two people to be a unit and the ways they can be arranged together with the group becomes 6!. Now, the two people considered as a unit can be arranged among themselves in 2! Ways. Hence 6! x 2! = 1440 is the required answer.

15. How many ways can 840 be written as the product of two numbers?

(a) 32 **(b) 16** (c) 6 (d) 18

Solution:

Factor of 840 = 2 x 2 x 2 x 3 x 5 x 7;

Total number of method to select the combination = (3 + 1) x (1 + 1) x (1 + 1) x (1 + 1) / 2! = 16

(here we should make attention in one step that a x b and b x a will be considered as 1 case)

HOME WORK

16. In how many ways can the letters of the English alphabet be arranged so that there are seven letters between the letters A and B?

(a) 31! x 2! (b) ${}^{24}P_7$ x 18! x 2 **(c) 36 x 24!** (d) None of these

Solution:

A and B can occupy the first and the ninth places, the second and the tenth places, the third and the eleventh place and so on… This can be done in 18 ways.

A and B can be arranged in 2 ways.

All the other 24 alphabets can be arranged in 24! Ways.

Hence the required answer = 2 x 18 x 24!

17. A driving license number has 6 digits (between 1 to 9). The first two digits are 12 in that order, the third digit is bigger than 6, the fourth one can be divided by 3 and the fifth digit is 3 times bigger than sixth one. How many license numbers can be made using this arrangement?

(a) 27 (b) 36 (c) 72 (d) 144

Solution:

First two digits are fixed, third digit can have 3 values 7, 8, 9.

4^{th} digit can have 3 values 3, 6, 9.

5^{th} and 6^{th} digit can have 3 combinations like 31, 62, 93.

So total combinations are 3 x 3 x 3 = 27

18. Each of the 11 letters A, H, I, M, O, T, U, V, W, X and Z appear same when looked at in a mirror. They are called symmetric letters. Other letters in the alphabet are asymmetric letters. How many three letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?

(a) 12870 (b) 12000

(c) 13730 (d) 15620

Solution:

There are a total of 11 symmetric letters, 15 asymmetric letters.

Total number of words possible (no repetition): 26 x 25 x 24 = 650 x 24 = 15600

Total number of words possible with only asymmetric letters: 15 x 14 x 13 = 210 x 13 = 2730

Total number of words with at least one symmetric letter: 15600 – 2730 = 12870

19. If all letters of the word "CHCJL" be arranged in an English dictionary, what will be the 50^{th} word?

(a) HCCLJ (b) LCCHJ

(c) LCCJH (d) JHCLC

Solution:

In the English Dictionary, the ordering of the words would be in alphabetical order. Thus, words starting with C would be followed by words starting with H, followed by words starting with J and finally words

starting with L. Words starting with C = 4! = 24; Words starting with H = 4! / 2! = 12 words. Words starting with J = 4! / 2! = 12 words. This gives us a total of 48 words. The 49^{th} and the 50^{th} words would start with L. The 49^{th} word would be the first word starting with L (=LCCHJ) and the 50^{th} word would be the 2^{nd} word starting with L – which would be LCCJH. Option (c) is correct.

20. In how many ways can a lock be opened if it has three-digit number lock and;

(i) The last digit is 9

(ii) Sum of the first two digits is less than or equal to the last digit (numbers are from 0 – 9)

(a) 45 **(b) 55** (c) 90 (d) 110

Solution:

10 ways when first digit is 0

9 ways when first digit is 1

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1 way when first digit is 9. Total = 10 + 9 + …… + 1 = 55.