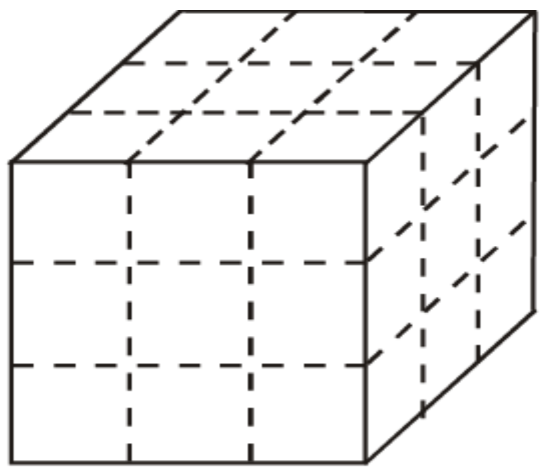
LOGICAL REASONING

MODULE 13 – CUBES

(Directions 1 – 2): A wooden cube is painted Blue on all the four adjoining sides and Green on two opposite sides i.e. top and bottom. It is then cut at equal distances at right angles four times vertically (top to bottom) and two times horizontally (along the sides) as shown in the figure where the dotted lines represent the cuts made. Study the diagram and answer the following questions:



1. How many cubes are formed in all?

(a) 16 (b) 24 **(c) 27** (d) 32

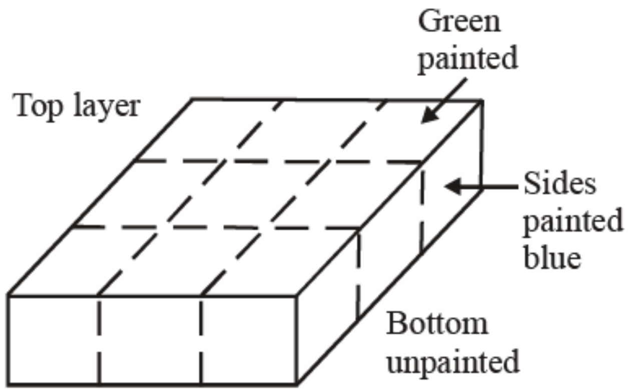
2. How many cubes will have one face painted only in Green?

(a) 1 **(b) 2** (c) 3 (d) 4

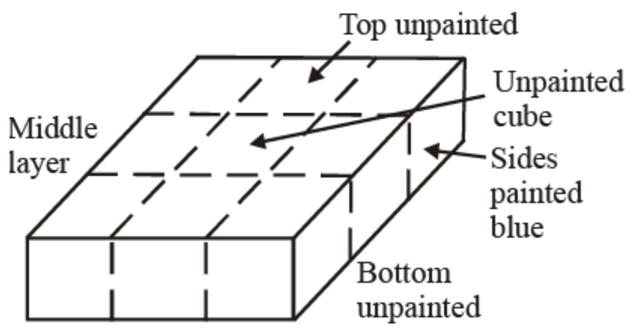
Solution:

The figure can be analyzed by dividing it into three horizontal layers:

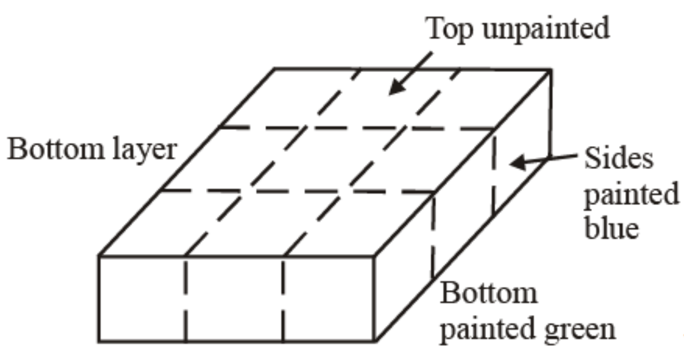
In the top layer, the central cube has only one face painted green; the four cubes at the corner have three faces painted (one face green and two faces blue). The remaining four cubes have two faces painted (one green and one blue).



In the middle layer, the central cube has no face painted, four cubes at the corner have two faces painted blue and the remaining four cubes have one face painted blue.



In the bottom layer, the central cube has one face painted green and four cubes at the corners have three faces painted (two blue and one green). The remaining four cubes have two faces painted (one blue and one green)



Answers:

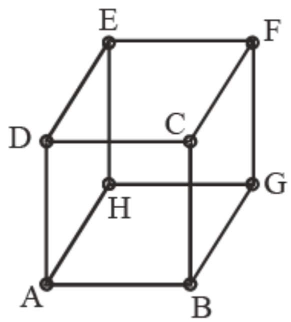
1. There are 9 cubes in each of the three layers. Thus, there are 27 cubes in all.

2. There is one (central) cube in the top layer and one (central) cube in the bottom layer which have one face painted only in Green.

3. A cube is painted red on two adjacent faces and on one opposite face, yellow on two opposite faces and green on the remaining face. It is then cut into 64 equal cubes. How many cubes have only one red coloured face?

(a) 16 **(b) 12** (c) 8 (d) 4

Solution:



Let faces ABCD, ABGH and CDEF are painted with red colour.

Faces BCFG and ADEH are painted with yellow and EFGH is painted with green colour.

Clearly the cubes which have only one red coloured face and all other faces uncoloured are the four central cubes at each of the three faces ABCD, ABGH and CDEF. Thus, there are 4 x 3 = 12 such cubes.

(Directions 4 – 5): A cube of side 10 cm is coloured red with a 2 cm wide green strip along all the sides on all the faces. The cube is cut into 125 smaller cubes of equal size. Answer the following questions based on this statement:

4. How many cubes have at least one face coloured?

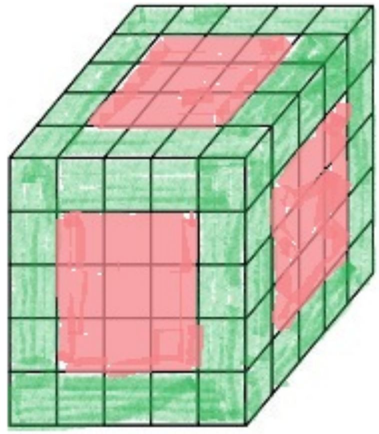
(a) 27 (b) 36 **(c) 98** (d) 127

5. How many cubes have at least two green faces each?

(a) 44 (b) 32 (c) 28 (d) 36

Solution:

Clearly, upon colouring the cube as stated and then cutting it into 125 smaller cubes of equal size we get a stack of cubes as shown in the following figure.



4. Let us calculate the number of cubes with no painting. By formula, $(n - 2)^3 = (5 - 2)^3 = 27$

Therefore, there are 125 – 27 = 98 cubes having at least one face coloured.

5. From the total cubes, Let us subtract the cubes with red painting and cubes with no painting.

125 - (9 x 6) - 27 = 44.

(Direction 6 – 9): One hundred and twenty-five cubes of the same size are arranged in the form of a cube on a table. Then a column of five cubes is removed from each of the four corners. All the exposed faces of the rest of the solid (except the face touching the table) are coloured red. Now, answer these questions based on the above statement:

6. How many small cubes are there in the solid after the removal of the columns?

(a) 125 **(b) 105** (c) 115 (d) 95

7. How many cubes do not have any coloured face?

(a) 24 (b) 28 (c) 32 **(d) 36**

8. How many cubes have only one red face each?

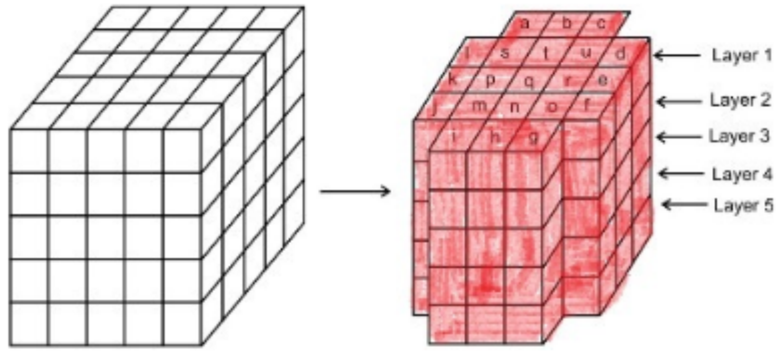
(a) 24 (b) 27 **(c) 25** (d) 32

9. How many cubes have two coloured faces each?

(a) 32 **(b) 36** (c) 40 (d) 44

Solution:

The following figure shows the arrangement of 125 cubes to form a single cube followed by the removal of 4 columns of five cubes each.



When the corner columns of the original cube are removed, and the resulting block is coloured on all the exposed faces (except the base) then we get the right hand side diagram. We labelled the various columns from a to u as shown in the figure

(6) Since out of 125 total number of cubes, we removed 4 columns of 5 cubes each, the remaining number of cubes = 125 - (4 x 5) = 125 - 20 = 105.

(7) Cubes with no painting lie in the middle. So cubes which are below the cubes named as s, t, u, p, q, r, m, n, o got no painting. Since there are 4 rows below the top layer, total cubes with no painting are (9 x 4) = 36.

(8) There are 9 cubes namaed as m, n, o, p, q, r, s, t and u in layer 1, and 4 cubes (in columns b, e, h and k) in each of the layers 2, 3, 4 and 5 got one red face. Thus, there are 9 + (4 x 4) = 25 cubes.

(9) The columns (a, c, d, f, g, i, j, l) each got 4 cubes in the layers 2, 3, 4, 5. Also in the layer 1, h, k, b, e cubes got 2 faces coloured. So total cubes are 32 + 4 = 36.

(Directions 10 – 11): Three adjacent faces of a cube are coloured blue. The cube is then cut (once horizontally and once vertically) to form four cuboids of equal size, each of these cuboids is coloured pink on all the uncoloured faces and is then cut (as before) into four cuboids of equal size.

10. How many cuboids have three faces coloured blue?

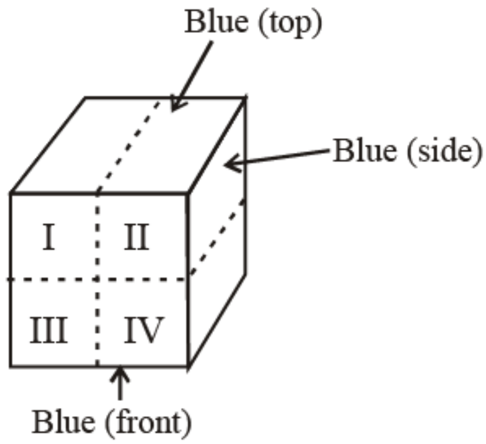
(a) 4 (b) 2 **(c) 1** (d) 0

11. How many cuboids have three faces coloured pink?

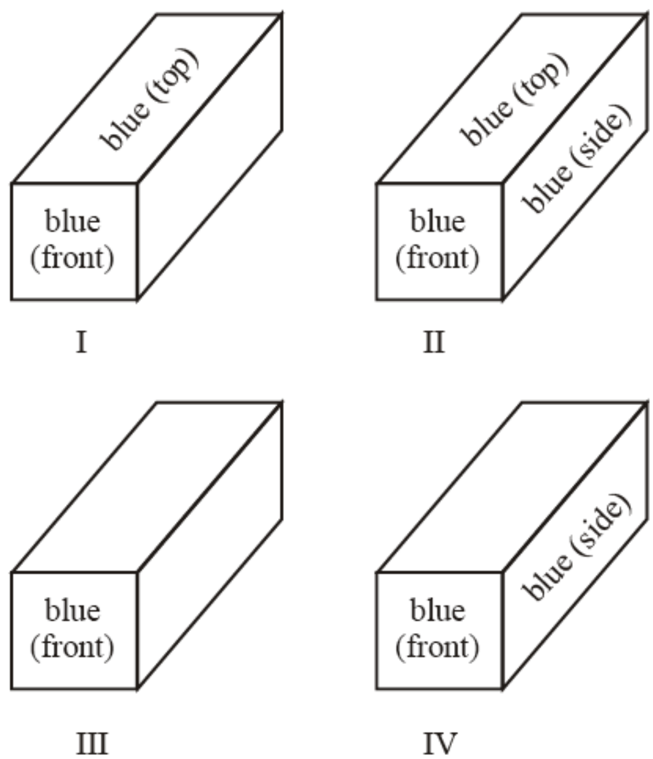
(a) 9 (b) 7 (c) 5 (d) 3

Solution:

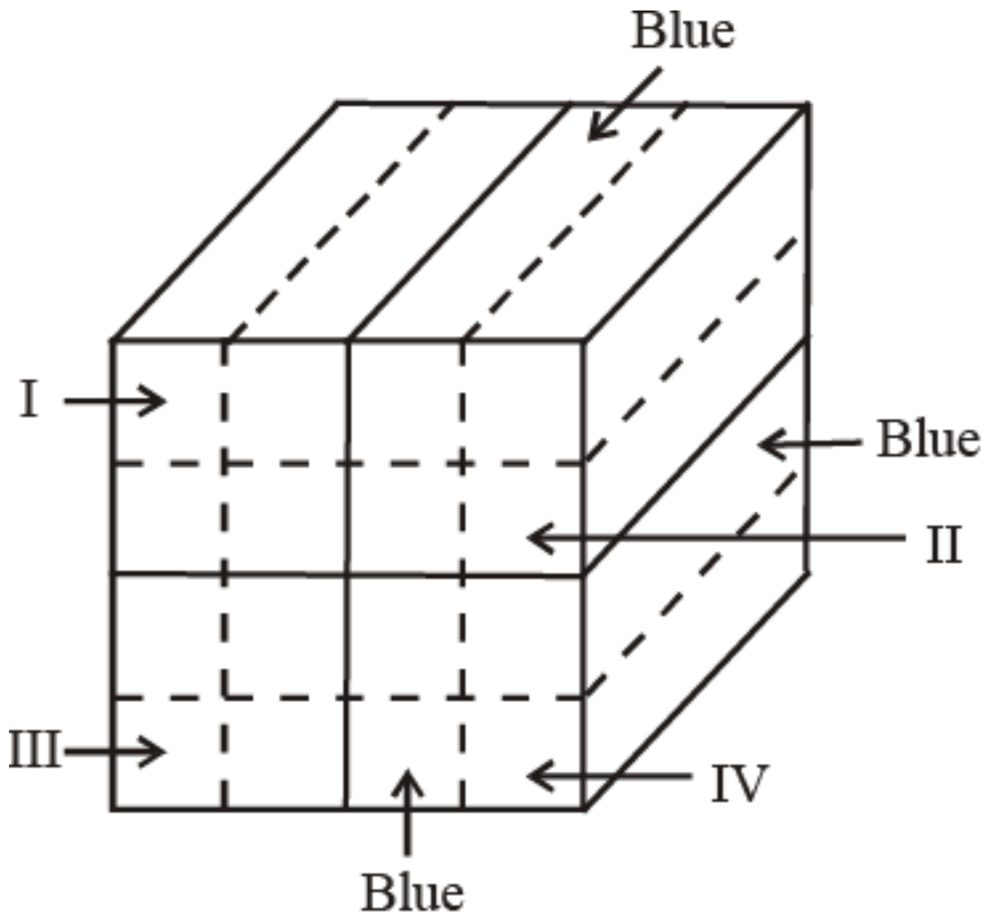
The adjoining figure shows the cube coloured and cut into four cuboids as stated in the question.



Four cuboids are obtained as shown below:



Now, all uncoloured faces of each cuboid are coloured with pink and then again cut each cuboid into four cuboids.



In set I and IV: 2 cuboids have 2 faces blue, 2 faces pink and 2 faces uncoloured each. 2 cuboids have 1 face blue, 3 faces pink and 2 faces uncoloured each.

In set II: 2 cuboids have 2 faces blue, 2 faces pink and 2 faces uncoloured each.

1 cuboid have 3 faces blue, 1 face pink and 2 faces uncoloured each.

1 cuboid has 1 face blue, 3 faces pink and 2 faces uncoloured each.

In set III: All the four cuboids have 1 face blue, 3 faces pink and 2 faces uncoloured each.

Answers:

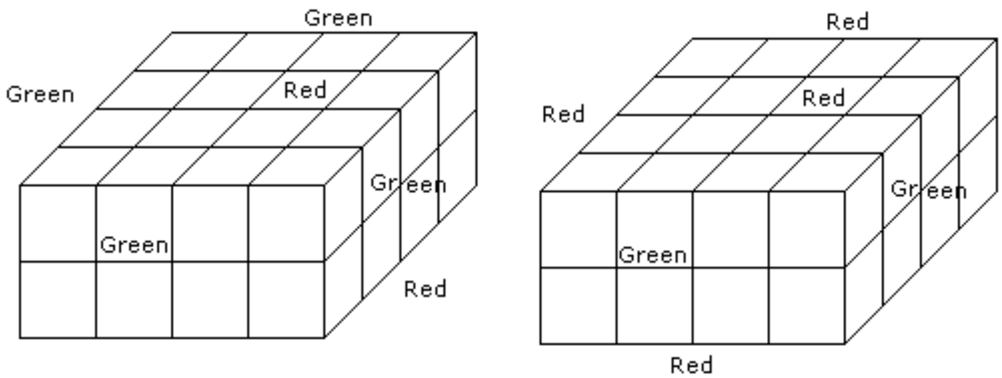
10. There is only one cuboid having three faces blue. This cuboid lies in set II.

11. There are 2 cuboids in set I, 1 cuboid in set II, 4 cuboids in set III and 2 cuboids in set IV having 3 faces pink each. Thus, there are 9 such cuboids.

12. A cube is cut in two equal parts along a plane parallel to one of its faces. One piece is then coloured red on the two larger faces and green on the remaining, while the other is coloured green on two smaller adjacent faces and red on the remaining. Each is then cut into 32 cubes of same size and mixed up. How many cubes have only one coloured face each?

(a) 32 (b) 8 **(c) 16** (d) 0

Solution:



(I) (II)

8 from (I) and 8 from (II)

Therefore 8 from each.

(Direction 13 – 15): A cube of each side 4 cm has been painted black, red and green on pairs of opposite faces. It is then cut into small cubes of each side 1 cm.

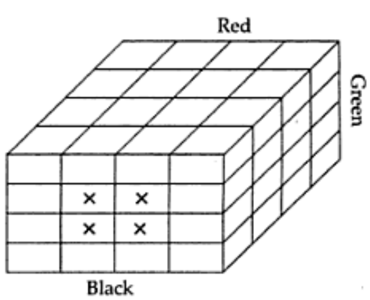
13. How many small cubes will be only red painted?

(a) 8 (b) 12 (c) 16 (d) 20

Solution:

No. of small cubes having only red paint

= 4 + 4 = 8



14. How many small cubes will have only two faces painted?

(a) 12 **(b) 24** (c) 32 (d) 36

Solution:

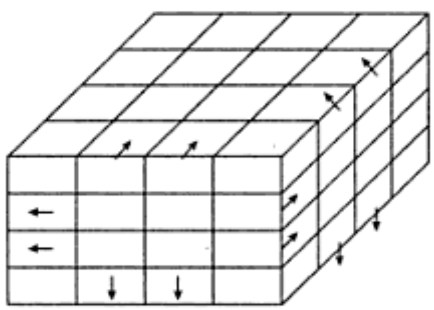
No. of small cubes having only two faces painted- From the figure it is clear that to each edge of the big cube 4 small cubes are connected and two out of them are situated at the corners of the big cube which have all the three faces painted. Thus, to each edge two small cubes are left which have two faces painted. As the total no. of edges in a cube are 12, hence the no. of small cubes with two faces coloured = 12×2 = 24.

(or)

No. of small cubes with two face coloured

= $(x - 2)$ x No. of edges

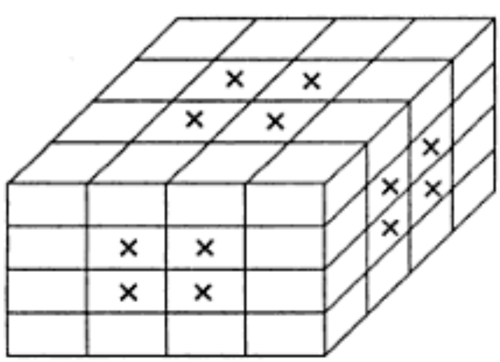
Where $x = \frac{side\ of\ big\ cube}{side\ of\ small\ cube}$



15. How many small cubes will have only one faces painted?

(a) 12 **(b) 24** (c) 32 (d) 36

Solution:



No. of small cubes having only one face painted- The cubes which are painted on one face only are the cubes at the centre of each face of the big cube. Since there are 6 faces in the big cube and each of the face of big cube there will be four such small cubes. Hence, in all there will be 6×4 i.e., 24 such small cubes.

or $(x-2)^2 \times 6$

HOME WORK

(Directions 16 – 19): A solid cube is painted purple on two adjacent sides and white on the sides opposite to the purple sides and black on the remaining sides. It is then cut into 64 cubes of equal size.

16. How many cubes have only one side coloured?

(a) 16 **(b) 24** (c) 28 (d) 32

17. How many cubes have at least one side as purple?

(a) 16 (b) 24 **(c) 28** (d) 32

18. How many cubes are there with one side black and the adjacent side either purple or white and painted on two sides only?

(a) 8 **(b) 16** (c) 24 (d) 32

19. How many cubes are there which are purple on one side and white on the opposite side?

(a) 0 (b) 4 (c) 8 (d) 16

Solution:

16. Cubes with one side coloured = $6\,(n-2)^2 = 24$ cubes

17. As there are two faces painted with purple, the 32 (2 faces of 16 cubes each) cubes on these faces will have purple colour. But as adjacent faces are painted in purple, 4 cubes would have been double counted. Removing them, the number of cubes with purple colour will be 28.

18. There are 8 edges (in the big cube), where black is painted on one side and purple or white on the adjacent side. Along each edge, there are 2 cubes with the required colour combination, totaling to 16 cubes.

19. Of the smaller cubes, no cube will have paint on opposite sides.

20. How many cuts should be made to get 125 small cubes out of a cube?

(a) 12 (b) 9 (c) 16 (d) 15

Solution:

Pieces: L x B x H = 5 x 5 x 5 = 125 small cubes

Cuts: L x B x H = 4 + 4 + 4 = 12 cuts

(Since, we know that ‘n’ cuts will give (n+1) number of pieces)