(2)

D (a) Initially → $\frac{1}{3}$ chance present one has money

$\frac{2}{3}$ chance it's \$ in Present 2 or 3

(b) Host opens present 2 and it's empty

If present 1 had the money ($\frac{1}{3}$ chance) switching loses

If present 3 has the money ($\frac{2}{3}$ chance) switching wins

$$E(\text{Switch}) = \frac{1}{3} \times 0 + \frac{2}{3} \times 1000$$

$$= 666.6 \text{ \$}$$

(4) Let $X = n$ with Probability $P(X=n) = \frac{6}{\pi^2 n^2}$.

~~$E(X) = \sum_{n=1}^{\infty} n \frac{6}{\pi^2 n^2} = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} = \infty$ Converging series.~~

$$P(X=n) = \frac{c}{n^p} \quad \text{if } p = 2.5$$

$$E(X) = \sum n \frac{c}{n^{2.5}} = c \sum \frac{1}{n^{1.5}} \text{ Converges}$$

$$E(X^2) = \sum n^2 \frac{c}{n^{2.5}} = c \sum \frac{1}{n^{0.5}} \text{ diverges}$$

PAGE NO.
DATE: / /

(b) $f(x) = \frac{c}{x^p}$ for $x \geq 1$ $p = 2.5$

$E(X) = c\int_1^\infty x^{-1.5}\, dx \to$ Converges

$E(X^2) = \int_1^\infty x^2 \frac{c}{x^{2.5}}\, dx = c\int_1^\infty x^{-0.5}\, dx \to$ diverge.

(c) $E(X) = 1$ but $E(e^{-X}) < \frac{1}{3}$

~~let $P(X=0) = 0.01$ $P(X=100) = 0.99$~~

~~$E(X) = 0.01\times 0 + 0.99\times 100 = 99 > 1$~~

$P(X = \ln 100) = p$ $P(X=0) = 1-p$ so

$E(X) = p\ln 100 + (1-p)\cdot 0 = 1$ $p = \frac{1}{\ln 100}$

$E(e^{-X}) = pe^{-\ln 100} + (1-p)\cdot 1$

$= p\times\frac{1}{100} + (1-p) = 1 - \frac{99}{100}p$

$\approx 0.27 < \frac{1}{3}$

PAGE NO.
DATE: / /

(6) $P\left(|X-Y| < \frac{d}{3}\right) = \frac{1}{d^2}\int_0^d \int_{x-\frac{d}{3}}^{x+\frac{d}{3}} \mathbb{1}_{[0,d]}(y)\, dy\, dx$

area b/w $y = x \pm \frac{d}{3}$ inside $[0,d]^2$

$P = 1 - \frac{2}{3} = \frac{1}{3}$

(8) Given $A_1 \ldots A_n$ independent.

$P\left(\bigcap_{i=1}^{n} A_i^c\right) = \prod_{i=1}^{n}(1-P(A_i)) \leq e^{-\sum P(A_i)}$

$1 - x \leq e^{-x} \Rightarrow \prod(1-x_i) \leq \prod e^{-x_i}$

$= e^{-\sum x_i}$

(10) let $X \geq 0$ with CDF $F(x)$

$E(X) = \int_0^\infty P(X > x)\, dx = \int_0^\infty (1-F(x))\, dx$

let $X(\omega) = \int_0^{X(\omega)} 1\, dx$

$X(\omega) = \int_0^\infty \mathbb{1}_{[0,X(\omega)]}(x)\, dx$

$E(X) = \int_\Omega \int_0^\infty \mathbb{1}_{[0,X(\omega)]}(x)\, dx\, dP(\omega) = \int_0^\infty \int_\Omega \mathbb{1}_{[0,X(\omega)]}(x)\, dP(\omega)\, dx$

$E(X) = \int_0^\infty P(X > x) = \int_0^\infty (1-F(x))\, dx$