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1) (a) $\{1, 2, 3, 4\}$ ← states

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0.5 & 0.5 & 0 & 0 \\ 2 & 0.25 & 0.75 & 0 & 0 \\ 3 & 0 & 0 & 0.25 & 0.75 \\ 4 & 0 & 0 & 0.75 & 0.25 \end{array}$$

(b) Recurrent state = [1, 2, 3, 4]
Transient → None

(c) for stationary Distribution

$$\pi Q = \pi \quad \& \quad \pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

first state Dist (supp a $\{1,2\}$)

$$\pi_1 = 0.5\pi_1 + 0.25\pi_2$$
$$\pi_2 = 0.5\pi_1 + 0.75\pi_2$$

$$\pi_1 = 0.5\pi_2 \qquad \pi_1 + \pi_2 = 1 \qquad \pi_1 = \frac{1}{3} \quad \pi_2 = \frac{2}{3}$$

$$\pi^{(1)} = \left[\frac{1}{3}, \frac{2}{3}, 0, 0\right]$$

Second stationary distribution (support on $\{3, 4\}$)

$$\pi_3 = 0.25\pi_3 + 0.75\pi_4 \qquad \pi_3 = \pi_4 = \frac{1}{2}$$
$$\pi_4 = 0.75\pi_3 + 0.25\pi_4$$

$$\pi^{(2)} = \left[0, 0, \frac{1}{2}, \frac{1}{2}\right]$$

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(2)

$$\begin{array}{c|cc} & W & L \\ \hline W & 0.8 & 0.2 \\ L & 0.3 & 0.7 \end{array}$$

$\pi = \pi P$ $\quad\quad \pi_W + \pi_L = 1$

$\pi_W = 0.8\pi_W + 0.3\pi_L \Rightarrow 0.2\pi_W = 0.3\pi_L$

$\pi_W = \frac{3}{2}\pi_L$

$\pi_W + \pi_L = 1$

$\pi_L = \frac{2}{5}$ $\quad \pi_W = \frac{3}{5}$

Team wins $\frac{3}{5}$ games in long run.

(b) $P(\text{Dine}) = \pi_W \cdot 0.7 + \pi_L \cdot 0.2 = \frac{3}{5} \times 0.7 + \frac{2}{5} \times 0.2$

$= 0.42 + 0.08$

$= 0.5$

(c) Same as expected no of trials till first success

$E[\text{Games till next Dine}] = \frac{1}{0.5} = 2$

(a) ~~Pcat~~ $C_1 \rightarrow$ Cat in room 1
$C_2 \rightarrow$ " " room 2

$$P_{Cat} = \begin{array}{c|cc} & C_1 & C_2 \\ \hline C_1 & 0.2 & 0.8 \\ C_2 & 0.8 & 0.2 \end{array}$$

Let stationary distribution be $\pi_c = [c_1, c_2]$

$$\pi_c P_{cat} = \pi_c \qquad c_1 + c_2 = 1$$

$$c_1 = 0.2c_1 + 0.8c_2 = \quad c_1 = c_2 = \frac{1}{2}$$

$$\pi_c = \left[\frac{1}{2}, \frac{1}{2}\right]$$

for Mouse $P_{mous} = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$

$$m_1 = 0.7m_1 + 0.6m_2 \qquad m_1 = \frac{2}{3} \quad m_2 = \frac{1}{3}$$

$$\pi_m = \left[\frac{2}{3}, \frac{1}{3}\right]$$

(b) Let Z_n = (Cat location, Mouse location)
possible joint state are (1,1) (1,2) (2,1) (2,2)

Since the Cat & Mouse moves independently so the probability of Moving to a new state depends only on current state

$$P(Z_{n+1} = (i', j') \mid Z_n = (i, j)) = P(\text{Cat move } i \rightarrow i') \cdot P(\text{Mou } j \rightarrow j')$$

So Z_n satisfy Markov property

Square Type	No of Sq	Moves
Corner	4	3
Edges	24	5
Interior ones	36	8

Since king choose uniformly from available legal moves the stationary probability of being on a square is proportional to number of available moves

Avail moves $= 4 \times 3 + 24 \times 5 + 36 \times 8 = 420$

Corner $\rightarrow$ prob per square $= \frac{3}{420}$

edge $\frac{5}{420}$

interior $\frac{8}{420}$

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(a) Yes, the stock price is recurrent. If stock reaches a certain price there is always a chance that it will return to same price again. Even though the stock is slightly more likely to go up than down, it mostly stays at same price. Also, because the market is open for only five hours and price change is in small steps, it won't move too far in either direction. So within the trading day, stock has enough time & chances to come back to any price it has visited before

(b) Yes, a stationary distribution exist. This means that over time stock price settles into a stable pattern of how often it takes each value. Since price can stay the same, move up, or move down, and all nearby prices are connected the system doesn't get stuck or break apart. Also, because market is open for a fixed time, the price remains within a reasonable range, allowing a stable long term pattern to form.