Discrete Structures for Computer Science

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What is Mathematics, really?

It's *not* just about numbers!

Mathematics is *much* more than that:

> Mathematics is, most generally, the study of any and all truths about any and all *well-defined* concepts.

But, these concepts can be *about* numbers, symbols, objects, images, sounds, *anything*!

So, what's *this* class (Discrete Structures for Computer Science) about?

A **set** is an unordered collection of objects called the **elements**.

For example, V = { a, e, i, o, u} is the set of all vowels in the English alphabet.

A **matrix** is a rectangular array of numbers arranged in m horizontal **rows** and n vertical **columns**.

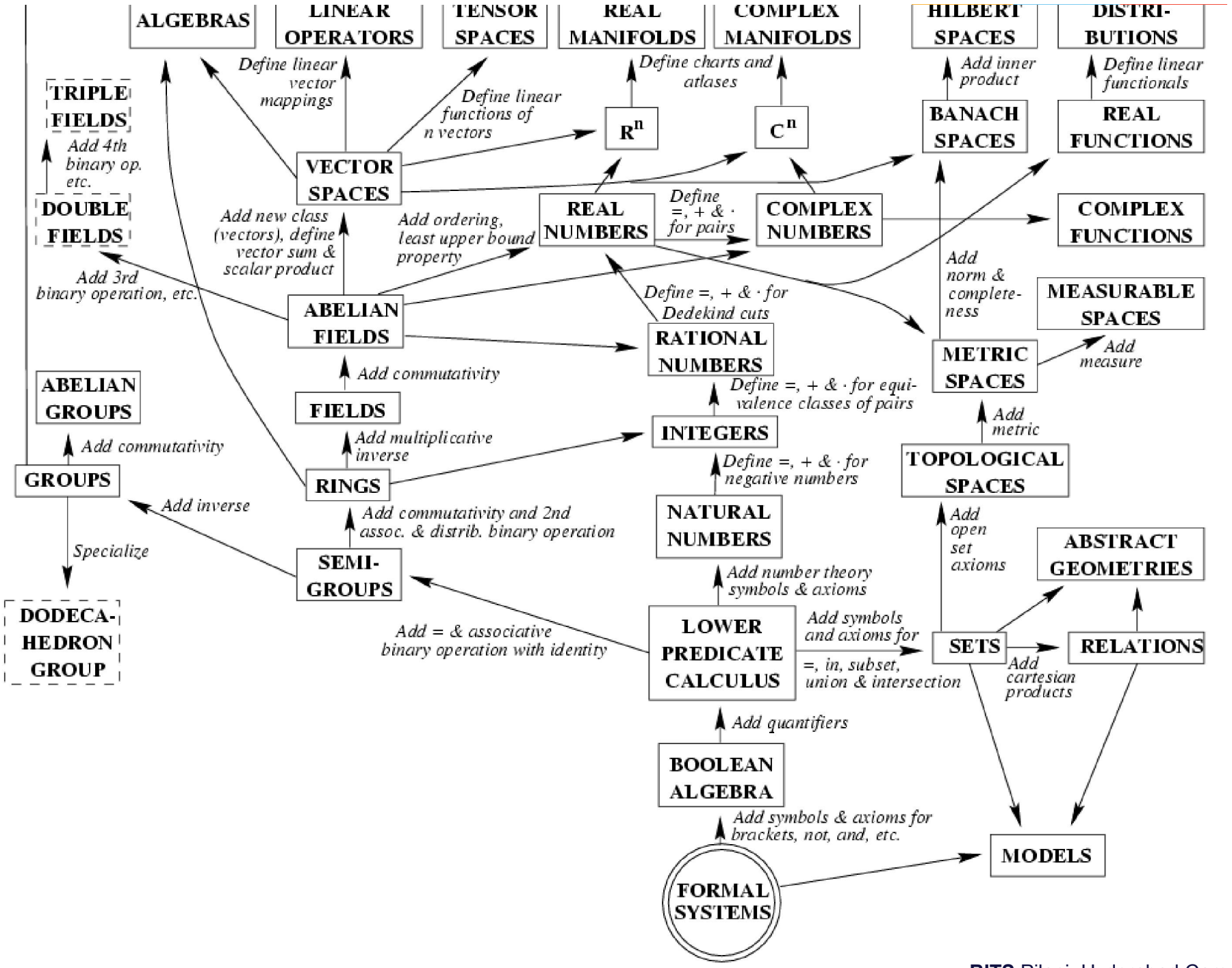
Sets, Matrices etc are all called **mathematical objects.**

Collection of mathematical objects with operations defined on them and the accompanying properties form a **mathematical structure or system.**

Example: The collection of sets with the operations of union, intersection, and complement and their accompanying properties is mathematical structure. We denote this structure by (sets, ∩, U, ').

So, what's *this* class (Discrete Structures for Computer Science) about?

- ✓ What are "discrete structures" anyway?
- ✓ "***Discrete***" (≠ "discreet"!) - Composed of distinct, separable parts. (Opposite of *continuous*.)
  discrete:*continuous* :: *digital*:*analog*
  Example - integers, people, house
- ✓ "***Discrete Mathematical Structures or Discrete structures***" - The abstract mathematical structures used to represent discrete objects and relationships between the objects
  Example – graphs, sets, relations
- ✓ "***Discrete Mathematics***" - The mathematical study of discrete objects and structures.


ALGEBRAS
LINEAR OPERATORS
TENSOR SPACES
REAL MANIFOLDS
COMPLEX MANIFOLDS
HILBERT SPACES
DISTRI-BUTIONS
Define linear vector mappings
Define linear functions of n vectors
Define charts and atlases
Add inner product
Define linear functionals
TRIPLE FIELDS
Add 4th binary op. etc.
DOUBLE FIELDS
VECTOR SPACES
R^n
C^n
BANACH SPACES
REAL FUNCTIONS
Add new class (vectors), define vector sum & scalar product
Add ordering, least upper bound property
REAL NUMBERS
Define =, + & · for pairs
COMPLEX NUMBERS
COMPLEX FUNCTIONS
Add 3rd binary operation, etc.
ABELIAN FIELDS
Define =, + & · for Dedekind cuts
Add norm & complete-ness
MEASURABLE SPACES
RATIONAL NUMBERS
METRIC SPACES
Add measure
ABELIAN GROUPS
Add commutativity
FIELDS
Define =, + & · for equi-valence classes of pairs
Add metric
Add multiplicative inverse
INTEGERS
TOPOLOGICAL SPACES
GROUPS
RINGS
Define =, + & · for negative numbers
Add inverse
Add commutativity and 2nd assoc. & distrib. binary operation
NATURAL NUMBERS
Add open set axioms
ABSTRACT GEOMETRIES
Specialize
SEMI-GROUPS
Add number theory symbols & axioms
DODECA-HEDRON GROUP
Add = & associative binary operation with identity
LOWER PREDICATE CALCULUS
Add symbols and axioms for =, in, subset, union & intersection
SETS
RELATIONS
Add cartesian products
Add quantifiers
BOOLEAN ALGEBRA
Add symbols & axioms for brackets, not, and, etc.
FORMAL SYSTEMS
MODELS

Why study this course?

- ✓ Information is stored and manipulated by computers in a discrete fashion. 0101101…
- ✓ Digital computers are based on discrete bits. Therefore, both a computer's
  – structure (circuits) and
  – operations (execution of algorithms)
  can be described by discrete math.
- ✓ The basis of all of computing is: *Discrete manipulations of discrete structures represented in memory.*
- ✓ Discrete Math is the basic language and conceptual foundation for all of computer science.

Discrete Math in Computer Science

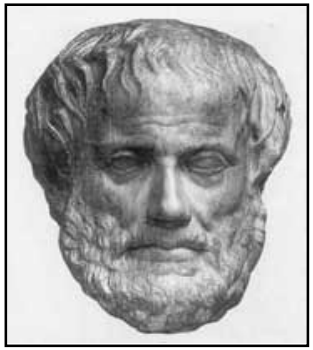
- Networking
- Database
- Image Processing
- Programming
- Languages
- Compilers & Interpreters
- Software Engineering
- Artificial Intelligence
- Computer Architecture
- Operating Systems
- Security & Cryptography
- Data Structures
- Algorithms
- Graphics & Animation

What else is remaining in computer science?

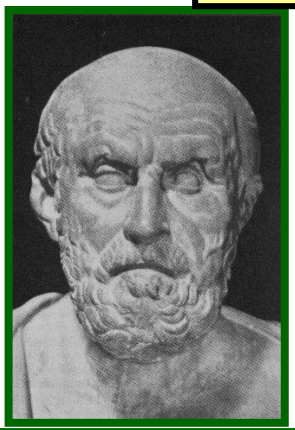
Quick Overview - Topics

Logic is a tool for working with elaborate *compound* statements.

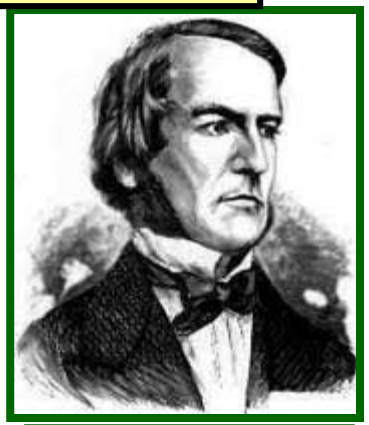
- Propositional Logic
- Predicate Logic
- Equivalences
- Logical Inference



Aristotle
(ca. 384-322 B.C.)



Chrysippus of Soli
(ca. 281 B.C. – 205 B.C.)



George Boole
(1815-1864)

Quick Overview - Topics

- Methods of proof
  - *Direct* proof: Assume p is true, and prove q.
  - *Indirect* proof: Assume $\neg q$, and prove $\neg p$.
  - *Vacuous* proof: Prove $\neg p$ by itself.
  - *Trivial* proof: Prove q by itself.
  - Proof by cases:
  - Proof by Contradiction
  - Proof by Induction
- Fundamental to all mathematical disciplines
- Useful for digital circuits, hardware design

Quick Overview - Topics

- ✓ Sets, Relations & Functions
  - Sets - Fundamental to all mathematical disciplines
  - Functions are heavily used in analysis of algorithms
  - Relations as concepts are building blocks modern databases
  - Equivalence Relations, Partially Ordered Sets, Lattice Theory
- ✓ Number Theory & Introduction to Cryptography
  - Get to rediscover the old reliable number theory and find out some surprising facts
  - Very useful in crypto-systems
- ✓ Recurrence Relations
  - ✓ Problems solved using recurrence

Quick Overview - Topics

innovate achieve lead

✓ Graph Theory

- Many clever data-structures for organizing information and making programs highly efficient are based on graph theory
- Very useful in describing problems in
  - Databases
  - Operating Systems
  - Networks
  - EVERY CS DISCIPLINE!!!!

✓ Counting and Combinatorics

- Compute your odds of winning lottery
- Important for predicting how long certain computer program will take to finish
- Pigeonhole principle

Books

- ✓ R1. Kolman, Busby, Ross and Rehman, Discrete Mathematical Structures for Computer Science, Pearson Education, 5th Edition, 2003.
- ✓ **R2. Kenneth H. Rosen : Discrete Mathematics and its Applications, Tata McGraw Hill, 5th Edition, 2004 Seventh Edition**
- ✓ R3. Goodaire & Parmenter : Discrete Mathematics & Graph Theory, Pearson Education, 2000
- ✓ R4. D.S.Malik and M.K.Sen, Discrete Mathematical Structures: Theory and Applications, Thomson, 2004.
- ✓ R5. C.L.Liu, Elements of Discrete Mathematics, 2nd Edition, McGraw Hill, 1986.

Notations

$\neg p$	$p \wedge q$	$p \oplus q$	$p \rightarrow q$	$p \Leftrightarrow q$	$\forall x\, P(x)$
$\exists x\, P(x)$	$\{a_1, \cdots, a_n\}$	$\mathbf{Z}, \mathbf{N}, \mathbf{R}$	$\therefore$	$\{x \mid P(x)\}$	$x \notin S$
$\varnothing$	$S \subseteq T$	$\lvert S \rvert$	$A \cup B$	$\overline{A}$	$\bigcap_{i=1}^{n} A_i$
$f : A \rightarrow B$	$f^{-1}(x)$	$f \circ g$	$\lfloor x \rfloor$	$\sum_{\alpha \in S} a_\alpha$	$\prod_{i=1}^{n} a_i$
O, Ω, Θ	$\min, \max$	$a \mid b$	$\gcd, \mathrm{lcm}$	mod	$a \equiv b \pmod{m}$
$(a_k \cdots a_0)_b$	$[a_{ij}]$	$\mathbf{A}^{\mathrm{T}}$	$\mathbf{A} \odot \mathbf{B}$	$\mathbf{A}^{[n]}$	$\binom{n}{r}$
$C(n; n_1, \cdots, n_m)$	$p(E \mid F)$	R^*	Δ	$[a]_R$	$\deg^+(v)$

Thank You!!