Mini Project on

Application of Z Transform

Submitted in partial fulfillment of the requirements of the degree of
Bachelor of Engineering in Computer Science of Engineering (AIML)

Submitted by

Ayush Khamkar - 21
Sameer Khan -22
Harshad Kolekar -23
Sahil Kudalkar -24

Under the guidance of:
Prof. Ramashankar Prajapati

DEPARTMENT OF computer science engineering

ENGINEERING VIVA INSTITUTE OF TECHNOLOGY (AIML)

At Shirgoan, Virar (East), Tal. Vasai, Dist. Palghar – 401305

2023-2024

Application of Z Transform

Group Number: 6

Ayush Khamkar - 21

Sameer Khan -22

Harshad Kolekar -23

Sahil Kudalkar -24

Acknowledgement

In the accomplishment of this project successfully, many people have helped us and bestowed their heart-warming blessings and support.

I am taking this time to thank them for their concerned and support I would like to express my special thanks of gratitude to our professor Mr. Ramashankar Prajapati as well as our principal Dr. Arun Kumar who gave us the opportunity to work on this project on the topic ‘Function Of Square Matrix’,which also helped us understanding this topic more clearly in which we came across many new things and different applications on Function Of Square Matrix which were previously unknown to us.

Secondly, I would like to thank our team for giving their 100% and doing the project with their most sincere work and parents for supporting us while accomplishing this project successfully.

Contents

Abstract

A **square matrix** is a fundamental concept in linear algebra. It is defined as a matrix with an equal number of rows and columns. When a square matrix has n rows and n columns, we say its order is $n \times n$. The number of elements in a square matrix is always a perfect square, determined by the product of the number of rows and columns.

In every square matrix, the number of rows and columns is equal. The sum of all principal diagonal elements in a square matrix is called the **trace** of the matrix. Determinants can only be computed for square matrices. If the determinant of a square matrix is zero, it is called a **singular matrix**; otherwise, it is **non-singular**. An **identity matrix** has ones on the principal diagonal and zeros elsewhere. The order of a square matrix and its transpose are the same. Various operations (addition, multiplication, inverse) apply to square matrices.

There are different types of square matrices:

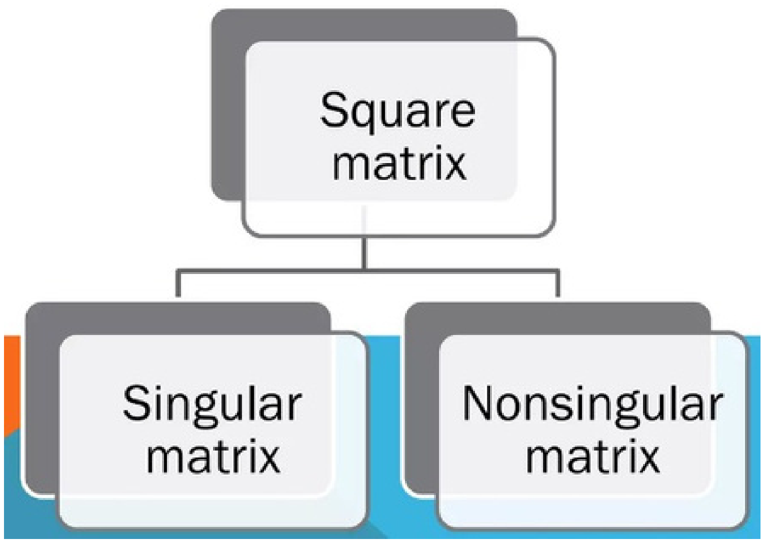
- **Singular Matrix**: If the determinant of a square matrix is zero, it is singular.
- **ScalarMatrix**:Principaldiagonalelementsareequal,andtherestarezeros.
- **IdentityMatrix**:Principaldiagonalelementsareones,andtherestarezeros.
- **SymmetricMatrix**:Thetransposeisthesameastheoriginalmatrix.

Square matrices find applications in various fields, including computer graphics, physics, and engineering. They represent linear transformations and serve as the building blocks for mathematical operations.

Introduction

A square matrix is an important format of a matrix and it has the perfect square number of elements. It has an equal number of rows and columns, and hence its order is of the form $n \times n$. All the matrix operations of transpose, determinant, adjoint, inverse, and the mathematical operations of matrices apply to a square matrix also.

A square matrix has special application in solving quadratic equations in two variables. Here we shall learn the different properties of a square matrix, and try to understand how to do the mathematical operations across these matrices.



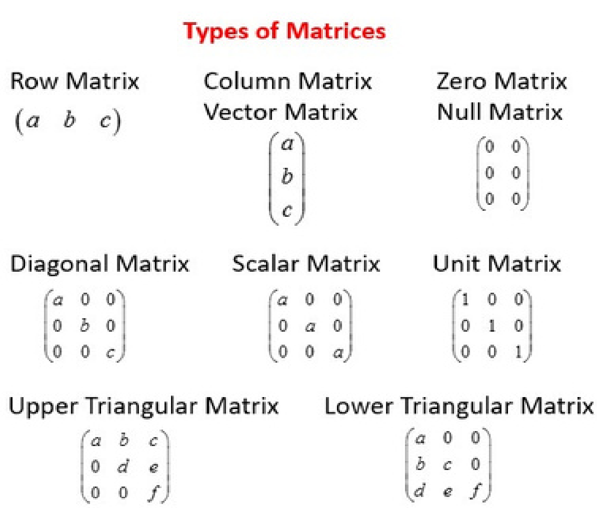
The matrix is also used in mathematical equations and can provide an approximation of complicated calculations. Matrices can be of different types, and invertible matrix, symmetric matrix, singular matrix, etc. are among the important ones.

Definition

A square matrix is a matrix with an equal number of rows and columns. Its order is of the form n x n. Also, the product of these rows and columns gives the number of elements in the square matrix. Hence the number of elements in it is always a perfect square number. A typical square matrix looks as follows.

$$A_{n\times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & . & . & a_{1n} \\ a_{21} & a_{22} & a_{23} & . & . & a_{2n} \\ a_{31} & a_{32} & a_{33} & . & . & a_{3n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & a_{n3} & . & . & a_{nn} \end{bmatrix}$$

The order of a square matrix that has “n” rows and “n” columns is “n × n.” The number of elements in a matrix can be determined by the product of the number of rows and columns in the matrix. So, the number of elements in a square matrix is always a perfect square number.



Eigenvalues and eigenvectors are essential concepts in linear algebra.

1. Eigenvalues
    - Eigenvalues are scalar values associated with linear transformations (usually represented by matrices).
    - They indicate how much an eigenvector is stretched or compressed during the transformation.
    - Eigenvalues help us understand stability, growth, and behavior of systems.
    - Mathematically, for a matrix A, an eigenvalue λ satisfies the equation:
    - $Av=\lambda v$
2. Eigenvectors
    - Eigenvectors are non-zero vectors that remain in the same direction (up to a scalar factor) after applying the transformation.
    - They represent special directions in the matrix.
    - To find eigenvectors, we solve the equation:
    - $Av=\lambda v$
    - Eigenvectors are crucial in solving differential equations, data science (PCA), and more.

Applications

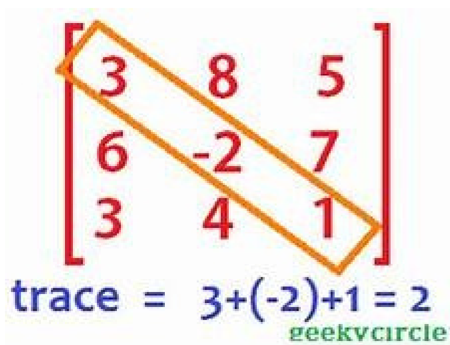
1. Linear Transformations
   - Square matrices play a crucial role in representing linear transformations. For example:
     - Shearing:Transformingobjectsalongaspecificaxis.
     - Rotation:Rotatingobjectsintwoorthreedimensions.
     - Scaling:Enlargingorshrinkingobjectsuniformly.
     - Reflection:Mirroringobjectsacrossalineorplane.
2. Solving Quadratic Equations
   - Square matrices find special application in solving quadratic equations involving two variables. These equations often arise in physics, engineering, and optimization problems.
3. Computer Graphics
   - In computer graphics, square matrices are used to project three-dimensional images onto two-dimensional screens.
   - Transformations like rotation, translation, and scaling are achieved using square matrices.
4. Physics and Engineering
   - Quantum mechanics, electromagnetism, and fluid dynamics rely on matrix representations.
   - Structural analysis, control systems, and circuit theory use square matrices for modeling physical systems.
5. Economics and Finance
   - Input-output models in economics use square matrices to study interdependencies between sectors.
   - Markov chains and stochastic processes involve square transition matrices.

6. Markov Chains
    - Markov chains model probabilistic transitions between states. The transition matrix is square and describes state probabilities.
7. Graph Theory
    - Adjacency matrices for graphs are square matrices. They represent connections between nodes (vertices) in networks.
8. Differential Equations
    - Systems of differential equations can be expressed as matrix equations using square matrices.
9. Symmetry Analysis
    - Symmetric matrices (where the transpose equals the original matrix) are used to study symmetry properties in mathematical structures.
10. Optimization and Machine Learning
    - Optimization algorithms often involve solving systems of linear equations using square matrices.
    - Principal Component Analysis (PCA) and eigenvalue problems rely on square matrices.

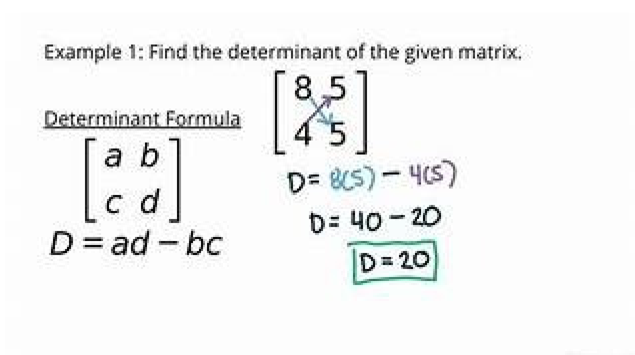
Properties

Certainly! Let’s explore the essential properties of **square matrices**:

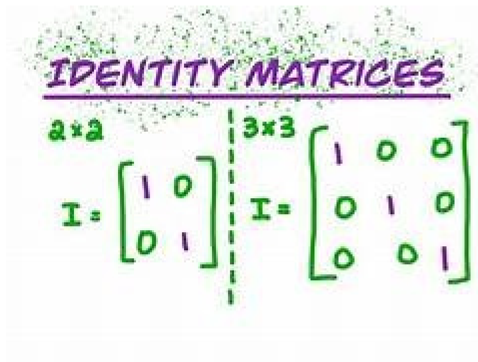
1. **Equal Rows and Columns**
   - A square matrix has the same number of rows and columns.
   - n If a matrix has rows and n columns, it is a square matrix of order $n \times n$.
2. **Trace of a Matrix**
   - The sum of all principal diagonal elements in a square matrix is called the **trace**.
   - For example,



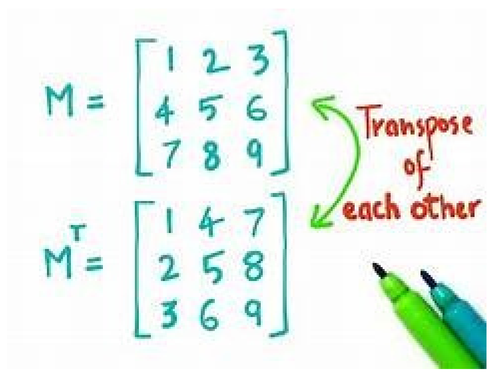
3. **Determinant**
   - Determinants can only be calculated for square matrices.
   - If the determinant of a square matrix is zero, it is called a **singular matrix**; otherwise, it is **non-singular**.



4. **Identity Matrix**
    - An identity matrix has ones on the principal diagonal and zeros elsewhere.



5. **Transpose**
    - The order of a square matrix and its transpose are the same.
    - If A is a square matrix, its transpose is denoted as A^T.



6. **Operations**:
    - We can perform various operations on square matrices, including addition, multiplication, and finding inverses.
7. **Types of Square Matrices**
    - **Singular Matrix**: Determinant is zero.
    - **Scalar Matrix**: Principal diagonal elements are equal, rest are zeros.
    - **Symmetric Matrix**: Transpose is the same as the original matrix.

Problems

1.Given two matrices A and B, find the product of AB if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$.

Solution:

To find the product of two matrices, we need to multiply each element of one matrix with the corresponding element in the other matrix and add the products. Thus, we get:

$$\Rightarrow AB = \begin{bmatrix} (1)(9)+(2)(6)+(3)(3) & (1)(8)+(2)(5)+(3)(2) & (1)(7)+(2)(4)+(3)(1) \\ (4)(9)+(5)(6)+(6)(3) & (4)(8)+(5)(5)+(6)(2) & (4)(7)+(5)(4)+(6)(1) \\ (7)(9)+(8)(6)+(9)(3) & (7)(8)+(8)(5)+(9)(2) & (7)(7)+(8)(4)+(9)(1) \end{bmatrix}$$

Simplifying the products, we get:

$$AB = \begin{bmatrix} 30 & 20 & 10 \\ 84 & 56 & 28 \\ 138 & 92 & 46 \end{bmatrix}$$

Thus, the product of matrices A and B is the matrix $AB = \begin{bmatrix} 30 & 20 & 10 \\ 84 & 56 & 28 \\ 138 & 92 & 46 \end{bmatrix}$.

2.Given a matrix A, find its transpose if $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

Solution:

To find the transpose of a matrix, we need to interchange its rows and columns. Thus, we get:

$$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Thus, the transpose of matrix A is the matrix obtained by interchanging its rows and columns, which is $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$.

3.Given a matrix A, find its determinant if $A = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 9 \\ 2 & 6 & 5 \end{bmatrix}$.

Solution:

To find the determinant of a matrix, we need to follow a specific formula that involves finding the sum of products of elements in specific arrangements. Using this formula for the matrix A, we get:

$$|A| = 3(55 - 69) - 1(15 - 64) + 4(16 - 52)$$

$$\Rightarrow |A| = 15 - 11 + 8$$

$$\Rightarrow |A| = 12$$

Thus, the determinant of matrix A is 12.

Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

Solution The matrix $A - \lambda I_3$ is obtained by subtracting λ from the diagonal elements of A. Thus

$$A - \lambda I_3 = \begin{bmatrix} 5-\lambda & 4 & 2 \\ 4 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{bmatrix}$$

The characteristic polynomial of A is $|A - \lambda I_3|$. Using row and column operations to simplify determinants, we get

$$|A - \lambda I_3| = \begin{vmatrix} 5-\lambda & 4 & 2 \\ 4 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & -1+\lambda & 0 \\ 4 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{vmatrix}$$

Eigenvalues and Eigenvectors for 3×3 Matrix

If Eigenvalues are $\lambda = 2, 3 \; and \; 4$

$$\left(\begin{bmatrix} 4 & -1 & 1 \\ -2 & 4 & 0 \\ -4 & 3 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Eigenvector is $\mathrm{X} = \begin{bmatrix} t \\ 2t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Conclusion

Square matrices are fundamental in linear algebra. They serve as the building blocks for mathematical transformations. Key properties include the trace (sum of diagonal elements), determinants, and identity matrices. Eigenvalues and eigenvectors play a crucial role, representing stability and scaling factors. Applications span physics, computer graphics, and more. Remember, square matrices are the backbone of mathematical transformations!

Bibliography

https://byjus.com/maths/square-matrix/

https://www.vedantu.com/maths/square-matrix

https://en.wikipedia.org/wiki/Square_matrix

https://www.cuemath.com/algebra/square-matrix/

https://www.geeksforgeeks.org/square-matrix/