

Review of Differentiation

Rules

1. **Constant:** $\frac{d}{dx} c = 0$
2. **Constant multiple:** $\frac{d}{dx} c f(x) = c f'(x)$
3. **Sum:** $\frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x)$
4. **Product:** $\frac{d}{dx} f(x)g(x) = f(x)g'(x) + g(x)f'(x)$
5. **Quotient:** $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
6. **Chain:** $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$
7. **Power:** $\frac{d}{dx} x^n = nx^{n-1}$
8. **Power:** $\frac{d}{dx} [g(x)]^n = n[g(x)]^{n-1} g'(x)$

Functions

Trigonometric:

9. $\frac{d}{dx} \sin x = \cos x$
10. $\frac{d}{dx} \cos x = -\sin x$
11. $\frac{d}{dx} \tan x = \sec^2 x$
12. $\frac{d}{dx} \cot x = -\csc^2 x$
13. $\frac{d}{dx} \sec x = \sec x \tan x$
14. $\frac{d}{dx} \csc x = -\csc x \cot x$

Inverse trigonometric:

15. $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
16. $\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$
17. $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
18. $\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$
19. $\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
20. $\frac{d}{dx} \csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$

Hyperbolic:

21. $\frac{d}{dx} \sinh x = \cosh x$
22. $\frac{d}{dx} \cosh x = \sinh x$
23. $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$
24. $\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$
25. $\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$
26. $\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x$

Inverse hyperbolic:

27. $\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{x^2+1}}$
28. $\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$
29. $\frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2}$
30. $\frac{d}{dx} \coth^{-1} x = \frac{1}{1-x^2}$
31. $\frac{d}{dx} \operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1-x^2}}$
32. $\frac{d}{dx} \operatorname{csch}^{-1} x = -\frac{1}{|x|\sqrt{x^2+1}}$

Exponential:

33. $\frac{d}{dx} e^x = e^x$
34. $\frac{d}{dx} a^x = a^x (\ln a)$

Logarithmic:

35. $\frac{d}{dx} \ln |x| = \frac{1}{x}$
36. $\frac{d}{dx} \log_a x = \frac{1}{x(\ln a)}$

Integral defined:

37. $\frac{d}{dx} \int_a^x g(t) dt = g(x)$
38. $\frac{d}{dx} \int_a^b g(x, t) dt = \int_a^b \frac{\partial}{\partial x} g(x, t) dt$

Brief Table of Integrals

1. $\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
2. $\int \frac{1}{u} du = \ln|u| + C$
3. $\int e^u du = e^u + C$
4. $\int a^u du = \frac{1}{\ln a} a^u + C$
5. $\int \sin u du = -\cos u + C$
6. $\int \cos u du = \sin u + C$
7. $\int \sec^2 u du = \tan u + C$
8. $\int \csc^2 u du = -\cot u + C$
9. $\int \sec u \tan u du = \sec u + C$
10. $\int \csc u \cot u du = -\csc u + C$
11. $\int \tan u du = -\ln|\cos u| + C$
12. $\int \cot u du = \ln|\sin u| + C$
13. $\int \sec u du = \ln|\sec u + \tan u| + C$
14. $\int \csc u du = \ln|\csc u - \cot u| + C$
15. $\int u \sin u du = \sin u - u \cos u + C$
16. $\int u \cos u du = \cos u + u \sin u + C$
17. $\int \sin^2 u du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$
18. $\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$
19. $\int \tan^2 u du = \tan u - u + C$
20. $\int \cot^2 u du = -\cot u - u + C$
21. $\int \sin^3 u du = -\frac{1}{3}(2 + \sin^2 u)\cos u + C$
22. $\int \cos^3 u du = \frac{1}{3}(2 + \cos^2 u)\sin u + C$
23. $\int \tan^3 u du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$
24. $\int \cot^3 u du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$
25. $\int \sec^3 u du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$
26. $\int \csc^3 u du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$
27. $\int \sin au \sin bu du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$
28. $\int \cos au \cos bu du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$
29. $\int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2}(a \sin bu - b \cos bu) + C$
30. $\int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2}(a \cos bu + b \sin bu) + C$
31. $\int \sinh u du = \cosh u + C$
32. $\int \cosh u du = \sinh u + C$
33. $\int \operatorname{sech}^2 u du = \tanh u + C$
34. $\int \operatorname{csch}^2 u du = -\coth u + C$
35. $\int \tanh u du = \ln(\cosh u) + C$
36. $\int \coth u du = \ln|\sinh u| + C$
37. $\int \ln u du = u \ln u - u + C$
38. $\int u \ln u du = \frac{1}{2}u^2 \ln u - \frac{1}{4}u^2 + C$
39. $\int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \frac{u}{a} + C$
40. $\int \frac{1}{\sqrt{a^2 + u^2}} du = \ln|u + \sqrt{a^2 + u^2}| + C$
41. $\int \sqrt{a^2 - u^2} du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1} \frac{u}{a} + C$
42. $\int \sqrt{a^2 + u^2} du = \frac{u}{2}\sqrt{a^2 + u^2} + \frac{a^2}{2}\ln|u + \sqrt{a^2 + u^2}| + C$
43. $\int \frac{1}{a^2 + u^2} du = \frac{1}{a}\tan^{-1} \frac{u}{a} + C$
44. $\int \frac{1}{a^2 - u^2} du = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$

Note: Some techniques of integration, such as integration by parts and partial fractions, are reviewed in the *Student Resource Manual* that accompanies this text.