Applications of Linear Equations

Example-7 (p-78, DGZ).

According to Kirchoff's second law, the sum of voltage drop across the inductor $\left(L\frac{di}{dt}\right)$ and the voltage drop across the resistor (iR) is the voltage drop of the battery.

$$L\frac{di}{dt} + Ri = E$$

$$\frac{1}{2}\frac{di}{dt} + 10i = 12$$

$$\frac{di}{dt} + 20i = 24$$

$$P(t) = 20.$$

$$\int P(t)\,dt = \int 20\,dt = 20t.$$

$$e^{\int P(t)dt} = e^{20t}.$$

$$e^{20t}\frac{di}{dt} + e^{20t}\,20i = e^{20t}\cdot 24.$$

$$\frac{d}{dt}\left[e^{20t}i\right] = 24e^{20t}.$$

$$\int \frac{d}{dt}\left(e^{20t} i\right) dt = 24 \int e^{20t}\, dt + c$$

$$e^{20t} i = 24 \frac{e^{20t}}{20} + c$$

$$i = \frac{6}{5} + c\, e^{-20t}$$

$$i(0) = 0$$

$$0 = \frac{6}{5} + c e^{0}$$

$$c = -\frac{6}{5}.$$

$$i = \frac{6}{5} - \frac{6}{5} e^{-20t}.$$

$$i = \frac{6}{5}\left(1 - e^{-20t}\right).$$

Example--2 (p-29, EK).

Use methodology of previous example and solve yourself.

Example-3 (p-30, EK)

Suppose that hormone level in the blood at any time t is y(t).

Suppose A is the average level of hormone present every time in the body.

The sinusoidal input of hormone $= B\cos\omega t$.

Hormone output $= Ky(t) = Ky$

$$\frac{dy}{dt} = A + B\cos\omega t - Ky$$

$$\frac{dy}{dt} + Ky = A + B\cos\omega t$$

$$e^{\int p(t)dt} = e^{\int K dt} = e^{Kt}$$

$$e^{Kt}\frac{dy}{dt} + Ke^{Kt}y = Ae^{Kt} + Be^{Kt}\cos\omega t$$

$$\frac{d}{dt}(e^{Kt}y) = Ae^{Kt} + Be^{Kt}\cos\omega t$$

$$\int \frac{d}{dt}(e^{Kt}y)dt = A\int e^{Kt}dt + B\int e^{Kt}\cos\omega t\,dt + c$$

$$e^{Kt}y = A\frac{e^{Kt}}{K} + BI + c$$

$$I = \int e^{Kt} \cos\omega t \, dt$$

$$= \cos\omega t \frac{e^{Kt}}{K} - \int \frac{e^{Kt}}{K} (-\sin\omega t)\, \omega \, dt$$

$$= \frac{e^{Kt}}{K} \cos\omega t + \frac{\omega}{K}\left[\sin\omega t \frac{e^{Kt}}{K} - \int \frac{e^{Kt}}{K} (\cos\omega t)\, \omega \, dt \right]$$

$$= \frac{e^{Kt}}{K} \cos\omega t + \frac{\omega}{K^2} e^{Kt} \sin\omega t - \frac{\omega^2}{K^2} I$$

$$I + \frac{\omega^2}{K^2} I = \frac{e^{Kt}}{K} \cos\omega t + \frac{\omega}{K^2} e^{Kt} \sin\omega t$$

$$I\left(\frac{\omega^2+K^2}{K^2}\right) = \frac{e^{Kt}}{K} \cos\omega t + \frac{\omega}{K^2} e^{Kt} \sin\omega t$$

$$I = \frac{K}{\omega^2+K^2} e^{Kt} \cos\omega t + \frac{\omega}{\omega^2+K^2} e^{Kt} \sin\omega t$$

$$e^{Kt} y = \frac{A}{K} e^{Kt} + \frac{BK}{\omega^2+K^2} e^{Kt} \cos\omega t + \frac{B\omega}{\omega^2+K^2} e^{Kt} \sin\omega t + c$$

$$y = \frac{A}{K} + \frac{B}{\omega^2+K^2} (K\cos\omega t + \omega \sin\omega t) + ce^{-Kt}$$

$$\because y(0) = 0$$

$$0 = \frac{A}{K} + \frac{B}{\omega^2+K^2}(K) + c \Rightarrow c = -\left(\frac{A}{K} + \frac{BK}{\omega^2+K^2}\right)$$

$$y = \frac{A}{K} + \frac{B}{\omega^2+K^2} (K\cos\omega t + \omega\sin\omega t) - \left(\frac{A}{K} + \frac{BK}{\omega^2+K^2}\right) e^{-Kt}$$

where, $\omega = \frac{2\pi}{24} = \frac{\pi}{12}$.