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PREFACE

This book is intended to accompany a text used in that first course in fluid mechanics which is required in all mechanical engineering and civil engineering departments, as well as several other departments. It provides a succinct presentation of the material so that the students more easily understand those difficult parts. If an expanded presentation is not a necessity, this book can be used as the primary text. We have included all derivations and numerous applications, so it can be used with no supplemental material. A solutions manual is available from the authors at MerleCP@sbcglobal.net.

We have included a derivation of the Navier–Stokes equations with several solved flows. It is not necessary, however, to include them if the elemental approach is selected. Either method can be used to study laminar flow in pipes, channels, between rotating cylinders, and in laminar boundary layer flow.

The basic principles upon which a study of fluid mechanics is based are illustrated with numerous examples, solved problems, and supplemental problems which allow students to develop their problem-solving skills. The answers to all supplemental problems are included at the end of each chapter. All examples and problems are presented using SI metric units. English units are indicated throughout and are included in the Appendix.

The mathematics required is that of other engineering courses except that required if the study of the Navier–Stokes equations is selected where partial differential equations are encountered. Some vector relations are used, but not at a level beyond most engineering curricula.

If you have comments, suggestions, or corrections or simply want to opine, please e-mail me at: merlecp@sbcglobal.net. It is impossible to write an error-free book, but if we are made aware of any errors, we can have them corrected in future printings. Therefore, send an email when you find one.

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Chapter 1

Basic Information

1.1 INTRODUCTION

Fluid mechanics is encountered in almost every area of our physical lives. Blood flows through our veins and arteries, a ship moves through water and water flows through rivers, airplanes fly in the air and air flows around wind machines, air is compressed in a compressor and steam expands around turbine blades, a dam holds back water, air is heated and cooled in our homes, and computers require air to cool components. All engineering disciplines require some expertise in the area of fluid mechanics.

In this book we will present those elements of fluid mechanics that allow us to solve problems involving relatively simple geometries such as flow through a pipe and a channel and flow around spheres and cylinders. But first, we will begin by making calculations in fluids at rest, the subject of fluid statics. The math requirement is primarily calculus but some differential equation theory will be used. The more complicated flows that usually are the result of more complicated geometries will not be presented in this book.

In this first chapter, the basic information needed in our study will be presented. Much of it has been included in previous courses so it will be a review. But, some of it should be new to you. So, let us get started.

1.2 DIMENSIONS, UNITS, AND PHYSICAL QUANTITIES

Fluid mechanics, as all other engineering areas, is involved with physical quantities. Such quantities have dimensions and units. The nine basic dimensions are mass, length, time, temperature, amount of a substance, electric current, luminous intensity, plane angle, and solid angle. All other quantities can be expressed in terms of these basic dimensions, e.g., force can be expressed using Newton's second law as

$$F = ma \tag{1.1}$$

In terms of dimensions we can write (note that F is used both as a variable and as a dimension)

$$F = M\frac{L}{T^2} \tag{1.2}$$

where F, M, L, and T are the dimensions of force, mass, length, and time. We see that force can be written in terms of mass, length, and time. We could, of course, write

$$M = F\frac{T^2}{L} \tag{1.3}$$

Units are introduced into the above relationships if we observe that it takes 1 N to accelerate 1 kg at 1 m/s^2 (using English units it takes 1 lb to accelerate 1 slug at 1 ft/sec^2), i.e.,

$$\mathrm{N} = \mathrm{kg{\cdot}m/s^2} \qquad \mathrm{lb} = \mathrm{slug\text{-}ft/sec^2} \tag{1.4}$$

These relationships will be used often in our study of fluids. Note that we do not use "lbf" since the unit "lb" will always refer to a pound of force; the slug will be the unit of mass in the English system. In the SI system the mass will always be kilograms and force will always be newtons. Since weight is a force, it is measured in newtons, never kilograms. The relationship

$$W = mg \tag{1.5}$$

is used to calculate the weight in newtons given the mass in kilograms, where $g = 9.81$ m/s^2 (using English units $g = 32.2$ ft/sec^2). Gravity is essentially constant on the earth's surface varying from 9.77 to 9.83 m/s^2.

Five of the nine basic dimensions and their units are included in Table 1.1 and derived units of interest in our study of fluid mechanics in Table 1.2. Prefixes are common in the SI system so they are presented in Table 1.3. Note that the SI system is a special metric system; we will use the units presented

Table 1.1 Basic Dimensions and Their Units

Quantity	Dimension	SI	Units	English	Units
Length l	L	meter	m	foot	ft
Mass m	M	kilogram	kg	slug	slug
Time t	T	second	s	second	sec
Temperature T	Θ	kelvin	K	Rankine	°R
Plane angle		radian	rad	radian	rad

Table 1.2 Derived Dimensions and Their Units

Quantity	Dimension	SI units	English units
Area A	L^2	m^2	ft^2
Volume $\forall$	L^3	m^3 or L (liter)	ft^3
Velocity V	L/T	m/s	ft/sec
Acceleration a	L/T^2	m/s^2	ft/sec^2
Angular velocity Ω	T^{-1}	s^{-1}	sec^{-1}
Force F	ML/T^2	kg·m/s^2 or N (newton)	slug-ft/sec^2 or lb
Density ρ	M/L^3	kg/m^3	slug/ft^3
Specific weight γ	M/L^2T^2	N/m^3	lb/ft^3
Frequency f	T^{-1}	s^{-1}	sec^{-1}
Pressure p	M/LT^2	N/m^2 or Pa (pascal)	lb/ft^2
Stress τ	M/LT^2	N/m^2 or Pa (pascal)	lb/ft^2
Surface tension σ	M/T^2	N/m	lb/ft
Work W	ML^2/T^2	N·m or J (joule)	ft-lb
Energy E	ML^2/T^2	N·m or J (joule)	ft-lb
Heat rate $\dot{Q}$	ML^2/T^3	J/s	Btu/sec

Table 1.2 Continued

Quantity	Dimension	SI units	English units
Torque T	ML^2/T^2	N·m	ft-lb
Power $\dot{W}$	ML^2/T^3	J/s or W (watt)	ft-lb/sec
Mass flux $\dot{m}$	M/T	kg/s	slug/sec
Flow rate Q	L^3/T	m^3/s	ft^3/sec
Specific heat c	$L^2/T^2\Theta$	J/kg·K	Btu/slug-°R
Viscosity μ	M/LT	$N{\cdot}s/m^2$	$lb\text{-}sec/ft^2$
Kinematic viscosity v	L^2/T	m^2/s	ft^2/sec

Table 1.3 SI Prefixes

Multiplication factor	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

in these tables. We often use scientific notation, such as 3×10^5 N rather than 300 kN; either form is acceptable.

We finish this section with comments on significant figures. In every calculation, well, almost every one, a material property is involved. Material properties are seldom known to four significant figures and often only to three. So, it is not appropriate to express answers to five or six significant figures. Our calculations are only as accurate as the least accurate number in our equations. For example, we use gravity as 9.81 m/s^2, only three significant figures. It is usually acceptable to express answers using four significant figures, but not five or six. The use of calculators may even provide eight. The engineer does not, in general, work with five or six significant figures. Note that if the leading numeral in an answer is 1, it does not count as a significant figure, e.g., 1248 has three significant figures.

EXAMPLE 1.1 Calculate the force needed to provide an initial upward acceleration of 40 m/s^2 to a 0.4-kg rocket.

Solution: Forces are summed in the vertical y-direction:

$$\begin{aligned}&\sum F_y = ma_y\\&F - mg = ma\\&F - 0.4\times9.81 = 0.4\times40\\&\therefore\ F = 19.92\,\text{N}\end{aligned}$$

Note that a calculator would provide 19.924 N, which contains four significant figures (the leading 1 does not count). Since gravity contained three significant figures, the 4 was dropped.

1.3 GASES AND LIQUIDS

The substance of interest in our study of fluid mechanics is a gas or a liquid. We restrict ourselves to those liquids that move under the action of a shear stress, no matter how small that shearing stress may be. All gases move under the action of a shearing stress but there are certain substances, like ketchup, that do not move until the shear becomes sufficiently large; such substances are included in the subject of rheology and are not presented in this book.

A force acting on an area is displayed in Fig. 1.1. A *stress vector* is the force vector divided by the area upon which it acts. The *normal stress* acts normal to the area and the *shear stress* acts tangent to the area. It is this shear stress that results in fluid motions. Our experience of a small force parallel to the water on a rather large boat confirms that any small shear causes motion. This shear stress is calculated with

$$\tau = \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A} \tag{1.6}$$

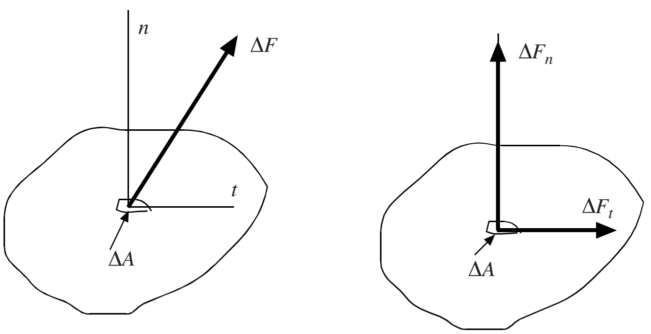


Figure 1.1 Normal and tangential components of a force.

Each fluid considered in our study is continuously distributed throughout a region of interest, that is, each fluid is a *continuum*. A liquid is obviously a continuum but each gas we consider is also assumed to be a continuum; the molecules are sufficiently close to one another so as to constitute a continuum. To determine whether the molecules are sufficiently close, we use the *mean free path*, the average distance a molecule travels before it collides with a neighboring molecule. If the mean free path is small compared to a characteristic dimension of a device (e.g., the diameter of a rocket), the continuum assumption is reasonable. In atmospheric air at sea level, the mean free path is approximately 6×10^{-6} cm and at an elevation of 100 km, it is about 10 cm. So, at high elevations, the continuum assumption is not reasonable and the theory of rarified gas dynamics is needed.

If a fluid is a continuum, the *density* can be defined as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} \tag{1.7}$$

where Δm is the infinitesimal mass contained in the infinitesimal volume ΔV. Actually, the infinitesimal volume cannot be allowed to shrink to zero since near zero there would be few molecules in the small volume; a small volume ϵ would be needed as the limit in Eq. (1.7) for the definition to be acceptable. This is not a problem for most engineering applications since there are 2.7×10^{16} molecules in a cubic millimeter of air at standard conditions.

So, with the continuum assumption, the quantities of interest are assumed to be defined at all points in a specified region. For example, the density is a continuous function of x, y, z, and t, i.e., $\rho = \rho(x,y,z,t)$.

1.4 PRESSURE AND TEMPERATURE

In our study of fluid mechanics, we often encounter pressure. It results from compressive forces acting on an area. In Fig. 1.2 the infinitesimal force ΔF_n acting on the infinitesimal area ΔA gives rise to the *pressure*, defined by

$$p = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} \tag{1.8}$$

The units on pressure result from force divided by area, that is, N/m^2, the pascal, Pa. A pressure of 1 Pa is a very small pressure, so pressure is typically expressed as kilopascals or kPa. Using English units, pressure is expressed as lb/ft^2 (psf) or lb/in^2 (psi). Atmospheric pressure at sea level is 101.3 kPa, or most often simply 100 kPa (14.7 lb/in^2). It should be noted that pressure is sometimes expressed as millimeters of mercury, as is common with meteorologists, or meters of water; we can use $p = \rho g h$ to convert the units, where ρ is the density of the fluid with height h.

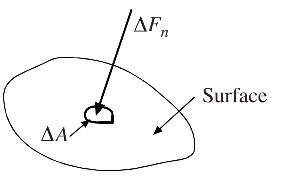


Figure 1.2 The normal force that results in pressure.

Pressure measured relative to atmospheric pressure is called *gage pressure*; it is what a gage measures if the gage reads zero before being used to measure the pressure. *Absolute pressure* is zero in a volume that is void of molecules, an ideal vacuum. Absolute pressure is related to gage pressure by the equation

$$p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atmosphere}} \tag{1.9}$$

where $p_{\text{atmosphere}}$ is the atmospheric pressure at the location where the pressure measurement is made; this atmospheric pressure varies considerably with elevation and is given in Table C.3 in App. C. For example, at the top of Pikes Peak in Colorado, it is about 60 kPa. If neither the atmospheric pressure nor elevation are given, we will assume standard conditions and use $p_{\text{atmosphere}} = 100$ kPa. Figure 1.3 presents a graphic description of the relationship between absolute and gage pressure. Several common representations of the *standard atmosphere* (at 40° latitude at sea level) are included in that figure.

We often refer to a negative pressure, as at B in Fig. 1.3, as a vacuum; it is either a negative pressure or a *vacuum*. A pressure is always assumed to be a gage pressure unless otherwise stated (in thermodynamics the pressure is assumed to be absolute). A pressure of -30 kPa could be stated as 70 kPa absolute or a vacuum of 30 kPa, assuming atmospheric pressure to be 100 kPa (note that the difference between 101.3 and 100 kPa is only 1.3 kPa, a 1.3% error, within engineering acceptability).

We do not define temperature (it requires molecular theory for a definition) but simply state that we use two scales: the Celsius scale and the Fahrenheit scale. The absolute scale when using temperature in degrees Celsius is the kelvin (K) scale and the absolute scale when using temperature in degrees Fahrenheit is the Rankine scale. We use the following conversions:

$$\begin{aligned} K &= {}^\circ C + 273.15 \\ {}^\circ R &= {}^\circ F + 459.67 \end{aligned} \tag{1.10}$$

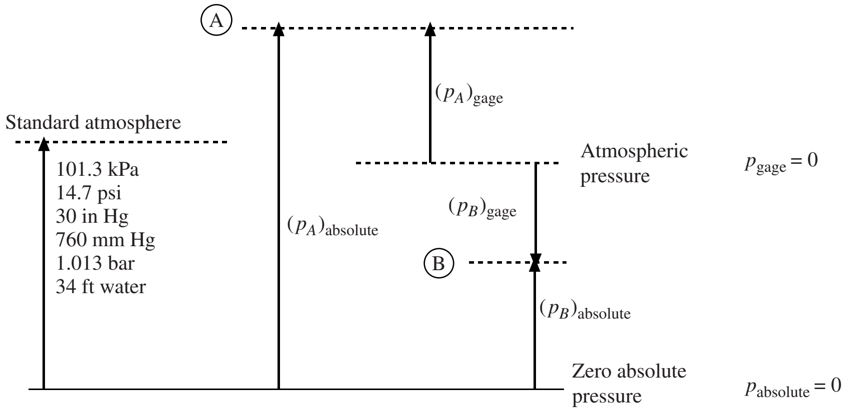


Figure 1.3 Absolute and gage pressure.

In engineering problems we use the numbers 273 and 460, which allows for acceptable accuracy. Note that we do not use the degree symbol when expressing the temperature in degrees kelvin nor do we capitalize the word "kelvin." We read "100 K" as 100 kelvins in the SI system (remember, the SI system is a special metric system).

EXAMPLE 1.2 A pressure is measured to be a vacuum of 23 kPa at a location in Wyoming where the elevation is 3000 m. What is the absolute pressure?

Solution: Use Appendix C to find the atmospheric pressure at 3000 m. We use a linear interpolation to find $p_{\text{atmosphere}} = 70.6$ kPa. Then,

$$p_{\text{abs}} = p_{\text{atm}} + p = 70.6 - 23 = 47.6\,\text{kPa}$$

The vacuum of 23 kPa was expressed as -23 kPa in the equation.

1.5 PROPERTIES OF FLUIDS

A number of fluid properties must be used in our study of fluid mechanics. Mass per unit volume, density, was introduced in Eq. (1.7). We often use weight per unit volume, the *specific weight* γ, related to density by

$$\gamma = \rho g \tag{1.11}$$

where g is the local gravity. For water, γ is taken as 9810 N/m^3 (62.4 lb/ft^3) unless otherwise stated. Specific weight for gases is seldom used.

Specific gravity S is the ratio of the density of a substance to the density of water and is often specified for a liquid. It may be used to determine either the density or the specific weight:

$$\rho = S\rho_{\text{water}} \qquad \gamma = S\gamma_{\text{water}} \tag{1.12}$$

As an example, the specific gravity of mercury is 13.6, which means that it is 13.6 times heavier than water. So, $\rho_{\text{mercury}} = 13.6 \times 1000 = 13\,600\,\text{kg/m}^3$, where we used the density of water to be 1000 kg/m^3, the value used for water if not specified.

Viscosity can be considered to be the internal stickiness of a fluid. It results in shear stresses in a flow and accounts for losses in a pipe or the drag on a rocket. It can be related in a one-dimensional flow to the velocity through a shear stress τ by

$$\tau = \mu \frac{du}{dr} \tag{1.13}$$

where we call du/dr a *velocity gradient*, where r is measured normal to a surface and u is tangential to that surface, as in Fig. 1.4. Consider the units on the quantities in Eq. (1.13): the stress (force divided by an area) has units of N/m^2 (lb/ft^2) so that the viscosity has the units $N{\cdot}s/m^2$ ($lb\text{-}sec/ft^2$).

To measure the viscosity, consider a long cylinder rotating inside a second cylinder, as shown in Fig. 1.4. In order to rotate the inner cylinder with the rotational speed Ω, a torque T must be applied. The velocity of the inner cylinder is $R\Omega$ and the velocity of the outer cylinder is zero. The velocity distribution in the gap h between the cylinders is essentially a linear distribution as shown, so that

$$\tau = \mu \frac{du}{dr} = \mu \frac{R\Omega}{h} \tag{1.14}$$

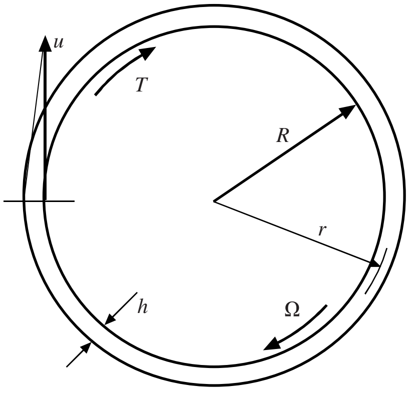


Figure 1.4 Fluid being sheared between two long cylinders.

We can relate the shear to the applied torque as follows:

$$\begin{aligned} T &= \text{stress} \times \text{area} \times \text{moment arm} \\ &= \tau \times 2\pi RL \times R \\ &= \mu \frac{R\Omega}{h} \times 2\pi RL \times R = 2\pi \frac{R^3 \Omega L \mu}{h} \end{aligned} \tag{1.15}$$

where the shear acting on the ends of the long cylinder has been neglected. A device used to measure the viscosity is a *viscometer*.

In this introductory book, we focus our attention on *Newtonian fluids*, those that exhibit a linear relationship between the shear stress and the velocity gradient, as in Eqs. (1.13) and (1.14), as displayed in Fig. 1.5. Many common fluids, such as air, water, and oil are Newtonian fluids. *Non-Newtonian fluids* are classified as *dilatants*, *pseudoplastics*, and *ideal plastics* and are also displayed.

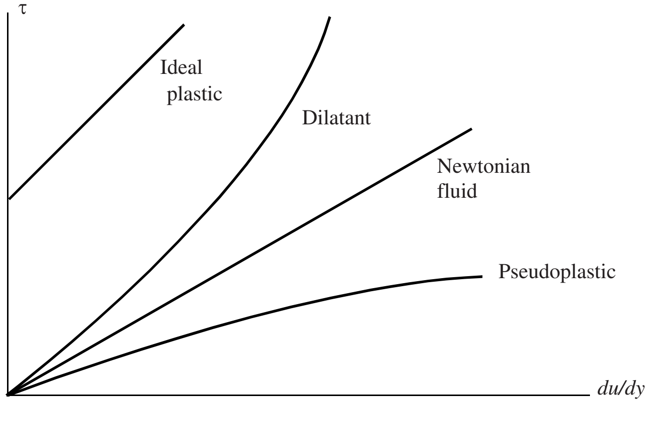


Figure 1.5 Newtonian and Non-Newtonian fluids.

A very important effect of viscosity is to cause the fluid to stick to a surface, the *no-slip condition*. If a surface is moving extremely fast, as a satellite entering the atmosphere, this no-slip condition results in very large shear stresses on the surface; this results in extreme heat which can burn up entering satellites. The no-slip condition also gives rise to wall shear in pipes resulting in pressure drops that require pumps spaced appropriately over the length of a pipe line transporting oil or gas.

Viscosity is very dependent on temperature. Note that in Fig. C.1 in App. C, the viscosity of a liquid decreases with increased temperature but the viscosity of a gas increases with increased temperature. In a liquid the viscosity is due to cohesive forces but in a gas it is due to collisions of molecules; both of these phenomena are insensitive to pressure so we note that viscosity depends on temperature only in both a liquid and a gas, i.e., $\mu = \mu(T)$.

The viscosity is often divided by density in equations, so we have defined the *kinematic viscosity* to be

$$\nu = \frac{\mu}{\rho} \tag{1.16}$$

It has units of m^2/s (ft^2/sec). In a gas we note that kinematic viscosity does depend on pressure since density depends on both temperature and pressure.

The volume of a gas is known to depend on pressure and temperature. In a liquid, the volume also depends slightly on pressure. If that small volume change (or density change) is important, we use the *bulk modulus B*:

$$\mathrm{B} = \mathcal{V} \left.\frac{\Delta p}{\Delta \mathcal{V}}\right|_T = \rho \left.\frac{\Delta p}{\Delta \rho}\right|_T \tag{1.17}$$

The bulk modulus has the same units as pressure. It is included in Table C.1 in App. C. For water at 20°C, it is about 2100 MPa. To cause a 1% change in the volume of water, a pressure of 21 000 kPa is needed. So, it is obvious why we consider water to be incompressible. The bulk modulus is also used to determine the speed of sound in water. It is given by

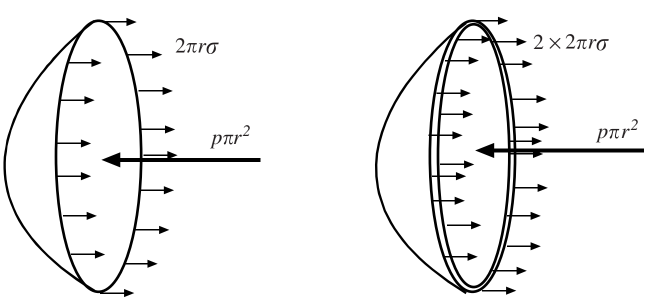
$$c = \sqrt{B/\rho} \tag{1.18}$$

This yields about $c = 1450$ m/s for water at 20°C.

Another property of occasional interest in our study is *surface tension* σ; it results from the attractive forces between molecules, and is included in Table C.1. It allows steel to float, droplets to form, and small droplets and bubbles to be spherical. Consider the free-body diagram of a spherical droplet and a bubble, as shown in Fig. 1.6. The pressure force inside the droplet balances the force due to surface tension around the circumference:

$$p\pi r^2 = 2\pi r\sigma$$

$$\therefore\ p = \frac{2\sigma}{r} \tag{1.19}$$



(*a*) (*b*)

Figure 1.6 Free-body diagrams of (*a*) a droplet and (*b*) a bubble.

Note that in a bubble there are two surfaces so that the force balance provides

$$p = \frac{4\sigma}{r} \tag{1.20}$$

So, if the internal pressure is desired, it is important to know if it is a droplet or a bubble.

A second application where surface tension causes an interesting result is in the rise of a liquid in a capillary tube. The free-body diagram of the water in the tube is shown in Fig. 1.7. Summing forces on the column of liquid gives

$$\sigma \pi D \cos\beta = \rho g \frac{\pi D^2}{4} h \tag{1.21}$$

where the right-hand side is the weight W. This provides the height the liquid will climb in the tube:

$$h = \frac{4\sigma \cos\beta}{\gamma D} \tag{1.22}$$

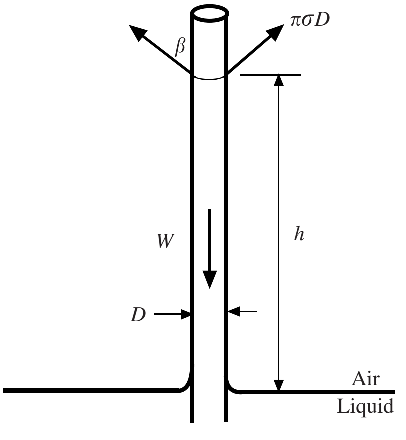


Figure 1.7 The rise of a liquid in a small tube.

The final property to be introduced in this section is vapor pressure. Molecules escape and reenter a liquid that is in contact with a gas, such as water in contact with air. The *vapor pressure* is that pressure at which there is equilibrium between the escaping and reentering molecules. If the pressure is below the vapor pressure, the molecules will escape the liquid; it is called *boiling* when water is heated to the temperature at which the vapor pressure equals the atmospheric pressure. If the local pressure is decreased to the vapor pressure, vaporization also occurs. This can happen when liquid flows through valves, elbows, or turbine blades, should the pressure become sufficiently low; it is then called *cavitation*. The vapor pressure is found in Table C.1 in App. C.

EXAMPLE 1.3 A $0.5\,\text{m} \times 2\,\text{m}$ flat plate is towed at 5 m/s on a 2-mm-thick layer of SAE-30 oil at 38°C that separates it from a flat surface. The velocity distribution between the plate and the surface is assumed to be linear. What force is required if the plate and surface are horizontal?

Solution: The velocity gradient is calculated to be

$$\frac{du}{dy} = \frac{\Delta u}{\Delta y} = \frac{5-0}{0.002} = 2500\,\text{m/(s·m)}$$

The force is the stress multiplied by the area:

$$F = \tau \times A = \mu \frac{du}{dy} \times A = 0.1 \times 2500 \times 0.5 \times 2 = 250\,\text{N}$$

Check the units to make sure the units of the force are newtons. The viscosity of the oil was found in Fig. C.1.

EXAMPLE 1.4 A machine creates small 0.5-mm-diameter bubbles of 20°C water. Estimate the pressure that exists inside the bubbles.

Solution: Bubbles have two surfaces leading to the following estimate of the pressure:

$$p = \frac{4\sigma}{r} = \frac{4 \times 0.0736}{0.0005} = 589\,\text{Pa}$$

where the surface tension was taken from Table C.1.

1.6 THERMODYNAMIC PROPERTIES AND RELATIONSHIPS

A course in thermodynamics and/or physics usually precedes a fluid mechanics course. Those properties and relationships that are presented in those courses that are used in our study of fluids are included in this section. They are of particular use when compressible flows are studied, but they also find application to liquid flows.

The *ideal gas law* takes the two forms

$$p\mathcal{V} = mRT \qquad \text{or} \qquad p = \rho RT \tag{1.23}$$

where the pressure p and the temperature T must be absolute quantities. The gas constant R is found in Table C.4 in App. C.

Enthalpy is defined as

$$H = m\tilde{u} + p\mathcal{V} \qquad \text{or} \qquad h = \tilde{u} + pv \tag{1.24}$$

where $\tilde{u}$ is the *specific internal energy*. In an ideal gas we can use

$$\Delta h = \int c_p \, dT \qquad \text{and} \qquad \Delta \tilde{u} = \int c_v \, dT \tag{1.25}$$

where c_p and c_v are the specific heats also found in Table C.4. The specific heats are related to the gas constant by

$$c_p = c_v + R \tag{1.26}$$

The ratio of specific heats is

$$k = \frac{c_p}{c_v} \tag{1.27}$$

For liquids and solids, and for most gases over relatively small temperature differences, the specific heats are essentially constant and we can use

$$\Delta h = c_p \Delta T \qquad \text{and} \qquad \Delta \tilde{u} = c_v \Delta T \tag{1.28}$$

For *adiabatic* (no heat transfer) *quasi-equilibrium* (properties are constant throughout the volume at an instant) *processes*, the following relationships can be used for an ideal gas assuming constant specific heats:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad \frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^k \tag{1.29}$$

The adiabatic, quasi-equilibrium process is also called an *isentropic process*.

A small pressure wave with a relatively low frequency travels through a gas with a wave speed of

$$c = \sqrt{kRT} \tag{1.30}$$

Finally, the *first law of thermodynamics* will be of use in our study; it states that when a *system*, a fixed set of fluid particles, undergoes a change of state from state 1 to state 2, its energy changes from

E_1 to E_2 as it exchanges energy with the surroundings in the form of work W_{1-2} and heat transfer Q_{1-2}. This is expressed as

$$Q_{1-2} - W_{1-2} = E_2 - E_1 \tag{1.31}$$

To calculate the heat transfer from given temperatures and areas, a course on heat transfer is required, so it is typically a given quantity in thermodynamics and fluid mechanics. The work, however, is a quantity that can often be calculated; it is a force times a distance and is often due to the pressure resulting in

$$\begin{aligned} W_{1-2} &= \int_{l_1}^{l_2} F\,dl \\ &= \int_{l_1}^{l_2} pA\,dl = \int_{\not{V}_1}^{\not{V}_2} p\,d\not{V} \end{aligned} \tag{1.32}$$

The energy E considered in a fluids course consists of kinetic energy, potential energy, and internal energy:

$$E = m\left(\frac{V^2}{2} + gz + \tilde{u}\right) \tag{1.33}$$

where the quantity in the parentheses is the specific energy e. (We use $\tilde{u}$ to represent specific internal energy since u is used for a velocity component.) If the properties are constant at an exit and an entrance to a flow, and there is no heat transferred and no losses, the above equation can be put in the form

$$\frac{V_2^2}{2g} + \frac{p_2}{\gamma_2} + z_2 = \frac{V_1^2}{2g} + \frac{p_1}{\gamma_1} + z_1 \tag{1.34}$$

This equation does not follow directly from Eq. (1.31); it takes some effort to derive Eq. (1.34). An appropriate text could be consulted, but we will derive it later in this book. It is presented here as part of our review of thermodynamics.

Solved Problems

1.1 Show that the units on viscosity given in Table 1.1 are correct using (*a*) SI units and (*b*) English units.

Viscosity is related to stress by

$$\mu = \tau \frac{dy}{du}$$

In terms of units this is

$$[\mu] = \frac{\text{N}}{\text{m}^2}\frac{\text{m}}{\text{m/s}} = \frac{\text{N}\cdot\text{s}}{\text{m}^2} \qquad [\mu] = \frac{\text{lb}}{\text{ft}^2}\frac{\text{ft}}{\text{ft/sec}} = \frac{\text{lb-sec}}{\text{ft}^2}$$

1.2 If force, length, and time are selected as the three fundamental dimensions, what are the dimensions on mass?

We use Newton's second law, which states that

$$F = ma$$

In terms of dimensions this is written as

$$F = M\frac{L}{T^2} \qquad \therefore\ M = \frac{FT^2}{L}$$

1.3 The mean free path of a gas is $\lambda = 0.225m/(\rho d^2)$, where d is the molecule's diameter, m is its mass, and ρ the density of the gas. Calculate the mean free path of air at 10 000 m elevation, the elevation where many commercial airplanes fly. For an air molecule $d = 3.7 \times 10^{-10}$ m and $m = 4.8 \times 10^{-26}$ kg.

Using the formula given, the mean free path at 10 000 m is

$$\lambda = 0.225 \times \frac{4.8 \times 10^{-26}}{0.4136(3.7 \times 10^{-10})^2} = 8.48 \times 10^{-7}\ \text{m or } 0.848\ \mu\text{m}$$

where the density was found in Table C.3.

1.4 A vacuum of 25 kPa is measured at a location where the elevation is 3000 m. What is the absolute pressure in millimeters of mercury?

The atmospheric pressure at the given elevation is found in Table C.3. It is interpolated to be

$$p_{\text{atm}} = 79.84 - \frac{1}{2}(79.84 - 61.64) = 70.7\,\text{kPa}$$

The absolute pressure is then

$$p = p_{\text{gage}} + p_{\text{atm}} = -25 + 70.7 = 45.7\,\text{kPa}$$

In millimeters of mercury this is

$$h = \frac{p}{\rho_{\text{Hg}}g} = \frac{45\,700}{(13.6 \times 1000)9.81} = 0.343\,\text{m or } 343\,\text{mm}$$

1.5 A flat 30-cm-diameter disk is rotated at 800 rpm at a distance of 2 mm from a flat, stationary surface. If SAE-30 oil at 20°C fills the gap between the disk and the surface, estimate the torque needed to rotate the disk.

Since the gap is small, a linear velocity distribution will be assumed. The shear stress acting on the disk will be

$$\tau = \mu\frac{\Delta u}{\Delta y} = \mu\frac{r\omega}{h} = 0.38 \times \frac{r(800 \times 2\pi/60)}{0.002} = 15\,900r$$

where the viscosity is found from Fig. C.1 in App. C. The shear stress is integrated to provide the torque:

$$T = \int_A r\,dF = \int_A r\tau 2\pi r\,dr = 2\pi\int_0^{0.15} 15\,900r^3\,dr = 10^5 \times \frac{0.15^4}{4} = 12.7\,\text{N}\cdot\text{m}$$

Note: The answer is not given to more significant digits since the viscosity is known to only two significant digits. More digits in the answer would be misleading.

1.6 Water is usually assumed to be incompressible. Determine the percentage volume change in 10 m^3 of water at 15°C if it is subjected to a pressure of 12 MPa from atmospheric pressure.

The volume change of a liquid is found using the bulk modulus of elasticity (see Eq. (1.17)):

$$\Delta V = -V\frac{\Delta p}{B} = -10 \times \frac{12\,000\,000}{214 \times 10^7} = -0.0561\,\text{m}^3$$

The percentage change is

$$\%\,\text{change} = \frac{V_2 - V_1}{V_1} \times 100 = \frac{-0.0561}{10} \times 100 = -0.561\%$$

This small percentage change can usually be ignored with no significant influence on results, so water is essentially incompressible.

1.7 Water at 30°C is able to climb up a clean glass of 0.2-mm-diameter tube due to surface tension. The water-glass angle is 0° with the vertical ($\beta = 0$ in Fig. 1.7). How far up the tube does the water climb?

The height that the water climbs is given by Eq. (1.22). It provides

$$h = \frac{4\sigma \cos\beta}{\gamma D} = \frac{4 \times 0.0718 \times 1.0}{(996 \times 9.81)0.0002} = 0.147\,\text{m or } 14.7\,\text{cm}$$

where the properties of water come from Table C.1 in App. C.

1.8 Explain why it takes longer to cook potatoes by boiling them in an open pan on the stove in a cabin in the mountains where the elevation is 3200 m.

Water boils when the temperature reaches the vapor pressure of the water; it vaporizes. The temperature remains constant until all the water is boiled away. The pressure at the given elevation is interpolated in Table C.3 to be 69 kPa. Table C.1 provides the temperature of slightly less than 90°C for a vapor pressure of 69 kPa, i.e., the temperature at which the water boils. Since it is less than the 100°C at sea level, the cooking process is slower. A pressure cooker could be used since it allows a higher temperature by providing a higher pressure inside the cooker.

1.9 A car tire is pressurized in Ohio to 250 kPa when the temperature is −15°C. The car is driven to Arizona where the temperature of the tire on the asphalt reaches 65°C. Estimate the pressure in the tire in Arizona assuming no air has leaked out and that the volume remains constant.

Assuming the volume does not change, the ideal gas law requires

$$\frac{p_2}{p_1} = \frac{mR\mathcal{V}_1 T_2}{mR\mathcal{V}_2 T_1} = \frac{T_2}{T_1}$$

$$\therefore\ p_2 = p_1 \frac{T_2}{T_1} = (250 + 100) \times \frac{423}{258} = 574\,\text{kPa abs or } 474\,\text{kPa gage}$$

since the mass also remains constant. (This corresponds to 37 lb/in^2 in Ohio and 70 lb/in^2 in Arizona.)

1.10 A farmer applies nitrogen to a crop from a tank pressurized to 1000 kPa absolute at a temperature of 25°C. What minimum temperature can be expected in the nitrogen if it is released to the atmosphere?

The minimum exiting temperature occurs for an isentropic process (see Eq. (1.29)), which is

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k} = 298 \times \left(\frac{100}{1000}\right)^{0.4/1.4} = 154\,\text{K or } -119^\circ\text{C}$$

Such a low temperature can cause serious injury should a line break and nitrogen impact the farmer.

Supplementary Problems

1.11 There are three basic laws in our study of fluid mechanics: the conservation of mass, Newton's second law, and the first law of thermodynamics. (*a*) State an integral quantity for each of the laws and (*b*) state a quantity defined at a point for each of the laws.

Dimensions, Units, and Physical Quantities

1.12 Verify the SI units presented in Table 1.2 for the following:

(*a*) Force (*b*) Specific weight (*c*) Surface tension
(*d*) Torque (*e*) Viscosity (*f*) Work

1.13 Verify the dimensions presented in Table 1.2 for the following:

(*a*) Force (*b*) Specific weight (*c*) Surface tension
(*d*) Torque (*e*) Viscosity (*f*) Work

1.14 Select the *F–L–T* system of dimensions and state the dimensions on the following:

(*a*) Force (*b*) Specific weight (*c*) Surface tension
(*d*) Torque (*e*) Viscosity (*f*) Work

1.15 An equation that provides the flow rate in an open channel is given by

$$Q = kAR^{\frac{2}{3}}S^{\frac{1}{2}}$$

where k is a constant, A is the area of the channel, R is a radius, and S is a slope. Determine both the dimensions and the SI units on k.

1.16 Express the following using powers rather than prefixes:

(*a*) 200 cm^2 (*b*) 500 mm^3 (*c*) 10 μm
(*d*) 32 MPa (*e*) 400 kN (*f*) 5 nN

1.17 Express the following using prefixes rather than powers:

(*a*) 2×10^{-8} m (*b*) 5×10^{8} m (*c*) 2×10^{-5} Pa
(*d*) 32×10^{8} Pa (*e*) 4×10^{-6} N (*f*) 8×10^{11} N

1.18 Quantities are often given in units that are unacceptable when using the SI system of units. Convert each of the following to acceptable SI units:

(*a*) 60 mi/h (*b*) 35 lb/in^2 (*c*) 2 g/cm^3
(*d*) 22 slug/h (*e*) 20 ft^3/min (*f*) 50 kW·h

1.19 What force is needed to accelerate a 1500-kg car at 3 m/s^2:

(*a*) on the horizontal? (*b*) on a 20° incline?

1.20 An astronaut weighs 850 N on earth. Calculate the weight of the astronaut on the moon, where $g = 5.4$ ft/sec^2.

1.21 Estimate the mean free path of air molecules, using information from Solved Problem 1.3, at an elevation of

(*a*) 750 m (*b*) 40 000 m (*c*) 80 000 m

Pressure and Temperature

1.22 A pressure of 28 kPa is measured at an elevation of 2000 m. What is the absolute pressure in

(*a*) kPa (*b*) lb/in^2 (*c*) mm of Hg (*d*) ft of water

1.23 A gage reads a vacuum of 24 kPa. What is the absolute pressure at

(*a*) sea level (*b*) 4000 m (*c*) 8000 m

1.24 The equation $p(z) = p_0 e^{-gz/RT_0}$ is a good approximation to the pressure in the atmosphere. Estimate the pressure at $z = 6000$ m using this equation and calculate the percent error using the more accurate value found in Table C.3. Assume $p_0 = 100$ kPa and $T_0 = 15°$C.

1.25 A pressure of 20 kPa and a shear stress of 80 Pa act on a 0.8-m^2-flat surface. Calculate the normal force F_n, the tangential shear force F_t, and the total force F acting on the surface. Also, calculate the angle the total force makes with respect to a normal coordinate.

1.26 A temperature of 20°C is measured at a certain location. What is the temperature in

(*a*) kelvins (*b*) degrees Fahrenheit (*c*) degrees Rankine

Properties of Fluids

1.27 A fluid mass occupies 2 m^3. Calculate the density, specific weight, and specific gravity if the fluid mass is

(*a*) 4 kg (*b*) 8 kg (*c*) 15 kg

1.28 A formula that provides a good estimate of the density in kg/m^3 of water is

$$\rho_{\text{water}} = 1000 - \frac{(T-4)^2}{180}$$

where the temperature T is in degrees Celsius. Use this formula and find the density of water at 80°C. What is the error?

1.29 The specific weight of a fluid is 11 200 N/m^3. Calculate the mass contained in 2 m^3

(*a*) Using the standard gravity.
(*b*) Using the maximum gravity on the earth's surface.
(*c*) Using the minimum gravity on the earth's surface.

1.30 The specific gravity of mercury is given by the formula

$$S_{\text{Hg}} = 13.6 - 0.0024T$$

where the temperature is in degrees Celsius. What is the specific weight of mercury at 45°C? Calculate the error if $S_{\text{Hg}} = 13.6$ were used at 45°C.

1.31 A viscometer, used to measure the viscosity of a liquid, is composed of two 12-cm-long concentric cylinders with radii 4 and 3.8 cm. The outer cylinder is stationary and the inner one rotates. If a torque of 0.046 N·m is measured at a rotational speed of 120 rpm, estimate the viscosity of the liquid. Neglect the contribution to the torque from the cylinder ends and assume a linear velocity profile.

1.32 Water at 20°C flows in a 0.8-cm-diameter pipe with a velocity distribution of $u(r) = 5(1 - r^2/16 \times 10^{-6})$ m/s. Calculate the shear stress on (*a*) the pipe wall, (*b*) at a radius where $r = 0.2$ cm, and (*c*) at the centerline of the pipe.

1.33 SAE-30 oil at 30°C fills the gap between a 40-cm-diameter flat disk rotating 0.16 cm above a flat surface. Estimate the torque needed to rotate the disk at

(*a*) 200 rpm (*b*) 600 rpm (*c*) 1200 rpm

1.34 A 2-m-long, 4-cm-diameter shaft rotates inside an equally long 4.02-cm-diameter cylinder. If SAE-10W oil at 25°C fills the gap between the concentric cylinders, determine the torque and horsepower needed to rotate the shaft at 1200 rpm.

1.35 A 0.1-m^3 volume of water is observed to be 0.0982 m^3 after a pressure is applied. What is that pressure?

1.36 How long would it take a small wave to travel under 22°C water a distance of 800 m?

1.37 The *coefficient of thermal expansion* α_T allows the expansion of a liquid to be determined using the equation $\Delta V = \alpha_T V \Delta T$. Calculate the decrease in 2 m^3 of 40°C water if the temperature is lowered by 10°C. What pressure would be needed to cause the same decrease in volume?

1.38 Estimate the pressure inside a droplet of 20°C water and a bubble of 20°C water if their diameters are

(*a*) 40 μm (*b*) 20 μm (*c*) 4 μm

1.39 How high would 20°C water climb in a 24-μm-diameter vertical capillary tube if it makes an angle of 20° with the wall of the tube?

1.40 Mercury makes an angle of 130° with respect to the vertical when in contact with clean glass. How far will mercury depress in a clean, 10-μm-diameter glass tube if $\sigma_{Hg} = 0.467$ N/m.

1.41 A steel needle of length L and radius r will float in water if carefully placed. Write an equation that relates the various variables for a floating needle assuming a vertical surface tension force.

1.42 Using the equation developed in Supplementary Problem 1.41, determine if a 10-cm-long, 1-mm-diameter steel needle will float in 20°C water. $\rho_{steel} = 7850$ kg/m^3.

1.43 Derive an equation that relates the vertical force T needed to just lift a thin wire loop from a liquid assuming a vertical surface tension force. The wire radius is r and the loop diameter is D. Assume $D \gg r$.

Thermodynamic Properties and Relationships

1.44 Two kilograms of 40°C air is contained in a 4-m^3 volume. Calculate the pressure, density, specific volume, and specific weight.

1.45 The temperature outside a house is –20°C and inside it is 20°C. What is the ratio of the density of the outside air to the density of the inside air? Would infiltration, which results from cracks around the windows, doors, and siding, etc., occur even with no wind causing a pressure difference?

1.46 A car with tires pressurized to 240 kPa (35 lb/in^2) leaves Phoenix with the tire temperature at 50°C. Estimate the tire pressure (in kPa and lb/in^2) when the car arrives in Alaska with a tire temperature of −30°C.

1.47 Estimate the mass and weight of the air contained in a classroom where Thermodynamics is taught. Assume the dimensions to be 3.2 m $\times$ 8 m $\times$ 20 m.

1.48 Calculate the weight of the column of air contained above a 1-m^2 area of atmospheric air from sea level to the top of the atmosphere.

1.49 A 100 kg body falls from rest from a height of 100 m above the ground. Calculate its maximum velocity when it hits the ground. (*a*) Use the maximum value for gravity, (*b*) use the minimum value for gravity, and (*c*) use the standard value for gravity. (The minimum value is at the top of Mt Everest and the maximum value is at bottom of the lowest trench in the ocean.)

1.50 Air expands from a tank maintained at 18°C and 250 kPa to the atmosphere. Estimate its minimum temperature as it exits.

1.51 Air at 22°C is received from the atmosphere into a 200 cm^3 cylinder. Estimate the pressure and temperature if it is compressed isentropically to 10 cm^3.

1.52 Two cars, each with a mass of 6000 kg, hit head on each traveling at 80 km/h. Estimate the increase in internal energy absorbed by the materials in each car.

1.53 A 6500-kg car is traveling at 90 km/h and suddenly brakes to a stop. If the four brake disks absorb all the energy, estimate the maximum increase in temperature of those disks, assuming the disks absorb the energy equally. The 0.7-cm-thick, 30-cm-diameter disks are made of steel. Use $\rho_{\text{steel}} = 7850\,\text{kg/m}^3$ and $(c_p)_{\text{steel}} = 0.5\,\text{kJ/kg}\cdot°\text{C}$.

1.54 Calculate the speed of sound in: (*a*) air at 0°C, (*b*) nitrogen at 20°C, (*c*) hydrogen at 10°C, (*d*) air at 100°C, and (*e*) oxygen at 50°C.

1.55 Lightning is observed and thunder is heard 1.5 s later. About how far away did the lightning occur?

Answers to Supplementary Problems

1.11 (*a*) The mass flux into a jet engine; the force of air on a window; the heat transfer through a wall. (*b*) The velocity V; the pressure p; the temperature T.

1.12 (*a*) $F = $ ma. N $=$ kg·m/s^2, etc.

1.13 (*a*) $F = $ ma. $F = ML/T^2$, etc.

1.14 (*b*) $\gamma = $ weight/volume $= F/L^3$, etc.

1.15 $L^{1/3}/T$, m$^{1/3}$/s

1.16 (*a*) $2 \times 10^{-2}\,\text{m}^2$ (*b*) $5 \times 10^{-7}\,\text{m}^3$ (*c*) $10^{-5}\,\text{m}$ (*d*) $32 \times 10^6\,\text{Pa}$ (*e*) $4 \times 10^5\,\text{N}$ (*f*) $5 \times 10^{-9}\,\text{N}$

1.17 (*a*) 20 nm (*b*) 500 Mm (*c*) 20 μm (*d*) 320 MPa (*e*) 4 μN (*f*) 800 GN

1.18 (*a*) 96.56 m/s (*b*) 241 kPa (*c*) 2000 kg/m^3 (*d*) 0.0892 kg/s (*e*) $1.573 \times 10^{-4}\,\text{m}^3/\text{s}$ (*f*) 80 MJ

1.19 (*a*) 4500 N (*b*) 9533 N

1.20 468 N

1.21 (*a*) 0.000308 mm (*b*) 0.0877 mm (*c*) 17.5 mm

1.22 (*a*) 107.5 (*b*) 15.6 (*c*) 806 (*d*) 36

1.23 (*a*) 77.3 kPa (*b*) 37.6 kPa (*c*) 11.65 kPa

1.24 49.1 kPa, 4.03%

1.25 16 kN, 64 N, 0.229°

1.26 293 K, 68°F, 528°R

1.27 (*a*) 2 kg/m^3, 19.62 N/m^3, 0.002 (*b*) 4 kg/m^3, 39.24 N/m^3, 0.004
(*c*) 7.5 kg/m^3, 73.6 N/m^3, 0.0075

1.28 968 kg/m^3, −0.4%

1.29 (*a*) 2283 kg (*b*) 2279 kg (*c*) 2293 kg

1.30 13.49, −0.8%

1.31 0.1628 N·s/m^2

1.32 (*a*) 2.5 N/m^2 (*b*) 1.25 N/m^2 (*c*) 0 N/m^2

1.33 (*a*) 7.2 N·m, 0.2 hp (*b*) 21 N·m, 1.81 hp (*c*) 43 N·m, 7.2 hp

1.34 0.88 N·m, 0.15 hp

1.35 37.8 MPa

1.36 0.539 s

1.37 −0.0076 m^3, 7.98 MPa

1.38 (*a*) 3680 Pa, 7360 Pa (*b*) 36.8 Pa, 73.6 Pa (*c*) 7.36 Pa, 14.72 Pa

1.39 1.175 m

1.40 −0.900 m

1.41 $2\sigma > \rho\pi r^2$

1.42 Yes

1.43 $\pi D(2\sigma + \gamma_{\text{wire}}\pi r^2)$

1.44 45 kPa, 0.5 kg/m^3, 2 m^3/kg, 4.905 N/m^3

1.45 1.158, yes

1.46 156 kPa, 22.7 lb/in^2

1.47 609 kg, 5970 N

1.48 100 kN

1.49 44.34 m/s, 44.20 m/s, 44.29 m/s

1.50 -69.6°C

1.51 6630 kPa, 705°C

1.52 1.48 MJ

1.53 261°C

1.54 (*a*) 331 m/s (*b*) 349 m/s (*c*) 1278 m/s (*d*) 387 m/s (*e*) 342 m/s

1.55 515 m