# Paper III (Operational Research)

# MSC (Computer Science) Semester- III 2022-23.

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To solve linear programming using R studio, we need to install lpsolve package Install.packages("lpsolve")

# PRACTICAL 1

# **GRAPHICAL METHOD USING R PROGRAMMING**

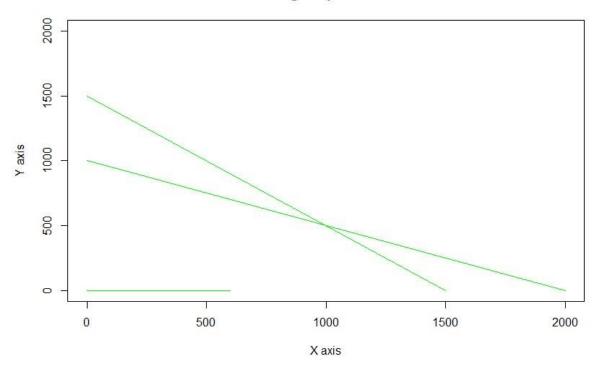
```
# R Program
#Find a geometrical interpretation and solution as well for the following LP
problem
\#Max z = 3x1 + 5x2
#subject to constraints:
#x1+2x2<=2000
#x1+x2<=1500
#x2<=600
#x1,x2>=0
# Load IpSolve
require(lpSolve)
## Set the coefficients of the decision variables -> C of objective function
C <- c(3,5)
# Create constraint martix B
A <- matrix(c(1, 2,
        1, 1,
        0, 1
), nrow=3, byrow=TRUE)
# Right hand side for the constraints
B <- c(2000,1500,600)
```

```
# Direction of the constraints
constranints direction <- c("<=", "<=", "<=")
# Create empty example plot
plot.new()
plot.window(xlim=c(0,2000), ylim=c(0,2000))
axis(1) axis(2)
title(main="LPP using Graphical method")
title(xlab="X axis") title(ylab="Y axis")
box() # Draw one line
segments(x0 = 2000, y0 = 0, x1 = 0, y1 = 1000, col = "green")
segments(x0 = 1500, y0 = 0, x1 = 0, y1 = 1500, col = "green")
segments(x0 = 0, y0 = 0, x1 = 600, y1 = 0, x1 
# Find the optimal solution optimum <-
lp(direction="max", objective.in = C,
const.mat = A, const.dir =
constranints direction, const.rhs = B,
all.int = T)
# Print status: 0 = success, 2 = no feasible solution
print(optimum$status)
# Display the optimum values for x1,x2
best_sol <- optimum$solution</pre>
names(best_sol) <- c("x1", "x2")
print(best sol)
# Check the value of objective function at optimal point
print(paste("Total cost: ", optimum$objval, sep=""))
```

```
[Workspace loaded from ~/.RData]
> # Right hand side for the constraints
> B < -c(2000, 1500, 600)
> # R Program
> # Load lpSolve
> require(lpSolve)
Loading required package: lpSolve
> ## Set the coefficients of the decision variables -> C
> C < -c(3,5)
> # Create constraint martix B
> A <- matrix(c(1, 2,
                1, 1,
                0, 1 +
), nrow=3, byrow=TRUE)
> # Right hand side for the constraints
> B < -c(2000, 1500, 600)
> # Direction of the constraints
> constranints_direction <- c("<=", "<=", "<=")
> # Create empty example plot
> #plot(2000, 2000, col = "white", xlab = "", ylab = "")
> plot.new()
> plot.window(xlim=c(0,2000), ylim=c(0,2000))
> axis(1)
> axis(2)
> title(main="LPP using Graphical method")
> title(xlab="X axis")
> title(ylab="Y axis")
> box()
> # Draw one line
> segments(x0 = 2000, y0 = 0, x1 = 0, y1 = 1000, col = "green")
> segments(x0 = 1500, y0 = 0, x1 = 0, y1 = 1500, col = "green")
> segments (x0 = 0, y0 = 0, x1 = 600, y1 = 0, col = "green")
>
> # Find the optimal solution
> optimum <- lp(direction="max",</pre>
                objective.in = C_{i}
                const.mat = A,
                const.dir = constranints direction,
                const.rhs = B_{i}
                all.int = T)
> # Print status: 0 = success, 2 = no feasible solution
> print(optimum$status)
[1] 0
> # Display the optimum values for x1,x2
> best sol <- optimum$solution >
names(best sol) <- c("x1", "x2")</pre>
```

```
> print(best_sol)
x1     x2 1000 500
>
> # Check the value of objective function at optimal point
> print(paste("Total cost: ", optimum$objval, sep=""))
[1] "Total cost: 5500"
```

# LPP using Graphical method



**PRACTICAL 2** 

# Simplex Method with 2 variables using Python

from scipy.optimize import linprog

#Max z=3x1+2x2

#subject to

#x1 + x2 <= 4

#x1 - x2 <=2

#x1,x2>=0

obj = [-3, -2]

lhs\_ineq = [[ 1, 1], # Red constraint left side

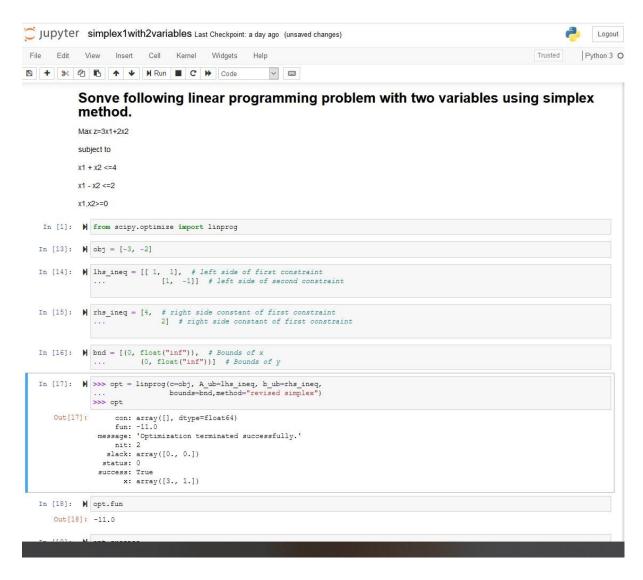
... [1, -1]] # Blue constraint left side

opt.x

```
rhs_ineq = [4, # Red constraint right side
... 2] # Blue constraint right side
bnd = [(0, float("inf")), # Bounds of x
... (0, float("inf"))] # Bounds of y

>>> opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
... bounds=bnd,method="revised simplex")
>>> opt

opt.fun
```



# **Simplex Method with 3 variables using Python**

from scipy.optimize import linprog

#Min z = x1-3x2+2x3

#subject to

#3x1-x2+3x3<=7

#-2x1+4x2<=12

#-4x1+3x2+8x3<=10

#x1,x2,x3>=0

obj = [1, -3, 2]

lhs\_ineq = [[ 3, -1, 3], # Red constraint left side

... [-2, 4, 0], # Blue constraint left side ...

[-4, 3, 8]] # Yellow constraint left side

rhs\_ineq = [7, # Red constraint right side

... 12, # Blue constraint right side ...

10] # Yellow constraint right side

bnd = [(0, float("inf")), # Bounds of x

... (0, float("inf")),

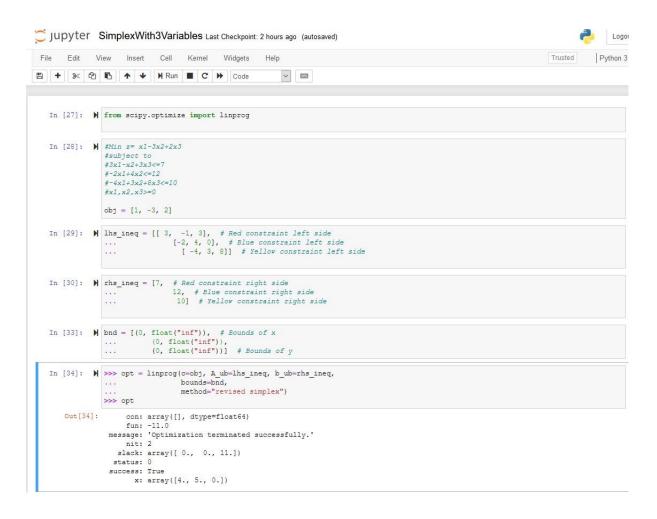
... (0, float("inf"))] # Bounds of y

>>> opt = linprog(c=obj, A\_ub=lhs\_ineq, b\_ub=rhs\_ineq,

... bounds=bnd,

... method="revised simplex")

>>> opt



# **Simplex Method with Equality Constraints Using Python**

```
from scipy.optimize import linprog
\#Max z=x+2y
#subject to
#2x+y<=20
#-4x+5y<=10
#-x+2y>=-2
\#-x+5y=15
#x,y>=0
obj = [-1, -2]
lhs_ineq = [[ 2, 1], # Red constraint left side
         [-4, 5], # Blue constraint left side
        [1, -2]] # Yellow constraint left
side
rhs_ineq = [20, # Red constraint right side
         10, # Blue constraint right side
         2] # Yellow constraint right side
lhs_eq = [[-1, 5]] # Green constraint left side rhs_eq =
[15]
       # Green constraint right side
bnd = [(0, float("inf")), # Bounds of x
      (0, float("inf"))] # Bounds of y
opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
          A_eq=lhs_eq, b_eq=rhs_eq, bounds=bnd,
...
```

... method="revised simplex")

Opt

## method ="revised simplex" solves linear programming problem using two phase simplex method.

```
con: array([0.]) fun: -
     16.8181818181817
message: 'Optimization terminated successfully.'
   slack: array([ 0. , 18.18181818, 3.36363636])
  status: 0
 success: True x: array([7.72727273]
        4.54545455])
 File Edit View Insert Cell Kernel Widgets Help
                                                                                   Trusted Python 3 O
 v 🖃
   In [1]: M from scipy.optimize import linprog
   In [2]: | #Max z=x+2y
            #subject to
#2x+y<=20
#-4x+5y<=10
           \#-x+2y>=-2
\#-x+5y=15
           \#x,y>=0
obj = [-1, -2]
   In [5]: M lhs_eq = [[-1, 5]] # Green constraint left side
   In [6]: M rhs_eq = [15] # Green constraint right side
   In [8]: M opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
                 A_eq=lhs_eq, b_eq=rhs_eq, bounds=bnd,
method="revised simplex")
   In [9]: N opt
            message: 'Optimization terminated successfully.'
nit: 3
             con: array([0.])
      Out [9]:
              nit: 3
slack: array([ 0. , 18.18181818, 3.36363636])
            status: 0
success: True
                x: array([7.72727273, 4.54545455])
```

**PRACTICAL 5 BigM Simplex Method using Python** 

Solve Following linear programming problem using Big M Simplex method.

```
Min z= 4x1 + x2 subjected to: 3x1 + 4x2 >= 20 x1 + 5x2 >= 15 x1, x2 >= 0
```

from scipy.optimize import linprog obj = [4, 1]

lhs\_ineq = [[ -3, -4], # left side of first constraint

... [-1, -5]] # right side of first constraint

rhs\_ineq = [-20, # right side of first constraint ...

-15] # right side of Second constraint

bnd = [(0, float("inf")), #Bounds of x1]

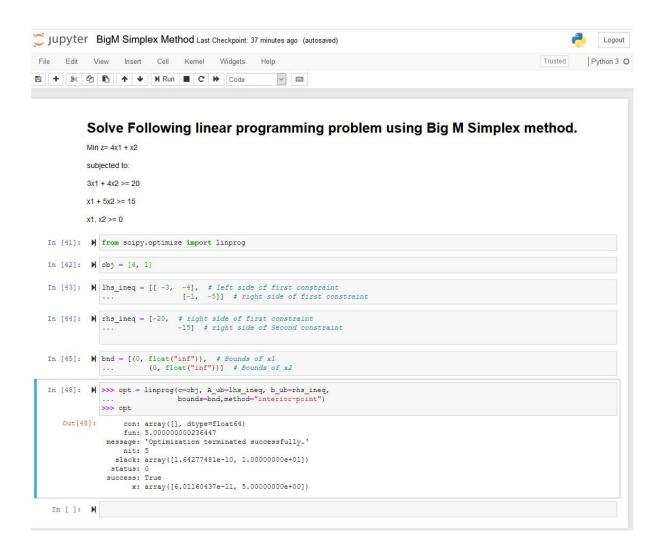
... (0, float("inf"))] # Bounds of x2

>>> opt = linprog(c=obj, A\_ub=lhs\_ineq, b\_ub=rhs\_ineq,

... bounds=bnd,method="interior-point")

>>> opt

## method =" interior-point" solves linear programming problem using default simplex method.



### **RESOURCE ALLOCATION PROBLEM BY SIMPLEX METHOD**

# Use SciPy to solve the resource allocation problem stated as follows:

Max 
$$z=20x1 + 12x2 + 40x3 + 25x4$$
 .......(profit) subjected to:  
 $x1 + x2 + x3 + x4 <= 50$  ----- (manpower)  
 $3x1 + 2x2 + x3$  <= 100 ----- (material A)  
 $x2 + 2x3$  <= 90 ----- (material B)

from scipy.optimize import linprog

rhs\_ineq = [50, # Manpower

... 100, # Material A

... 90] # Material B

opt = linprog(c=obj, A\_ub=lhs\_ineq, b\_ub=rhs\_ineq,
... method="revised simplex")

Opt



```
Use SciPy to solve the resource allocation problem stated as follows:
          subjected to:
             x1 + x2 + x3 + x4 <= 50 ----- (manpower)
             3x1 + 2x2 + x3 <= 100 ----- (material A)
                     x2 + 2x3 <= 90 -----(material B)
                 x1, x2, x3, x4 >= 0
In [12]: | from scipy.optimize import linprog
In [13]: M obj = [-20, -12, -40, -25]
                                            #profit objective function
In [14]: M lhs_ineq = [[1, 1, 1, 1], # Manpower ... [3, 2, 1, 0], # Material A ... [0, 1, 2, 3]] # Material B
In [17]: M opt
              con: array([], dtype=float64)
fun: -1900.0
message: 'Optimization terminated successfully.'
   Out[17]:
                 slack: array([ 0., 40., 0.])
               status: 0
success: True
                    x: array([ 5., 0., 45., 0.])
          The result tells you that the maximal profit is 1900 and corresponds to x_1 = 5 and x_2 = 45. It's not profitable to
          produce the second and fourth products under the given conditions. You can draw several interesting conclusions here:
          The third product brings the largest profit per unit, so the factory will produce it the most.
          The first slack is 0, which means that the values of the left and right sides of the manpower (first) constraint are the same. The factory produces 50 units per day, and that's its full capacity.
         The second slack is 40 because the factory consumes 60 units of raw material \lambda (15 units for the first product plus 45 for the third) out of a potential 100 units.
          The third slack is 0, which means that the factory consumes all 90 units of the raw material B. This entire amount
          is consumed for the third product. That's why the factory can't produce the second or fourth product at all and can't produce more than 45 units of the third product. It lacks the raw material B.
          opt.status is 0 and opt.success is True, indicating that the optimization problem was successfully solved with the
```

#### **INFEASIBILITY IN SIMPLEX METHOD**

# Solve following linear programming problem using Simplex method

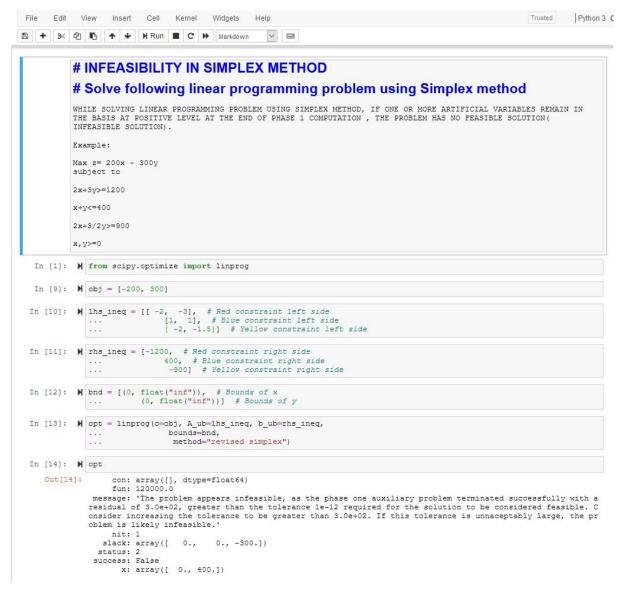
WHILE SOLVING LINEAR PROGRAMMING PROBLEM USING SIMPLEX METHOD, IF ONE OR MORE ARTIFICIAL VARIABLES REMAIN IN THE BASIS AT POSITIVE LEVEL AT THE END OF PHASE 1 COMPUTATION, THE PROBLEM HAS NO FEASIBLE SOLUTION( INFEASIBLE SOLUTION).

```
Example: Max z=
200x - 300y
subject to
2x+3y>=1200
x+y<=400
2x+3/2y>=900
x,y>=0
from scipy.optimize import linprog
obj = [-200, 300]
lhs ineq = [[ -2, -3], # Red constraint left side
         [1, 1], # Blue constraint left side ...
[-2, -1.5]] # Yellow constraint left side
rhs_ineq = [-1200, # Red constraint right side
         400, # Blue constraint right side ...
-900] # Yellow constraint right side
bnd = [(0, float("inf")), # Bounds of x
      (0, float("inf"))] # Bounds of y
opt = linprog(c=obj, A_ub=lhs_ineq, b_ub=rhs_ineq,
```

bounds=bnd,

### ... method="revised simplex")

opt



#### PRACTICAL 8 DUAL SIMPLEX METHOD

##SOLVE FOLLOWING LINEAR PROGRAMMING PROBLEM USING DUAL SIMPLEX METHOD USING R PROGRAMMING

```
# Max z=40x1+50x2

#subject to

#2x1 + 3x2 <= 3

#8x1 + 4x2 <= 5

# x1, x2>=0
```

# Import IpSolve package

library(lpSolve)

# Set coefficients of the objective function

f.obj <- c(40, 50)

# Set matrix corresponding to coefficients of constraints by rows

# Do not consider the non-negative constraint; it is automatically assumed f.con <matrix(c(2, 3,

$$8, 4)$$
, nrow =  $2$ , byrow = TRUE)

# Set unequality signs

# Set right hand side coefficients

# Final value (z)

# Variables final values

lp("max", f.obj, f.con, f.dir, f.rhs)\$solution

### # Sensitivities

Ip("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$sens.coef.from Ip("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$sens.coef.to

# Dual Values (first dual of the constraints and then dual of the variables)
# Duals of the constraints and variables are mixed lp("max",
f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$duals

# Duals lower and upper limits

Ip("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$duals.from Ip("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)\$duals.to

```
##SOLVE FOLLOWING LINEAR PROGRAMMING PROBLEM USING DUAL SIMPLEX METHOD USING R
PROGRAMMI
> # Max z=40x1+50x2
> #subject to
> #2x1 + 3x2 <= 3
> #8x1 + 4x2 <= 5
> \# x1, x2>=0
> # Import lpSolve package
> library(lpSolve)
> # Set coefficients of the objective function
> f.obj <- c(40, 50)
> # Set matrix corresponding to coefficients of constraints by rows
> # Do not consider the non-negative constraint; it is automatically assumed
> f.con <- matrix(c(2, 3,
                    8, 4), nrow = 2, byrow = TRUE)
> # Set unequality signs
> f.dir <- c("<=",
```

```
"<=")
>
> # Set right hand side coefficients
> f.rhs <- c(3,
             5)
>
> # Final value (z)
> lp("max", f.obj, f.con, f.dir, f.rhs)
Success: the objective function is 51.25
> # Variables final values
> lp("max", f.obj, f.con, f.dir, f.rhs)$solution
[1] 0.1875 0.8750
> # Sensitivities
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$sens.coef.from
[1] 33.33333 20.00000
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$sens.coef.to
[1] 100 60
> # Dual Values (first dual of the constraints and then dual of the variables)
> # Duals of the constraints and variables are mixed > lp("max",
f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals
[1] 15.00 1.25 0.00 0.00
> # Duals lower and upper limits
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals.from
[1] 1.25e+00 4.00e+00 -1.00e+30 -1.00e+30
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)$duals.to
[1] 3.75e+00 1.20e+01 1.00e+30 1.00e+30
```

>

## **PRACTICAL 9 TRANSPORTATION PROBLEM**

##sOLVE FOLLOWING TRANSPORTATION PROBLEM IN WHICH CELL ENTRIES REPRESENT UNIT COSTS USING R PROGRAMMING.

# "Customer 1", "Customer 2", "Customer 3", "Customer 4" SUPPLY 2 #sUPPLIER 1 10 20 11 15 #sUPPLIER 1 12 7 9 20 25 #sUPPLIER 1 4 14 16 18 10 #DEMAND 5 15 15 15

# Import lpSolve package library(lpSolve)

# Set transportation costs matrix

costs <- matrix(c(10, 2, 20, 11,

12, 7, 9, 20,

4, 14, 16, 18), nrow = 3, byrow = TRUE)

# Set customers and suppliers' names

colnames(costs) <- c("Customer 1", "Customer 2", "Customer 3", "Customer 4")
rownames(costs) <- c("Supplier 1", "Supplier 2", "Supplier 3")</pre>

# Set unequality/equality signs for suppliers

row.signs <- rep("<=", 3)

# Set right hand side coefficients for suppliers

row.rhs <- c(15, 25, 10)

```
# Set unequality/equality signs for customers
```

```
col.signs <- rep(">=", 4)
```

# Set right hand side coefficients for customers

```
col.rhs <- c(5, 15, 15, 15)
```

# # Final value (z)

TotalCost <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)

## # Variables final values

lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)\$solution
print(TotalCost)

```
> ##solve following transportation problem in which cell entries represent unit costs us
>
            "Customer 1", "Customer 2", "Customer 3", "Customer 4" SUPPLY
> #sUPPLIER 1 10
                              2
                                              20
                                                            11
                                                                        15
                             7
> #sUPPLIER 1
                 12
                                              9
                                                            20
                                                                        25
> #SULLET
> #SUPPLIER 1
                             14
                                                                        10
                4
                                             16
                                                            18
                 5
                             15
                                              15
                                                            15
> #DEMAND
> # Import lpSolve package
> library(lpSolve)
> # Set transportation costs matrix
> costs <- matrix(c(10, 2, 20, 11,
                    12, 7, 9, 20,
                    4, 14, 16, 18), nrow = 3, byrow = TRUE)
> # Set customers and suppliers' names
> colnames(costs) <- c("Customer 1", "Customer 2", "Customer 3", "Customer 4")</pre>
> rownames(costs) <- c("Supplier 1", "Supplier 2", "Supplier 3")</pre>
> # Set unequality/equality signs for suppliers
> row.signs <- rep("<=", 3)</pre>
> # Set right hand side coefficients for suppliers
> row.rhs <- c(15, 25, 10)
```

```
>
> # Set unequality/equality signs for customers
> col.signs <- rep(">=", 4)
> # Set right hand side coefficients for customers
> col.rhs <- c(5, 15, 15, 15)
> # Final value (z)
> TotalCost <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)
> # Variables final values
> lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)$solution
[,1] [,2] [,3] [,4] [1,]
                        0 5 0 10
[2,] 0 10 15 0
      5 0 0
[3,]
> print(TotalCost)
Success: the objective function is 435
```

>

# **PRACTICAL 10**

## **ASSIGNMENT PROBLEM**

#SOLVE FOLLOWING ASSIGNMENT PROBLEM REPRESENTED IN FOLLOWING MATRIX USING R PROGRAMMING

```
# Assignment Problem
```

```
# JOB1 JOB2 JOB3

#W1 15 10 9

#W2 9 15 10

#W3 10 12 8
```

# Import IpSolve package

library(lpSolve)

# Set assignment costs matrix

costs <- matrix(c(15, 10, 9,

```
9, 15, 10,
10, 12 ,8), nrow = 3, byrow = TRUE)
```

# # Print assignment costs matrix

costs

# Final value (z)

lp.assign(costs)

#### # Variables final values

Ip.assign(costs)\$solution

```
> #SOLVE FOLLOWING ASSIGNMENT PROBLEM REPRESENTED IN FOLLOWING MATRIX USING R PROGRAMMIN
> # Assignment Problem
      JOB1 JOB2 JOB3
15 10 9
> #
> #W1 15 10
> #W2
        9
               15
                      10
> #W3 10 12
> # Import lpSolve package
> library(lpSolve)
> # Set assignment costs matrix
> costs <- matrix(c(15, 10, 9,
                  9, 15, 10,
                  10, 12 ,8), nrow = 3, byrow = TRUE)
> # Print assignment costs matrix
> costs
    [,1] [,2] [,3]
[1,] 15 10 9
      9 15 10
[2,]
[3,] 10 12 8
> # Final value (z)
> lp.assign(costs)
Success: the objective function is 27
> # Variables final values
> lp.assign(costs)$solution
[,1] [,2] [,3] [1,] 0
1
   0
```

[2,] 1 0 0 [3,] 0 0 1

>