Linear Algebra Hw8

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Problem 1

Proof. False. Consider T that rotates counterlclokwise by 90 degrees. We have $T(x, y) = (-y, x)$. So $T^2 = -I$. So our matrix is,

$$-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

But if T is an upper-triangular matrix with respect to some basis then T would have an eigenvalue but we see that T has no eigenvalues because it is the rotation matrix. □

Problem 9

Proof. If B is a n by n matrix and T is our linear map given by $Tx = Bx$. We see that the elements of T with respect to $e_1, \ldots, e_n$ of C^n is B. So we can find a basis $v_1, \ldots, v_n$ of C^n such that the matrix is upper-transigular. So let $A = M(I, (v_1, \ldots, v_n), (e_1, \ldots, e_n))$ then we see that $A^{-1}BA$ is the upper traingular matrix $M(T, (v_1, \ldots, v_n))$ □