Linear Algebra Hw8

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Problem 1

Proof. (1). We have T^2 has eigenvalue 9. Which means that

$$T^2 v = 9v$$

$$(T^2 - 9I)v = 0$$

$$(T + 3I)(T - 3I)v = 0$$

So we have either $Tv = 3Iv = 3v$ which implies that 3 is an eigenvalue or $Tv = -3Iv = -3v$ which implies that -3 is an eigenvalue.
(2). Assume eigenvalue is either 3 or -3. So we have,

$$\begin{aligned} Tv &= 3v \\ T(Tv) &= T(3v) = 3T(v) \\ T^2 v &= 9v \end{aligned}$$

which means that 9 is an eigenvalue of T^2
Similarly we have if -3 is an eigenvalue of T,

$$\begin{aligned} Tv &= -3v \\ T(Tv) &= T(-3v) = -3T(v) \\ T^2 v &= -3 \cdot -3v = 9v \end{aligned}$$

□

Problem 6

Proof. Let e_1, e_2 be the standard basis, so we have, $T(e_1) = e_2$ and $T^2 e_1 = -e_1$.
Now we know that,

$$c_0 e_1 + c_1 T e_1 = -T^2 e_1$$

$e_0 e_1 + c_1 e_1 = e_1$
has the unique sol of $c_0 = 1$ and $c_1 = 0$. So the minimum polynomial would be,

$$p(t) = 1 + t^2$$

□