Linear Algebra HW10

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Problem 1

Proof. (1). We have $T^4 = I$. The minimal polynomial is,

$$z^4 - 1 = 0$$

whose factors are,

$$z^4 - 1 = (z+1)(z-1)(z+i)(z-i)$$

So the roots $\lambda_1, \ldots, \lambda_4$ are all distinct which means that the operator T is diagonalizable.
(2). We have $T^4 = T$ whose characteristic equation is,

$$z^4 - z$$

which can be factored as,

$$z^4 - z = z(z^3 - 1) = z((z-1)(z^2 + 1 + z))$$

And we know that $z^2 + 1 + z$ has zeroes as $\frac{-1 \pm \sqrt{-3}}{2}$
Which means that we have four distinct λ which implies that T is diagonizable.
(3). Let T be defined as $T(a, b) = (b, 0)$. We see that $T^2 = T^4$. However the matrix for this defined on e_1, e_2 is just $(01; 00)$. The only eigenvalue is 0 so we cannot diagonalize it. $\square$

Chapter 6

Problem 3

Proof. (a). We have f defined as $f((x_1, x_2), (y_1, y_2)) = |x_1 y_1| + |x_2 y_2|$
But we see that $f((-1, 0), (1, 0)) = 1$ but $f((1, 0), (1, 0)) = -1$
So its not homogeneous hence cannot be an inner product.
(b). We have $f((x_1, x_2, x_3), (y_1, y_2, y_2)) = x_1 y_1 + x_3 y_3$
But we see that $f((0, 1, 0), (0, 1, 0)) = 0$
So its not definite hence is not an inner product. $\square$