MATH 4320 HW03

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September 9, 2024

Problem 3

(a). $lim_{z \to z_o} \frac{1}{z^n} (z_0 \neq 0)$

We know,

$$lim_{z \to z_o} \frac{1}{z^n} = \frac{lim_{z \to z_0} 1}{lim_{z \to z_0} z^n} = \frac{1}{z_0^n}$$

(b). $lim_{z \to i} \frac{iz^3 - 1}{z + i}$

We know,

$$lim_{z \to i} \frac{iz^3 - 1}{z + i} = \frac{lim_{z \to z_0} iz^3 - 1}{lim_{z \to z_0} z + i} = \frac{i^4 - 1}{2i} = \frac{0}{2i} = 0$$

(c). $lim_{z \to z_0} \frac{P(z)}{Q(z)}$

We know,

$$lim_{z \to z_0} \frac{P(z)}{Q(z)} = \frac{lim_{z \to z_0} P(z)}{lim_{z \to z_0} Q(z)} = \frac{P(z_0)}{Q(z_0)}$$

Problem 7

Using the definition of limits we know that, $\forall \varepsilon > 0, \exists \delta$ such that.

$$|f(z) - w_0| < \varepsilon \text{ whenever } 0 < z - z_0 < \delta.$$

We know from the triangle inequality that,

$$|a - b| \geq ||a| - |b||.$$

Using this we can say,

$$||f(z)| - |w_0|| \leq |f(z) - w_0|.$$

So we have,

$$||f(z)| - |w_0|| < \varepsilon.$$

Now using the definition of limits once again, we get $\forall \varepsilon > 0, \exists \delta$ the same as before, such that

$$||f(z)| - |w_0|| < \varepsilon \text{ whenever } 0 < |z - z_0| < \delta.$$

This shows that we can write,

$$\lim_{z \to z_0} |f(z)| = |w_0|.$$

Problem 10

(a). $\lim_{z\to\infty} \frac{4z^2}{(z-1)^2} = 4$

We know that,

$$\lim_{z\to 0} f\left(\frac{1}{z}\right) = w_0.$$

then,

$$\lim_{z\to\infty} f(z) = w_0.$$

So we can say,

$$\lim_{z\to\infty} \frac{4z^2}{(z-1)^2} = \lim_{z\to 0} \frac{\frac{4}{z^2}}{(\frac{1}{z}-1)^2}.$$

$$= \lim_{z\to 0} \frac{4}{(1-z)^2}.$$

$$= \frac{4}{1-0} = 4.$$

(b). $\lim_{z\to 1} \frac{1}{(z-1)^3} = \infty$

Using theorem we know that,

$$\lim_{z\to z_0} f(z) = \infty \text{ if } \lim_{z\to z_0} \frac{1}{f(z)} = 0.$$

$$\lim_{z\to 1} \frac{(z-1)^3}{1} = \frac{(1-1)^3}{1} = 0.$$

So using the theorem we get,

$$\lim_{z\to 1} \frac{1}{(z-1)^3} = \infty.$$

(c). $\lim_{z\to\infty} \frac{z^2+1}{z-1} = \infty$ Using theorem we know that,

$$\lim_{z\to\infty} f(z) = \infty \text{ if } \lim_{z\to 0} \frac{1}{f(\frac{1}{z})} = 0.$$

Using this,

$$\lim_{z\to 0} \frac{\frac{1}{z}-1}{\frac{1}{z^2}+1} = \frac{1-1}{1+1} = 0.$$

So,

$$\lim_{z\to\infty} \frac{z^2+1}{z-1} = \infty.$$