Linear Algebra HW12

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November 26, 2024

Problem 1

Proof. Let $v \in span(v_1, \ldots, v_m)^\perp$. This means that $v \in span(V_1, \ldots, v_m)$ so $< v, v_k >= 0$ for every $k \in \{1, \ldots, m\}$.
If thats the case we have $v \in \{v_1, \ldots, v_m\}^\perp$ and $(span(v_1, \ldots, v_m))^\perp \subseteq \{v_1, \ldots, v_m\}^\perp$.
Now let $v \in \{v_1, \ldots, v_m\}^\perp$ and let $a_1v_1, \ldots, a_mv_m \in span(v_1, \ldots, v_m)$. We see that,

$$< v, a_1v_1, \ldots, a_mv_m >= \overline{a_1} < v, v_1 > + \cdots + \overline{a_m} < v, v_m >= 0$$

This shows that $v \in span(v_1, \ldots, v_m)^\perp$. This means that $\{v_1, \ldots, v_m^\perp \subseteq span(v_1, \ldots, v_m)^\perp$.
So both the results shows us that they are equal. $\square$

Chapter 7, Problem 1

We have,

$$< (w_1, \ldots, w_n), T^*(z_1, \ldots, z_n)) =< T(w_1, \ldots, w_n), (z_1, \ldots, z_n) >$$

$$=< (0, w_1, \ldots, w_{n-1}), (z_1, \ldots, z_n) >$$

$$= w_1z_2 + \cdots + w_{n-1}z_n$$

$$=< (w_1, \ldots, w_n), (z_2, \ldots, z_n, 0) >$$

Hence we show that $T^*(z_1, \ldots, z_n) = (z_2, \ldots, z_n, 0)$