Linear Alebgra HW06

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We know that $T(v_1) = A_{1,1}w_1 + \cdots + A_{n,1}w_m$. We need to show there exists a basis of W such that all values except for possibly $A_{1,1}$ is zero.
Consider the case when $T(v_1) = 0$ then we have $A_{1,1} = \cdots = A_{n,1} = 0$ for any arbitrary basis of W.
If $T(v_1) \neq 0$ then consider all of $A_{2,1}, \dots, An, 1 = 0$ except for $A_{1,1}$ (the element in the first column and row). In this case consdier,

$$T(v_1) = w_1$$

where w_1 is an arbitrary vector in W. Now we can extend w_1 to a basis of W. So in both cases we have a basis of W, $w_1, \dots, w_n$ such that only possibly the first row first column element is zero.

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Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and let $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$
We get,

$$AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

and

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So $AB \neq BA$

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Proof. We need to show $(AB)_{j,.} = A_{j,.}B$
We know that,

$$(AB)_{j,k} = A_{j,.}B_{.,k}$$

So let j be fixed then that means for each value in a given row we multiply $A_{j,.}$ with the curresponding column of B. Which means that,

$$(AB)_{j,k} = A_{j,.}B_{.,k}$$

becuse the curresponding columns of B gives us B itself. $\square$