Linear Algebra HW11

Aamod Varma

November 19, 2024

Problem 7

Proof. We have

$$||au + bv||^2 = a^2||u||^2 + 2ab < u, v > +b^2||v||^2$$

and

$$||bu + av||^2 = b^2||u||^2 + 2ab < u, v > +a^2||v||^2$$

So if they are equal then we have,

$$a^2||u||^2 + b^2||v||^2 = b^2||u||^2 + a^2||v||^2$$

Or,

$$(a^2 - b^2)(||u||^2 - ||v||^2) = 0$$

For all $a, b \in R$ for this to be true we need,

$$||u||^2 = ||v||^2 \Rightarrow ||u|| = ||v||$$

$\square$

Problem 11

Proof. We have $(1, 2) = u + v$ and $< u, v >= 0$. So,

$$< (1, 2), u >=< u, u >$$

If $u = (k, 3k)$ we have,

$$k + 6k = k^2 + 9k^2$$

$$10k^2 - 7k = 0$$

$$k(10k - 7) = 0$$

So we can take $k = \frac{7}{10}$ which gives us $u = (\frac{7}{10}, \frac{21}{10})$ and $v = (\frac{3}{10}, -\frac{1}{10})$ $\square$