MATH 4320 HW04

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Problem 3

(a). We can write

$$P(z) = a_0 + a_1 z + \cdots + a_n z^n.$$

Using results from section 20 we know that for $f(z), g(z)$

$$(f(z) + g(z))' = f'(z) + g'(z).$$

Using this idea we can write the polynomial as,

$$P'(z) = (a_0 + a_1 z + \dots)' + (a_n z^n)'.$$

Similary we can apply this for each functino as follows,

$$P'(z) = \frac{d}{dx}(a_0) + \frac{d}{dx}(a_1 z) + \dots \frac{d}{dx}(a_n z^n).$$

We know that $\frac{d}{dx} z^n = nz^{n-1}$

So we can write,

$$P'(z) = 0 + a_1 + 2a_2 z + \cdots + na_n z^{n-1}.$$

(b). We need to find the coefficients, $a_0, a_1, \dots, a_n$. To do this we need to remove the z term that is multiplied it and set the polynomial to 0 to isolate our coefficient.

So to find any a_n for $n \geq 1$ we first derive $P(z), n$ times.

$$P^n(z) = (n)(n-1)(n-2)\dots(1)a_n z^0 + (n+1)(n)\dots(2)a_{n+1}z^1 + \dots.$$

$$P^n(z) = (n!)a_n + \frac{(n+1)!}{1!}a_{n+1}z^1 + \frac{(n+2)!}{2!}a_{n+2}z^2 + \dots.$$

$$P^n(0) = (n!)a_n + 0 + \cdots + 0.$$

$$a_n = \frac{P^n(0)}{n!}.$$

For $n = 0$ it is obvious that we can plug in $z = 0$ to get,

$$a_0 = P(0).$$

Problem 1

(a). $f(z) = \bar{z}$;

We need to satisfy the Cauchy-Remann equations for $f'(z)$ to be defined, so,

$$u_x = v_y, u_y = -v_x.$$

We have, $z = x + iy$ and $\bar{z} = x - iy$. So, $u(x, y) = x, v(x, y) = -y$

We can write,

$$u_x = 1, u_y = 0.$$

$$v_x = 0, v_y = -1.$$

So we can see that, $u_x \neq v_y$ which means that our derivate $f'(z)$ cannot exist.

(b). $f(z) = z - \bar{z}$

We have, $z = x + iy$ so $f(z) = (x + iy) - (x - iy) = 2iy$

We can write, $u(x, y) = 0, v(x, y) = 2y$ So,

$$u_x = 0, u_y = 0.$$

$$v_x = 0, v_y = 2.$$

We can see that, $u_x \neq v_y$, so the derivative, $f'(z)$ cannot exist.

(c). $f(z) = 2x + ixy^2$

We have, $f(z) = 2x + ixy^2$

We can write, $u(x, y) = 2x, v(x, y) = xy^2$

So,

$$u_x = 2, u_y = 0.$$

$$v_x = y^2, v_y = 2xy.$$

We see that for this to be satisfied we need,

$$2 = 2xy \text{ and } y^2 = 0.$$

But if $y^2 = 0$ then $y = 0$ So $2xy = 0 \neq 2$

So for no values of x, y will the equation be satisfied. Hence our derivative, $f'(z)$ cannot exist.

(d). $f(z) = e^x e^{-iy}$

If $z = x + iy$ and $f(z) = e^x e^{-iy}$ we can write,

$$f(z) = e^x(\cos(y) - i\sin(y)) = e^x\cos(y) - ie^x\sin(y).$$

So, $u(x, y) = e^x\cos(y), v(x, y) = -e^x\sin(y)$

We have,

$$u_x = e^x\cos(y), u_y = -e^x\sin(y).$$

$$v_x = -e^x\sin(y), v_y = -e^x\cos(y).$$

So we need, $e^x\cos(y) = -e^x\cos(y)$ and $e^x\sin(y) = e^x\sin(y)$

As $e^x \neq 0$ we have, $cos(y) = 0$ and $sin(y) = 0$

However there exist no value y to satisfy both these conditions, hence our derivative $f'(z)$ cannot exist.

Problem 8