True/False Questions MATH-2551

Question: True or False: If x and y are continuous functions of t on an interval I, then the equations x = x(t) and y = y(t) are called parametric equations and t is called the parameter.

Answer: True

Explanation: The statement "If x and y are continuous functions of t on an interval I, then the equations x = x(t) and y = y(t) are called parametric equations and t is called the parameter" is indeed TRUE.

Parametric equations are a way to describe the relationship between two variables, typically denoted as x and y, in terms of a third variable, often represented by t. This third variable is called the parameter. In this context, x and y are functions of t, meaning that their values change as t varies within the interval I.

To illustrate this concept, consider a curve in the plane defined by the equations:

x(t) = 2t^2 + 3t y(t) = t^3 - 4t

Here, x and y are both functions of the parameter t. As t changes within some interval I, the values of x and y change accordingly, describing a specific curve in the plane.

In summary, parametric equations are used to describe the relationship between two variables (x and y) as they vary with respect to a third variable (t), where t is the parameter.

Question: True or False: A parabola is the set of all points whose distance from a fixed point, called the focus, is greater than the distance from a fixed line, called the directrix.

Answer: False

Explanation: A parabola is actually the set of all points whose distance from a fixed point, called the focus, is equal to the distance from a fixed line, called the directrix. The focus is situated on the axis of symmetry, and the directrix is perpendicular to this axis. This means that every point on the parabola has the same distance from the focus as it does from the directrix.

Question: An ellipse can be defined as the set of all points for which the difference between their distances from two fixed points is constant, rather than their sum. True or False?

Answer: False

Explanation: An ellipse can be defined as the set of all points in a plane that are equidistant from a fixed point called the foci and a fixed line, known as the major axis. The difference between the distance from each point to the foci is not constant; instead, it varies along the ellipse's length. A more accurate definition would be that an ellipse is the set of points where the sum of the distances to two fixed points (the foci) is constant. This is why the answer is FALSE.

Question: A hyperbola is the set of all points where the sum of their distances from two fixed points (the foci) is constant. True or False?

Answer: False

Explanation: A hyperbola is actually the set of all points where the absolute difference of their distances from two fixed points (the foci) is constant, not the sum. This can be seen by considering

the standard form of a hyperbola equation: (x-h)^2/a^2 - (y-k)^2/b^2 = 1. The distance formula shows that the distance from a point on the hyperbola to one focus is equal to c, and the distance to the other focus is equal to d, where c and d are constants related to the foci and the center of the hyperbola. Therefore, the sum of these distances would not be constant, but rather their absolute difference would be constant, making this statement false.

Question: If e=1, then the conic is a _____.
Answer: True
Explanation: When e=1, the conic is a circle. This is because in polar coordinates, the equation of a conic section takes the form $(r - e \cos \theta)^2 + (e \sin \theta)^2 = 1$. If e=1, this simplifies to $r^2 = 1$, which is the equation of a circle with radius 1 centered at the origin.

Question: True or False: A vector is a quantity with only magnitude, but no direction.
Answer: False
Explanation: A vector is a mathematical object that has both magnitude (or length) and direction. In fact, one of the fundamental properties of vectors is that they can be described in terms of both their size (magnitude) and orientation (direction).

To illustrate this, consider a displacement vector representing the distance between two points on a map. The vector's magnitude would represent the actual distance traveled, while its direction would indicate the path taken to get from one point to another.

In contrast, a quantity with only magnitude but no direction would not be a vector in the classical sense. For example, a scalar value like the number 5 has magnitude (it is five units large), but it does not have direction.

In summary, vectors are characterized by both magnitude and direction, whereas quantities with only magnitude but no direction do not fit the definition of a vector.

Question: Vectors that are perpendicular to each other are equivalent vectors if they have the same magnitude and direction. True or False?
Answer: False
Explanation: The statement is false because two perpendicular vectors can have different directions even if they have the same magnitude. In other words, just having the same length and being at right angles to each other does not mean they are equivalent.

For example, consider two vectors (2, 3) and (-2, -3). Both have a magnitude of sqrt(13), which is the same for both vectors. Additionally, their dot product is zero, indicating that they are perpendicular. However, they are pointing in opposite directions, so they are not equivalent.

In conclusion, having the same magnitude and being perpendicular does not imply equivalence between two vectors.

Question: When a scalar is multiplied by a vector, the magnitude of the resulting vector is determined by the magnitude of the scalar and the original vector, but if the scalar is positive it keeps the same direction as the original vector.
Answer: True
Explanation: When a scalar is multiplied by a vector, the resulting vector's magnitude is determined by the product of the scalar and the original vector's magnitude. This is because the scalar is a numerical value that scales the entire vector equally in all directions.

If the scalar is positive, it does not change the direction of the vector. The resulting vector still points in the same direction as the original vector. This is because a positive scalar simply stretches or compresses the original vector without reversing its direction.

Question: When constructing the sum of two vectors, placing the initial point of one vector at the terminal point of another results in the vector sum having an initial point that coincides with the terminal point of the first vector. True or False?

Answer: False

Explanation: When constructing the sum of two vectors, placing the initial point of one vector at the terminal point of another does not result in the vector sum having an initial point that coincides with the terminal point of the first vector.

Instead, consider a simple example where we have two vectors A and B. If the initial point of A is placed at the terminal point of B, this creates a new vector C. The initial point of C will be the same as the initial point of A, not the terminal point of B. This can be visualized by imagining two arrows on a graph: if you place the starting point of one arrow at the end point of another, you get a third arrow that has its own unique starting and ending points.

In summary, placing the initial point of one vector at the terminal point of another when constructing the sum does not produce an outcome where the resulting vector's initial point coincides with the terminal point of the original vector.

Question: The vector with initial point (0,0) and terminal point (x,y) can be written in component form as v=<x,y>.

Answer: True

Explanation: The statement "The vector with initial point (0,0) and terminal point (x,y) can be written in component form as v=<x,y>" is indeed TRUE.

This is because the standard convention for writing a vector in component form is to use angle brackets "<" and ">" around the coordinates of the vector. In this case, since the initial point is at the origin (0,0), the vector's components are simply the differences between the terminal point (x,y) and the initial point.

So, the vector from (0,0) to (x,y) can be written in component form as v=<x,y>, which means that the x-component of the vector is simply x and the y-component is simply y. This makes sense because moving a distance of x units in the positive x-direction and y units in the positive y-direction brings us from the origin to the point (x,y).

Question: Is it true that if k is a scalar, then the result of multiplying vector v by k results in a new vector with x-coordinate equal to k times the original x-coordinate?

Answer: True

Explanation: When multiplying a vector v by a scalar k, we are essentially stretching or shrinking the original vector in some direction. The x-coordinate of this new vector is indeed k times the original x-coordinate.

To see why, consider the components of the vectors involved. Let v = (x1, y1) be the original vector and let k be the scalar. Then, the product kv can be written as (kx1, ky1).

The x-coordinate of this new vector is kx1, which is simply the original x-coordinate multiplied by the scalar k. This makes sense because if you were to stretch or shrink a vector in only the x-direction, the x-coordinate would increase or decrease proportionally.

So, it is indeed true that multiplying a vector v by a scalar k results in a new vector with an x-coordinate equal to k times the original x-coordinate.

Question: The 3D rectangular coordinate system consists of three perpendicular axes: the x-axis, the y-axis, the z-axis, and an origin at the point of intersection (0) of the axes, which is often denoted by ℝ3. True or False?

Answer: True

Explanation: The 3D rectangular coordinate system consists of three perpendicular axes: the x-axis, the y-axis, and the z-axis. The origin (0) is the point where these axes intersect, often denoted by $\mathbb{R}^3$. This is indeed true because in a 3D coordinate system, we need three dimensions to define a unique position in space. The x, y, and z axes serve as the fundamental directions for measuring distances from the origin, allowing us to pinpoint any location in 3D space.

Question: A sphere is defined as the set of points in space that are equidistant from a fixed point, which is called the center of the sphere.

Answer: True

Explanation: A sphere is indeed defined as the set of points in space that are equidistant from a fixed point, which is called the center of the sphere. This definition captures the fundamental property of a sphere: every point on its surface is at an equal distance, known as the radius, from the central point.

In three-dimensional space, this means that any given point on the surface of the sphere lies along a radial line passing through the center, with all such lines having the same length. This equidistant property is what distinguishes spheres from other shapes, like ellipses or cylinders, which have varying distances between their centers and surfaces.

This definition not only accurately describes the geometric nature of a sphere but also provides a clear foundation for understanding its various properties, such as volume, surface area, and curvature.

Question: Is the dot product of vectors u and v calculated as the sum of the products of their corresponding components?

Answer: True

Explanation: The dot product of vectors u and v is calculated as the sum of the products of their corresponding components. This can be expressed mathematically as:

dot product = (u1 * v1) + (u2 * v2) + ... + (un * vn)

where ui and vi are the components of vectors u and v, respectively. In other words, you multiply each component of vector u by its corresponding component in vector v, then add up all these products to get the dot product.

Question: The direction angles of a nonzero vector are formed by the vector and its own coordinate axes, rather than the coordinate axes themselves. True or False?

Answer: False

Explanation: The direction angles of a nonzero vector are formed by drawing lines from the terminal point of the vector to the origin, intersecting with each coordinate axis. The angle between this line and the positive direction of the axis is the direction angle.

For example, consider a 2D vector (3,4) in the standard Cartesian plane. To find the direction

angles, draw lines from the terminal point (3,4) to the origin (0,0). These lines intersect with the x-axis at point (3,0) and the y-axis at point (0,4).

The angle between this line and the positive x-axis is the direction angle along the x-axis. Similarly, the angle between this line and the positive y-axis is the direction angle along the y-axis.

This process forms the direction angles of the vector with respect to its own coordinate axes, not the other way around as stated in the question.

Question: The vector projection of v onto u is given by projuv = (u · v) / ——u——, where u · v represents the dot product of u and v.

Answer: True

Explanation: The vector projection of v onto u, denoted by projuv, is a vector that represents the component of v in the direction of u. It can be calculated using the following formula:

projuv = (u · v) / ——u——

This formula makes sense because it scales the dot product of u and v by the magnitude of u to get a vector that points in the same direction as u.

The dot product, u · v, gives us the amount of " Agreement" between the two vectors. When you multiply this value by the reciprocal of the magnitude of u, ——u——, you effectively normalize the result to create a unit vector pointing in the same direction as u. This is exactly what we want for the projection.

Therefore, it is true that the formula for the vector projection of v onto u is given by projuv = (u · v) / ——u——.

Question: When a constant force is applied to an object, the work done by the force F, acting at an angle θ from the line of motion, is given by W = F·PQ→=‖F‖‖PQ→‖cosθ. True or False?

Answer: True

Explanation: The given expression for work W is indeed correct.

When a constant force F is applied to an object, the work done by the force is calculated as the dot product of the force vector F and the displacement vector PQ. This can be written as:

W = F·PQ→=‖F‖‖PQ→‖cosθ

The magnitude of the force (‖F‖) is multiplied by the magnitude of the displacement (‖PQ→‖), and then this product is further modified by the cosine of the angle θ between the force vector F and the displacement vector PQ. This factor accounts for the fact that only a portion of the force vector, parallel to the displacement vector, actually contributes to the work done.

Therefore, the given expression accurately represents the work done by a constant force acting at an angle θ from the line of motion.

Question: Is it true that the cross product u×v of two 3D vectors u and v is given by the equation: (u2v3−u3v2)i−(u1v3−u3v1)j+(u1v2−u2v1)k?

Answer: True

Explanation: The cross product u×v of two 3D vectors u and v is indeed given by the equation: (u2v3-u3v2)i - (u1v3-u3v1)j + (u1v2-u2v1)k

This formula can be derived by using the definition of the cross product as a determinant, which states that for two 3D vectors u = (u1, u2, u3) and v = (v1, v2, v3), their cross product is:

u×v = —i j k— = — i j k —— u1 u2 u3 —— v1 v2 v3 —

By expanding this determinant along the first row, we get:

u×v = (u2v3-u3v2)i - (u1v3-u3v1)j + (u1v2-u2v1)k

This formula is commonly used in vector calculus and 3D geometry to find the cross product of two vectors.

Question: The triple scalar product of vectors u, v, and w is equal to u · (v × w).u · (v × w).

Answer: True

Explanation: The triple scalar product of vectors u, v, and w is defined as $(u \cdot v) \times w$. By definition, the dot product u · (v × w) is equal to —u— —v × w— $\cos(\theta)$, where θ is the angle between u and v × w.

Similarly, the expression u · (v × w).u · (v × w) can be rewritten as —u—^{2} —v × w—^{2} $\cos^2(\theta)$.

Since $(u \cdot v) \times w = (u \cdot (v \times w))n$, where n is a unit normal vector perpendicular to both v and w, we have:

$(u \cdot v) \times w = (u \cdot (v \times w)).(v \times w)$

where the dot product is taken between u and (v × w). This equals —u—^{2} —v × w—^{2} $\cos^2(\theta)$, which matches the expression above.

Therefore, the triple scalar product of vectors u, v, and w indeed equals u · (v × w).u · (v × w).

Question: True or False: Torque is equal to the cross product of the position vector and the force vector.

Answer: True

Explanation: Torque is indeed equal to the cross product of the position vector and the force vector. This can be seen by considering a rigid object that is rotating around a fixed axis. The torque, τ, acting on the object can be thought of as the rotational equivalent of the force, F, that would produce this rotation if the object were not constrained.

Let r be the position vector from the pivot point to the point where the force is applied, and let F be the force vector at that point. Then, the cross product of r and F gives us a vector that is perpendicular to both r and F, which lies in the plane defined by these two vectors. This vector represents the rotational tendency or torque acting on the object.

Mathematically, this can be written as:

$\tau = r \times F$

This equation shows that the magnitude of the torque depends not only on the force but also on its distance from the pivot point, which is captured by the position vector r. This is why the cross product of the two vectors provides a complete description of the rotational tendency acting on the object.

Question: The vector equation of a plane is n·PQ→=0n·PQ→=0. The scalar equation of a plane containing point P with normal vector n can also be expressed as ax+by+cz+d=0 where d=−ax0−by0−cz0 true or false

Answer: True

Explanation: The given vector equation of a plane is n·PQ→=0, where n is the normal vector to the plane and PQ→ is any vector in the plane.

The scalar equation of this plane can be found by taking the dot product of both sides with a vector parallel to the x-axis. Let's call this vector parallel to the x-axis as i. Then, we get:

n·PQ→=0 (n·i)•(PQ→)=0

Now, simplify the expression:

ai·x + bi·y + ci·z = 0

where a, b, and c are the components of the normal vector n.

This is equivalent to the scalar equation ax+by+cz+d=0 where d=-ax0-by0-cz0. Hence, the statement is true.

Question: A set of lines that are parallel to a given line and pass through a given curve is known as _____.

Answer: True

Explanation: The statement is indeed TRUE.

A set of lines that are parallel to a given line and pass through a given curve is known as an asymptote. Asymptotes are lines or curves that a function approaches as the input values get arbitrarily large, but never actually touches. In this case, since the lines are parallel to a given line and pass through a given curve, they can be considered asymptotes of that curve.

In other words, as one moves along the given curve, the distance between the curve and the set of lines gets arbitrarily small, but never exactly zero. This concept is crucial in mathematics, particularly in calculus, where it's used to study limits and behavior of functions near singularities or points of discontinuity.

Question: The traces of a surface are the cross-sections created when the surface intersects a plane parallel to one of the coordinate planes. True or False?

Answer: True

Explanation: The statement is true.

In mathematics, the trace of a surface refers to the intersection of that surface with a plane parallel to one of the coordinate planes. This means that if we take a surface in 3D space and intersect it with a plane that is parallel to either the x-y plane, y-z plane, or x-z plane, we get a cross-section of the original surface.

For instance, if we have a surface in 3D space and we intersect it with a plane parallel to the x-y plane, the resulting cross-section would be a curve lying entirely in that plane. Similarly, if we intersect the same surface with a plane parallel to the y-z plane, we get another curve, and so on.

The key point here is that the trace of a surface is always a 2D object (a curve or a region), whereas the original surface was a 3D object. The trace can be thought of as a "shadow" of the surface in the coordinate plane.

Question: Quadric surfaces can be expressed in an equation with only quadratic terms, meaning no higher-order powers of x, y, or z are present. True or False?

Answer: True

Explanation: A quadric surface is a three-dimensional shape that can be expressed as an equation in x, y, and z where each term has degree two (quadratic) and only quadratic terms are present. This means the highest power of any variable (x, y, or z) is 2.

In other words, the equation for a quadric surface takes the form:

ax^2 + by^2 + cz^2 + dxy + exz + fy = 0

where a, b, c, d, e, and f are constants. This equation does not contain any terms with higher powers of x, y, or z, such as cubic (x^3), quartic (x^4), etc.

Since the highest power is always two, quadric surfaces can be graphed using a 3D coordinate system and can have various shapes including ellipsoids, hyperboloids, paraboloids, and more.

Question: In the cylindrical coordinate system, a point in space is represented by an ordered triple (r,θ,z), where:

True or False: In this representation, z refers to the Cartesian z-coordinate of the point's projection in the xy-plane.

Answer: False

Explanation: In the cylindrical coordinate system, a point in space is represented by an ordered triple (r,θ,z), where:

* r represents the radial distance from the z-axis * θ represents the angular displacement from the positive x-axis * z represents the height above or below the xy-plane

So, in this representation, z refers to the height of the point above or below the xy-plane, not its Cartesian z-coordinate. Therefore, the statement is False.

Question: In the spherical coordinate system, a point is represented by an ordered triple where ρ represents the angle between the positive z-axis and line segment OP–, θ represents the distance from the origin to the point, and φ represents the same angle used to describe the location in cylindrical coordinates. True or False?

Answer: False

Explanation: The spherical coordinate system represents a point in space by an ordered triple (ρ, θ, φ), where ρ is the radial distance from the origin, θ is the polar angle measured from the positive z-axis, and φ is the azimuthal angle measured from the x-axis.

In this system, ρ does not represent the angle between the positive z-axis and line segment OP. Instead, ρ is a measure of the point's distance from the origin.

θ represents the angle between the positive z-axis and the line segment OP. This is because θ is measured from the positive z-axis.

φ represents the same angle used to describe the location in cylindrical coordinates. In spherical coordinates, φ is the azimuthal angle, which measures the direction of a point from the x-axis.

Since ρ does not represent the angle between the positive z-axis and line segment OP, the statement is False.

Question: A two-dimensional vector-valued function is defined by a single real-valued function of one variable.

Answer: False

Explanation: A two-dimensional vector-valued function is typically defined as a function that assigns to each input (or independent variable) a unique output that is itself a 2D vector. In other words, it's a function of the form f(x,y) = (u(x,y), v(x,y)), where u and v are real-valued functions of two variables.

On the other hand, a single real-valued function of one variable is simply a function of the form g(x) that assigns to each input x a unique output y that is a real number.

Clearly, these two concepts are distinct, as a 2D vector-valued function can't be reduced to a single real-valued function of one variable. For instance, the function f(x,y) = (x^2 + y^2, xy) cannot be written in the form g(x), whereas the function h(x) = x^2 is a single real-valued function of one variable.

Therefore, the statement "A two-dimensional vector-valued function is defined by a single real-valued function of one variable" is false.

Question: A vector-valued function r approaches the limit L as t approaches a, written $\lim_{t \to a} r(t) = L$, provided that as t gets arbitrarily close to a, the distance between r(t) and L becomes infinitesimely small. True or False?

Answer: True

Explanation: Here's a concise and detailed explanation:

A vector-valued function r(t) approaches the limit L as t approaches a, denoted by $\lim_{t \to a} r(t) = L$, if and only if for any positive real number ε, there exists a positive real number δ such that for all t satisfying $|t-a| < \delta$, the distance between r(t) and L is less than or equal to ε.

In other words, as t gets arbitrarily close to a (i.e., within a distance of δ from a), the distance between r(t) and L becomes infinitesimally small, which is equivalent to saying that r(t) becomes arbitrarily close to L. This definition captures the intuitive notion of "getting closer and closer" to the limit as t approaches a.

Therefore, the statement is true: $\lim_{t \to a} r(t) = L$ implies that as t gets arbitrarily close to a, the distance between r(t) and L becomes infinitesimally small.

Question: For a vector-valued function r(t) to be continuous at point t=a, it is TRUE that:

r(a) exists, the limit as t approaches a of r(t) exists, and the limit as t approaches a of r(t) equals r(a).

Answer: True

Explanation: A vector-valued function r(t) is said to be continuous at point t=a if its limit as t approaches a exists and is equal to the value of the function at that point, i.e. r(a).

For a vector-valued function, continuity means that not only does the magnitude of the vector approach a certain value, but also the direction it points approaches the same direction.

Therefore, for r(t) to be continuous at t=a, all three conditions must hold:

1. The value of the function at that point, r(a), exists. 2. The limit as t approaches a of r(t) exists, meaning the vector-valued function does not have any "jumps" or discontinuities in its behavior near t=a. 3. The value of this limit is equal to the value of the function at that point, i.e. r(a).

In other words, if we imagine approaching the point t=a from either side (i.e. as t approaches a- or as t approaches a+), the vector-valued function should not change direction or magnitude abruptly; it should instead smoothly "approach" its value at t=a.

Thus, the statement is TRUE: for r(t) to be continuous at t=a, all three conditions must hold.

Question: True or False: The derivative of a vector-valued function r(t) is defined as $r'(t) = \lim_{\Delta t \to 0} \frac{r(t+\Delta t) - r(t)}{\Delta t}$, provided the limit exists.

Answer: True

Explanation: Here is a concise explanation for the answer:

The derivative of a vector-valued function r(t) is indeed defined as $r'(t) = \lim_{\Delta t \to 0} \frac{r(t+\Delta t) - r(t)}{\Delta t}$, provided the limit exists. This definition is an extension of the concept of a derivative from scalar-valued functions to vector-valued functions.

In essence, it measures the rate of change of the position vector r(t) at a given point t. The derivative is also often referred to as the velocity or tangent vector of the curve defined by r(t).

Note that this definition requires the limit to exist, meaning that as Δt approaches 0, the difference quotient $(r(t+\Delta t)-r(t))/\Delta t$ gets arbitrarily close to a fixed value, which is the derivative. If the limit does not exist, then the function is not differentiable at that point.

Question: The principal unit tangent vector at t is defined to be T(t) = r'(t) ——r'(t)——, provided ——r'(t)—— $\neq$ 0, if and only if the tail of any vector v that is a tangent vector to the curve C at point r(t) is placed at point r(t).

Answer: False

Explanation: The principal unit tangent vector T(t) is indeed defined as r'(t) ——r'(t)——, provided ——r'(t)—— $\neq$ 0. This definition states that the tail of any vector v that is a tangent vector to the curve C at point r(t) should be placed at point r(t).

In other words, if we draw any vector v that is parallel to and passing through the point r(t), its starting point or tail should be located at the point r(t). This ensures that the direction of v is consistent with being tangent to the curve C at that particular point. The normalization by ——r'(t)—— ensures that T(t) has a length of 1, making it a unit vector. Therefore, the statement is actually TRUE.

Question: When evaluating the indefinite integral of a vector-valued function, it is true that $\int[f(t)i+g(t)j+h(t)k]dt=[\int f(t)dt]i+[\int g(t)dt]j+[\int h(t)dt]k$.

Answer: True

Explanation: The statement is indeed true. When evaluating the indefinite integral of a vector-valued function F(t) = f(t)i + g(t)j + h(t)k, we can treat each component separately. This means that:

$\int[f(t)i+g(t)j+h(t)k]dt= (\int f(t)dt)i + (\int g(t)dt)j + (\int h(t)dt)k$

This is because the i, j, and k components are independent of each other. The integration operation only depends on the function being integrated and not on any specific direction or component. As a result, we can integrate each component separately and then combine them using the same basis vectors (i, j, and k) to get the desired result.

Question: True or False: The curvature of a curve is equal to the magnitude of its first derivative with respect to arc-length parameter.

Answer: True

Explanation: The statement is indeed TRUE.

To understand why, consider a smooth curve C in 3-dimensional space. The curvature of this curve at any point p is defined as the reciprocal of the osculating circle's radius at that point. This osculating circle is the best-fitting circular arc to the original curve at that point.

Now, let s be the arc-length parameter of the curve C. We can define the unit tangent vector T(s) and the principal normal vector N(s) along the curve. The first derivative with respect to arc-length parameter, denoted as C'(s), is simply the derivative of the position vector r(s) with respect to s.

At any point p on the curve, the osculating circle's radius can be calculated using the following formula:

1 / k = —T x N—

where k is the curvature and T x N denotes the cross product between T and N.

Differentiating this expression with respect to s gives us:

k' = (d/ds)—T x N— + —T x N——dT/ds—N

Now, recall that C'(s) = d/ds r(s). We can rewrite this as:

C'(s) = T(s)

Using the fact that T is a unit vector, we get:

dT/ds = 0

Substituting this back into the expression for k', we see that:

k' = —T x N—

This implies that the magnitude of the first derivative with respect to arc-length parameter, C'(s), is equal to the curvature k. Hence, the statement that the curvature of a curve is equal to the magnitude of its first derivative with respect to arc-length parameter is indeed TRUE.

Question: Is it true that the binormal vector at t is defined as $T(t) \times N(t)$, where $N(t)=T'(t)\|T'(t)\|$?

Answer: False.

Explanation: The binormal vector is indeed defined as $N(t) = T'(t) \times (T'(t) / ——T'(t)——)$. This is because the binormal vector is perpendicular to both the tangent and normal vectors, and it's direction is given by the cross product of these two vectors. So, the statement "$T(t) \times N(t)$" is incorrect, as it would not produce a vector perpendicular to $T(t)$ and $N(t)$.

Question: The acceleration vector of an object is defined as its velocity at that moment in time, not as the derivative of its velocity.

Answer: False

Explanation: The acceleration vector of an object is actually defined as the derivative of its velocity, not its velocity at that moment in time. This is because acceleration is a measure of how rapidly the velocity of an object changes, and taking the derivative of velocity does just that - it measures the rate of change of velocity.

In other words, acceleration is not simply a snapshot of velocity at a particular moment, but rather a description of how that velocity is changing over time. This is why we typically represent acceleration using the second derivative symbol (such as "a" or "acc"), and take its value to be the rate at which the velocity is changing.

For example, if an object's velocity increases from 2 meters per second to 4 meters per second in a certain amount of time, then its acceleration would be measured by taking the derivative of that change - i.e., how quickly did its velocity increase? This would give us a value for acceleration that describes not just what the velocity was at one moment, but also how it's changing over time.

Question: A function of two variables maps each ordered pair (x,y) in its domain to a unique real number z, but the range is not necessarily the set of all real numbers.

Answer: True

Explanation: The statement is indeed TRUE.

A function of two variables maps each ordered pair (x,y) in its domain to a unique real number z. This means that for every input (x,y), the function produces exactly one output value z. However, this does not imply that the range of the function consists of all possible real numbers. The range can be any subset of real numbers, including finite sets or intervals.

For example, consider the function $f(x,y) = x^2 + y^2$. This function maps each ordered pair (x,y) in its domain to a unique non-negative real number z. However, the range is not all real numbers because it only includes non-negative values.

Question: Is a level curve of a function of two variables for a given value c defined as the set of points satisfying the equation f(x,y)=c?

Answer: True

Explanation: A level curve of a function f(x,y) for a given value c is indeed defined as the set of points satisfying the equation f(x,y)=c.

To see why, consider a function f(x,y) that maps two-dimensional input values (x,y) to real numbers. A level curve of f at height c is essentially the set of all points in the xy-plane where f takes on the value c. This means that every point on the level curve has an x and y coordinate such that plugging those values into the function f(x,y)=c holds true.

In other words, a level curve is a contour of constant height for the given function. It's a set of points in the plane where the function takes on a specific value, and this definition captures the essence of what we mean by a level curve.

So, to summarize: yes, a level curve of a function f(x,y) for a given value c is defined as the set of points satisfying the equation f(x,y)=c.

Question: A vertical trace of the function f(x,y) can be either the set of points that solves the equation f(a,y)=z for a given constant x=a, or f(x,b)=z for a given constant y=b.

Answer: True

Explanation: A vertical trace of the function f(x,y) refers to the set of points that satisfies the equation f(a,y)=z for a given constant x=a, or f(x,b)=z for a given constant y=b.

The first case, f(a,y)=z, represents a vertical slice through the function at a fixed value of x=a. This is equivalent to looking at the function's behavior as y varies while keeping x fixed.

The second case, f(x,b)=z, represents a horizontal slice through the function at a fixed value of y=b. This is equivalent to looking at the function's behavior as x varies while keeping y fixed.

In both cases, the resulting set of points constitutes a vertical trace of the original function. Since this can be achieved by either fixing x or y and varying the other variable, it is indeed true that a vertical trace of the function f(x,y) can be obtained through either approach.

Question: A level surface of a function of three variables is defined to be the set of points satisfying the equation f(x,y,z)=c, given the function f(x,y,z) and a number c in the range of f.

Answer: True

Explanation: A level surface of a function of three variables is defined to be the set of points satisfying the equation f(x,y,z)=c, given the function f(x,y,z) and a number c in the range of f. This statement is indeed true.

To explain this concept further, consider a function f(x,y,z) that assigns a real number value to each point (x,y,z) in 3-dimensional space. The set of points satisfying the equation f(x,y,z)=c, where c is some constant, forms a surface in 3-dimensional space. This surface is called a level surface or an isosurface. It represents all points that have the same value or "level" according to the function.

For example, consider the function f(x,y,z) = x^2 + y^2 + z^2, which assigns to each point in 3-dimensional space its distance from the origin (0,0,0). The set of points satisfying the equation f(x,y,z)=c, where c is some positive constant, forms a sphere with radius sqrt(c).

The statement claims that this concept applies not just to specific functions and constants, but generally to any function f(x,y,z) and number c in its range. This means that for any given function and any given value of c within the range of the function, there exists a level surface defined by the equation f(x,y,z)=c.

Question: Is an open disk of radius δ centered at point (a,b) defined as {(x,y)—(x-a)^2+(y-

b)^2<δ^2}?

Answer: True

Explanation: The open disk of radius δ centered at point (a,b) is indeed defined as {(x,y)— (x-a)^2 + (y-b)^2 < δ^2}. This is a common and straightforward definition in mathematics, particularly in the context of real analysis and topology.

In essence, this equation describes all points (x,y) that are within a distance δ from the point (a,b). The radius δ is the maximum distance between (x,y) and (a,b), and the inequality ensures that all points closer to (a,b) than δ units away are included in the disk. This definition provides a clear and intuitive way to visualize an open disk in the plane, making it a fundamental concept in many areas of mathematics.

Question: True or False: For any given positive value ε, there exists a corresponding positive value δ such that for all points (x,y) in a circle of radius δ centered at (a,b), the difference between f(x,y) and its limit L is less than ε.

Answer: True

Explanation: The statement is indeed TRUE.

For any given positive value ε, we can find a corresponding positive value δ such that for all points (x,y) in a circle of radius δ centered at (a,b), the difference between f(x,y) and its limit L is less than ε.

To prove this, let's first consider a circle of radius δ centered at (a,b). This circle can be represented as {(x,y) — (x-a)^2 + (y-b)^2 ≤ δ^2}.

Since the function f(x,y) is continuous at its limit L, we know that for any positive ε, there exists a corresponding positive value δ such that —f(x,y)-L— < ε whenever (x-a)^2 + (y-b)^2 ≤ δ^2.

This means that for all points (x,y) in the circle of radius δ centered at (a,b), the difference between f(x,y) and its limit L is less than ε. Therefore, the statement is TRUE.

Question: Is it true that a point P0 is called an interior point of S if every δ disk centered around P0 contains points only inside S?

Answer: False

Explanation: A point P0 is called an interior point of S if every ε disk centered around P0 contains points only inside S. In other words, an interior point is a point where there exists some positive distance ε such that any disk of radius ε and center P0 lies entirely within S.

For example, consider the set S to be the unit circle in the plane. Then, any point on the circumference of the circle (like (1, 0)) is not an interior point because if you shrink a δ disk centered around it down to a sufficiently small size, it will contain points outside of S as well.

On the other hand, the origin (0, 0) is an interior point because any ε disk centered around it contains points only inside S.

Question: Is an open set in $\mathbb{R}2\mathbb{R}2$ always defined as every point of the subset being an interior point? True or False?

Answer: True

Explanation: An open set in $\mathbb{R}^2$ is indeed always defined as every point of the subset being an interior point. This means that if a set S is open, then for any given point p in S, there exists some positive distance ε such that the sphere centered at p with radius ε is entirely contained within S.

In other words, if a set contains all its boundary points (i.e., its limits or frontier), it cannot be considered open because these boundary points do not have any neighborhood (a set of all points that are closer to the point than some fixed distance) that is also contained within the set. Therefore, an open set in $\mathbb{R}^2$ is always defined as every point of the subset being an interior point, and this definition ensures that the set does not contain its boundary points.

Question: Is an open set S in $\mathbb{R}2\mathbb{R}2$ connected if it cannot be represented as the union of two or more disjoint, nonempty open subsets?

Answer: True

Explanation: A connected set in $\mathbb{R}2\mathbb{R}2$ is one that cannot be partitioned into two or more disjoint, nonempty subsets. In the case of an open set S in $\mathbb{R}2\mathbb{R}2$, this means that if it cannot be represented as the union of two or more disjoint, nonempty open subsets, then S must be connected.

This follows because any such representation would imply that S is disconnected, since each of these subsets would be a separate component. But then, S would not be connected, which contradicts our assumption. Therefore, we conclude that an open set in $\mathbb{R}2\mathbb{R}2$ cannot be represented as the union of two or more disjoint, nonempty open subsets if and only if it is connected.

Question: True or False: For any given positive value ε, there exists a suitably small distance δ such that for every point (x,y) within δ units of the boundary point (a,b), the function f(x,y) is no more than ε away from its limit L as (x,y) approaches (a,b).

Answer: False.

Explanation: The statement is false because it attempts to establish a uniform continuity property for the function f(x,y) around the point (a,b), which may not hold true in general.

Consider a simple example where f(x,y) is equal to 1 if the point (x,y) lies inside a certain region, and 0 otherwise. In this case, as we approach the boundary point (a,b), f(x,y) does not have a limit L as claimed by the statement. Instead, it takes on only two distinct values: 0 and 1.

However, for any given positive ε, there exists a small distance δ such that for every point (x,y) within δ units of the boundary point (a,b), we can make f(x,y) be arbitrarily close to either 0 or 1 by choosing δ sufficiently small. This means that f(x,y) is not ε away from its limit L as (x,y) approaches (a,b).

Question: For a function f to be continuous at (a, b), it is TRUE that:
f exists at (a, b) AND its limit as (x, y) approaches (a, b) exists AND this limit equals f(a, b).

Answer: True

Explanation: The statement is indeed TRUE.

For a function f to be continuous at (a, b), it must satisfy three conditions:

1. The function exists at the point (a, b). This means that as x approaches a and y approaches b, the function produces a specific output value f(a, b). 2. The limit of the function as (x, y) approaches (a, b) exists. In other words, the function's behavior is well-defined as we approach the point (a, b). This means that the function doesn't suddenly jump or change in some way that would make it difficult to determine its value at (a, b). 3. The limit of the function as (x, y) approaches (a, b) equals f(a, b). This is the most important condition - the actual value of the function at (a, b) must match the limiting behavior as we approach that point.

If all three conditions are met, then we can say that the function is continuous at (a, b), meaning that it has no "gaps" or "jumps" in its behavior. This continuity is essential for many mathematical

and physical applications, as it ensures that small changes in input lead to corresponding small changes in output.

Question: True or False: A $\delta\delta$ ball in three dimensions consists of all points in $\mathbb{R}3$ lying at a distance of less than δ from a given point, with the distance calculated as the sum of the absolute differences between corresponding coordinates.

Answer: False

Explanation: A $\delta\delta$ ball in three dimensions consists of all points in $\mathbb{R}3$ lying within a certain distance from a given point, where this distance is calculated as the maximum (not sum) of the absolute differences between corresponding coordinates. Specifically, a point (x,y,z) lies in the $\delta\delta$ ball if and only if —x-a—, —y-b—, and —z-c— are all less than δ for some constants a, b, and c. This is not equivalent to considering the sum of the absolute differences, as claimed by the false statement.

Question: True or False: The partial derivative of a function f(x,y) with respect to x is defined as the limit of (f(x+h,y)-f(x,y))/h as h approaches 0, where h is a small change in x.

Answer: True

Explanation: The partial derivative of a function f(x,y) with respect to x is indeed defined as the limit of (f(x+h,y)-f(x,y))/h as h approaches 0, where h is a small change in x.

In other words, the partial derivative measures how the function changes when x changes while keeping y constant. This is a fundamental concept in multivariable calculus and is used extensively in many branches of mathematics and science.

To see why this definition makes sense, consider a small change dx in the input x. The corresponding change df in the output f(x,y) can be approximated by the difference quotient (f(x+dx,y)-f(x,y))/dx. As dx approaches 0, this difference quotient approaches the partial derivative of f with respect to x.

The use of h instead of dx is simply a matter of convention and does not change the underlying mathematical concept. Both dx and h represent a small change in the input x, and both are used to define the partial derivative.

Question: The partial derivative of a function of three variables f(x,y,z) with respect to x is defined as the limit of the average rate of change of f as the increment in x approaches zero.

Answer: False

Explanation: The partial derivative of a function f(x,y,z) with respect to x is actually defined as the limit as h approaches 0 of the difference quotient:

(f(x+h,y,z)-f(x,y,z))/h

This represents the instantaneous rate of change of f with respect to x, and it's the primary way we define partial derivatives in multivariable calculus. The phrase "average rate of change" doesn't quite apply here, as we're looking at a single point rather than an interval.

So, the correct answer is actually True.

Question: The tangent plane to a surface at a point is determined by the direction of all curves passing through that point.

Answer: False

Explanation: The tangent plane to a surface at a point is determined by the direction of those

curves that are tangential to the surface at that point, not just any curve passing through the point. In other words, only those curves that have the same slope and curvature as the original surface at that point contribute to the determination of the tangent plane. This is because the tangent plane represents the best linear approximation of the surface at that point, and only tangential curves provide information about this local behavior. Non-tangential curves passing through the point do not capture this essential aspect of the surface's behavior, so they do not influence the determination of the tangent plane.

Question: The equation of the tangent plane to a surface defined by z = f(x,y) at point P0 = (x0, y0) is given by:

z = f(x0, y0) + fx(x0, y0)(x-x0) + fy(x0, y0)(y-y0).

True or False?

Answer: True

Explanation: The equation of the tangent plane to a surface defined by z = f(x,y) at point P0 = (x0, y0) is given by:

z = f(x0, y0) + fx(x0, y0)(x-x0) + fy(x0, y0)(y-y0).

This statement is true.

To prove this, we can use the definition of a tangent plane as the limiting position of the planes that pass through the point P0 and are parallel to the lines that intersect P0. The equation above represents the equation of the plane that passes through the point P0 and has the direction vectors (fx(x0,y0), 1) and (fy(x0,y0), 1). This is the tangent plane at P0.

Question: Is the equation for the linear approximation of a function f(x,y) at the point (x0,y0) given by L(x,y)=f(x0,y0)+fx(x0,y0)(y−y0)+fy(x0,y0)(x−x0)?

Answer: False

Explanation: The equation for the linear approximation of a function f(x,y) at the point (x0,y0) is actually given by:

L(x,y)=f(x0,y0)+$\partial f/\partial y$(x0,y0)(y−y0)+$\partial f/\partial x$(x0,y0)(x−x0)

The reason it's not f(x0,y0)+fx(x0,y0)(y−y0)+fy(x0,y0)(x−x0) is that the partial derivatives $\partial f/\partial x$ and $\partial f/\partial y$ are used instead of fx and fy. This is because fx and fy represent the slopes of the function in the x and y directions at the point (x0,y0), whereas the partial derivatives give us a more accurate representation of the linear approximation.

Question: True or False: A function f(x,y) is differentiable at a point P(x0,y0) if, for every point (x,y) in a small neighborhood around P, it can be written as a linear combination of x and y plus an error term E that approaches zero as (x,y) gets arbitrarily close to P.

Answer: True

Explanation: A function f(x,y) is differentiable at a point P(x0,y0) if and only if it can be written as a linear combination of x and y, plus an error term E that approaches zero as (x,y) gets arbitrarily close to P. This means that for every point (x,y) in a small neighborhood around P, there exist real numbers a, b, c such that:

f(x,y) = ax + by + E

where the size of E decreases as (x,y) gets closer to P. In other words, the error term E must be "negligible" or "arbitrarily small" in some sense. This property is essential for differentiability because it allows us to define a linear approximation of the function at point P, which is crucial in

many areas of mathematics and science, such as optimization, physics, and engineering.

Question: The differential dz of z=f(x,y) at (x0,y0) is defined as dz=fx(x0,y0)dx+fy(x0,y0)dy. True or False?

Answer: True

Explanation: Here is a concise explanation for the true statement:

The differential dz of z=f(x,y) at (x0,y0) indeed follows the definition dz=fx(x0,y0)dx+fy(x0,y0)dy. This is because the total derivative of f with respect to x at (x0,y0) is given by fx(x0,y0), and similarly for y.

In other words, when we take a small change dx in x and a small change dy in y about the point (x0,y0), the resulting change dz in z is proportional to the partial derivatives of f with respect to x and y at that point. The proportionality constants are fx(x0,y0) and fy(x0,y0), respectively.

This definition makes sense because it allows us to compute the linearization of a function around a given point, which is an important concept in multivariable calculus.

Question: A function f(x,y,z) is differentiable at a point P(x0,y0,z0) if for any points (x,y,z) near PP, it can be written as the sum of linear terms in x-x0, y-y0, and z-z0 plus an error term E that approaches zero as (x,y,z) approaches P.

Answer: True

Explanation: The statement is indeed TRUE.

A differentiable function f(x,y,z) at a point P(x0,y0,z0) means that it can be approximated by a linear combination of the differences (x-x0), (y-y0), and (z-z0) plus an error term E, which approaches zero as the points (x,y,z) approach P.

In other words, for any small changes Δx, Δy, and Δz in the variables x, y, and z, respectively, around the point P, we can write:

$f(x+\Delta x, y+\Delta y, z+\Delta z) \approx f(x0, y0, z0) + \partial f/\partial x * \Delta x + \partial f/\partial y * \Delta y + \partial f/\partial z * \Delta z + E$

where the partial derivatives $\partial f/\partial x$, $\partial f/\partial y$, and $\partial f/\partial z$ are evaluated at the point P. The error term E is such that it approaches zero as (Δx, Δy, Δz) approaches (0, 0, 0), i.e., as the points get arbitrarily close to P.

This concept of differentiability is fundamental in calculus and has far-reaching implications for many areas of mathematics, physics, engineering, and other sciences.

Question: The directional derivative of f(x,y) in the direction of u=cosθi+sinθj is given by Duf(a,b)=limh→0f(a+hcosθ,b+hsinθ)−f(a,b)h, provided the limit exists. True or False?

Answer: True

Explanation: The directional derivative of f(x,y) in the direction of u=cosθi+sinθj is given by Duf(a,b)=limh→0f(a+hcosθ,b+hsinθ)−f(a,b)h, provided the limit exists. This is indeed True.

To understand why, recall that the directional derivative measures the rate at which f(x,y) changes in the direction specified by the unit vector u. In this case, we are considering a small change Δx = hcosθ and Δy = hsinθ, which corresponds to moving a small distance h along the direction u.

The expression Duf(a,b) calculates the difference quotient (f(a+hcosθ,b+hsinθ)-f(a,b))/h, which approximates the rate of change in f(x,y) as we move in the direction u. As h approaches 0, this difference quotient converges to the directional derivative, provided it exists.

In other words, the given formula for Duf(a,b) is a precise mathematical representation of the

concept of taking a small step in the direction u and measuring how much f(x,y) changes as a result. This is why we say that the formula defines the directional derivative, and it is indeed True.

Question: The gradient of a function f(x,y) is a vector that contains only the x-component and no y-component. True or False?

Answer: False

Explanation: The gradient of a function f(x,y) is actually a vector that contains both x- and y-components, not just the x-component.

To understand why, let's consider the definition of the gradient. The gradient of a function f at a point (x0,y0) is given by:

∇f(x0,y0) = (∂f/∂x)(x0,y0), (∂f/∂y)(x0,y0)

This vector has both x- and y-components, specifically the partial derivatives of f with respect to x and y evaluated at the point (x0,y0).

The gradient represents the direction in which the function increases most rapidly. In this sense, it contains information about both the x- and y-directions. Therefore, the statement that the gradient is a vector containing only the x-component and no y-component is false.

Question: Is the gradient of a function f defined as the vector ∇f = fxi + fyj + fz k?

Answer: True

Explanation: The gradient of a function f is indeed defined as the vector ∇f = fxi + fyj + fz k, where xi, yj, and zk are unit vectors in the x, y, and z directions respectively. This definition arises from the concept of directional derivatives, which describe the rate at which the function changes as we move in a particular direction.

In Cartesian coordinates, the gradient ∇f is a vector that points in the direction of the steepest ascent of the function f and has magnitude equal to the rate of change of the function along that direction. In other words, it describes the maximum rate at which the function increases as we move in a specific direction.

The expression ∇f = fxi + fyj + fz k can be interpreted as a sum of partial derivatives, where each partial derivative represents the rate of change of the function in one of the coordinate directions. This allows us to calculate the gradient of a function at any point in space and use it for various applications in mathematics, physics, engineering, and other fields.

Question: Is the directional derivative of a function f(x,y,z) in the direction of unit vector u = cosαi + cosβj + cosγk given by the formula Duf(x0,y0,z0)=limt$\rightarrow$0f(x0+tcosα,y0+tcosβ,z0+tcosγ)$-$f(x0,y0,z0)t?

Answer: True

Explanation: The directional derivative of a function f(x,y,z) in the direction of unit vector u = cosαi + cosβj + cosγk is indeed given by the formula:

Duf(x0,y0,z0)=limt$\rightarrow$0f(x0+tcosα,y0+tcosβ,z0+tcosγ)$-$f(x0,y0,z0)t

This formula represents the rate of change of the function f at the point (x0, y0, z0) in the direction specified by the unit vector u. The limit as t approaches 0 ensures that we are considering infinitesimal changes in the input variables x, y, and z.

In essence, this formula gives us the derivative of the function in the specific direction defined by the unit vector u, which is a fundamental concept in multivariable calculus.

Question: Is a critical point of a function f(x,y) necessarily the point where both fx(x0,y0) and

fy(x0,y0) are equal to zero?

Answer: False

Explanation: A critical point of a function f(x,y) is a point where the partial derivatives fx(x0,y0) and fy(x0,y0) are both equal to zero. This is not necessarily true, however, because a critical point can also be a saddle point.

A saddle point is a point where the Hessian matrix (the matrix of second partial derivatives) has both positive and negative eigenvalues. In this case, the first partial derivatives fx and fy may not be equal to zero at the critical point, but the function still has a saddle-shaped behavior around that point.

For example, consider the function $f(x,y) = x^2 - y^2$. The critical point is (0,0), where both fx(0,0) = 0 and fy(0,0) = 0. However, this function does not have a minimum or maximum at the critical point, but rather a saddle shape.

Question: A function f that is defined and continuous on an open set has a local maximum at (x0,y0) if f(x0,y0) is greater than or equal to its value at any point within some disk centered at (x0,y0). True or False?

Answer: True

Explanation: The statement is indeed TRUE.

A function f that is defined and continuous on an open set has a local maximum at (x0,y0) if its value at (x0,y0) is greater than or equal to its value at any point within some disk centered at (x0,y0).

To see this, recall the definition of continuity: for every $\varepsilon > 0$, there exists $\delta > 0$ such that —f(x0,y0) - f(x,y)— $< \varepsilon$ whenever —(x,y) - (x0,y0)— $< \delta$.

Now suppose f has a local maximum at (x0,y0). Then for any $\varepsilon > 0$, we can find a $\delta > 0$ such that f(x0,y0) $\geq$ f(x,y) whenever —(x,y) - (x0,y0)— $< \delta$.

Since the function is defined on an open set and is continuous at (x0,y0), it follows that f(x0,y0) $\geq$ f(x,y) for all (x,y) within some disk centered at (x0,y0). This implies f(x0,y0) is a local maximum.

In summary, the statement is true because if a function has a local maximum at a point and is continuous on an open set, its value at that point must be greater than or equal to its values at nearby points within some disk centered at that point.

Question: Is the point where both partial derivatives are zero, but the function does not have a local extremum at that point considered a saddle point?

Answer: True

Explanation: The point where both partial derivatives are zero is known as a critical point. At this point, the function's behavior can be one of three types: local maximum, local minimum, or saddle point.

A local extremum occurs when the function has a relative maximum or minimum at the critical point. This means that every small perturbation away from the critical point will result in a value that is either greater than (for a maximum) or less than (for a minimum) the value of the function at the critical point.

In contrast, a saddle point occurs when the function has both a relative maximum and a relative minimum along different directions. This means that if you move away from the critical point in one direction, the function will decrease, but moving away in another direction will cause it to increase. The function does not have a local extremum at a saddle point because there is no consistent direction for the function to be increasing or decreasing.

Therefore, the statement "the point where both partial derivatives are zero, but the function does not have a local extremum at that point is considered a saddle point" is true.

Question: Is the double integral of a function f(x,y) over a rectangular region RR in the xy-plane defined as the limit of a sum of values of f at selected points xi*,yj* multiplied by the area ΔA?

Answer: True

Explanation: The double integral of a function f(x,y) over a rectangular region RR in the xy-plane is defined as the limit of a sum of values of f at selected points (xi*,yj*) multiplied by the area ΔA. This definition makes sense because it allows us to approximate the double integral using a Riemann sum.

Consider a rectangular region RR with length L and width W, bounded by the lines x = a, x = b, y = c, and y = d. We can divide this region into small sub-regions, or rectangles, of size ΔA = LΔx * WΔy, where Δx and Δy are small changes in the x- and y-coordinates respectively.

The double integral of f over RR is then defined as:

$\int\int$f(x,y) dx dy = lim (Δx → 0, Δy → 0) Σ [f(xi*,yj*) * ΔA]

where the sum is taken over all the sub-regions. The limit represents the process of refining the partition of RR into smaller and smaller rectangles.

By using this definition, we can evaluate double integrals using various techniques, such as iterated integration or polar coordinates. This definition also provides a solid foundation for further exploration of double integrals in calculus and their applications in mathematics and other fields.

Question: When evaluating an iterated integral, the order of integration can be reversed if the function being integrated is continuous over the rectangular region, regardless of whether the limits of integration are fixed or variable. True or False?

Answer: False.

Explanation: When evaluating an iterated integral, reversing the order of integration is only possible if the function being integrated is absolutely continuous over the rectangular region. This means that the function must be continuous and its derivative must also be continuous.

If the limits of integration are fixed, this condition is sufficient to ensure that the order of integration can be reversed. However, if the limits of integration are variable, additional conditions must be met.

For example, consider a function f(x,y) that is continuous over the region [a,b]x[c,d], but its derivative with respect to x is not continuous at some point (x0,y0). In this case, reversing the order of integration would not be valid because the iterated integral would not converge.

Therefore, the statement "the function being integrated is continuous over the rectangular region, regardless of whether the limits of integration are fixed or variable" is insufficient to guarantee that the order of integration can be reversed.

Question: The area of the region R is given by A(R) = $\int\int$R 1 dA is TRUE.

Answer: False

Explanation: The area of a region R is given by A(R) = $\int\int$R 1 dA is FALSE.

This statement is incorrect because the integral $\int\int$R 1 dA represents the volume of the solid region R, not its area. The correct formula for the area of a region R would depend on the specific shape and coordinate system used to define R.

Question: Is the average value of a function over a region calculated by integrating the function over that region?

Answer: True

Explanation: The average value of a function f(x) over a region R is indeed calculated by integrating f(x) over that region. This is because the average value represents the balance point, where the sum total of the function's values is equal to its mean.

To calculate the average value, we first need to find the area of the region under consideration. Then, we integrate the function f(x) with respect to x, effectively summing up the function's values over the entire region. The result is the total value of the function over the region, which can be divided by the area to get the average value.

Formally, if R is a region in the plane and f(x) is a function defined on R, then the average value of f(x) over R is given by:

Average Value = (1/A) * $\int$[R] f(x) dx

where A is the area of the region R. This formula illustrates that integrating the function f(x) over the region R indeed calculates its average value.

Question: A region in the (x,y) plane is of Type I if it lies between two horizontal lines and the graphs of two continuous functions.

Answer: False

Explanation: The given statement claims that a region in the (x,y) plane is of Type I if it lies between two horizontal lines and the graphs of two continuous functions.

This assertion is false. A region can indeed be bounded by two horizontal lines, which would define its upper and lower bounds on the y-axis, and also by the graphs of two continuous functions, defining its left and right boundaries along the x-axis. This would satisfy the given conditions for a Type I region.

However, this does not mean that all regions meeting these criteria are necessarily of Type I. The classification as Type I typically requires additional constraints on the nature of the bounding curves or lines, which have not been specified in the original statement.

In summary, while it is possible to find regions that satisfy the given conditions, they may not be classified as Type I without further specifications or constraints.

Question: The area of a plane-bounded region is defined as the double integral $\iint$D1dA, which means that true or false?

Answer: True

Explanation: The area of a plane-bounded region is defined as the double integral $\iint$D1dA, which means that it is indeed true.

In mathematics, the concept of area is used to measure the amount of space inside a shape. For a region bounded by a curve in the plane, this area can be calculated using the double integral. The double integral represents the accumulation of small pieces of area over the entire region.

The notation $\iint$D1dA means that we integrate the function 1 (which is equivalent to a constant) with respect to the variables x and y over the region D. This integration process essentially adds up all the tiny areas within the region, giving us the total area enclosed by the boundary curve.

In essence, the statement "The area of a plane-bounded region is defined as the double integral $\iint$D1dA" accurately reflects the mathematical concept of calculating the area of a region using a double integral.

Question: The average value of a function f(x,y) over a plane-bounded region D is equal to 1/A(D) $\iint$D f(x,y) dA, where A(D) represents the area of the region D.

Answer: True

Explanation: The statement is indeed TRUE.

The average value of a function f(x,y) over a plane-bounded region D is defined as the ratio of the integral of the function over the region to the area of the region. Mathematically, this can be represented as:

Average Value = (1/A(D)) $\iint$D f(x,y) dA

where A(D) represents the area of the region D.

This formula makes intuitive sense: if you were to divide the total value of a function over a region by the area of that region, you would get an average value. The integral in the numerator captures this total value, and the denominator provides the scale to convert it into an average.

So, for any plane-bounded region D and function f(x,y), we have:

Average Value = (1/A(D)) $\iint$D f(x,y) dA

Which is why the statement "The average value of a function f(x,y) over a plane-bounded region D is equal to 1/A(D) $\iint$D f(x,y) dA" is TRUE.

Question: Is it true that a joint density function of two continuous random variables must satisfy f(x,y) $\geq$ 0 and $\iint$R2f(x,y)dA = 1?

Answer: True

Explanation: The joint density function of two continuous random variables X and Y must satisfy two properties:

Firstly, f(x,y) $\geq$ 0 for all (x,y) in the support of X and Y. This is because a density function cannot take on negative values - it's a probability distribution!

Secondly, $\iint$R2f(x,y)dA = 1, where R2 represents the region of support of X and Y. This ensures that the total probability mass is equal to 1, which is a fundamental property of a probability distribution.

These two conditions are necessary and sufficient for a function f(x,y) to be a joint density function of X and Y.

Question: Two random variables XX and YY are independent if their joint density function can be written as the product of their individual density functions.

Answer: True

Explanation: The statement is indeed TRUE.

Two random variables X and Y are independent if their joint density function can be written as the product of their individual density functions, denoted by f(x) and g(y), respectively. This means that the probability of a particular outcome for X and Y occurring together is equal to the product of the probabilities of each variable occurring individually.

In other words, the independence of X and Y implies that the information contained in one random variable does not provide any additional information about the other random variable. This is often represented mathematically as:

f(x, y) = f(x) * g(y)

This property is a fundamental concept in probability theory and statistics, and it has important implications for data analysis, modeling, and decision-making.

Question: In probability theory, the expected values E(X) and E(Y) are considered to be the most likely outcomes of the events, with the formulas for their calculation being:

E(X)=∬Sxf(x,y)dAandE(Y)=∬Syf(x,y)dA.

True or False?

Answer: False

Explanation: The correct calculation of expected values E(X) and E(Y) in probability theory is:

E(X)=∫x*f(x)dxdyandE(Y)=∫y*f(x)dxdy

Where f(x) is the joint probability density function (pdf) of X and Y.

These formulas represent the average value of a random variable over its entire range, weighted by the probability of each outcome.

In contrast, the original formulas given in the question:

E(X)=∬Sxf(x,y)dAandE(Y)=∬Syf(x,y)dA

are incorrect, as they imply that expected values are being calculated for a 2D region S rather than individual random variables X and Y.

Therefore, the answer is: False.

Question: True or False: The double integral of a function f(r,θ) over a polar rectangular region R in the r-θ plane is defined as the limit of the sum of values of the function at the points (rij*,θij*) multiplied by the area ΔA, where m and n approach infinity.

Answer: True

Explanation: The double integral of a function f(r,θ) over a polar rectangular region R in the r-θ plane is indeed defined as the limit of the sum of values of the function at the points (rij*,θij*) multiplied by the area ΔA, where m and n approach infinity.

This definition is based on the concept of Riemann sums, which are used to approximate definite integrals. In this case, we are dealing with a double integral in polar coordinates, which can be viewed as a sum of values of f(r,θ) multiplied by the area ΔA of small rectangular regions (or "cells") that make up the region R.

The idea is to divide the region R into m*n cells, where each cell has an area ΔA = (rij+1 - rijn)(θij+1 - θij). Then, we evaluate the function f(r,θ) at the points (rij*,θij*) that lie within each cell and multiply this value by the area ΔA of that cell. This gives us a sum of m*n terms, where each term is the product of the function value and the area of the corresponding cell.

As m and n approach infinity, these sums converge to the double integral of f(r,θ) over the region R. The limit defines the value of the double integral as the accumulation of all these values weighted by the areas of the cells. This definition is equivalent to the more familiar definition of a double integral in Cartesian coordinates, but it is particularly useful when working with polar coordinates and regions defined in terms of r and θ.

Question: The triple integral of a function f(x,y,z) over a rectangular box B is defined as: True or False?

Answer: True

Explanation: The triple integral of a function f(x,y,z) over a rectangular box B is indeed defined as True.

This definition states that for a given function f(x,y,z) and a rectangular box B with dimensions [a,b] in the x-direction, [c,d] in the y-direction, and [e,f] in the z-direction, the triple integral of f over B can be expressed as:

$\int[a,b] \int[c,d] \int[e,f]$ f(x,y,z) dxdydz

This represents the volume under the surface defined by f(x,y,z) within the rectangular box B. The order of integration is typically written in the xyz-order, with dx being integrated first, followed by dy, and finally dz. This definition allows us to compute volumes, masses, and other physical quantities for various functions and shapes in three-dimensional space.

Question: True or False: The triple integral of a continuous function f(r,θ,z) in cylindrical coordinates can be defined as the limit of a triple Riemann sum for any sample point (r,θ,z) in the cylindrical subbox Bijk.

Answer: False

Explanation: The triple integral of a continuous function f(r,θ,z) in cylindrical coordinates cannot be defined as the limit of a triple Riemann sum for any sample point (r,θ,z) in the cylindrical subbox Bijk. This is because the Riemann sum would require dividing the region into smaller rectangular boxes, which are not compatible with the cylindrical coordinate system.

In cylindrical coordinates, the region of integration can be divided into smaller cylindrical shells or washers, but these shapes cannot be approximated by rectangles. Therefore, a different approach is needed to define and evaluate the triple integral in terms of Riemann sums. This typically involves dividing the region into small cylinders or conical frustums, and then summing up the volume elements weighted by the function f.

Question: Is the triple integral in spherical coordinates defined as the limit of a triple Riemann sum?

Answer: True

Explanation: The triple integral in spherical coordinates is indeed defined as the limit of a triple Riemann sum. This can be seen by considering the definition of a triple integral:

$\int\int\int$ f(x,y,z) dxdydz = lim (Δx $\to$ 0, Δy $\to$ 0, Δz $\to$ 0) Σ [f(x_i,y_j,z_k) $\times$ (Δx)(Δy)(Δz)]

where the sum is taken over all points in the region of integration, and f(x,y,z) is the function being integrated.

When switching to spherical coordinates (r,θ,φ), this integral can be rewritten as:

$\int\int\int$ f(r,θ,φ) r^2 sin(θ) dr dθ dφ = lim (Δr $\to$ 0, $\Delta\theta$ $\to$ 0, $\Delta\varphi$ $\to$ 0) Σ [f(r_i,θ_j,φ_k) $\times$ (Δr)($\Delta\theta$)($\Delta\varphi$)]

where the sum is taken over all points in the region of integration, and f(r,θ,φ) is the function being integrated.

This can be seen as a triple Riemann sum, where the area elements are given by r^2 sin(θ) dr dθ dφ. This shows that the triple integral in spherical coordinates is indeed defined as the limit of a triple Riemann sum.

Question: True or False: The moments of inertia about the yz-plane, xz-plane, and xy-plane are Ix = $\int\int\int$Q(x2+y2)ρ(x,y,z)dV, Iy = $\int\int\int$Q(y2+z2)ρ(x,y,z)dV, Iz = $\int\int\int$Q(z2+x2)ρ(x,y,z)dV.

Answer: False

Explanation: The moments of inertia about the yz-plane, xz-plane, and xy-plane are given by:

Ix = $\int\int\int$Q(x2+y2)ρ(x,y,z)dV Iy = $\int\int\int$Q(y2+z2)ρ(x,y,z)dV Iz = $\int\int\int$Q(z2+x2)ρ(x,y,z)dV

These expressions are actually correct for a solid object with density ρ(x, y, z), where Q is some function of position (x, y, z).

To see this, recall that the moment of inertia about an axis is given by:

I = $\iiint(x2+y2+z2)\rho(x,y,z)dV$

for an axis parallel to one of the coordinate axes.

Now, when we want to find the moments of inertia about the yz-plane (xz-plane or xy-plane), we need to rotate our original axis system so that it is aligned with the plane of interest.

In this case:

* For Ix, we would like to find the moment of inertia about the x-axis. This corresponds to rotating our coordinate system by 90 degrees around the yz-plane, which doesn't change the z-coordinate at all. * For Iy, we need to rotate it by 90 degrees around the xz-plane, so that the xz-plane becomes parallel to the new x-axis and the original yz-plane is now perpendicular to this axis. In this case, the x2 terms become y2 terms and vice versa. * Similarly, for Iz, we need to rotate our coordinate system by 90 degrees around the xy-plane.

In each of these cases, when we write down the expression for I in terms of (x', y', z'), where (x', y', z') is the new set of coordinates that we have rotated into, we see that the term that becomes important in this particular rotation is precisely the one corresponding to our desired plane. For instance, in the case of Iz, it's the z2 terms that become important when we rotate by 90 degrees around the xy-plane.

So, these expressions are actually correct for finding the moments of inertia about the yz-plane, xz-plane, and xy-plane.

Question: A one-to-one transformation is defined as:

True or False?

Answer: True

Explanation: A one-to-one transformation is defined as a function that has exactly one pre-image for each of its images, meaning that every output corresponds to exactly one input. In other words, if we know the output of the function, we can uniquely determine the input.

This means that if we apply the same input twice, we will always get the same output, and vice versa: if we know the output, there is only one possible input that could have produced it. This property allows us to define a bijective correspondence between the domain and range of the function, which is essential in many areas of mathematics, such as calculus, algebra, and geometry.

Therefore, our answer is indeed TRUE.

Question: The Jacobian of the C1C1 transformation T(u,v) is defined by its determinant, which is equal to $(\partial x/\partial u) * (\partial y/\partial v) - (\partial x/\partial v) * (\partial y/\partial u)$. True or False?

Answer: False

Explanation: The Jacobian of the C1C1 transformation T(u,v) is actually defined by its determinant, which is equal to $(\partial x/\partial u) * (\partial y/\partial v) + (\partial x/\partial v) * (\partial y/\partial u)$, not minus. This can be verified by considering the chain rule and the definition of a Jacobian matrix.

Question: The Jacobian determinant of three variables is defined as the absolute value of the product of the partial derivatives of x, y, and z with respect to u, v, and w in either order.

Answer: False

Explanation: The Jacobian determinant of three variables is actually defined as the signed value of the product of the partial derivatives, not absolute value. This means that the order in which the variables are listed matters, unlike the definition given in the question. The correct formula is:

J = ∂x/∂u * ∂y/∂v * ∂z/∂w - ∂x/∂v * ∂y/∂u * ∂z/∂w

The sign of the determinant depends on the relative order of the variables. This is in contrast to the statement that the Jacobian determinant can be any value, true or false.

Question: A vector field in $\mathbb{R}2$ is defined by assigning a one-dimensional vector to each point of its domain, whereas a vector field in $\mathbb{R}3$ is defined by assigning a three-dimensional vector to each point of its domain.

Answer: False

Explanation: The statement is actually True.

In $\mathbb{R}2$ (two-dimensional space), a vector field is indeed defined by assigning a one-dimensional vector, typically an arrow in the plane, to each point of its domain. This one-dimensional vector has both magnitude and direction, but only two degrees of freedom (direction).

Similarly, in $\mathbb{R}3$ (three-dimensional space), a vector field is defined by assigning a three-dimensional vector, again represented as an arrow in 3D space, to each point of its domain. This three-dimensional vector has magnitude, direction, and three degrees of freedom.

So, the statement accurately describes the nature of vector fields in both $\mathbb{R}2$ and $\mathbb{R}3$ domains.

Question: A vector field FF in $\mathbb{R}2$ or $\mathbb{R}3$ is a gradient field if there exists a scalar function ff such that $\nabla f = F$. Is this statement True?

Answer: True

Explanation: The statement "A vector field FF in $\mathbb{R}2$ or $\mathbb{R}3$ is a gradient field if there exists a scalar function ff such that $\nabla f = F$" is indeed True.

To see why, recall that the gradient of a function f at a point x is given by $\nabla f(x) = (\partial f/\partial x1,$ ..., $\partial f/\partial xn)$, where n is the dimension of the space. If we have a vector field FF on $\mathbb{R}n$, then $FF(x) = \nabla f(x)$ for some function f if and only if $F(x) = \nabla f(x)$. This is because both sides represent the same thing - the directional derivative of f in the direction given by FF.

So, if there exists a scalar function ff such that $\nabla ff = F$, then it follows that FF is a gradient field. Conversely, if FF is a gradient field, then we can define ff as the function whose gradient is FF, and $FF(x) = \nabla ff(x)$. This shows that the existence of ff implies that FF is a gradient field, and vice versa.

In summary, the statement is True because it characterizes a gradient field in terms of its relation to a scalar function, which is equivalent to saying that $FF(x) = \nabla ff(x)$ for some ff.

Question: The scalar line integral of a function along a smooth curve is defined as the limit of the sum of values of the function evaluated at points on the curve, where the points are chosen to approximate the curve with increasing precision.

Answer: True

Explanation: The scalar line integral of a function along a smooth curve is indeed defined as the limit of the sum of values of the function evaluated at points on the curve, where the points are chosen to approximate the curve with increasing precision.

In more detail, let's consider a continuous function f(x,y) and a smooth curve C in the plane. We can divide the curve into small sub-intervals and evaluate the function at each point within these intervals. The sum of these values is the Riemann sum. As we make the sub-intervals smaller and smaller (i.e., as our approximation of the curve becomes more precise), the Riemann sum approaches a limiting value, which is the scalar line integral of f along C.

This definition allows us to generalize the concept of definite integrals from one-dimensional functions to two-dimensional curves. It also provides a framework for evaluating and computing scalar line integrals of various types of functions and curves.

Question: True or False: The vector line integral of a vector field along an oriented smooth curve is equivalent to the sum of infinitesimal contributions from each point on the curve, in the limit where the number of points increases without bound?

Answer: True

Explanation: The vector line integral of a vector field along an oriented smooth curve is indeed equivalent to the sum of infinitesimal contributions from each point on the curve, in the limit where the number of points increases without bound.

To see why this is true, consider a small segment of the curve, say from point P to point Q. The vector line integral can be approximated by the dot product of the vector field at some point within that segment and the length of the segment itself. This can be written as:

$\int (F(x) \cdot dR)$

where F is the vector field, x is a point on the curve, and dR is an infinitesimal displacement vector along the curve.

As we make the segment smaller and smaller, the approximation becomes better and better. In the limit where the number of points increases without bound, the sum of these infinitesimal contributions converges to the original vector line integral.

In other words, the original integral can be broken down into an infinite number of small pieces, each contributing a tiny amount to the overall value of the integral. This is the essence of the Riemann sum, which is used to approximate definite integrals in calculus.

The beauty of this equivalence lies in its simplicity and elegance. It provides a powerful tool for evaluating vector line integrals by breaking them down into smaller, more manageable pieces.

Question: The flux of F across C is determined by integrating the vector product CF along the curve C, weighted by the magnitude of the normal vector n(t) at each point on the curve.

Answer: True

Explanation: The statement is indeed true. The flux of vector field F across curve C is given by the surface integral:

$\int (CF) \cdot n \, ds$

where n(t) is the normal vector to the curve at each point t, and ds is the infinitesimal length element along the curve.

This can be rewritten as:

$\int ((F \cdot n) \cdot t) \, —t— \, dt$

where —t— is the magnitude of the tangent vector t to the curve. The dot product F·n represents the component of F in the direction perpendicular to the curve at each point, which is precisely what we want for the flux.

So, the integral indeed calculates the weighted sum of these components as one traverses the curve C, with the weight being the magnitude —t— of the normal vector.

Question: A curve C is a closed curve if and only if there exists a parameterization r(t) such that it traverses C exactly once and returns to its starting point.

Answer: True

Explanation: The statement "A curve C is a closed curve if and only if there exists a parameterization r(t) such that it traverses C exactly once and returns to its starting point" is indeed true.

To see why, consider the following. If a curve C has a parameterization r(t) that traverses C exactly once and returns to its starting point, then we can define a new parameter t' = t + T, where T is the total length of the curve. This new parameter will still traverse the entire curve, but now it will start at the same point where it ended originally.

Conversely, if a curve C is closed, then there exists a parameterization r(t) that traverses C exactly once and returns to its starting point. To see this, let r(t) be any parameterization of the curve, and define s(t) = r(2t/T), where T is the total length of the curve. Then s(t) will traverse the entire curve exactly once and return to its starting point, so C is a closed curve.

In summary, the two statements are equivalent because they both describe the existence of a parameterization that traverses the curve once and returns to its starting point.

Question: Is a simply connected region in 2D necessarily one that is both connected and has no holes?

Answer: True

Explanation: A simply connected region in 2D is necessarily one that is both connected and has no holes because its fundamental group, $\pi 1(X)$, is trivial. This means that every loop in the region can be continuously shrunk to a point without leaving the region. Since the region is connected, there are no separate components for loops to get stuck in. Furthermore, if the region had holes, some loops would have to stay outside these holes, contradicting the definition of simply connected. Therefore, the statement is true: a simply connected region in 2D must be both connected and have no holes.

Question: A vector field is independent of path if it has the same value when integrated along any two paths in its domain that have the same endpoints.

Answer: True

Explanation: A vector field is said to be independent of path if the line integral of the vector field along any two paths in its domain that have the same endpoints is equal. In other words, the value of the line integral does not depend on which specific path between those endpoints we take.

This means that if we were to calculate the work done by the force of the vector field as we move from one point to another, the answer would be the same regardless of whether we took a straight line, a curved line, or even a zigzagging path. The value of the line integral is purely determined by the endpoints and not at all by the specific route taken.

This property is often referred to as path independence. It's an important concept in physics and engineering, as it allows us to define quantities like electric potential and magnetic potential without having to specify a particular path.

Question: Is the divergence of a vector field F in $\mathbb{R}3\mathbb{R}3$ defined as Px+Qy+Rz=∂P∂x+∂Q∂y+∂R∂z?

Answer: True

Explanation: The divergence of a vector field F in $\mathbb{R}^3$ is indeed defined as:

Px + Qy + Rz = ∂P/∂x + ∂Q/∂y + ∂R/∂z

This definition states that the divergence of a vector field F at a point (x, y, z) is equal to the partial derivative of the x-component P with respect to x, plus the partial derivative of the

y-component Q with respect to y, plus the partial derivative of the z-component R with respect to z.

Question: The curl of a vector field is a scalar field. True or False?

Answer: False

Explanation: The curl of a vector field is actually a pseudoscalar field, not a scalar field. This means that it can be either a scalar or a vector depending on the coordinate system used to describe it.

In Cartesian coordinates, the curl of a vector field F = (F1, F2, F3) is defined as:

curl(F) = (∂F3/∂y - ∂F2/∂z), (∂F1/∂z - ∂F3/∂x), (∂F2/∂x - ∂F1/∂y)

As you can see, the curl is a vector field, not a scalar. It has both magnitude and direction.

In other coordinate systems, such as spherical or cylindrical coordinates, the curl can also be a pseudoscalar. This means that it can be expressed in terms of scalars and vectors, but it's still not a pure scalar value.

So, to summarize: the curl of a vector field is not a scalar field, but rather a pseudoscalar field that can be either a scalar or a vector depending on the coordinate system used.

Question: The parameter domain of a surface parameterization is the set of points in the uv-plane that cannot be substituted into r.

Answer: False

Explanation: The parameter domain of a surface parameterization is actually the set of points in the uv-plane that can be substituted into r, not cannot. The parameter domain is the region in which the parameters u and v are defined, and it is used to determine the values of r that correspond to each point (u,v) in this region. In other words, the parameter domain specifies which values of u and v are valid for the surface parameterization, and all points outside of this region cannot be substituted into r.

Question: Is a regular parameterization one where ru×rv is never zero for any point (u,v) in the parameter domain?

Answer: False

Explanation: A regular parameterization is defined as one where the Jacobian determinant, —ru×rv—, is non-zero everywhere in the parameter domain. This does not imply that ru×rv is never zero for any point (u,v) in the parameter domain, but rather that it is non-zero almost everywhere.

In other words, a regular parameterization can have isolated points where ru×rv is zero, as long as these points are countable and do not form a set of positive measure. Therefore, the statement "ru×rv is never zero for any point (u,v) in the parameter domain" is false.

Question: Is a surface parameterization smooth if it does not have any points where both partial derivatives with respect to u and v are simultaneously zero?

Answer: False

Explanation: A surface parameterization is said to be smooth if it is continuous up to second derivatives, meaning that its partial derivatives exist and are continuous at every point in the parameter domain. The condition you mentioned, where neither ∂f/∂u nor ∂f/∂v is zero simultaneously, ensures the existence of partial derivatives, but does not guarantee their continuity.

For instance, consider a surface defined by the equation f(u,v) = u^2 v. At the point (0,0), both $\partial f/\partial u$ and $\partial f/\partial v$ are zero, yet the surface is smooth at this point because the second partial derivatives exist and are continuous.

Therefore, the statement "a surface parameterization is smooth if it does not have any points where both partial derivatives with respect to u and v are simultaneously zero" is False.

Question: The surface area of a smooth parameterized surface is calculated using an integral that involves the magnitude of the cross product between the partial derivatives of the surface with respect to its parameters, where the partial derivatives are continuous.

Answer: True

Explanation: The surface area of a smooth parameterized surface is calculated using an integral that involves the magnitude of the cross product between the partial derivatives of the surface with respect to its parameters, where the partial derivatives are continuous. This is true.

To calculate the surface area of a surface S(u,v), where u and v are parameters, we need to integrate the magnitude of the cross product of two vectors: ∇u and ∇v. These vectors represent the partial derivatives of the surface with respect to its parameters u and v respectively. The magnitude of this cross product is denoted as ——∇u x ∇v——.

The condition that the partial derivatives are continuous means that there are no discontinuities or singularities in the surface, ensuring a well-defined integral. This allows us to compute the surface area by evaluating the integral:

A = $\int\int$——∇u x ∇v——(du dv)

over the region of definition of the parameters u and v.

This formula is widely used in various fields such as computer graphics, engineering, and physics to calculate the surface areas of complex shapes.

Question: Is the surface integral of a scalar-valued function over a piecewise smooth surface equal to the limit as m and n approach infinity of the sum of the values of the function at discrete points Pij times the area ΔSij of the corresponding small surface elements?

Answer: True

Explanation: The surface integral of a scalar-valued function f over a piecewise smooth surface S is defined as:

$\int\int$f dS = $\int[\int$f —∇s—] ds

where the inner integral is taken over each small surface element ΔS, and ∇s is the normal vector to the surface.

The right-hand side can be rewritten as a sum of the values of f at discrete points Pij times the area ΔSij of the corresponding small surface elements:

$\int\int$f dS = lim (m,n)$\rightarrow\infty$ Σ [f(Pij) —ΔSij—]

where m and n are the number of subdivisions in the x and y directions, respectively.

This is because as the size of the small surface elements approaches zero, the sum of their areas ΔSij converges to the surface area ds. Similarly, the values of f at these points Pij converge to the value of f at each point on S.

Therefore, we can conclude that:

$\int\int$f dS = lim (m,n)$\rightarrow\infty$ Σ [f(Pij) —ΔSij—]

which is true.

Question: True or False: The surface integral of a continuous vector field F over an oriented surface S is equal to the dot product of F and the unit normal vector N?

Answer: True

Explanation: The surface integral of a continuous vector field F over an oriented surface S is indeed equal to the dot product of F and the unit normal vector N.

This is because the surface integral represents the amount of "flux" that passes through the surface, where flux is defined as the amount of F that "flows" into or out of the surface. The unit normal vector N gives us the direction perpendicular to the surface at each point, allowing us to measure this flux correctly. When we take the dot product of F and N, we are essentially projecting F onto N, giving us the component of F that is parallel to the surface and therefore contributes to the total flux.

In other words, if F is a vector field pointing in some direction, taking its dot product with the unit normal vector N effectively gives us the amount of "volume" under F's projection onto the surface. This volume is precisely what we're trying to measure when calculating the surface integral. So, it makes perfect sense that the surface integral would equal the dot product of F and N.

Question: A second-order differential equation is linear if it can be written in the form: a2(x)y" + a1(x)y' + a0(x)y = r(x), where a2(x), a1(x), a0(x), and r(x) are real-valued functions, and a2(x) is not identically zero.

Answer: True

Explanation: The given definition of a second-order differential equation as linear is correct. A second-order differential equation is considered linear if it can be written in the form:

a2(x)y" + a1(x)y' + a0(x)y = r(x)

where a2(x), a1(x), a0(x), and r(x) are real-valued functions, and a2(x) is not identically zero.

This definition encompasses all possible linear second-order differential equations. The presence of the constant coefficients and the polynomial functions in front of y", y', and y ensures that the equation remains linear.

In this context, linear means that the highest power of the dependent variable (in this case, y) appears only to the first degree, with no products of y or its derivatives. This characteristic is distinct from non-linear differential equations where higher powers of the dependent variable are present.

Question: A set of functions is considered linearly dependent if there exist constants c1, c2, ..., cn that are not all zero, such that their combination equals 0 for every value in a given interval, but it is not necessarily equal to 0 at all values. True or False?

Answer: False

Explanation: A set of functions is considered linearly dependent if there exist constants c1, c2, ..., cn that are not all zero, such that their combination equals 0 for every value in a given interval. This means that the linear combination of these functions takes on the value 0 at every point within this interval.

The statement "but it is not necessarily equal to 0 at all values" actually highlights the correct definition of linear dependence. A set of functions is said to be linearly dependent if there exists a non-trivial (non-zero) linear combination that is identically zero, i.e., the combination equals 0 for all values in its domain.

So, the statement "it is not necessarily equal to 0 at all values" actually confirms our understanding of linear dependence. It does not mean that the functions are only equal to 0 within a certain interval, but rather it means that the condition for linear dependence is met if there exists an interval where the combination equals 0 for every value.

Therefore, the answer to this true or false question is False.

Question: The characteristic equation of the differential equation ay"+by'+cy=0 is $a\lambda 2+b\lambda +c=0$. True or False?

Answer: True

Explanation: The characteristic equation of the differential equation ay"+by'+cy=0 is indeed a quadratic equation in λ, which can be written as:

$a\lambda 2 + b\lambda + c = 0$

This is because the general form of a second-order linear homogeneous ordinary differential equation (ODE) with constant coefficients is given by:

ay" + by' + cy = 0

The characteristic equation is obtained by setting $y = e^{\lambda t}$ and differentiating to obtain the derivative terms. This leads to:

$a(e^{\lambda t})" + b(e^{\lambda t})' + c(e^{\lambda t}) = a\lambda 2e^{\lambda t} + be^{\lambda t} + ce^{\lambda t} = 0$

Simplifying, we get:

$a\lambda 2e^{\lambda t} = 0$

Since $e^{\lambda t}$ is not zero, the only way for this equation to hold is if $a\lambda 2 = 0$. This can be rewritten as $a\lambda 2 + 0\lambda + c = 0$, which is the characteristic equation of the original differential equation.

Question: A particular solution to a differential equation contains arbitrary constants.

Answer: False

Explanation: A solution to a differential equation typically contains particular values of constants that satisfy the equation's conditions. These constant values are determined by the initial and boundary conditions imposed on the problem, which means they are not arbitrary but rather specific and dependent on the given constraints.

In other words, the constants in a differential equation's solution do not have inherent arbitrariness; instead, they emerge from the solution process as a consequence of applying the equation's conditions. This is in contrast to an algebraic equation where the variables may contain arbitrary constants that can be freely chosen.