LECTURE 12

LECTURE NOTES: FEBRUARY 14, 2003

OUTLINE

REVIEW

- Problem Solving Methods for Non-Ideal Operational Amplifier Circuits
- Problem Solving Methods for Ideal Operational Amplifier Circuits
- PSpice Simulation Methods for Operational Amplifier Circuits

UPDATED HOMEWORK ASSIGNMENT 6

HOMEWORK DUE FEBRUARY 24, 2003 BEFORE CLASS:

1. Textbook Problem 5.6 — a) –3.1mA
2. Textbook Problem 5.12 — a) –4V, b) $-2.5V \leq v_b \leq -1.3V$
3. Textbook Problem 5.20 — a) 6.4μW, b) 0.4μW, c) 16
4. Textbook Problem 5.27 — a) 4.2V, b) $-771mV \leq v_C \leq 1371mV$
5. Textbook Problem 5.38: — a) $v_O = -9v_g/(1+10/A)$ (b) –3.24V, c) –3.60V, d) 190
   Note that this operational amplifier in non-ideal
6. Textbook Problem 6.7: — a) $A_1 = 0.175A$, $A_2 = -0.125$, b) t = 152.43 μs
7. PSPice Problem 2: — Solve the Textbook Design Problem 5.30

Check the Web Site for general update and hints on homework

OPERATIONAL AMPLIFIER CIRCUITS

- Inverting Current Amplifier for Photodiode Optical Communication Applications
  - Effective Input Resistance
  - Analytical Problem Solution
    - Ideal Operational Amplifier
    - Non-Ideal Operational Amplifier
  - PSpice Analysis
- High Gain Inverting Current Amplifier with Reduced Resistor Values
  - Design
  - PSpice Analysis
- Applications: Operational Voltage Adder
  - Analytical Problem Solution
    - Ideal Operational Amplifier
  - PSpice Analysis
- Applications: Differential Voltage Amplifier
  - Wireline Communication
  - Biomedical Instrumentation
  - Analytical Problem Solution
    - Ideal Operational Amplifier
    - Non-Ideal Operational Amplifier
    - PSpice Analysis
- Unity Gain Voltage Amplifier Application
  - Analytical Problem Solution
    - Ideal Operational Amplifier
    - Non-Ideal Operational Amplifier
  - Role of the Open Loop Input Resistance
  - Switching Inverting/NonInverting Amplifier Unity Gain Amplifier
    - PSpice Simulation
- The Operational Current Source
  - Analytical Problem Solution
    - Ideal Operational Amplifier
    - Non-Ideal Operational Amplifier
  - Output resistance – Thevenin equivalent

APPLICATIONS: DIFFERENTIAL VOLTAGE AMPLIFIER

- One of the most important electronic circuits is the diffential voltage amplifier.
- On Wednesday, we discussed communications applications for Differential Amplifiers. In particular, we discussed the property that Differential Amplifiers may be used to implement balanced transmission systems.
- The Differential Amplifier allows noise and interfering signals to be removed from balanced transmission systems.
- Differential Amplifier methods are employed in telephony and digital data transmission systems.
- Differential principles are also important in measurement systems.
- For example, in biomedical instrumentation, one of the most important measurements is that of the electrocardiogram, or ECG. This is a measure of the potential generated by the heart's natural pacemaker system. ECG measurements can indicate normal or abnormal operation and forecast future heart disease or failure.
- ECG signals are generated as differential signals. However, their amplitude is low and the output resistance of the ECG signal source is large. Therefore, ECG signals are interfered with by signal sources in the environment.
- As for communication systems, the ECG is a typical measurement problem that benefits from the Differential Amplifier.
- Typical Differential Amplifier systems may reject noise and "common mode" signals that are a greater than the desired signal by a factor of 1000 or more.
- Lets examine the Differential Amplifier circuit, shown in Figure 1.

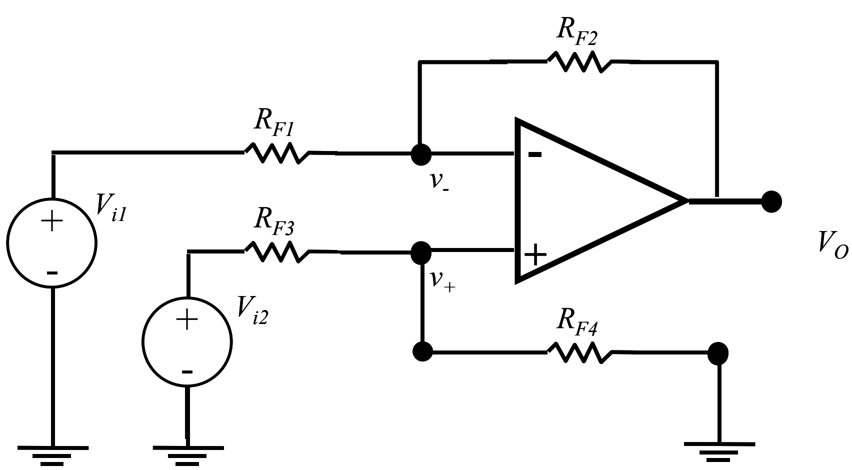


Figure 1. The Operational Differential Amplifier

- Now, lets compute the response of the Differential Amplifier assuming an ideal

Operational Amplifier.

- First, we see that we have Essential Nodes at the Inverting and Non-Inverting input terminals and at the Ground Reference.

- So, the Node Voltage equation at the Inverting input is:

$$\frac{V_{i1}-v_-}{R_{F1}}-\frac{v_- - V_O}{R_{F2}}=0$$

- And, the Node Voltage equation at the Non-Inverting input is:

$$\frac{V_{i2}-v_+}{R_{F3}}-\frac{v_+}{R_{F4}}=0$$

- However, as we always must assume for the Ideal Operational Amplifier

$$v_+ = v_-$$

- Also, lets consider the special case where $R_{F3} = R_{F1}$ and $R_{F4} = R_{F2}$

- Then, with the condition on the input voltages and the condition on the resistor values, we have.

$$\frac{V_{i1}-v_+}{R_{F1}}-\frac{v_+ - V_O}{R_{F2}}=0$$

- and

$$\frac{V_{i2}-v_+}{R_{F1}}-\frac{v_+}{R_{F2}}=0$$

- Subtracting these, we have

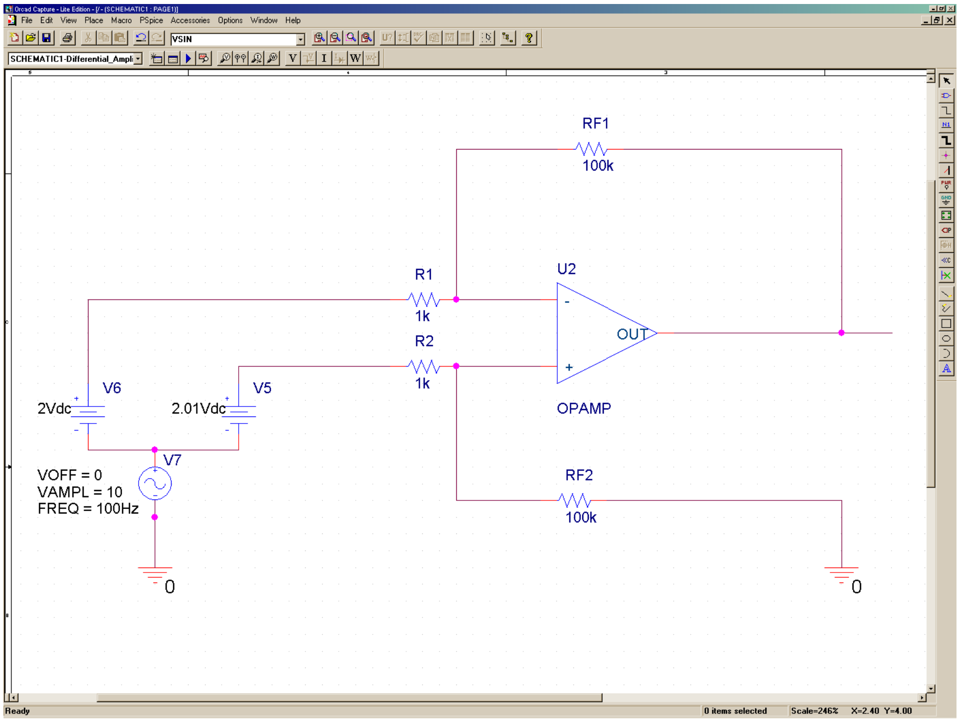
$$\frac{V_{i1}-v_+}{R_{F1}}-\frac{v_+ - V_O}{R_{F2}}-\frac{V_{i2}-v_+}{R_{F1}}+\frac{v_+}{R_{F2}}=\frac{V_{i1}-V_{i2}}{R_{F1}}-\frac{-V_O}{R_{F2}}=0$$

- Finally, this provides a transfer function:

$$V_O=\frac{R_{F2}}{R_{F1}}\left(V_{i2}-V_{i1}\right)$$

- Note that we have a purely differential amplifier. Its output does not depend on either input voltage independently, only on the input voltage difference.

- Also, there is a Gain factor simply equal to the ratio of the Feedback to Input Resistors.
- Lets discuss the intuition regarding the nature of this feedback circuit.
- Also, lets test the properties of this amplifier using PSpice.
- Here is the Differential Amplifier schematic that we will be using here.



OPERATIONAL CURRENT SOURCE

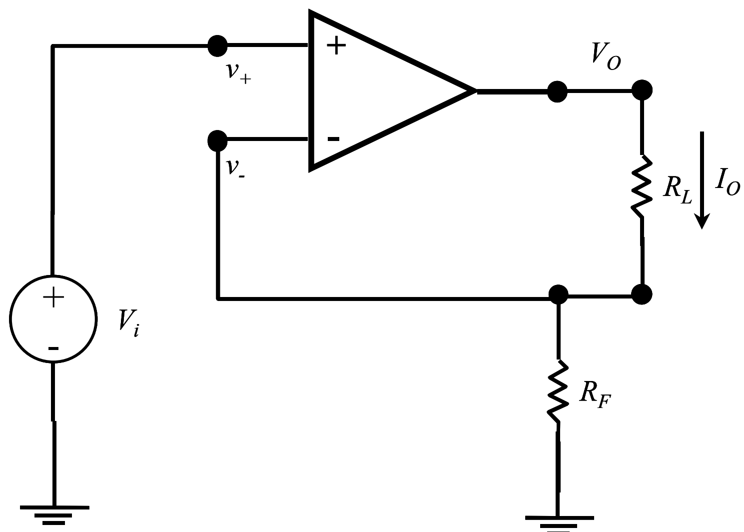
- We have examined voltage input and voltage output amplifiers, as well as current input, voltage output amplifiers.
- Frequently, Current Sources are required, for example, to bias the laser diode in an optical communication system.
- Lets now examine a voltage input, current output amplifier. This is sometimes referred to as the Operational Current Source.
- Now, before proceeding, we can discuss the attributes of current sources.
- An ideal current source should supply a specified current into a load resistor for any value of the

load resistor.

- Now, according to the theory of source equivalents, we may model a current source as a Norton Equivalent, consisting of an ideal current source in parallel with a resistor. Now, the performance of a current source is determined by the magniture of the Norton resistance. This must be as large as possible.

- We will analyze this.

- Lets examine the Operational Current Source using a Non-Ideal Operational Amplifiers. The circuit is shown below.



- Now, this circuit is similar to our Non-Inverting Voltage Amplifier. However, for this circuit, our objective is not to establish a specified output voltage, rather our objective is to establish a specified current, I_O, in the Load Resistor, R_L.

- Now, we can proceed with our problem solving procedure. We will use the Non-Ideal amplifier model with a finite open loop gain, A_V.

- Now, first since the amplifier is Non-Ideal, we follow can write down:

$$V_O = A_V(v_+ - v_-)$$

- Also, we can proceed to write the Node Voltage equation at the Inverting Input.

$$\frac{V_O - v_-}{R_L} - \frac{v_-}{R_F} = 0$$

- Now, also, we examine the Non-Inverting input and the input signal,

$$V_i = v_+$$

- But, also we wish to solve for I_O in terms of V_i
- And,

$$I_O = \frac{V_O - v_-}{R_L}$$

- Also, combining the first and third equations, first we have:

$$v_- = v_+ - \frac{V_O}{A_V} = V_i - \frac{V_O}{A_V}$$

- So, combining these results for the Inverting Input and

$$I_O - \frac{V_i - \dfrac{V_O}{A_F}}{R_F} = 0$$

- Finally, this provides the desired result:

$$I_O = \frac{V_i}{R_F}\left(1 - \frac{V_O/V_i}{A_V}\right)$$

- And, we can also solve for the ratio of output to input voltage. We can just adapt the result from our Lecture of May 13.

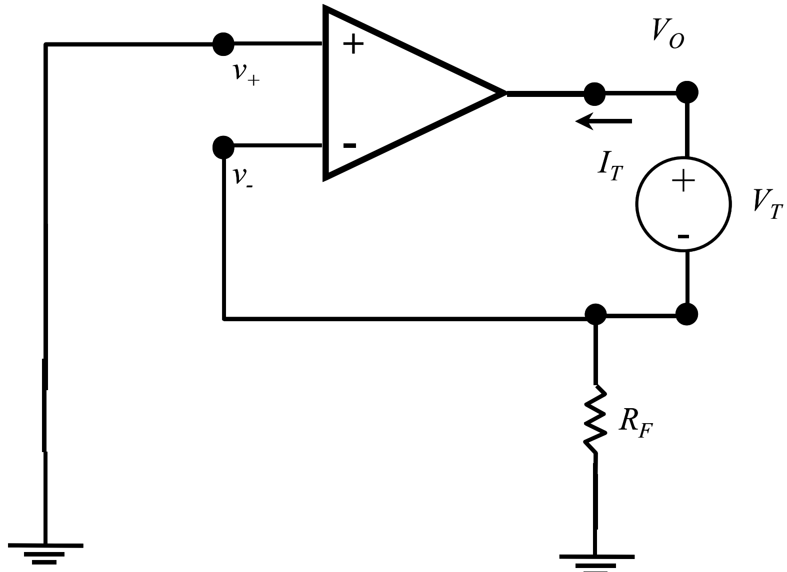
$$\frac{V_o}{V_i} = \frac{1}{\left[\dfrac{R_{F2}}{(R_{F1}+R_{F2})} + \left(\dfrac{1}{A_V}\right)\left(1 + \dfrac{R_O}{R_{F1}+R_{F2}}\right)\right]}$$

- Now, in the limit of large gain, we obtain the Operational Current Source result:

$$I_O = \frac{V_i}{R_F}$$

- Lets examine this with PSpice and observe the independence of Output Current on Load Resistor value for differing values of Open Loop Gain.

- Now, it is important for us to analyze the Norton Source Equivalent for this circuit.
- We will first replace the Voltage Source at the input by a short circuit.
- We will now introduce a Test Voltage at the Current Source Output, in place of the Load Resistor.
- This will be a new circuit, shown below.



- Now, we can write down a new Node Voltage Equation at the Inverting Terminal.

$$-I_T - \frac{v_-}{R_F} = 0$$

- And, of course, since $V_i = 0$., then

$$V_O = A_V(-v_-)$$

- And, V_T is now the voltage drop between V_O and v_-, or

$$v_- = -v_O/A_V = -(V_T + v_-)/A_V$$

- and

$$v_- = -V_T/(1+A_V)$$

- So, our Node Voltage Equation becomes

$$-I_T - \frac{-V_T \, {}^{1}\!/\!_{(1+A_V)}}{R_F} = 0$$

- Now, the Norton Equivalent Resistance is defined as V_T/I_T, so this is just

$$R_N = R_F\left(1 + A_V\right)$$

- This results can provide very high performance in our current source. For example, if we consider that Open Loop Gain may be 10^6 and R_F may be 1kΩ, then $R_N = 10^9\Omega$!

- Lets examine this behavior with PSpice.

- Here is our PSpice circuit for computing the Norton Equivalent Resistance:

- We will examine the performance of this circuit for varying Open Loop Gains.

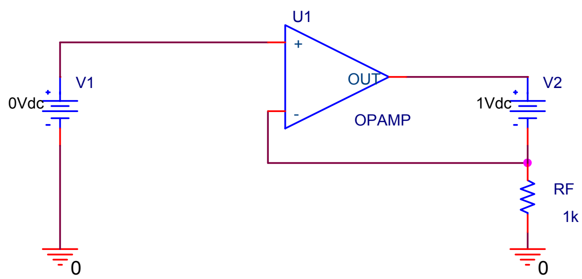


Figure 2. PSpice Schematic for computation of Norton Source Equivalent for the Operational Current Source

OPERATIONAL AMPLIFIER REVIEW

- The Operational Amplifier System
- Principles of Negative Feedback
- Fundamental Operational Amplifier Circuits
    - Unity Gain Voltage Amplifier
    - Non-Inverting Voltage Amplifier
    - Voltage Summing Operational Amplifier
    - Differential Amplifier
    - Operational Current Source
- Problem Solving Methods for Non-Ideal Operational Amplifier Circuits
- Problem Solving Methods for Ideal Operational Amplifier Circuits
- PSpice Simulation Methods for Operational Amplifier Circuits

NEW TOPICS: CIRCUITS WITH REACTIVE ELEMENTS

- Our study of circuits in EE10 began with the fundamental Kirchoff Laws for currents at a node and voltages around a path.
- Then, we used the properties of resistors defined by Ohm's Law to analyze circuits consisting of independent and dependent current and voltage sources and resistors.
- This allows us to analyze static circuit systems, where current and voltage signals remain constant.
- Now, for virtually every electrical engineering circuit application, we are interested in both static and dynamic response of circuits.
- When current and voltage change with time, we encounter the motion of charge (creating time varying electric fields) and changing currents (creating time varying magnetic fields.)
- We will observe that all electronic components are affected by these phenomena. We will find that we must characterize the Capacitance and Inductance of circuit structures.
- We will see that Capacitance and Inductance may be used to create circuit elements that temporarily may store energy and can be used to implement filters, oscillators, memory elements (as in Dynamic RAM), and other critical circuit systems.
- Capacitance and Inductance may appear in circuits as parasitic effects to be avoided. For example, capacitance appears in the signal lines of digital logic circuits, causing speed limitations and power dissipation that is undesired.

MAGNETIC INDUCTION

- One of the most important discoveries in engineering and science in the 1800's was the principle of magnetic induction.
- Magnetic induction is the process by which a current carrying conductor induces a magnetic field and also a current in neighboring circuit elements. This was discovered by Joseph Henry in 1831 in the U.S. and by Michael Faraday in England, independently.
- This was a first *unification theory,* unifying electricity and magnetism. Ultimately, this principle lead to the understanding of electromagnetism by Maxwell and Einstein's theories later that rested on the constant speed of light predicted by Maxwell.
- But, the application of circuits was changed fundamentally by this discovery. Magnetic induction permitted "action at a distance" and later, the inductor circuit element.
- An inductor is a device that generates a magnetic field, internal to the element, by action of the current flowing through the device.
- Michael Faraday, a chemist and physicist, discovered that a time varying current in an inductor produces a voltage across the inductive element.

INDUCTOR CIRCUIT ELEMENTS

- Inductors are composed of generally low resistance conductors arranged in a pattern to create the largest magnetic field per unit current. This magnetic field may remain trapped within the inductor structure, or be distributed over large volumes surrounding the inductor.

- Some inductors are fabricated with materials containing high permeability values to increase the effective inductance.

- Inductors may be incorporated into integrated circuits via conductor coils created on the circuit. The growth of wireless technology has lead to a major increase in technology investment in the properties and fabrication of high performance inductors.

- Inductors may also appear in a parasitic fashion.

- Specifically, a single, straight conductor carries an inductance values since its generates a magnetic field when current is flowing in the conductor. This parasitic conductance is often a major limitation in circuit systems. We will review an example today.

- Inductors may also be arranged to link their magnetic fields to other inductors. A mutual inductance may be created and this enables a new class of four terminal circuit elements – transformers.

- An inductor circuit element is drawn:

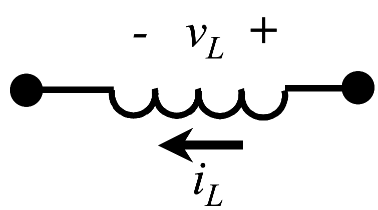


Figure 3. The Inductor circuit symbol.

CURRENT-VOLTAGE RELATIONSHIPS IN THE INDUCTOR CIRCUIT ELEMENT

Faraday's Law states that:

$$\boxed{v_L = L\frac{di_L}{dt}}$$

- The two terminal inductor provides a linear current-voltage characteristic that, unlike the resistor characteristic, depends on the time derivative of current.

- This definition uses the Passive Sign Convention. Specifically, the current reference direction is in the direction of the voltage drop.

- Lets examine this with a PSpice simulation.

- Consider a current of the form,

$$i_L = I_O \sin(\omega t)$$

- Inductor voltage must be:

$$v_L = L\omega I_O \cos(\omega t)$$

- Also, we should note the phase relationship between current and voltage.

$$v_L =$$

- Here is a PSpice simula

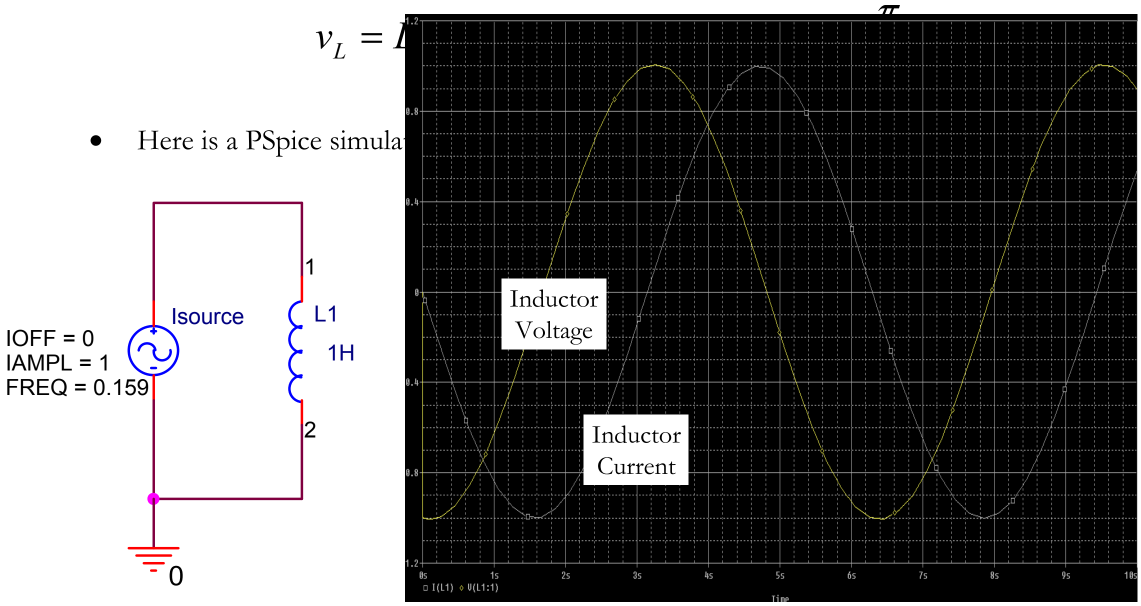


Figure 4. Inductor and Current Source circuit. The sin inductor current source amplitude is 1A. The frequency is (1/2π) Hz = 0.159Hz. *The Inductor current is the sin function, whereas the voltage is the cosine function.*

- There is a special note here regarding the polarity of currents. For a PSpice ISIN current source, we have labels showing + and – terminals on the Source. These indicated that when the value of the current is positive as indicated by a positive value in a Probe Output (for example I(Isource) is positive) then current is *entering* the + terminal and *exiting* the – terminal.

- For the Inductor or Resistor, current plotted as I(L1) or I(Rtest) is positive when current *enters* Terminal 1 and *exits* Terminal 2.

- With this, lets check the amplitude, polarity of voltage, and the dependence of the voltage on time.

- In many applications, the large change in voltage induced by changes in current lead to significant design challenges.

- For example, in the backplane of a high speed workstation PC, large currents must flow in the long conductors in the "PCI" bus. This bus, carrying data at speeds greater than 100 MHz, must support changes in current over times as short as 1 – 5 nanoseconds.

- The effective inductance of the conductors may be 100 nano-Henry with series resistance values of 0.1 Ω

- Here are simulation results for a system that raises the current from 0 to 10mA through a 100 nano-Henry inductor over a period of 2nS. (Here the current is 0 from the beginning of simulation, through 10nS. Then the current rises to 10mA.

- Note that there is a large voltage "transient" during this period.

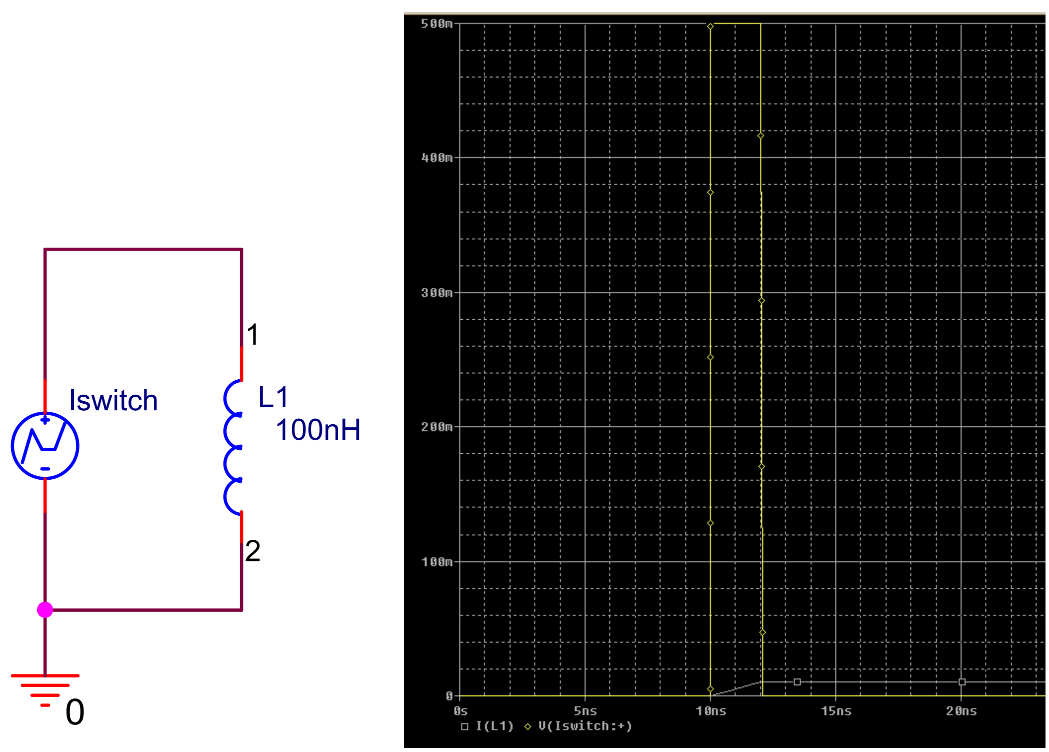


Figure 5. Circuit used to demonstrate the concept of inductive voltage transients.

- The presence of the voltage transient can be so large that signal distortion is severe.
- The ability to create a large amplitude transient is used in automotive ignition systems to create 50-kilovolt amplitude transients for creation of electric discharge at spark plug gaps.
- Now, in addition to computing voltage drops across the inductor in the presence of time dependent inductor current., we must also compute inductor current.
- Lets consider the differential equation that describes the inductor current voltage relationship. We can write this down by multiplying both sides of the Faraday's Law equation by the differential time, dt. Then,

$$vdt = Ldi$$

- To determine current at any point in time, we must integrate voltage with respect to time.
- So, current will depend on the initial value of current present at a point in time and on an integral over time. This will be:

$$\int_{i(t=t_O)}^{i(t)} di = \frac{1}{L}\int_{t_O}^{t} v(t')dt'$$

- Or,

$$i(t) = i(t_O) + \frac{1}{L}\int_{t_O}^{t} v(t')dt'$$

- Consider the example, where, we have an inductor voltage of the form,

$$v_L = V_O \sin(\omega t)$$

- Also, lets consider an example where at $t = 0$, $i_L = 0$.
- Then, inductor current must be:

$$i(t) = i(t_O) + \frac{1}{L}\int_{t_O}^{t} V_O \sin(\omega t')dt' = 0 + -\frac{V_O}{\omega L}\cos(\omega t)$$

- Lets also examine this behavior with PSpice using the following circuit. Note the presence of the series resistor here.

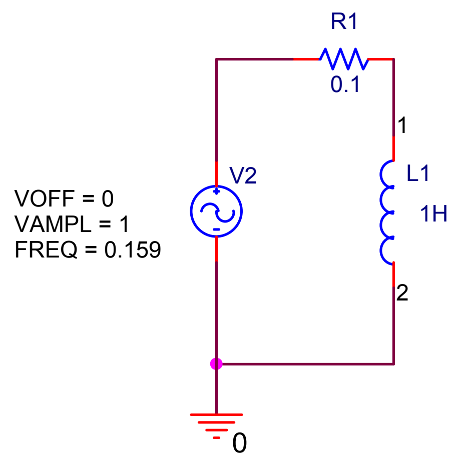


Figure 6. Voltage Source and Inductor circuit. Note the presence of the small resistor. This is required to allow PSpice to perform its DC Bias analysis - which would otherwise encounter infinite currents.

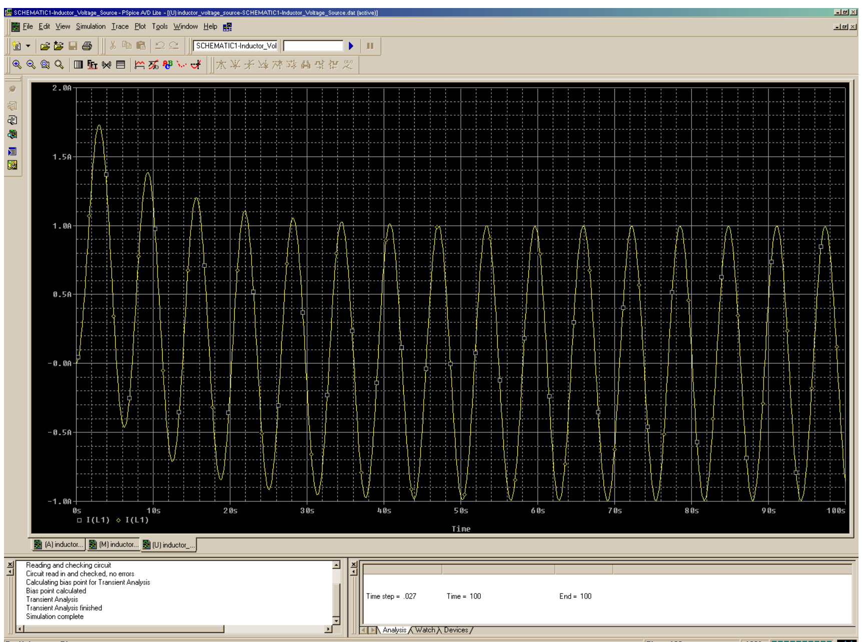


Figure 7. PSpice results for the circuit above. Note the initial transient behavior.

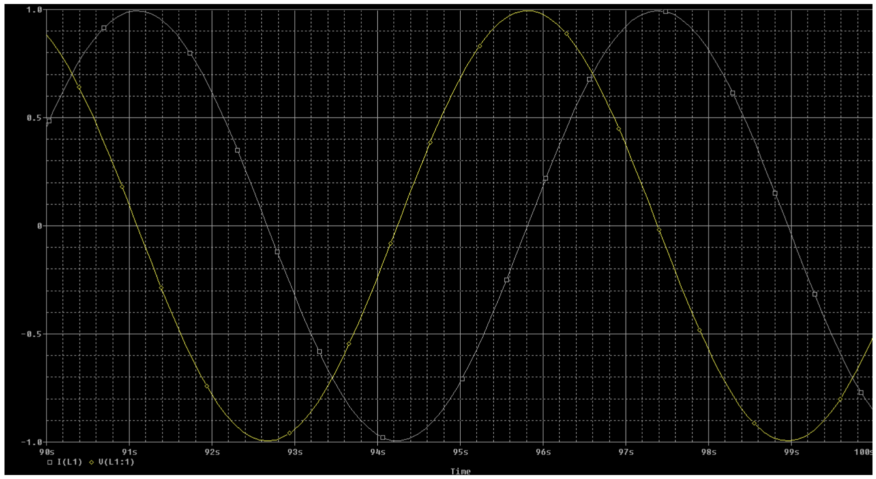


Figure 8. The PSpice simulation results above, now showing Current and Voltage over the interval from 90s to 100s. The voltage is the yellow characteristic with a maximum near 90s.

POWER DISSIPATION IN INDUCTOR CIRCUIT ELEMENTS

- Now, the inductor may be composed of conductors having eseentially zero resistance. Thus, we would expect that for static current, that the inductor would support no voltage drop and the power dissipation in the inductor, the current voltage product should always vanish.

- However, for time varying current and voltage, the magnetic field created by the inductor is varying. Thus, the energy in the magnetic field is changing and this change in energy must be supplied by energy sources in circuits.

- The power dissipation in inductors is via energy lost to magnetic fields. In contrast, the power dissipation in resistors is due to energy loss due to heating.

- Now, the we can compute power dissipation for the Inductor using the Passive Sign Convention. Just as for the Resistor where we could compute power in terms of the resistance and voltage or current, here we can express power in terms of Inductance, and current or voltage. We will encounter either a differential or integral expression.

$$p = iv = i \cdot L \frac{di}{dt}$$

- Or,

$$p = iv = v \cdot \left(i(t_O) + \frac{1}{L} \int_{t_O}^{t} v(t')dt' \right)$$

ENERGY STORAGE IN INDUCTOR CIRCUIT ELEMENTS

- Now, when a current flows in an inductor, and a magnetic field is created , a potential energy is associated with this field. The energy in this field may, in turn be absorbed by the inductor and the collapse of this field induces an inductor current.

- Frequently, it is useful to quote the energy, W(t), stored in an inductor for a specified current flow. This is a useful quantity in the design of circuits associated with radio frequency, wireless systems, for example.

- We can compute this by integrating the inductor power dissipation over time since,

$$p \equiv \frac{dW}{dt}$$

- So, we will consider an inductor system where we increase current from a zero value (where stored energy must be zero) to a finite value, I. Then we will compute energy at this value, I.

- First, we note that

$$p \equiv \frac{dW}{dt} = iL\frac{di}{dt}$$

- So, we can multiply by differential time and get the differential equation:

$$dW = iLdi$$

- Now, we can integrate from an operating point where $i = 0$ and $W=0$ to the value where $i = I$

$$\int_{W=0}^{W} dW' = 0 + L\int_{i=0}^{i=I} idi$$

- And evaluating this is:

$$W = \frac{1}{2}LI^2$$

CAPACITANCE AND THE DISPLACEMENT CURRENT

- Early in the history of electronics, it was discovered that when two conducting objects are separated when a potential difference is applied, there is a separation of charges in the objects. Also, there is an electric field between the objects.
- An arrangement of two parallel plates, separated by a small gap, was referred to as a capacitor.
- This field, as for the magnetic field of inductors, contains potential energy and allows capacitors to be energy storage elements.
- It was also discovered that when current entered one capacitor plate in a circuit, an equal current exited the other.
- This was a mystery since there was no intervening medium. Again, an apparent "action at a distance" was observed.
- James Clerk Maxwell developed the notion of a displacement current to describe this.
- The history of this area contains several major errors in concept. For example, we may still use this notion of displacement current for all circuit work, although it does not explain the phenomena entirely.
- Heaviside, a physicist was one of the first to understand that transmission lines (like telephone lines) contained inductor and capacitance distributed along their length.
- In about 1900 when telephone systems were first being deployed, this distributed capacitance and inductance limited the length of telephone lines by attentuating signals.
- Heaviside invented the periodic loading coil, intended to balance the distributed capacitance. This is used today throughout the telephony system.
- However, his theories were rejected by Kelvin. This delayed the introduction of the telephone by 20 years.

CAPACITOR CIRCUIT ELEMENTS

- As we proceed, we will examine many interesting circuits and important applications that rely on capacitors. We may highlight a few issues here.
- Capacitors are composed of structures that may include large area planar conductors separated by microscopically thin insulators. The capacitor electric field may remain trapped within the capacitor structure.
- Some capacitors are fabricated with complece material systems containing high dielectric

constant values to increase the effective capacitance. These may exploit novel material properties of even electrochemical properties.

- Capacitors may be directly incorporated into integrated circuits by introducing metal or semiconductor planar structures. The definition of capacitor geometry may be very precise, thus, circuit systems are often designed to exploit the ratio of the areas of inductors.

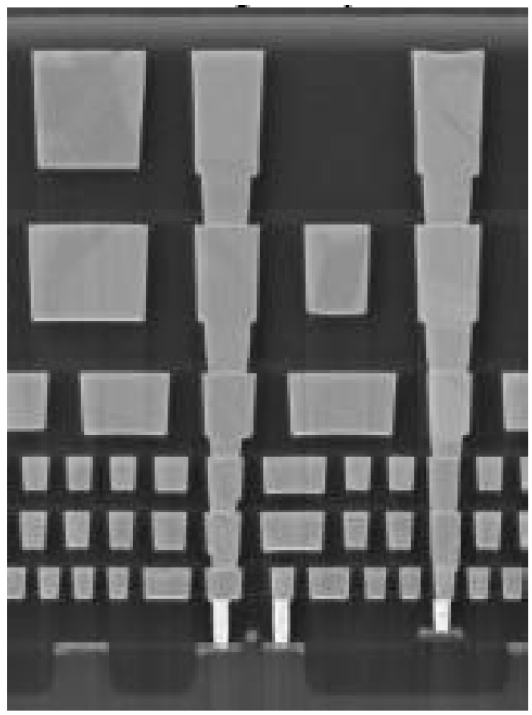


Figure 9. Microscopic cross-sectional view of Intel 0.13 micron design rule microprocessor process showing 6 layers of copper metallization.

- The parasitic effects of capacitors are generally more serious than the effects of inductance. Consider the structure of Figure 8 where capacitance exists between each of the conductors

- Capacitive energy dissipation is the primary source of power dissipation in modern microprocessors and digital systems. This may easily account for over 100W of power dissipation in a high end workstation system.

- A large investment is always underway to decrease parasitic capacitance and increased desired "gate" capacitance in integrated circuits via development of new materials and structures.

- The capacitor circuit element is drawn:

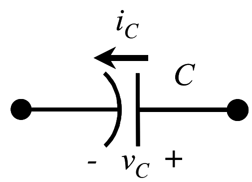


Figure 10. The Capacitor circuit symbol.

CURRENT-VOLTAGE RELATIONSHIPS IN THE CAPACITOR CIRCUIT ELEMENT

- Maxwell's displacement current law describes the current in a capacitor element. When the voltage across a capacitor element changes, charge is displaced according to the electric field produced. Current is produced by the time rate of change of charge and therefore voltage.

$$\boxed{i_C = C\frac{dv_C}{dt}}$$

- The two terminal capacitor provides a linear current-voltage characteristic that depends on the time derivative of current. It is a dual of the inductor. The units of capacitance are in Farads.

- This definition also uses the Passive Sign Convention, as for the Inductor and Resistor. Specifically, the current reference direction is in the direction of the voltage drop.

- Lets examine this with a PSpice simulation.

- Consider a voltage of the form,

$$v_C = V_O \sin(\omega t)$$

- Capacitor current must be:

$$i_C = C\omega V_O \cos(\omega t)$$

- Here is a PSpice simulation:

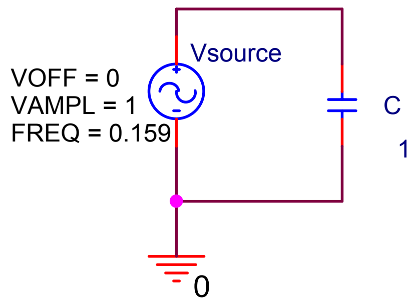


Figure 11. Circuit used for demomonstrating capacitor simulation and response

- Note, ***please*** read the Section in Tutorial I on units, note in particular here, the units associated with specifying the capacitor. The absence of a unit symbol implies that the units are Farads.

- There is a special note here regarding the polarity of currents, just as we encountered above. For a PSpice VSIN voltage source, we have labels showing + and – terminals on the Source. These indicated that when the value of the current is positive as indicated by a positive value in a Probe Output (for example I(Vsource) is positive) then current is entering the + terminal and exiting the – terminal.

- For the Capacitor, Inductor, or Resistor, current plotted as I(C1) or I(Rtest) is positive when current *enters* Terminal 1 and *exits* Terminal 2.

- With this, lets check the amplitude, polarity of voltage, and the dependence of the voltage on time.

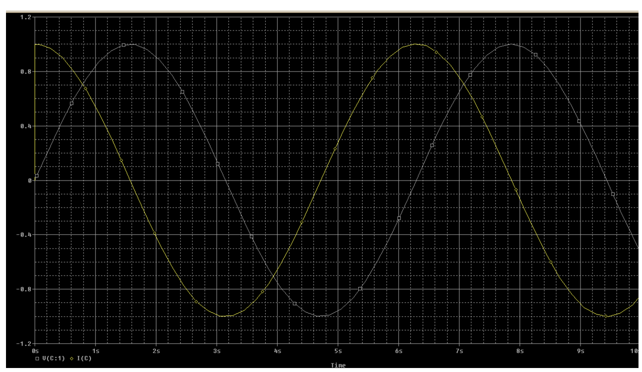


Figure 12. PSpice results for the circuit above. The Capacitor voltage is the sin function, whereas the current is the cosine function.

- Also, we should note the phase relationship between current and voltage for the capacitor.

$$v_C = V_O \sin(\omega t)$$

- and

$$i_C = C\omega V_O \cos(\omega t) = C\omega V_O \sin(\omega t + \frac{\pi}{2})$$

- For the Inductor, we developed a differential equation to relate current to the time integral of applied voltage.

- For the Capacitor, we will need a differential equation that describes the capacitor voltage dependence on the time integral of current. As for the Inductor, we can write this down by multiplying both sides of the Displacement Current Law equation by the differential time, dt. Then,

$$idt = Cdv$$

- To determine voltage at any point in time, we must integrate voltage with respect to time.

- So, voltage will depend on the initial value of the capacitor charge present and the capacitor voltage at a point in time and on an integral over time of the current and its contribution to the charge. Thus, we will have:

$$\int_{v(t=t_O)}^{v(t)} dv = \frac{1}{C}\int_{t_O}^{t} i(t')dt'$$

- Or,

$$v(t) = v(t_O) + \frac{1}{C}\int_{t_O}^{t} i(t')dt'$$

- Consider the example, where, we have an inductor voltage of the form,

$$v_L = V_O \sin(\omega t)$$

- Also, lets consider an example where at $t = 0$, $v_C = 0$.

- Then, inductor current must be:

$$v(t) = v(t_O) + \frac{1}{C}\int_{t_O}^{t} I_O \sin(\omega t')dt' = 0 + -\frac{I_O}{\omega C}\cos(\omega t)$$

- Lets also examine this behavior with PSpice using the following circuit.

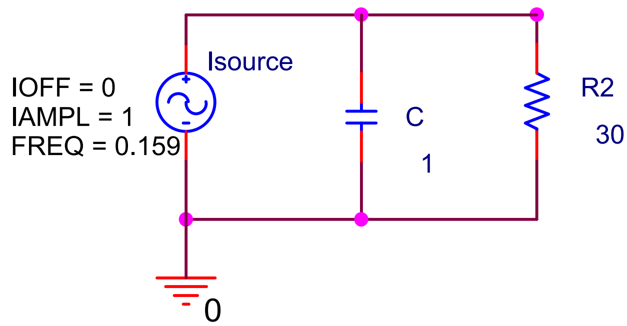


Figure 13. Circuit used for demonstrating the voltage dependence on capcitor current. Note the presence of the parallel resistor

- Note the presence of the parallel resistor here. This is a dual of the series resistor present in the Inductor case.
- Again, PSpice analysis must include a step of computation of the current and voltage values at Time=0.
- Now, since the current source is ideal, and the capacitor has infinite resistance for static, applied voltage, then the voltage drop across the capacitor is initially undefined. Therefore, we apply this parallel resistor to establish a zero potential. It must be small since it also effects the time response of this circuit, as we will discuss.

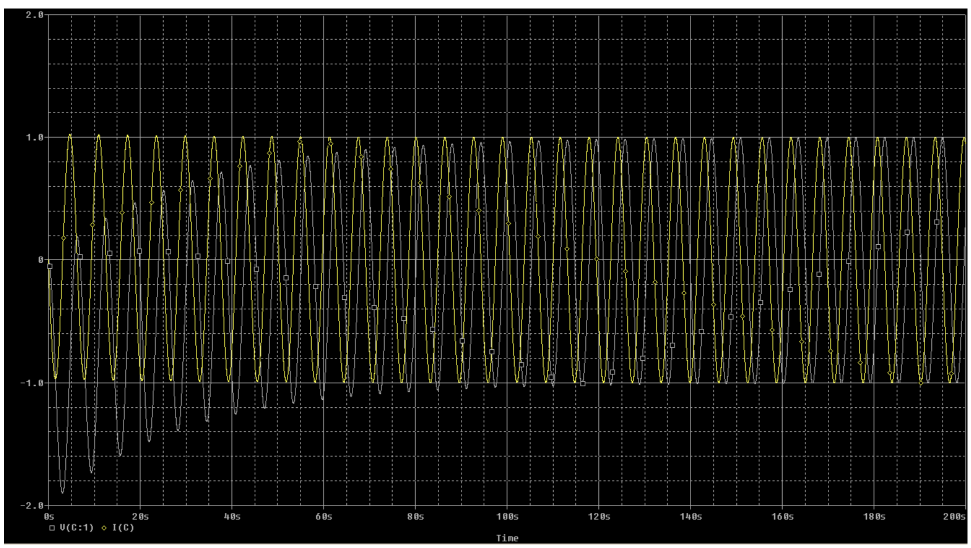


Figure 14. PSPice simulation result for the circuit above. Note the long term response characteristic and the phase relationship between current and voltage.

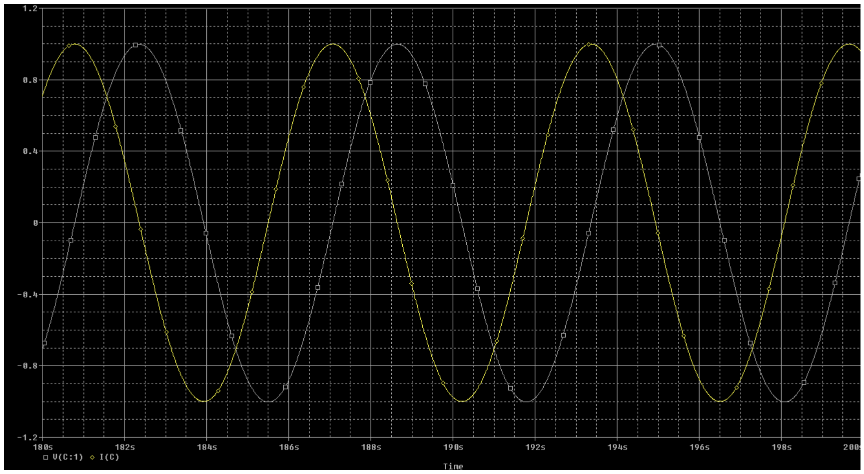


Figure 15. The PSpice simulation results over the interval from 180 to 200s. Note the current and voltage phase relationship. The Current is the yellow characteristic with a maximum near 181sec.

- We will continue with discussion of power and energy in capacitive circuits and also the role of capacitive power dissipation in digital systems.