EE102 - Practice Midterm Exam

Rules:

- **You have 1 hour and 40 minutes.**
- **Only this exam booklet and <u>One Sheet</u> of notes may be on your desk.**
- **NOT allowed: lecture notes, homeworks, calculators,...**
- **Answer each question in the space provided. EXPLAIN your reasoning. Simply writing down the answer is not adequate.**

Problem 1 [15 pts]

For the function

$$f(t) = 2(t+1)[u(t+1) - u(t)] + (2t - t^2)u(t)u(2-t) + u(t-3).$$

Sketch $f(t)$ and $\frac{df}{dt}$, and give an analytic formula for the latter in its simplest form.

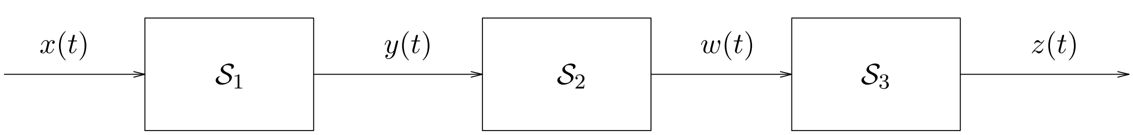
Problem 2 [15 pts]

Given a linear, time-invariant system with impulse response function

$$h(t) = u(t)e^{-t},$$

find the response to $x(t) = u(-t)e^{t}$.

Problem 3 [20 pts]



Consider the cascade interconnection of the figure, where $\mathcal{S}_1$ and $\mathcal{S}_3$ are LTI, causal systems, and $\mathcal{S}_2$ is defined by the relationship

$$w(t) = e^{t}y(t).$$

(a) Is $\mathcal{S}_2$ LTI, causal?

(b) We are told that

- The impulse response of $\mathcal{S}_3$ is $h_3(t) = \delta(t) - u(t)$.
- Applying the input $x(t) = e^{-t}u(t)$, the overall output is $z(t) = tu(t)$.

Find the impulse response $h_1(t)$ of the first system.

Problem 4 [25 pts]

Consider the system described by the input-output relationship $y(t) = |x(t)|$.

a) Is the system (i) linear? (ii) time invariant? (iii) causal?

b) We apply the input $x(t) = u(t)\sin(t)$; sketch $y(t)$ and also the difference $z(t) = y(t) - y(t - \pi)$.

c) Find the Laplace transform $Y(s)$ for the output $y(t)$ in part b), and its DOC.
Hint: It may help to work with $z(t)$, and express it in terms of $x(t)$ and $x(t - \pi)$.

Problem 5 [25 pts]

Consider the differential equation defined for $t \geq 0$,

$$\frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = te^{-t}, \qquad y(0) = \alpha, \quad \frac{dy(t)}{dt}(0) = \beta.$$

(a) Find the Laplace transform $Y(s)$ as a function of α, β.

(b) Compute the initial and final values $\lim_{t\to 0+} y(t)$, $\lim_{t\to +\infty} y(t)$.
Do they depend on α, β?

(c) Now take $\alpha = 0$, $\beta = 1$. Find the solution $y(t)$ for $t \geq 0$.