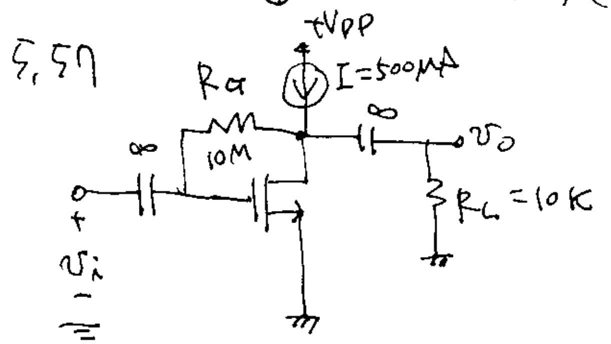
HW#8 Solution (Prob. sets)

5.57



$V_D = 2V$, $|V_t| = 0.9$, $V_A = 50V$

Voltage gain $\frac{v_o}{v_i} = -g_m \cdot (r_o \| R_L)$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{2 \cdot I_D}{V_{GS} - V_{TH}}$$

In this circuit, $V_D = V_G = 2V$ ($\because I_G = 0$ so no voltage drop across R_G)

$$\Rightarrow g_m = \frac{2 \cdot 500\mu}{2 - 0.9} = 9.09 \times 10^{-4} \ (A/V)$$

$$r_o = \frac{V_A}{I_D} = 100k\Omega, \quad (r_o \| R_L) = 9.09 k\Omega$$

$$\boxed{\therefore \frac{v_o}{v_i} = -8.2645}$$

If I increased to 1mA, consider

$$I_D = K \cdot (V_{gs} - V_t)^2 \times (1 + \frac{V_{DS}}{V_A})$$

For $I_D = 500\mu A$, $V_{gs} = 2$, we can find K.

$$\therefore K = \frac{I_D}{(V_{gs} - V_t)^2 \cdot (1 + \frac{V_{DS}}{V_A})} = \frac{500\mu A}{(2 - 0.9)^2 (1 + \frac{2}{50})} = 3.97 \times 10^{-4} \ (A/V^2)$$

Hence, for $I_D = 1mA$, new $V_D = V_D'$ is

$\Rightarrow 1mA \cong K (V_D' - V_t)^2$ (ignored V_A term for simplicity)

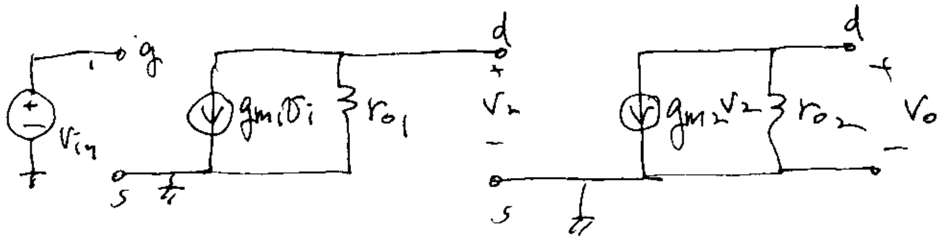
$$\Rightarrow \boxed{V_D = 2.48}$$

with $V_D = 2.48$ & $I_D = 1mA$, by following same way to find the gain,

$$\boxed{A_V = -10.5057}$$

5.75 small-signal model

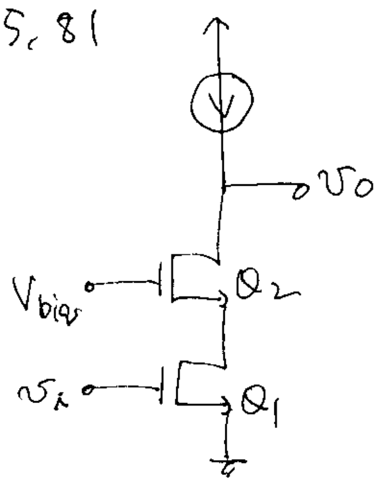
notice that small signal model for NMOS & PMOS are the same.



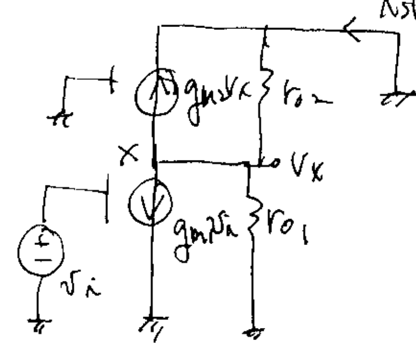
$$\frac{v_2}{v_i} = -g_{m1} \cdot r_{o1}, \quad \frac{v_o}{v_2} = -g_{m2} \cdot r_{o2}$$

$$\boxed{\therefore A_v = \frac{v_o}{v_i} = (g_{m1} r_{o1})(g_{m2} r_{o2})}$$

5.81



a) short-circuit ~~transconductance~~ $\triangleq \frac{i_{short}}{v_i} = G_{m,short}$



$$\rightarrow i_{short} = -\frac{v_x}{r_{o2}} - g_{m2}v_x = -\left(g_{m2} + \frac{1}{r_{o2}}\right)v_x \quad \text{—①}$$

$$\rightarrow \text{KCL at } X: \quad g_{m1}v_i + \frac{v_x}{r_{o1}} + \frac{v_x}{r_{o2}} + g_{m2}v_x = 0$$

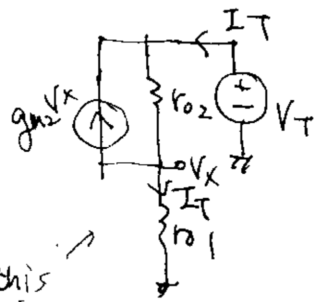
$$\Rightarrow v_x = \frac{-g_{m1}v_i}{\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{m2}\right)} \quad \text{—②}$$

combining ① & ② gives

$$i_{short} = \left(g_{m2} + \frac{1}{r_{o2}}\right) \cdot \frac{1}{\left(g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}\right)} g_{m1}v_i$$

$$\simeq g_{m1}v_i \quad \left(\because \frac{1}{r_{o1}} \simeq 0\right) \quad \therefore \boxed{\frac{i_{short}}{v_i} = g_{m1}}$$

(b)



this current source is removed because $V_{gs1}=0$

$$\Rightarrow V_T = (I_T + g_{m2}V_X)r_{o2} + I_T \cdot r_{o1}$$
$$V_X = I_T \cdot r_{o1}$$

$$\Rightarrow V_T = I_T(g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2})$$

$$\boxed{\therefore R_{OUT} = g_{m2}r_{o1}r_{o2} + r_{o1} + r_{o2}}$$

(c) $$\boxed{\frac{v_o}{v_i} = G_{m,short} \cdot R_{OUT} \simeq g_{m1}(g_{m2}r_{o1}r_{o2})}$$

$$\boxed{\text{without } Q_2, \ \frac{v_o}{v_i} = g_{m1}r_{o1}}$$

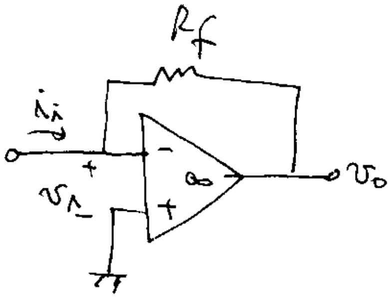
Therefore, gain is boosted by the factor of $g_{m2}r_{o2}$

2.8

	v_o/v_i	R_{in}
(a)	$-\frac{100}{10} = -10$	$10k\Omega$
(b)	$-\frac{100}{10} = -10$	$10k\Omega$
(c)	$-\frac{100}{10} = -10$	$10k\Omega$
(d)	$-\frac{100}{10} = -10$	$10k\Omega$
(e)	$0 \left(= -\frac{0}{10}\right)$	$10k\Omega$
(f)	$-\infty \left(= -\frac{100k}{0}\right)$	0Ω

2.16

(a)



For $A = \infty$,

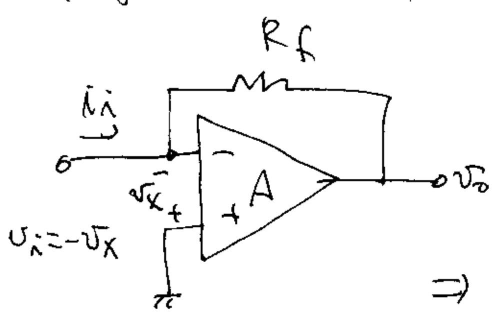
$v_i = 0$ ($\because v(+) = v(-)$ if $A = \infty$)

Hence,

$$i_i = \frac{0 - v_o}{R_f} \Rightarrow \boxed{R_m = \frac{v_o}{i_i} = -R_f}$$

$$\boxed{R_{in} = \frac{v_i}{R_i} = 0}$$

(b) if gain is finite, $v_o = A \cdot v_x$ ... ①



and, $i_i = \dfrac{-v_x - v_o}{R_f}$ ... ②

From ①, ②

$$\Rightarrow i_i = \frac{-(\frac{1}{A} + 1) v_o}{R_f}$$

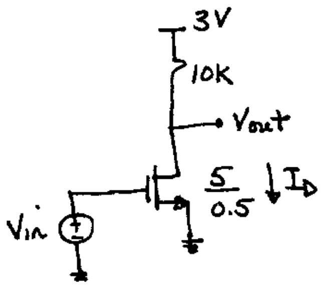
$$\boxed{\therefore R_m = \frac{v_o}{i_i} = -\frac{R_f}{1 + \frac{1}{A}}}$$

$R_{in} = \dfrac{v_i}{i_i}$, $\quad i_i = \dfrac{-v_x - A v_x}{R_f} = \dfrac{(1+A)}{R_f} \cdot v_i$

$$\boxed{\therefore R_{in} = \frac{R_f}{1+A}}$$

Pspice solution.

1.



$$V_{out} = 3 - 10K \cdot I_D \qquad (1)$$

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{in} - V_t) \qquad (2)$$

at the edge of saturation, $V_{out} = V_{in} - V_t$

$$\Rightarrow V_{in} - V_t = 3 - 10K\, I_D = 3 - 10^4 \cdot \frac{1}{2}\mu_n C_{ox} \frac{W}{L}(V_{in} - V_t)$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-12}\ F/m}{10^{-7}\ m} = 345.2 \times 10^{-6}\ F/m^2$$

$$\mu_n = 600\ \frac{cm^2}{S.V} \quad \text{(From spice model)}$$

$$\Rightarrow \mu_n C_{ox} = 600 \times 10^{-4} \times 345.2 \times 10^{-6} = 20.7 \times 10^{-6}\ A/V^2$$

$$V_{in} - V_t = 3 - 10^4 \times \frac{1}{2} \times 20.7 \times 10^{-6} \times 10\,(V_{in} - V_t)^2$$

$$\Rightarrow V_{in} - V_t = 1.29 \qquad ;\ V_t = 0.7 \text{ (from SPICE model)}$$

$$\Rightarrow \boxed{V_{in} = 1.99V}$$

This is very close to the simulated value.

In Simulation, the intersection of V_{out} & $V_{in} - V_t$ occurs at $V_{in} = 1.99V$

2

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*Common-Source MOS Amplifier Circuit

** Circuit Description **

* Power Supplies
Vdd Vdd 0 DC 3V
Vss Vss 0 DC 0V

* Input Voltage
Vin in 0 DC 0V AC 1V

* Common-Source Circuit
* MOS D G S B
M0 out in Vss Vss nch L=0.5u W=5u
Rd Vdd out 10k

* Model Definitions
* TOX = Oxide thickness in Meters
* U0 = Surface mobility in cm2/Vs
* VT0 = Zero-bias threshold voltage in V
* LAMBDA = channel-length modulation in 1/V
* CBD = base-drain junction capacitance
* CBS = base-source junction capacitance
.model nch nmos (LEVEL=1 TOX=1e-7 U0=600 +VTO=0.7 LAMBDA=0.02 CBD=20fF
+CBS=20fF)

** Analysis Request **
* DC Analysis (SOURCE START STOP INCREMENT)
.dc Vin 0V 3V 5mV

** Output Request **
.probe

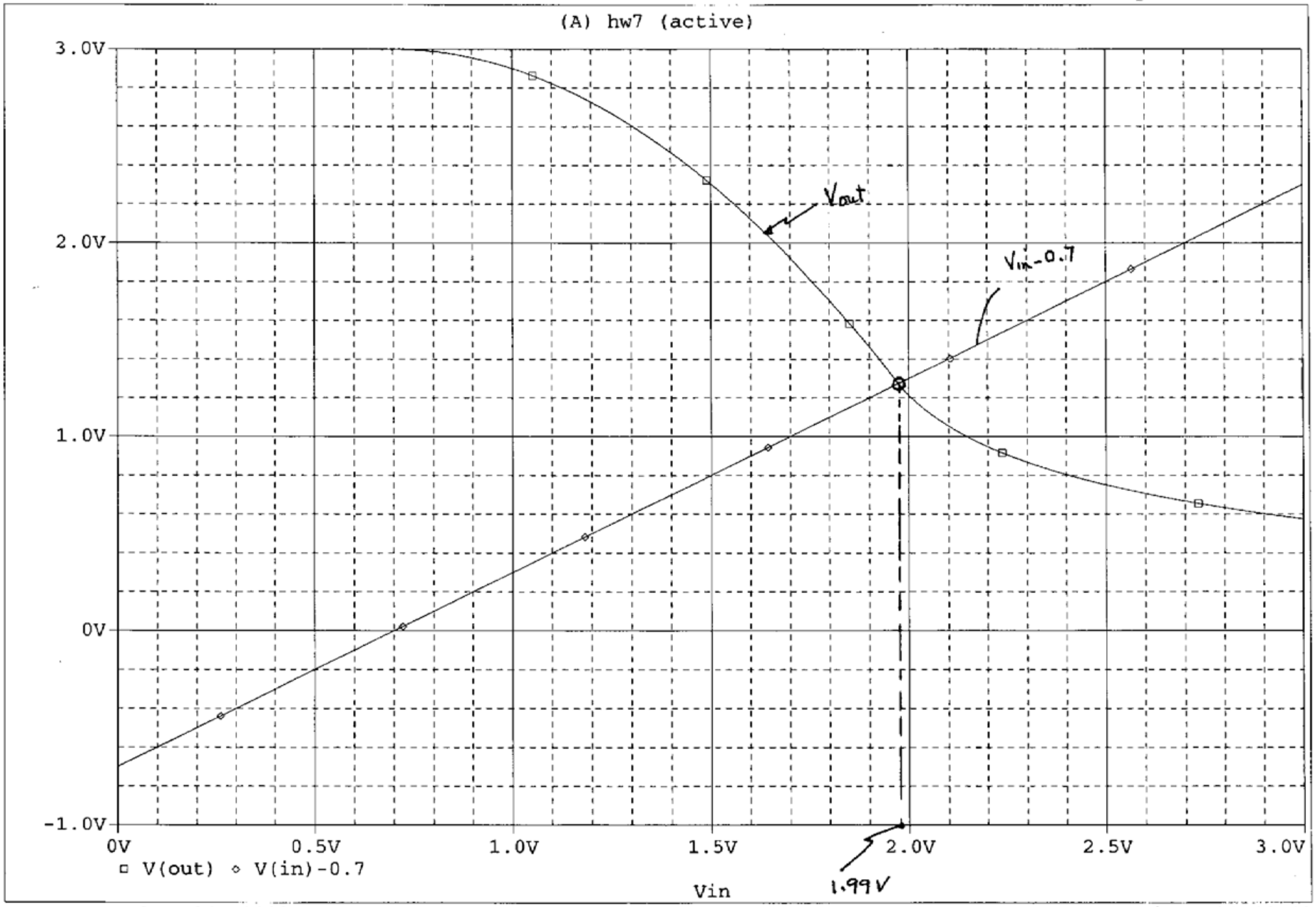
.end
```

3

*Common-Source MOS Amplifier Circuit

Date/Time run: 12/04/02 14:37:42

Temperature: 27.0



Date: December 04, 2002

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