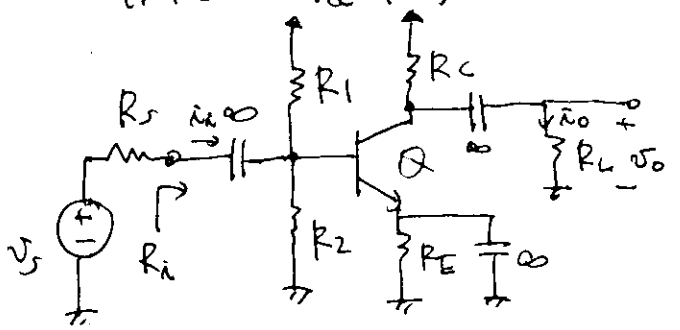
HW #6 Solution

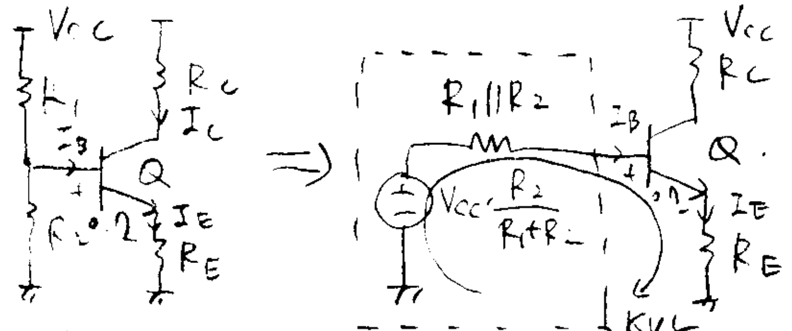
4.78



$\beta = 100$, $V_A = 100\,(V)$

$R_1 = 27k$, $R_2 = 15k$, $R_E = 1.2k$, $R_C = 2.2k$, $R_s = 10k$, $R_L = 2k$

① D.C. Analysis (∞ capacitor = open circuit)



(a)

Thevenin Equivalent of V_{CC}, R_1 & R_2 in (a)

By assuming Q is in active region,

KVL : $V_{CC} \cdot \frac{R_2}{R_1+R_2} = (R_1 \| R_2)\cdot I_B + 0.7 + I_E \cdot R_E$, $I_B = \frac{I_C}{\beta}$, $I_E = \frac{I_C}{\alpha}$

$= \left[(R_1 \| R_2)\frac{1}{\beta} + \frac{R_E}{\alpha} \right] \cdot I_C + 0.7$

$$\Rightarrow I_C = \frac{V_{CC} \cdot \frac{R_2}{R_1+R_2} - 0.7}{(R_1 \| R_2)\frac{1}{\beta} + \frac{R_E}{\alpha}} = 1.92\,mA$$

$$\boxed{I_E = \frac{I_C}{\alpha} = 1.94\,mA}$$

Hence, $g_m = \frac{I_C}{V_T} = 0.0769\ (A/V)$

$r_\pi = \frac{\beta}{g_m} = 1.3\,k\Omega$

$r_o = \frac{V_A}{I_C} = 52\,k\Omega$

Check!

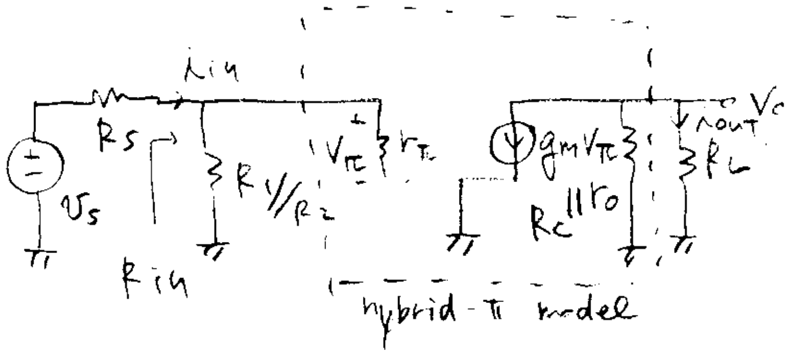
$V_B = V_{CC} \cdot \frac{R_2}{R_1+R_2} = 3.21$

$V_C = V_{CC} - I_C R_C = 4.77$

$V_{BC} = -1.55 < 0$, so

Q is in active region

② small-signal Analysis (∞ capacitor = short circuit, D.C. sources = zero element)



From the law of voltage divider,

$$V_\pi = V_s \cdot \frac{R_1 || R_2 || r_\pi}{R_1 || R_2 || r_\pi + R_s}$$

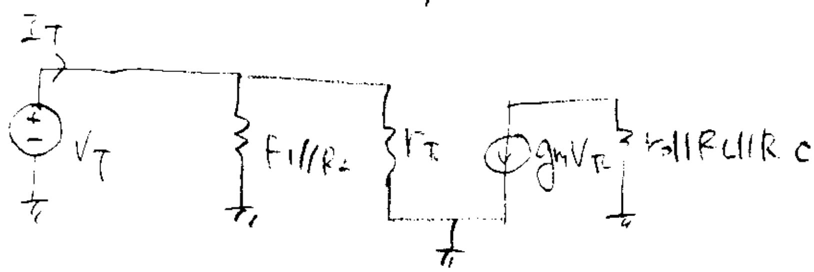
$$V_o = -g_m V_\pi (r_o || R_L || R_c)$$

$$= -g_m \cdot \frac{R_1 || R_2 || r_\pi}{R_1 || R_2 || r_\pi + R_s} \cdot (r_o || R_L || R_c) \cdot V_s$$

$$\Rightarrow \boxed{\frac{V_o}{V_s} = -8.13}$$

To Find R_{in}, note that R_{in} is Thevenin Equivalent Resistance at the (specified) input port with output open-circuited

We use test-voltage method to Find R_{in}. Also, R_{in} is small-signal quantity, so we use small signal model



$$\Rightarrow \frac{V_T}{I_T} = R_1 || R_2 || r_\pi$$

$$\boxed{R_{in} = 1.15 k\Omega}$$

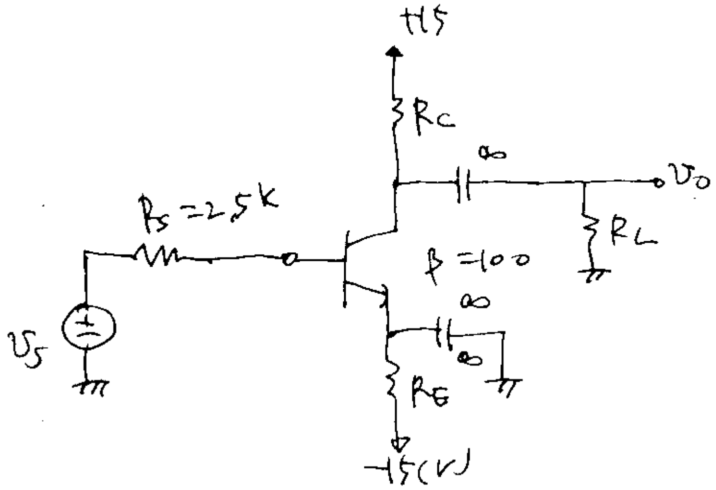
To Find current gain (i_o/i_{in}),

$$i_{in} = \frac{V_\pi}{R_1 \| R_2 \| r_\pi}$$

$$i_{out} = -g_m V_\pi \cdot \frac{R_c \| r_o}{R_c \| r_o + R_L}$$

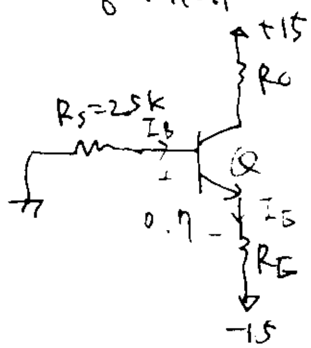
$$\frac{i_{out}}{i_{in}} = -\frac{g_m \cdot R_c \| r_o}{R_c \| r_o + R_L} \cdot (R_1 \| R_2 \| r_\pi) = \boxed{-45.24}$$

4.82



(a) D.C. analysis

Sin v_s has zero-average, D.C. value of $v_s = 0$
so equivalent ckt for D.C. analysis is as follows



by assuming Q is in active,

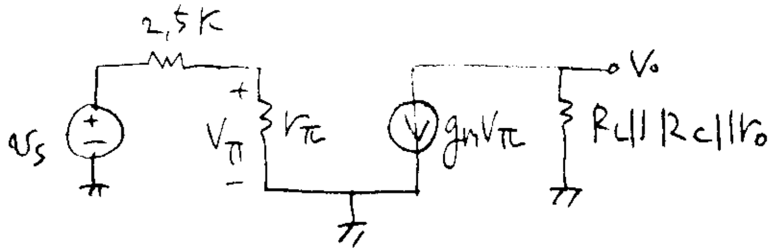
$$\begin{cases} 0 = R_S \cdot I_B + 0.7 + I_E \cdot R_E - 15 \\ I_E = 1\,mA, \quad I_B = \dfrac{I_E}{\beta + 1} \end{cases}$$

$$\Rightarrow \boxed{R_E = 14.3 k\Omega}$$

(b)
$$\begin{cases} V_C = 15 - I_C \cdot R_C = 5(V) \\ I_C = \alpha \cdot I_E = \dfrac{\beta}{\beta + 1} \cdot I_E \end{cases}$$

$$\Rightarrow R_C = 10 k\Omega$$

(c)



$$g_m = \frac{I_C}{V_T} = 40\ mA/V$$

$$r_\pi = \frac{\beta}{g_m} = 2.5k\Omega$$

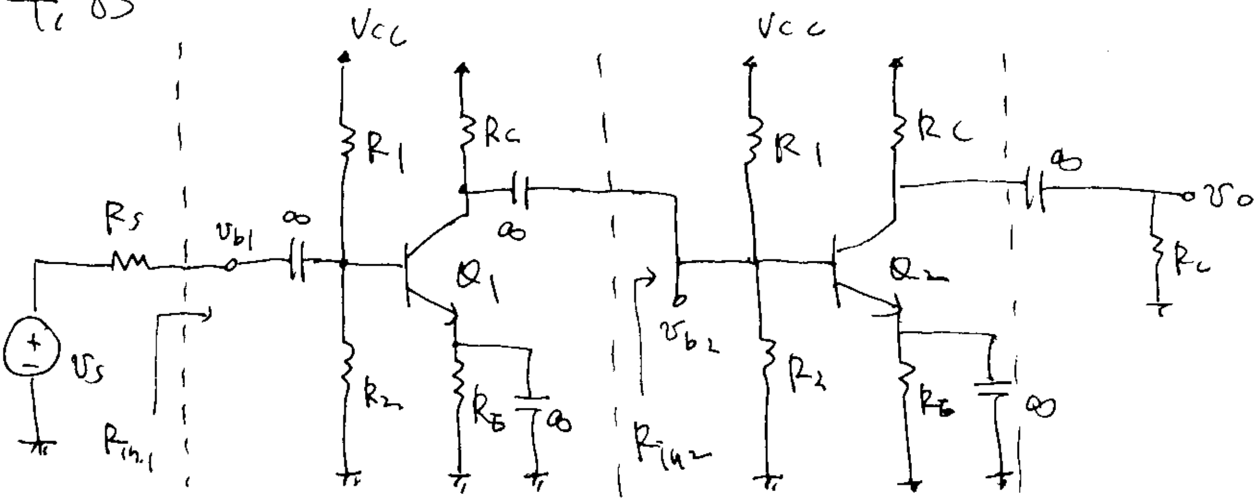
$$V_\pi = \frac{r_\pi}{r_\pi + 2.5k} \cdot v_s$$

$$\Rightarrow V_o = -g_m V_\pi \left(R_L \| R_C \| r_o\right)$$

$$= -\frac{r_\pi}{r_\pi + 2.5k} \cdot g_m \cdot \left(R_L \| R_C \| r_o\right) \cdot v_s$$

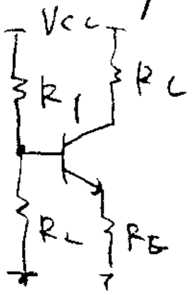
$$\Rightarrow \boxed{A_v = \frac{V_o}{v_s} = -64.5}$$

4.83



$V_{CC} = 15$, $R_1 = 100k$, $R_2 = 47k$, $R_E = 3.9k$, $R_C = 6.8k$, $\beta = 100$

a) D-C analysis



→ two identical stages for DC analysis

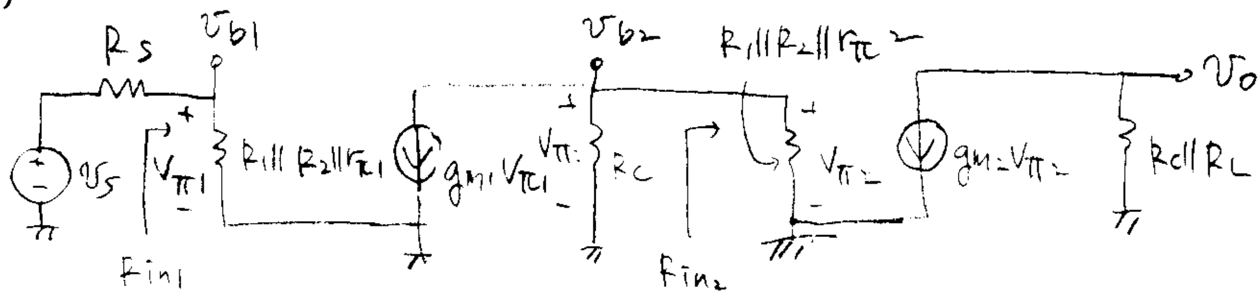
From P4, Q3,

$$I_C = \frac{V_{CC} \cdot \left(\frac{R_2}{R_1 \| R_2}\right) - 0.7}{(R_1 \| R_2)\frac{1}{\beta} + R_E/\alpha} = 0.96\,mA$$

$$V_C = V_{CC} - R_C \cdot I_C = 8.46\,(V)$$

$\boxed{I_C = 0.96\,mA \quad V_C = 8.46\,(V) \quad \text{for both } Q_1, Q_2}$

b)



c) $R_{in1} = R_1 \| R_2 \| r_{\pi 1}$

$= \boxed{2.4k}$

$$\frac{v_{b1}}{v_s} = \frac{R_1 \| R_2 \| r_{\pi 1}}{R_S + R_1 \| R_2 \| r_{\pi 1}} = \boxed{0.3247}$$

(d) $R_{in2} = R_1 \| R_2 \| r_{\pi 2} = \boxed{2.4k}$

$v_{b2} = -g_{m1} V_{\pi 1}(R_C \| R_{in2})$, $V_{\pi 1} = v_{b1}$

$$\Rightarrow \frac{v_{b2}}{v_{b1}} = -g_{m1} \cdot (R_C \| R_{in2}) = \boxed{-68.3}$$

e) $R_L = 2K$,

$v_o = -g_{m2} \cdot V_{\pi 2} \cdot R_C \| R_L$, $v_{\pi 2} = v_{b2}$

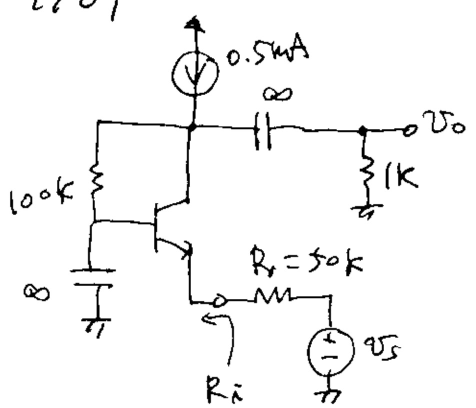
$\Rightarrow \frac{v_o}{v_{b2}} = -g_{m2} R_C \| R_L = \boxed{-59.45}$

f) $\frac{v_o}{v_s} = \frac{v_{b1}}{v_s} \cdot \frac{v_{b2}}{v_{b1}} \cdot \frac{v_o}{v_{b2}}$

$= 0.3247 \times (-68.3) \times (-59.45)$

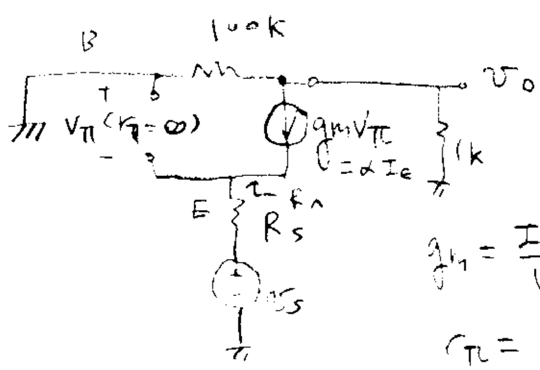
$= \boxed{1319}$

4.89



assume $\beta \to \infty$ & $\alpha \to 1$

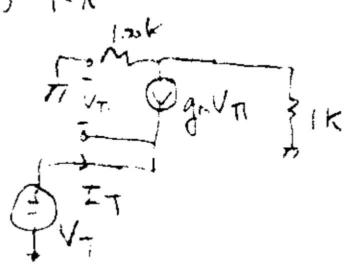
Then, small-signal model is,



$g_m = \frac{I_C}{V_T} = \frac{I_E}{V_T}$ ($\because \alpha = 1$)

$r_\pi = \beta / g_m = \infty$

i) R_i



Since $V_\pi = -V_T$,

$I_T = g_m V_T$

$\Rightarrow R_{in} = \frac{V_T}{I_T} = \frac{1}{g_m} = \boxed{50}$

ii) v_o / v_s

$v_o = -g_m V_\pi \cdot (1k \| 100k)$

$V_\pi = -\frac{R_{in}}{R_{in} + R_s} \cdot v_s$

$\Rightarrow \frac{v_o}{v_s} = +g_m (1k \| 100k) \left(\frac{R_{in}}{R_s + R_{in}}\right)$

$= \boxed{9.9}$