SPECIAL ANNOUNCEMENTS

1) Midterm Date Planned: Monday February 10

2) Please Check Web Site for Tutorials on Problem Solving and Specific Problems

3) Correction on Homework Solutions:

4) Problem 4.22, solution should read:

a) v_O = 50V, b) $P_{Absorbed}$ = 31.875W,

c) $P_{Absorbed}$ (3A)= -150W, d) $P_{Absorbed}$ (80V)= -120W

LECTURE NOTES: JANUARY 27, 2003

OUTLINE

REVIEW

1. Dependent Sources

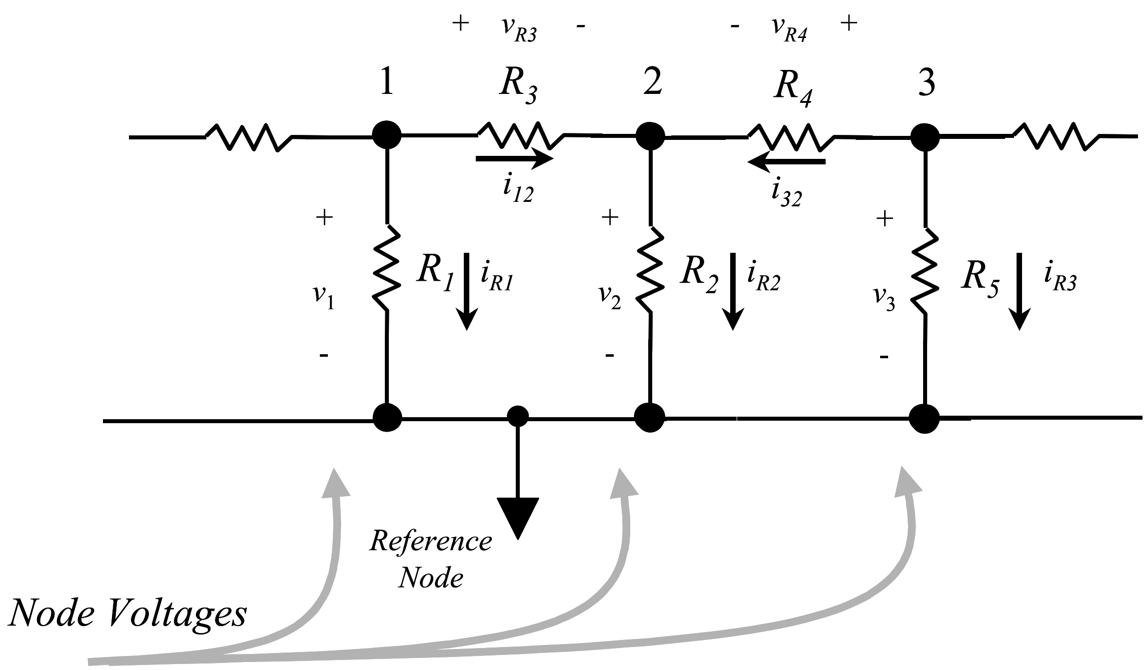
2. Node Voltage Methods for Circuits with Dependent Sources

3. Simplified Node Voltage Solution for Circuits Containing Voltage Sources Linking Essential Nodes.

4. Mesh Current Analysis Methods

5. Mesh Current Analysis Methods for Circuits with Dependent Sources

TUTORIAL: EXAMPLES OF TYPICAL CIRCUIT STRUCTURES AND CORRESPONDING NODE VOLTAGE EQUATIONS



$v_{R3} = v_1 - v_2$

$i_{12} = v_{R3}/R_3$

$v_{R4} = v_3 - v_2$

$i_{32} = v_{R4}/R_4$

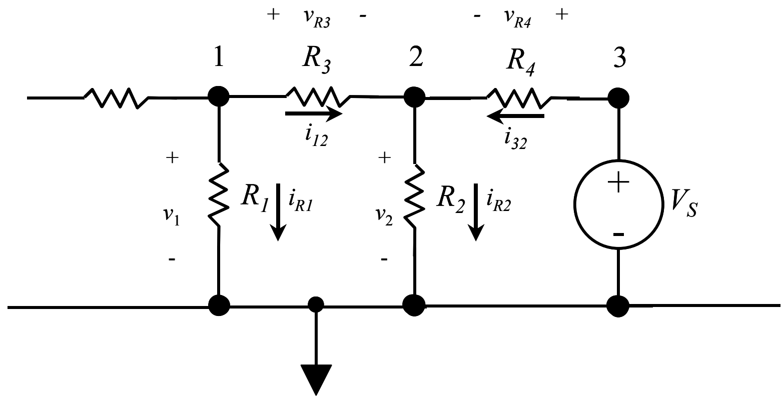
v_2 is the voltage drop across R2

$i_{R2} = v_{R2}/R_2$

Kirchoff's Current Law at Node 2: $i_{12} + i_{32} - i_{R2} = 0$

or

$(v_1 - v_2)/R_3 + (v_3 - v_2)/R_4 - v_2/R_2 = 0$



$v_{R4} = v_3 - v_2 = V_S - v_2$

$i_{32} = v_{R4}/R_4$

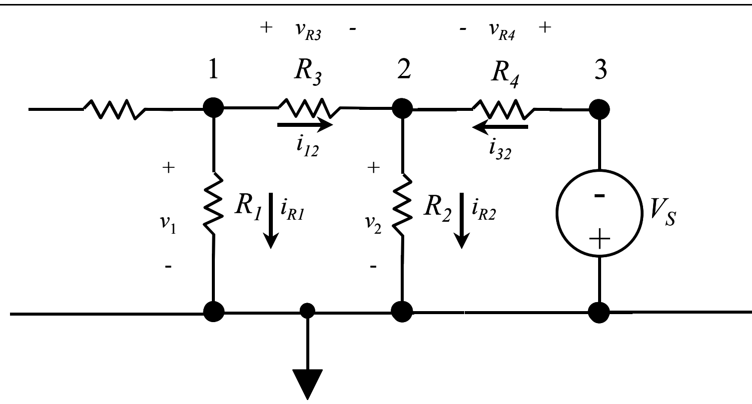
v_2 is the voltage drop across R2

$i_{R2} = v_{R2}/R_2$

Kirchoff's Current Law at Node 2: $i_{12} + i_{32} - i_{R2} = 0$

or

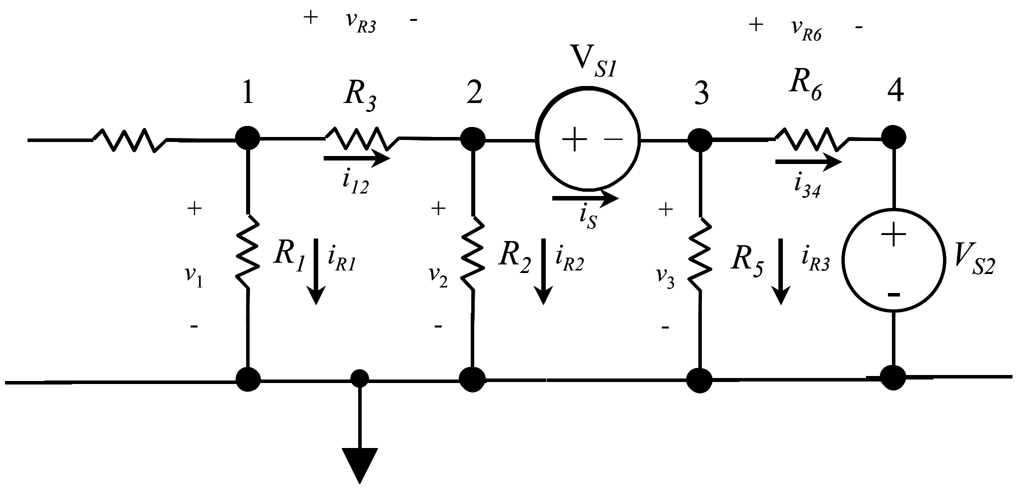
$(v_1 - v_2)/R_3 + (V_S - v_2)/R_4 - v_2/R_2 = 0$



Kirchoff's Current Law at Node 2: $i_{12} + i_{32} - i_{R2} = 0$

or

$(v_1 - v_2)/R_3 + (-V_S - v_2)/R_4 - v_2/R_2 = 0$



Kirchoff's Current Law at Node 2: $i_{12} - i_S - i_{R2} = 0$

or

$$(v_1 - v_2)/R_3 - i_S - v_2/R_2 = 0$$

Kirchoff's Current Law at Node 3: $i_S - i_{34} - i_{R3} = 0$

or

$$i_S - (v_3 - V_{S2})/R_6 - v_3/R_5 = 0$$

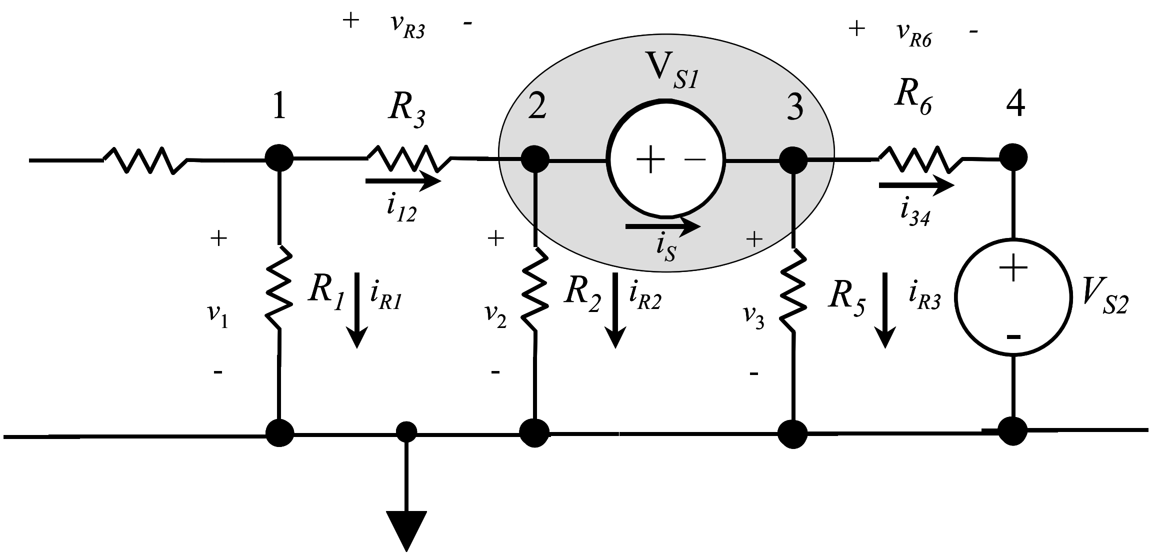
combining we have:

$$(v_1 - v_2)/R_3 - v_2/R_2 - (v_3 - V_{S2})/R_6 - v_3/R_5 = 0$$

Now, we are apparently missing information at this point. If the branch 2-3 was occupied by a resistor, then Ohm's Law would provide a condition on the voltage drop v_{23}. This would have provided enough information to solve for current and voltage values. But, the voltage source produces V_S *independent* of its current. We thus have no local condition that we can immediately write down relating this current and voltage.

But, we have a constraint condition!

$$v_2 - v_3 = V_{S1}$$



Kirchoff's Current Law at Node 2: $i_{12} – i_S – i_{R2} = 0$

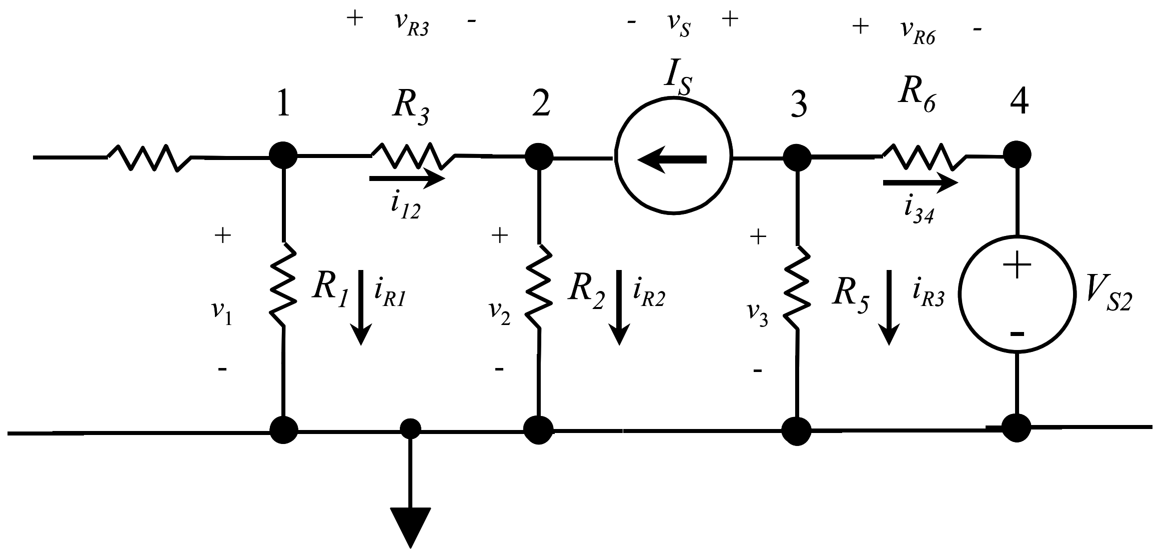
Kirchoff's Current Law at Node 3: $i_S – i_{34} – i_{R3} = 0$

Introduce a SuperNode

$$(v_1 – v_2)/R_3 - v_2/R_2 - (v_3 – V_{S2})/R_6 – v_3/R_5 = 0$$

and a constraint condition:

$$v_2 – v_3 = V_{S1}$$



Kirchoff's Current Law at Node 2: $i_{12} + I_S - i_{R2} = 0$

or

$$(v_1 - v_2)/R_3 + I_S - v_2/R_2 = 0$$

Now, this provides all the information needed for currents to the *left* of node 2. Since the current source *defines* the current from node 2 to node 3.

We proceed to solve this problem with the additional equations that may result due to circuit components to the left of node 1. We will derive equations that define v_2 and v_3.

For example, at node 3 we will have a set of equations that are independent of those at node 2.

Kirchoff's Current Law at Node 3: $- I_S - i_{34} - i_{R5} = 0$

$$-I_S - (v_3 - V_{S2})/R_6 - v_3/R_5 = 0$$

In fact, this provides enough information to solve for v_3 directly (since Is is known and I_S defines all current flowing from 3 to 2). This effectively *isolates* node 3 from 2 for the purposes of node voltage computation.

$$v_3 = (V_{S2} - I_S R_6)R_5/(R_5 + R_6)$$

SIMPLIFIED MESH CURRENT SOLUTIONS FOR SPECIAL CIRCUIT STRUCTURES

Certain arrangment of dependent and independent voltage sources reduces the complexity of the circuit solution for Node Voltage Methods.

Correspondingly, Certain arrangement of dependent and independent current sources reduces the complexity of the circuit solution for *Mesh Current* Methods.

When a branch includes a current source, then one less equation is required to specify the circuit, than would otherwise be required.

We will develop a new rule:

> ***When a current source appears in a branch,*** then this branch may be ignored and a "***supermesh***" may be used for the purposes of computing voltages for the KVL sum.

We will illustrate this now.

EXAMPLE PROBLEM WITH INDEPENDENT SOURCES FOR MESH CURRENT ANALYSIS

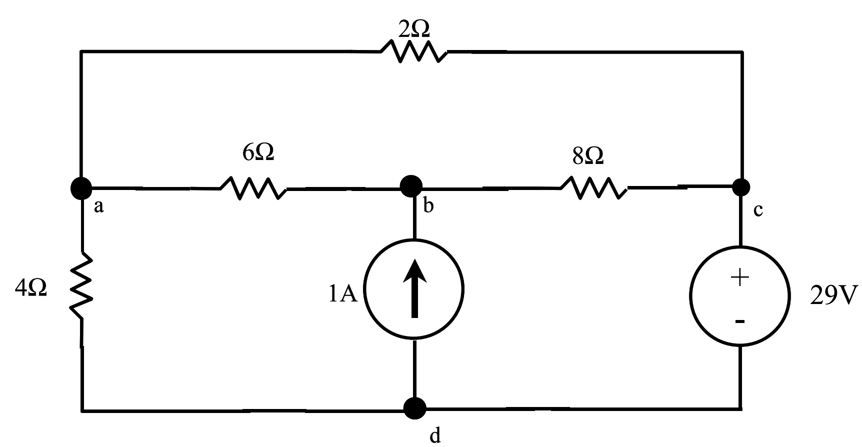


Figure 1. An Example Problem for illustrating the Super Mesh.

- This problem is similar to the problem we solved previously to illustrate the Mesh Current method.
- However, this circuit has been modified to insert a current source ***connecting to nodes.***

- We will proceed to solve for the currents and voltage values in this circuit. Lets follow our procedures.

1) List the information that is requested by the problem.

   a. We wish to obtain the power dissipation in the sources.

2) Examine the circuit to determine the approach for a solution

   a. Determine if a circuit simplification may be accomplished using an equivalent circuit, for example, a parallel, series, delta, or Y, circuit structure.

   b. None is recommended.

3) List known values of circuit variables

   a. Source potential

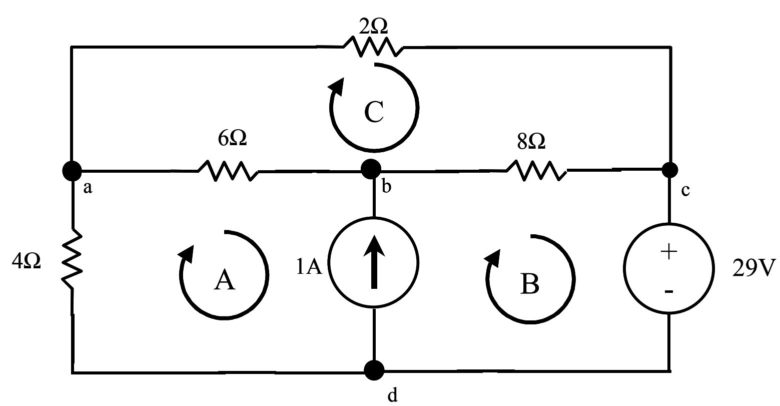
4) Identify and label the N_E Essential Nodes and N_{EB} Essential Branches

   a. N_E: a, b, c, d

   b. N_{EB}: All branches

5) Draw and label *Mesh Currents*

   a. This is shown below.



6) Use KVL and Ohm's Law to write down an equation for each Mesh, writing the equations in terms of the resistances, and Mesh Currents.

   This KVL sum will have to include the *unknown* voltage across the 1A current source.

   a. Mesh Current A) $-4(i_A) - 6(i_A - i_C) - (\boldsymbol{v_b - v_d}) = 0$
   a. Mesh Current B) $(\boldsymbol{v_b - v_d}) - 8(i_B - i_C) - 29\text{V} = 0$
   b. Mesh Current C) $-6(i_C - i_A) - 2(i_C) - 8(i_C - i_B) = 0$

7) Include any Dependent Source constraint equations in terms of Mesh Currents

   a. There are none.

8) Write down N_{EB} - (N_E – 1) equations.

   a. $N_{EB} - (N_E - 1) = 6 - 3 = 3$

   b. Note this contrasts with ***10 equations*** by the conventional method.

9) Solve the set of equations.

   a. Mesh Current A) $-4(i_A) - 6(i_A - i_C) - (\boldsymbol{v_b - v_d}) = 0$
   b. Mesh Current B) $(\boldsymbol{v_b - v_d}) - 8(i_B - i_C) - 29V = 0$
   c. Mesh Current C) $-6(i_C - i_A) - 2(i_C) - 8(i_C - i_B) = 0$

10) Adding, 9a) and 9b) we have:

   a. $-4(i_A) - 6(i_A - i_C) - (\boldsymbol{v_b - v_d}) + (\boldsymbol{v_b - v_d}) - 8(i_B - i_C) - 29V = 0$

   or

   b. $-10i_A - 8i_B + 14i_C - 29V = 0$

10) Rearranging 9c), we have

   a. $6i_A + 8i_B - 16i_C = 0$

10) Now, we have one more constraint equation due to the 1 A current source. This is that the difference between i_B and i_A must equal the 1A source current.

   a. $i_B - i_A = 1A$, or, $i_B = i_A + 1A$

   Then, 10c) and 11a) become

   a. $-10i_A - 8(i_A + 1A) + 14i_C - 29V = 0$

   b. $-18i_A + 14i_C - 37V = 0$

   c. $14i_A - 16i_C + 8 = 0$

11) Solving, we find, i_A = -5.21A, and i_B = -4.21A, and i_C = -4.06A

12) So, the current through the 25V source as indicated by i_B is in the direction that corresponds to the Passive Sign Convention. Thus, the power delivered is $P = -iv = -(-2A)(29V) = -58W$

Now, lets examine the step that occurred at 10). Note that we introduced the unknown potential across the current source dividing two meshes.

Consider a new circuit where we have suppressed the current source from the *Mesh Current* calculation.:

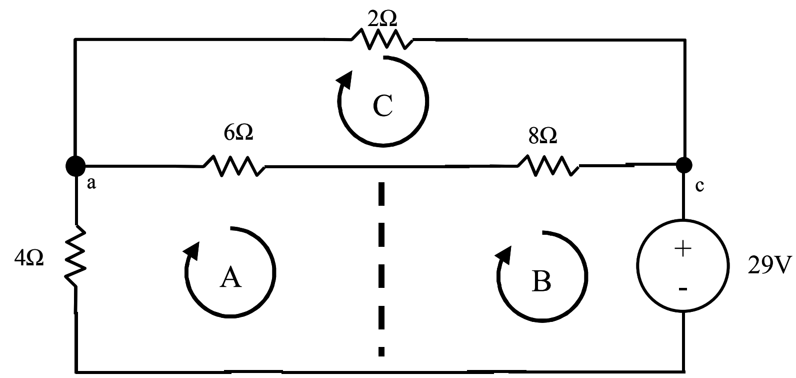


Figure 2. The Current Source is removed to enable the Mesh Current calculation using the Super Mesh concept.

When a current source appears in a branch, then this branch may be ignored and a "***supermesh***" may be used for the purposes of computing voltages for the KVL sum. The currnent source value must, later be introduced to constrain the current differences between adjoining mesh currents.

Lets proceed to solve this problem via the Supermesh approach:

We will write down the Mesh Equations for Meshes A and B together, along with Mesh C.

a. Mesh Current A-B) $-4(i_A) - 6(i_A - i_C) - 8(i_B - i_C) - 29\text{V} = 0$

b. Mesh Current C) $-6(i_C - i_A) - 14(i_C) - 8(i_C - i_B) = 0$

- Now, note that we have obtained the same pair of equations that was obtained above, after having manipulated three equations.
- We can solve this problem using only this pair of equations, and only the constraint:

  a. $i_B - i_A = 1\text{A}$

- So, the presence of a current source constrains a current difference, and reduces the number of simultaneous equations needed to describe a circuit. This is analagous to the role of a series voltage source in a Node Voltage equation.
- ***When a current source appears in a branch,*** then this branch may be ignored and a "***supermesh***" may be used for the purposes of computing voltages for the KVL sum. This reduces the number of equations in our circuit solution by one. The currnent source value must, later be introduced to constrain the current differences between adjoining mesh currents.

THE PRINCIPLE OF SUPERPOSITION

- The Principle of Superposition will be a useful method for our circuit analysis in the future and also will enable us to understand the concept of Equivalent Circuits.
- Background
    - Hermann von Helmholtz developed the critical idea of Superposition.
    - Consider any circuit composed of resistors (that respond according to Ohm's Law) and Dependent Sources (that respond to controlling signals via linear, first order equations).
    - Then, as we know, if we apply a voltage source to two terminals of this circuit, a current will be observed. In addition, the variation in the current at these terminals will be *linearly* related (proportional) to variations in the applied voltage.
    - Or, if we apply a current source to two terminals of this circuit, a voltage will be observed. In addition, the variation in the current at these terminals will be *linearly* related (proportional) to variations in the applied current.
    - This applies for any number of applied sources.
    - This behavior follows the fundamental Principle of Superposition that is based on the response of a system to the superimposing of sources on the system.
    - The Principle of Superposition states:

The voltage and current response of a linear network to a number of independent sources is the sum of the responses obtained by applying each independent source once, while setting all other independent sources to zero.

- For the definition of the principle of Superposition for sources states that a zero value voltage source is equivalent to a short circuit, and a zero current value current source is equivalent to an open circuit.
- Thus, when we solve a circuit problem, using the Principle of Superposition, we will calculate the response of the circuit for each source applied, individually, with other current sources replaced by open circuits and voltage sources replaced by closed (short circuits).
- We will examine an example problem, below.

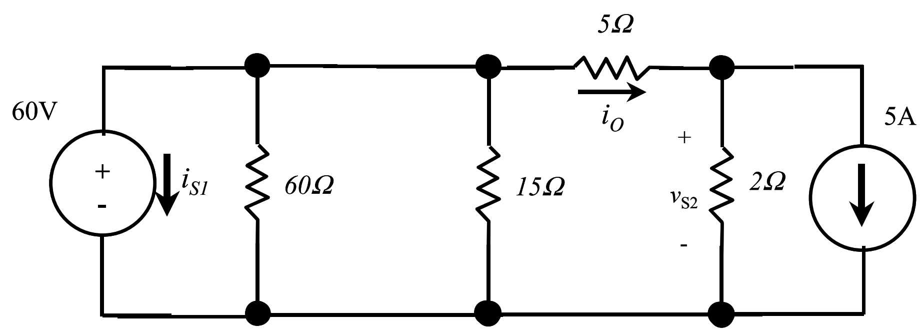


Figure 3. A circuit to be used to demonstrate the Principle of Superposition.

- This circuit problem is similar to one we had solved in Lecture 4, using the Node Voltage method. The results are that:

$$i_{S1} = -15\text{A},\ vS_2 = 10\text{V and } i_O = (v_1 - v_2)/5 = (60\text{ V} - 10\text{V})/5\Omega = 10\text{A}$$

- Now, lets use the Superposition Principle to solve this problem for v_O.

- Our procedure will be listed next.

PROBLEM SOLVING PROCEDURES USING THE SUPERPOSITION PRINCIPLE

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1. List the information that is requested by the problem.
2. Examine the circuit to determine the approach for a solution
3. Determine if a circuit simplification may be accomplished using an equivalent circuit, for example, a parallel, series, delta, or Y, circuit structure.
4. List known values of circuit variables
5. List unknown values of circuit variables.
6. Choose one Independent Source to remain unmodified and and replace all others with "zero" elements (open, or short circuit whether current, or voltage source, respectively.)
7. Solve for the unknown variables and enter the unknown variable values into sums, respectively for each unknown variable.
8. Continue choosing each Independent Source, in turn, and replacing all others with "zero" elements (open, or short circuit whether current, or voltage source, respectively.)
9. Complete the sum of variable values for each unknown until all sources have been selected. The final sum for each variable is the desired value.
10. Note that for every step where sources are eliminated, we may use any method we choose for solving for the unknown values.

- The Superposition Method is quite powerful for application to circuits containing many sources since this permits, often, a circuit system with a relatively simple set of circuit equations to solve.

- Now, let us proceed to solve the problem of Figure 3 with this method. Lets first select the voltage source. We must, therefore, replace the current source with an open circuit. This yields this circuit:

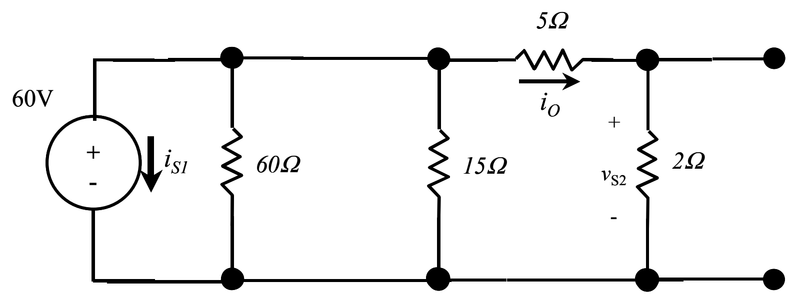


Figure 4. Example Problem with 60V source selected and remaining Current Source set to its zero value

- Now, we can easily solve for i_O. We can see that the 5 and 2Ω resistors present a series load to the 60V source.

- So, the first value of the sum for i_o is (60/7) A. This will be entered as a positive value in the sum since this current is computed to be in the reference direction for i_o

- Now, lets select the current source and replace the voltage source by a zero valued voltage source (a short circuit).

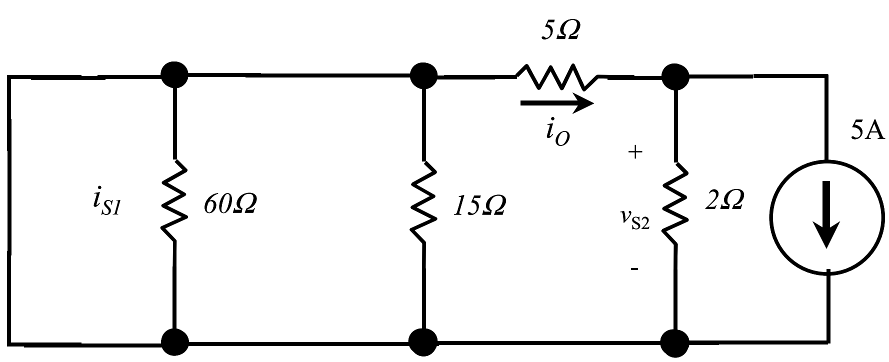


Figure 5. Example Problem with 5A source selected and remaining Voltage Source set to its zero value

- Now, we can again easily solve for i_O. We can see that the 5 and 2Ω resistors present a current divider to the 5A source.

- Thus, i_O = (2/7)(5A) = (10/7)A. Note that the sign is again positive. Here we must carefully observe the directions of reference and supply currents, as always.

- So, the second value of the sum for i_o is (10/7) A. This will be entered as a positive value in the sum since this current is computed to be in the reference direction for i_o

- Now, the total current found after using Superposition on both sources is

$$i_O = (70/7)\text{A} = 10\text{A}$$

- Of course, this is in agreement with the previous Node Voltage calculation.

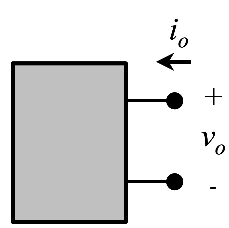
EQUIVALENT CIRCUITS: SOURCE EQUIVALENTS

- During our last three weeks, we have examined several methods for replacing complex resistor circuit constructions with simplified equivalent circuits, for example series or parallel equivalent circuits.

- Also, we saw an example of a source transformation, the Δ to Y and Y to Δ transformations of resistor circuits.

- Many amplifier, interface circuit, and power source systems are described effectively in terms of equivalent circuits where internal voltage sources, current sources, and resistors are replaced by a simplified circuit.

- We are particularly interested in circuits that are presented to us as two terminal circuit elements and we are interested in characterizing the circuit behavior at the two terminals.



- We would like to replace all of the complexity of the internal circuit structure with either an equivalent voltage or current source and appropriate equivalent resistors.

- The motivation for equivalent circuits is to enable rapid calculation, circuit understanding and design intuition.

- Lets consider the voltage that is produced by an arbitrary arrangment of linear resistors and sources.

- Superposition states that the voltage across any terminal pair is a linearly-weighted sum of voltage contributions from all elements.

- Lets consider a terminal pair across which the voltage is V. Then,

- We can separate all of the terms in the defining equations to be a sum of voltages due to voltage sources and a sum of voltages due to resistances and currents.

$$V = \sum_{\text{All Sources, k}} V_k + \sum_{\text{All Resistors, l}} I_l R_l$$

- The first term is referred to as an Equivalent Voltage Source.

- Now, if we were to set all sources to zero, and then apply a voltage V, to the two terminals, then a current, I would flow.

- This current I, is an Equivalent Current source and the resistance, V/I forms an Equivalent Resistance.

- Our circuit with its two terminals, may be represented, therefore by one of two alternatives Equivalent Circuits:

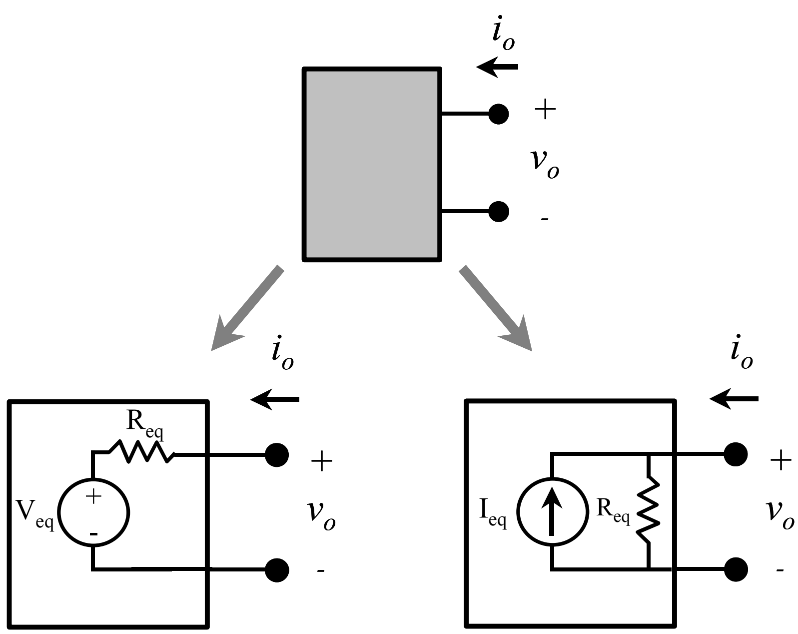


Figure 6. Replacement of a Linear Circuit System with either a Voltage Souce Equivalent or Current Source Equivalent Circuit containing an Equivalent Resistor

- The observation that any circuit of linear sources and passive elements may be replaced by equivalent elements was first made by Helmholtz in 1853.

- Thevenin, a French telegraph engineer, drew attention to the circuit on the left hand of Figure 6. This equivalent circuit is now called the Thevenin Equivalent.

- Edward Norton, an engineer at Bell Laboratories, is credited with the Norton Equivalent circuit shown at the right of Figure 6.

- Summary:

    - The Thevenin and Norton Equivalent Circuits will replace a linear two terminal circuit structure with simply either:

        - For a Thevenin Equivalent: A Thevenin Equivalent Voltage Source, V_{Th} , in series with a Thevenin Equivalent Resistance R_{Th} .

        - For a Norton Equivalent: A Norton Equivalent Current Source, I_N , in parallel with a Norton Equivalent Resistance R_N.

        - Also, for all circuits, $R_N = R_{Th}$

DERIVING THEVENIN AND NORTON EQUIVALENT CIRCUITS

- We may define a procedure for determining the values of Equivalent Sources and Resistances. These will be our procedures:

PROCEDURES FOR THEVENIN EQUIVALENT CIRCUITS

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance R_{AB}. This resistance will equal R_{TH}

4) Return all Independent Sources to their original values and compute the voltage value corresponding to the voltage drop from A to B, V_{AB} (with Nodes A and B open-circuited). This voltage is the Thevenin Equivalent voltage, $V_{AB} = V_{Th}$

5) You may compute this voltage using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

6) Draw the new Thevenin Equivalent Circuit.

PROCEDURES FOR NORTON EQUIVALENT CIRCUITS

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance R_{AB}. This resistance will equal R_N

4) Return all Independent Sources to their original values and then apply a short circuit at the terminals A and B. Compute the current value from A to B, I_{AB} (with Nodes A and B short-circuited). This current is the Norton Equivalent Current, $I_{AB} = I_N$

5) You may compute this current using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

6) Draw the new Norton Equivalent Circuit using this I_N and R_N. Note polarities for I_N.

- We will consider examples:

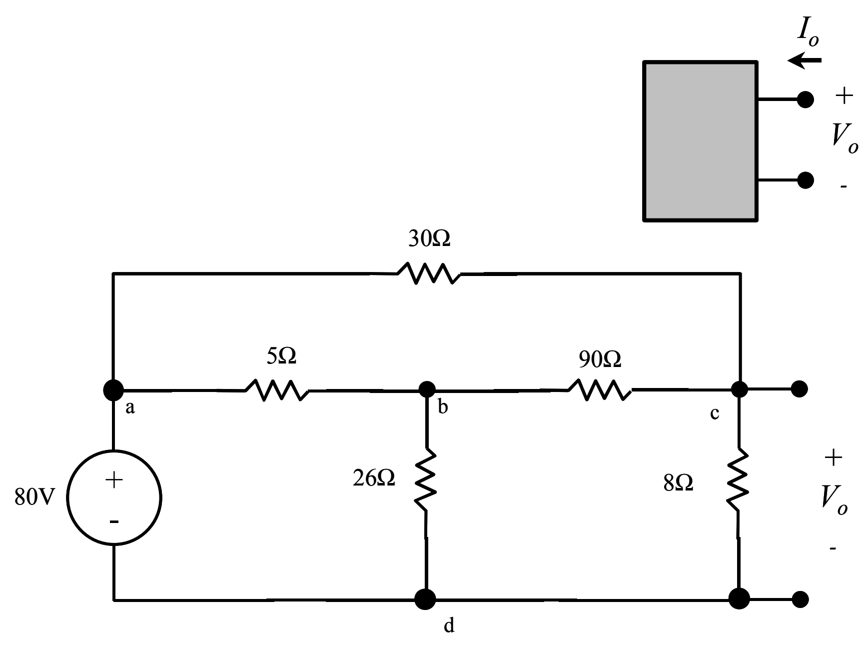


Figure 7. An example problem for demonstrating Thevenin and Norton Equivalent Circuits We have previously used other methods to solve this circuit problem.

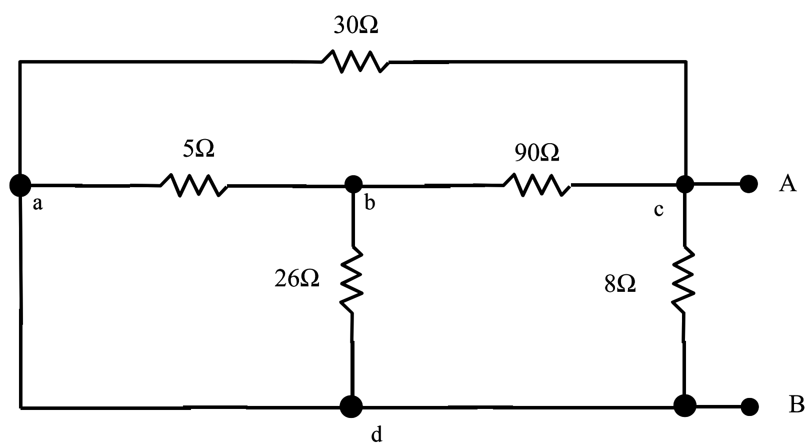
- Let use the procedures for computing the Thevenin Equivalent Circuit.

- Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

  a. We see that these terminals are labled V_O and are equal to v_4 .

- Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

  a. We redraw this circuit now.



- Compute the resistance R_{AB}. This resistance will equal R_{TH}

  a. Now, this computation is straightforward. We observe that the 30Ω and 8Ω resistor connect directly in parallel and with Nodes A and B. Also, the 5Ω and 26Ω resistors are in parallel, and themselves in series with the 90Ω resistor. Finally, this latter combination is in parallel with the 8Ω and 30Ω resistors.

  b. Computing, $R_{AB} = R_{TH} = 5.92\Omega$

- Return all Independent Sources to their original values and compute the voltage value corresponding to the voltage drop from A to B, V_{AB} (with Nodes A and B open-circuited). This voltage, $V_{AB} = V_{Th}$

- You may compute this voltage using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

  a. Here we use Mesh Current methods, of Lecture 6. Mesh Currents are attractive in that we have a limited number of equations and we are required to compute one variable that directly equals a Mesh Current.

  a. Draw and label *Mesh Currents*

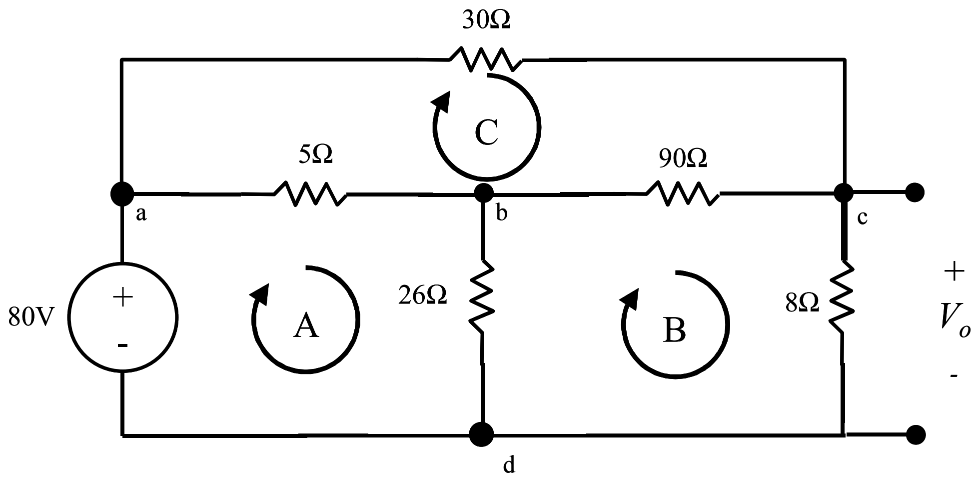


Figure 8. Using Mesh Currents to solve for V_O

b. Use KVL and Ohm's Law to write down an equation for each Mesh, writing the equations in terms of the resistances, and Mesh Currents.

   i. Mesh Current A) $80 - 5(i_A - i_C) - 26(i_A - i_B) = 0$
   i. Mesh Current B) $-26(i_B - i_A) - 90(i_B - i_C) - 8(i_B) = 0$
   ii. Mesh Current C) $-5(i_C - i_A) - 30(i_C) - 90(i_C - i_B) = 0$

c. Include any Dependent Source constraint equations in terms of Mesh Currents

   i. There are none.

d. Write down N_{EB} - (N_E – 1) equations.

   i. $N_{EB} - (N_E - 1) = 6 - 3 = 3$

e. Solve the set of equations.

   i. $80 - 31i_A - 5i_C - 26i_B = 0$

   ii. $26i_A - 124i_B + 90i_C = 0$

   iii. $5i_A + 90i_B - 125i_C = 0$

f. Solving these,

   i. $i_A = 5\text{A}$

   ii. $i_B = 2.5\text{A}$

   iii. $i_C = 2\text{A}$

- Now, since we have solved for the current through the 8Ω resistor, we can compute V_O.

$V_O = i_B\,(8\Omega) = 20V$.

- Draw the new Thevenin Equivalent Circuit.

a. This becomes:

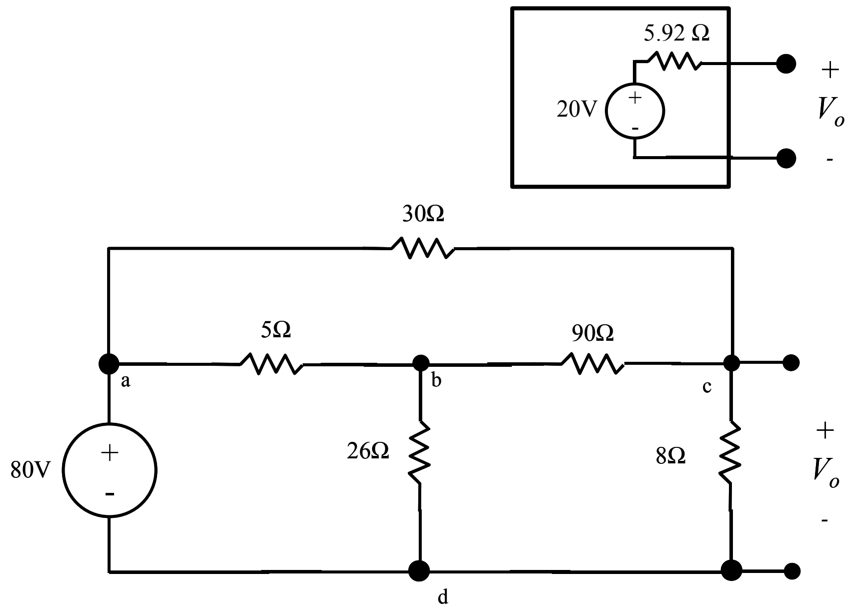


Figure 9. The Original Circuit and its Thevenin Equivalent

TO BE CONTINUED: COMPUTATION OF THE NORTON EQUIVALENT AND THE TREATMENT OF DEPENDENT SOURCES.
