Chapter 7

Exercise 7.4 The state diagram for this exercise is shown in Figure **??** on page **??**.

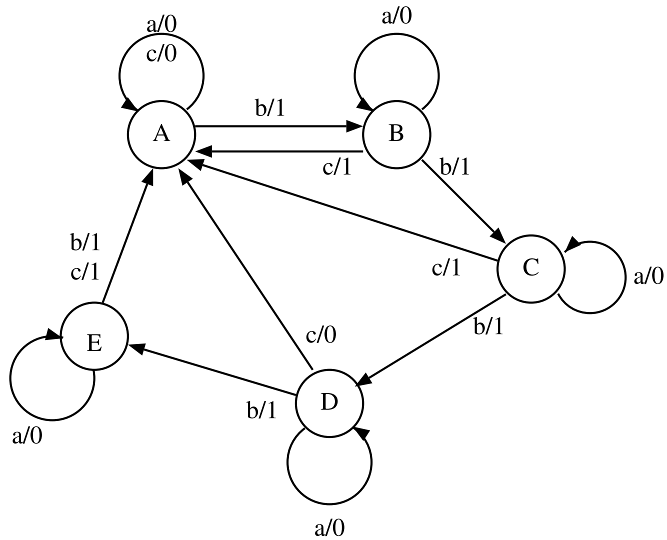

Figure 7.1: State Transition Diagram of Exercise 7.4

Exercise 7.6 The state table for this exercise is:

PS	Input		
	$x=0$	$x=1$	
0	0	1	1
1	1	2	0
2	2	3	1
3	3	4	0
4	4	5	1
5	5	6	0
6	6	0	1
	NS		Output (z)

From where we obtain the following state transition and output functions:

$$s(t+1) = (s(t) + x(t)) \bmod 7$$

$$z(t) = (s(t) + 1) \bmod 2$$

The system is a modulo-7 counter whose output is 1 whenever the number of 1's in the input modulo 7 is even

Exercise 7.10 The input has four values; we represent it by the bit-vector (x_1, x_0) with the following code: $a = 00, b = 01, c = 10$, and $d = 11$.

There are six states; we represent each one by the bit vector (y_2, y_1, y_0) and each state S_i correspond to a code shown in the next table (arbitrary):

State	(y_2, y_1, y_0)
S_0	001
S_1	010
S_2	000
S_3	011
S_4	100
S_5	101

The output z has five integer values. We use binary code for these integers, (z_2, z_1, z_0).

The corresponding state table is

PS	$x = 00$	$x = 01$	$x = 10$	$x = 11$
000	001,011	010,001	000,000	000,000
001	011,100	010,001	001,000	001,000
010	010,000	010,000	010,000	101,000
011	011,000	011,000	101,000	011,000
100	100,000	100,000	100,000	010,000
101	000,010	001,001	100,001	101,000
	$NS(Y_2Y_1Y_0), Output(z)$			

To obtain switching expressions, Karnaugh maps can be used. Since there are five variables, it is convenient to use 4-variable maps and include the 5th variable in the cells.

Y_2

		x_0	$x_1 x_0$	x_1
	y_2	y_2	0	y_2
y_0	0	0	y_2	y_2
$y_1 y_0$	0	0	0	1
y_1	0	0	1	0

Y_1

		x_0	$x_1 x_0$	x_1
	0	y_2'	y_2'	0
y_0	y_2'	y_2'	0	0
$y_1 y_0$	1	1	1	0
y_1	1	1	0	1

Y_0

		x_0	$x_1 x_0$	x_1
	y_2'	0	0	0
y_0	y_2'	y_2	1	y_2'
$y_1 y_0$	1	1	1	1
y_1	0	0	1	0

z_2

		x_0	$x_1 x_0$	x_1
	0	0	0	0
y_0	y_2'	0	0	0
$y_1 y_0$	0	0	0	0
y_1	0	0	0	0

z_1

		x_0	$x_1 x_0$	x_1
	y_2'	0	0	0
y_0	y_2	0	0	0
$y_1 y_0$	0	0	0	0
y_1	0	0	0	0

z_0

		x_0	$x_1 x_0$	x_1
	y_2'	y_2'	0	0
y_0	0	1	0	y_2
$y_1 y_0$	0	0	0	0
y_1	0	0	0	0

The expressions are:

$$Y_2 = x_1x_0y_1y_0' + x_1x_0'y_1'y_0 + x_1'y_1'y_0'y_2 + x_0'y_1'y_0'y_2 + x_1y_1'y_0y_2$$

$$
\begin{aligned}
Y_1 &= x_1' y_1 + y_1 y_0 x_0 + y_1 y_0' x_0' + y_0 x_1' y_2' + y_1' y_0' x_0 y_2' \\
Y_0 &= x_1' x_0' y_1' y_2' + y_0' y_0 y_2' + x_1' x_0 y_1' y_0 y_2 + y_1 y_0 + y_0 x_1 x_0 + y_1 x_1 x_0 \\
z_2 &= x_1' x_0' y_1' y_0 y_2' \\
z_1 &= x_1' x_0' y_2' y_1' y_0' + x_1' x_0' y_2 y_1' y_0 \\
z_0 &= y_1' y_0 x_1 x_0 y_2 + y_2' y_1' y_0 x_1' + y_1' y_0 x_1' x_0
\end{aligned}
$$

Exercise 7.12

The number of possible input sequences starting at time $t_1 = 5$ and going up to $t_2 = 16$ would be:

$$2^{16-5+1} = 2^{12} \text{ possible input sequences}$$

As it's possible to have the system in two different states at time t_1, S_3 or S_1, for the same input sequence it's possible to get two possible sequence pairs. So, the number of sequence pairs needed to describe the system would be:

$$2 \times 2^{12} = 2^{13}$$

Calling the states by their number (e.g. S_2 is called 2), three possible sequence pairs are:

t	5	6	7	8	9	10	11	12	13	14	15	16
$x(t)$	1	1	0	0	1	1	1	0	1	1	0	1
$s(t)$	3	3	3	1	2	2	2	2	0	1	3	1
$z(t)$	c	c	c	a	b	b	b	b	a	a	c	a

t	5	6	7	8	9	10	11	12	13	14	15	16
$x(t)$	1	1	0	1	1	0	1	1	0	0	1	0
$s(t)$	3	3	3	1	3	3	1	3	3	1	2	2
$z(t)$	c	c	c	a	c	c	a	c	c	a	b	b

t	5	6	7	8	9	10	11	12	13	14	15	16
$x(t)$	1	1	0	1	1	1	0	0	1	1	1	1
$s(t)$	1	3	3	1	3	3	3	1	2	2	2	2
$z(t)$	a	c	c	a	c	c	c	a	b	b	b	b

Exercise 7.16

Based on the outputs for each state we get the first partition P_1 as:

$$P_1 = (A, D, E)(B, F, G)(C, H)$$

Let's call (A,D,E) as group 1, (B,F,G) as group 2 and (C,H) as group 3.
We can construct a table representing the next group for each state transition:

	group 1			group 2			group 3	
	A	D	E	B	F	G	C	H
0	2	2	2	3	3	3	3	3
1	3	3	3	1	1	1	1	1

From the table we can see that the columns for each group of states are the same, and so, the states in each group are also 2-equivalent. $P_2 = P_1$. Renaming the states in group 1 as α, the states in group 2 as β and in group 3 as γ, we can represent the reduced sequential system as:

PS	$Input$	
	$x = 0$	$x = 1$
α	$\beta, 0$	$\gamma, 0$
β	$\gamma, 1$	$\alpha, 1$
γ	$\gamma, 0$	$\alpha, 1$

Exercise 7.24

Since two different patterns are generated, the system consists of two independent pattern generators, let's call these systems A and B. The corresponding state diagram is given in Figure **??**. The states $S_A i$ belongs to system A and states $S_B i$ belongs to system B.

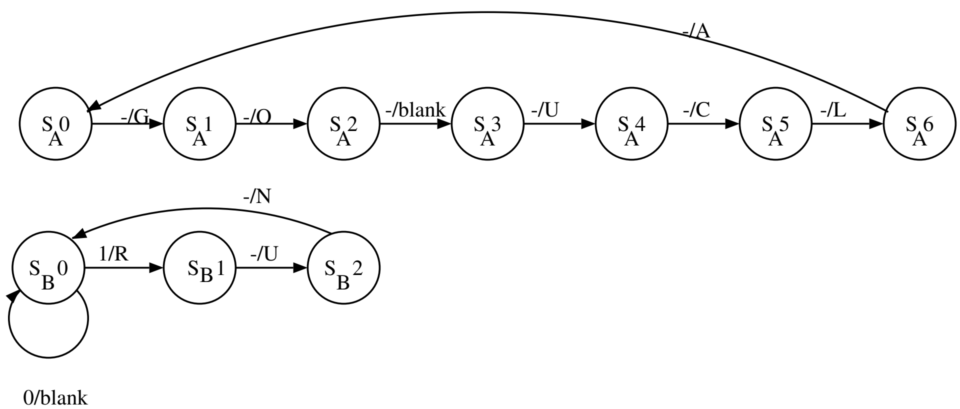

Figure 7.2: State diagram of Exercise 7.24