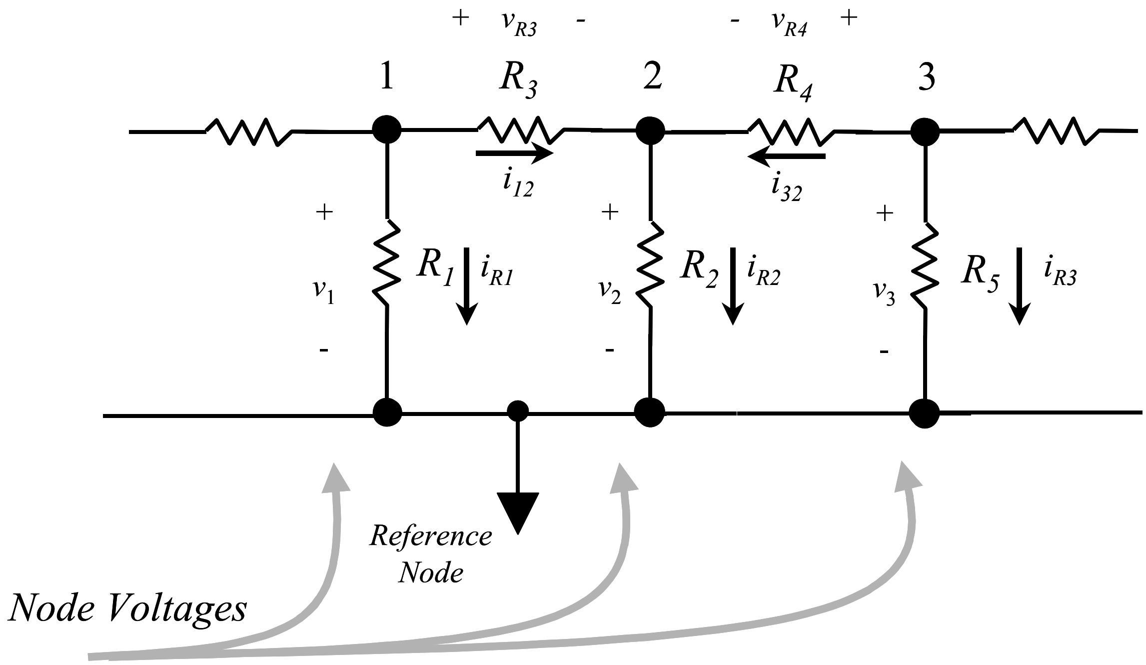
Tutorial: Typical Circuit Structures and Corresponding Node Voltage Equations

For this Tutorial, we will consider the problem of constructing the node voltage equations using Kirchoff's Current Law for virtually *all* the common forms of sources and resistors combinations that may appear between nodes.

We will form node voltage equations at Essential Nodes (where, by definition, three or more elements share a node).

What elements can exist between Essential circuit nodes:

1) Case 1: One resistor (or one resistor equivalent). (If resistors appear in series or parallel between Essential Nodes, then they will be replaced by one resistor equivalent).
    a. This resistor can be connected between two Essential Nodes (see Page 2)
    b. or, between an Essential Node and the Reference Node (see Page 2)
    c. Please note the procedure for constructing the Node Voltage Equations.
2) Case 2: One Voltage Source between an Essential Node and the Reference Node with two polarities(see Page 8)
    a. This eliminates the need for one Node Voltage Equation – since the Node Voltage is known.
    b. Please note the procedure for constructing the Node Voltage Equations.
    c. The Node Voltage equation at the node to which the source is connected is simply that this node voltage equals the source voltage !
3) Case 3: One resistor (or one resistor equivalent) and one Voltage Source
    a. This combination can be connected between two Essential Nodes with two polarities (see Page 4 and 5)
    b. or, between an Essential Node and the Reference Node with two polarities (equivalent to the circuits on Page 3)
    c. Please note the procedure for constructing the Node Voltage Equations.
4) Case 4: One Voltage Source
    a. This combination can be connected between two Essential Nodes with two polarities (see Page 6)
    b. This can be solved directly of via a Super Node (see Page 7)
    c. **Please note the constraint condition.**
5) Case 5: One Current Source with our without an additional series resistor.
    a. This can be connected between two Essential Nodes, or
    b. Or, between an Essential Node and the Reference.
    c. Here, we simply incorporate the current source value into our KCL equation evaluated at each node to which the current source is connected.
    d. (As we have explained, this cannot be replaced by a Supernode. A Supernode is a construct applied only in the event of a voltage source where the node voltage difference does not explicitly determine current flow, as is the case for resistors.).



$v_{R3} = v_1 - v_2$

$i_{12} = v_{R3}/R_3$

$v_{R4} = v_3 - v_2$

$i_{32} = v_{R4}/R_4$

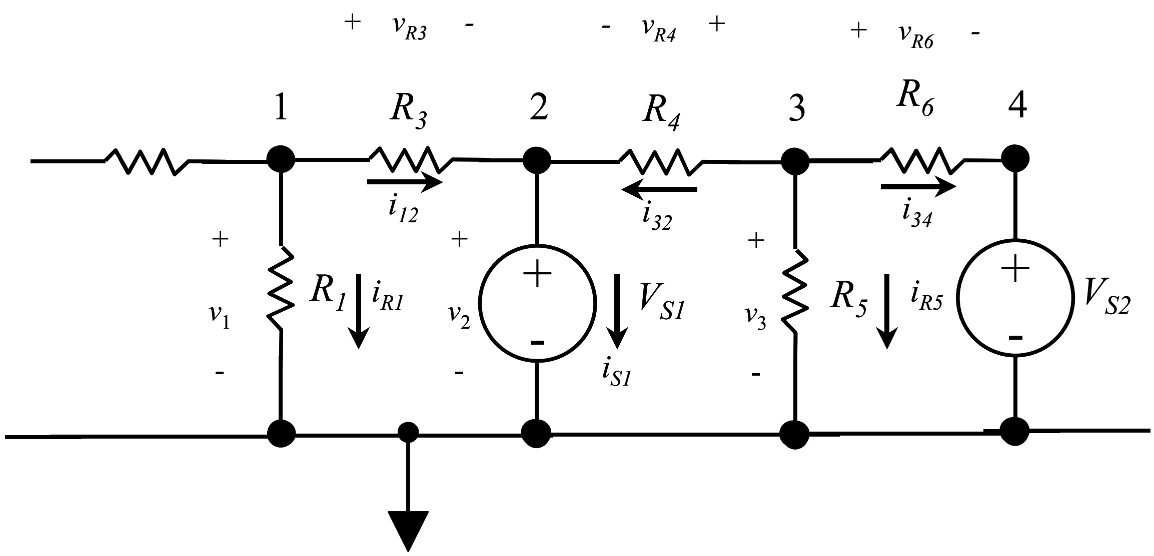
v_2 is the voltage drop across R2

$i_{R2} = v_{R2}/R_2$

Kirchoff's Current Law at Node 2: $i_{12} + i_{32} - i_{R2} = 0$

or

$(v_1 - v_2)/R_3 + (v_3 - v_2)/R_4 - v_2/R_2 = 0$



The Node Voltage Equation at Node 2 is simply:

$$\mathbf{v_2 = V_{S1}}$$

Also,

$$i_{12} = v_{R3}/R_3$$

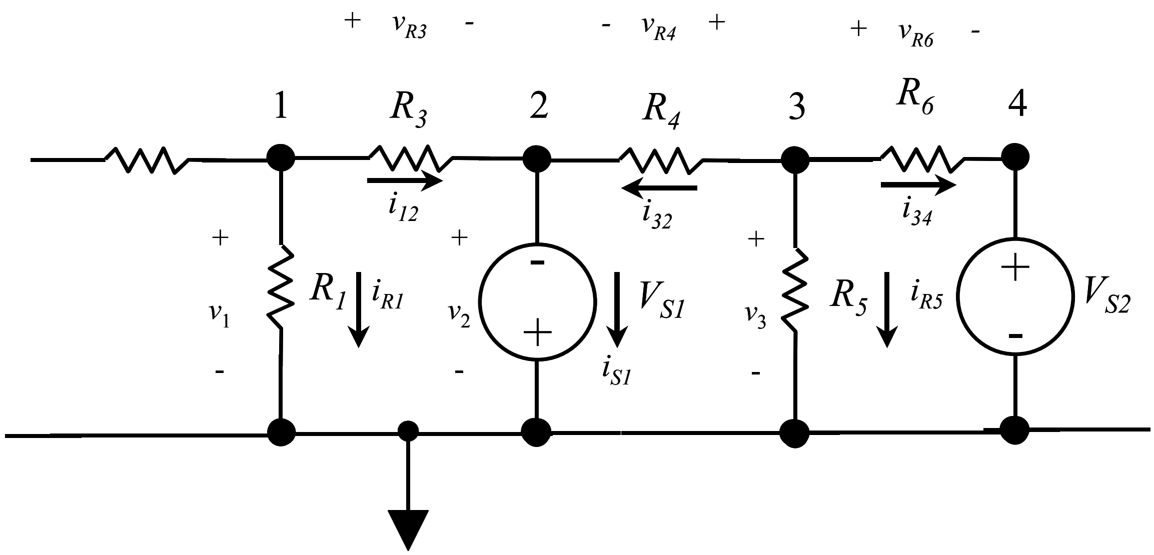
$$v_{R4} = v_3 - v_2 = v_3 - V_{S1}$$

$$i_{32} = v_{R4}/R_4$$

Kirchoff's Current Law at Node 3: $-i_{32} - i_{34} - i_{R5} = 0$

or

Node Voltage Equation at Node 3: $-(v_3 - V_{S1})/R_4 - (v_3 - V_{S2})/R_6 - v_3/R_5 = 0$



The Node Voltage Equation at Node 2 is simply:

$\mathbf{v_2 = -V_{S1}}$

Also,

$i_{12} = v_{R3}/R_3$

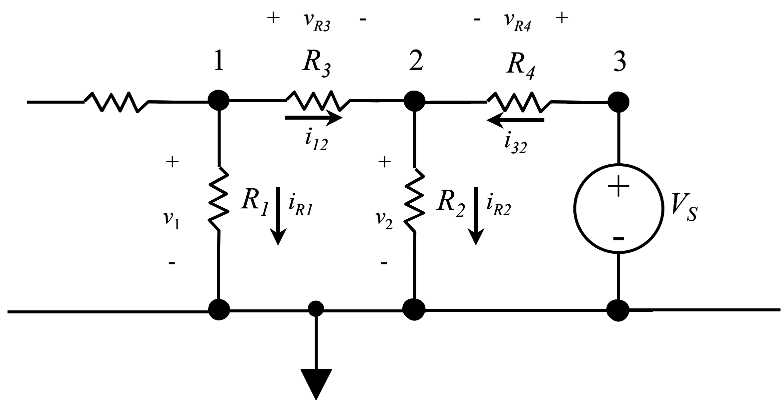
$v_{R4} = v_3 - v_2 = v_3 + V_{S1}$

$i_{32} = v_{R4}/R_4$

Kirchoff's Current Law at Node 3: $-i_{32} - i_{34} - i_{R5} = 0$

or

Node Voltage Equation at Node 3: $-(v_3 + V_{S1})/R_4 - (v_3 - V_{S2})/R_6 - v_3/R_5 = 0$



Consider the Essential Node, 2.

$v_{R4} = v_3 - v_2 = V_S - v_2$

$i_{32} = v_{R4}/R_4$

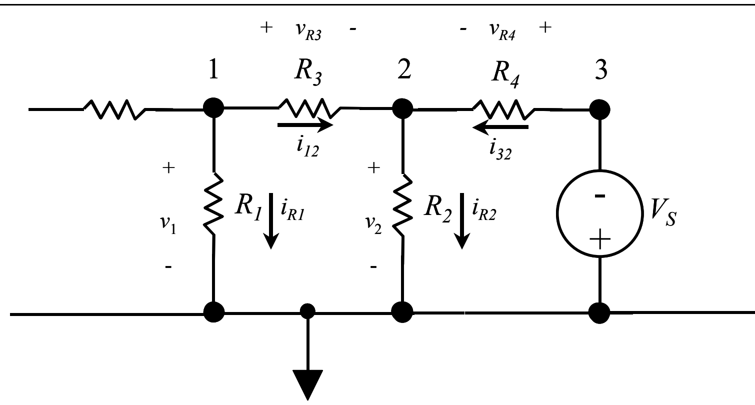
v_2 is the voltage drop across R2

$i_{R2} = v_{R2}/R_2$

Kirchoff's Current Law at Node 2: $i_{12} + i_{32} - i_{R2} = 0$

or

$$(v_1 - v_2)/R_3 + (V_S - v_2)/R_4 - v_2/R_2 = 0$$

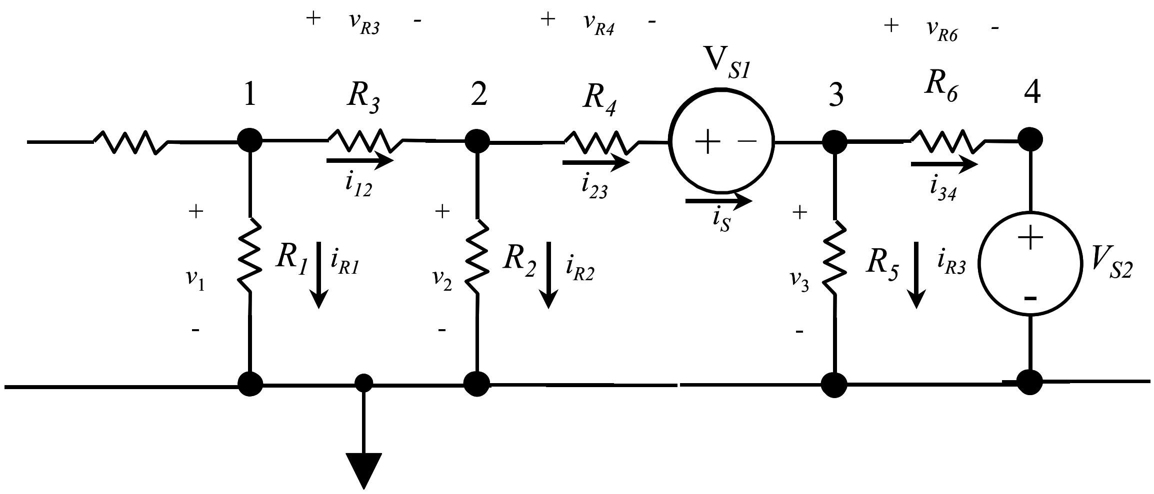


Considering a reversed polarity for the voltage souce:

Kirchoff's Current Law at Node 2: $i_{12} + i_{32} - i_{R2} = 0$

or

$$(v_1 - v_2)/R_3 + (-V_S - v_2)/R_4 - v_2/R_2 = 0$$



$$v_{R3} = v_1 - v_2$$
$$i_{12} = v_{R3}/R_3$$

Solve for V_{R4} to obtain i_{23} via Ohm's Law. Find this via KVL!. KVL involving a loop from Reference to Node 2 to Node 3 and back to Reference:

$$v_2 - v_{R4} - V_{S1} - v_3 = 0$$

So,

$$v_{R4} = v_2 - v_3 - V_{S1}$$

Now, $i_{23} = v_{R4}/R_4$

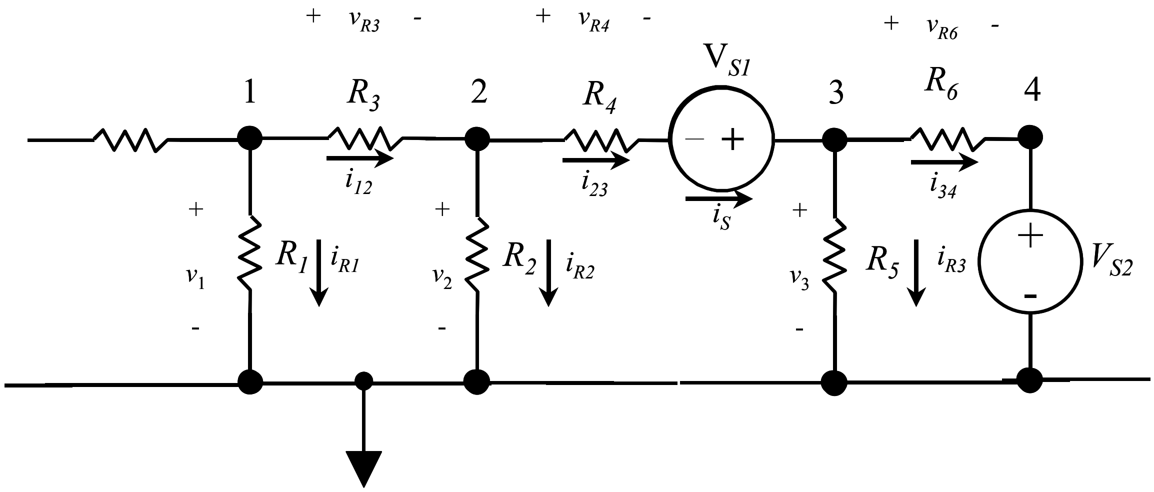
Also, v_2 is the voltage drop across R2

So, $i_{R2} = v_{R2}/R_2$

Kirchoff's Current Law at Node 2: $i_{12} - i_{23} - i_{R2} = 0$

or

$$(v_1 - v_2)/R_3 - (v_2 - v_3 - V_{S1})/R_4 - v_2/R_2 = 0$$



Now, reversing polarity of the voltage source between nodes 2 and 3 with respect to the previous circuit.

$$v_{R3} = v_1 - v_2$$
$$i_{12} = v_{R3}/R_3$$

Solve for V_{R4} to obtain i_{23} via Ohm's Law. Find this via KVL!. KVL involving a loop from Reference to Node 2 to Node 3 and back to Reference:

$$v_2 - v_{R4} + V_{S1} - v_3 = 0$$

So,

$$v_{R4} = v_2 - v_3 + V_{S1}$$

Now, $i_{23} = v_{R4}/R_4$

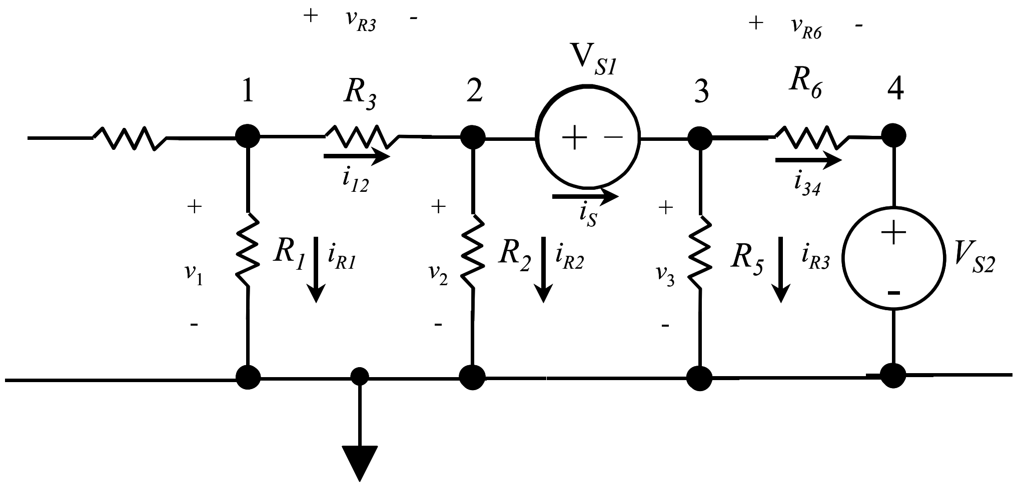
Also, v_2 is the voltage drop across R2

So, $i_{R2} = v_{R2}/R_2$

Kirchoff's Current Law at Node 2: $i_{12} - i_{23} - i_{R2} = 0$

or

$$(v_1 - v_2)/R_3 - (v_2 - v_3 + V_{S1})/R_4 - v_2/R_2 = 0$$



Kirchoff's Current Law at Node 2: $i_{12} - i_S - i_{R2} = 0$

or

$$(v_1 - v_2)/R_3 - i_S - v_2/R_2 = 0$$

Kirchoff's Current Law at Node 3: $i_S - i_{34} - i_{R3} = 0$

or

$$i_S - (v_3 - V_{S2})/R_6 - v_3/R_5 = 0$$

combining we have:

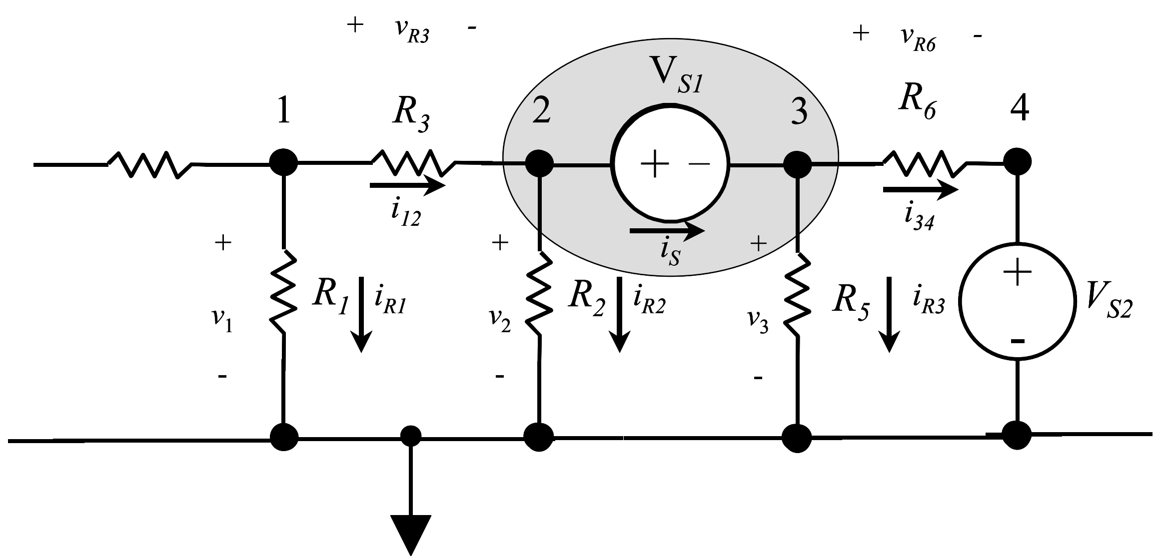
$$(v_1 - v_2)/R_3 - v_2/R_2 - (v_3 - V_{S2})/R_6 - v_3/R_5 = 0$$

Now, we are apparently missing information at this point. If the branch 2-3 was occupied by a resistor, then Ohm's Law would provide a condition on the voltage drop v_{23}. This would have provided enough information to solve for current and voltage values. But, the voltage source produces V_S *independent* of its current. We thus have no local condition that we can immediately write down relating this current and voltage.

But, we have a constraint condition!

$$\mathbf{v_2 - v_3 = V_{S1}}$$

We will proceed to use the equation derived from the combination of the Node 2 and Node 3 equations along with this constraint to solve for circuit variable values.



Kirchoff's Current Law at Node 2: $i_{12} - i_S - i_{R2} = 0$

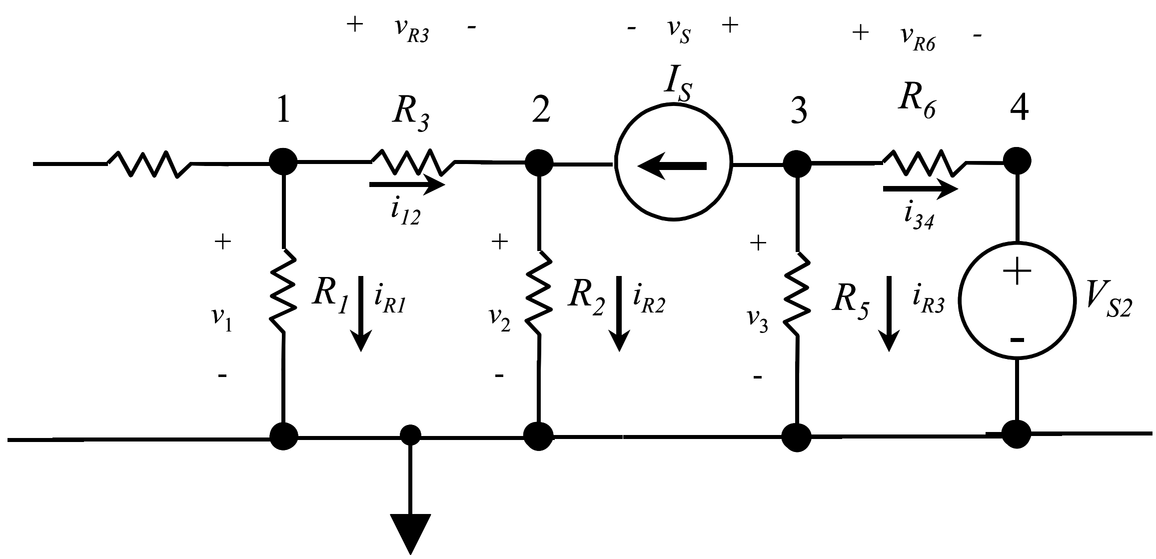
Kirchoff's Current Law at Node 3: $i_S - i_{34} - i_{R3} = 0$

Introduce a SuperNode

$$(v_1 - v_2)/R_3 - v_2/R_2 - (v_3 - V_{S2})/R_6 - v_3/R_5 = 0$$

and a constraint condition:

$$\mathbf{v_2 - v_3 = V_{S1}}$$



Kirchoff's Current Law at Node 2: $i_{12} + I_S - i_{R2} = 0$

or

$$(v_1 - v_2)/R_3 + I_S - v_2/R_2 = 0$$

Now, this provides all the information needed for currents to the *left* of node 2. Since the current source *defines* the current from node 2 to node 3.

We proceed to solve this problem with the additional equations that may result due to circuit components to the left of node 1. We will derive equations that define v_2 and v_3.

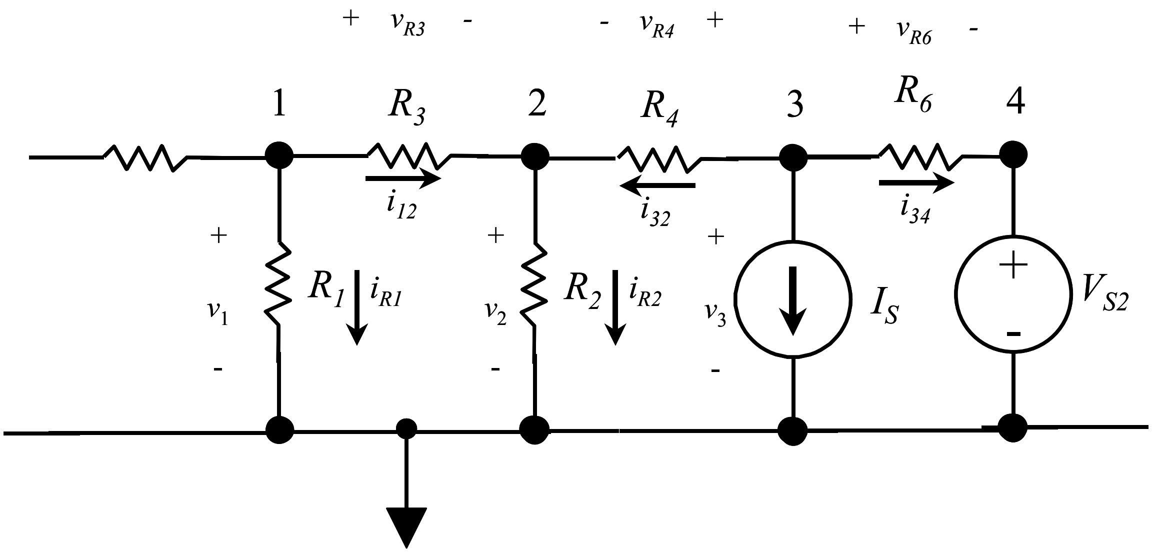
For example, at node 3 we will have a set of equations that are independent of those at node 2.

Kirchoff's Current Law at Node 3: $- I_S - i_{34} - i_{R5} = 0$

$$-I_S - (v_3 - V_{S2})/R_6 - v_3/R_5 = 0$$

In fact, this provides enough information to solve for v_3 directly (since Is is known and I_S defines all current flowing from 3 to 2). This effectively *isolates* node 3 from 2 for the purposes of node voltage computation.

$$v_3 = (V_{S2} - I_S R_6)R_5/(R_5 + R_6)$$



Kirchoff's Current Law at Node 3: $-i_{32} - I_S - i_{34} = 0$

or

$$(v_3 - v_2)/R_4 - I_S - (v_2 - V_{S2})/R_6 = 0$$