EE 102 # Homework #2 Solutions **Winter 2004**

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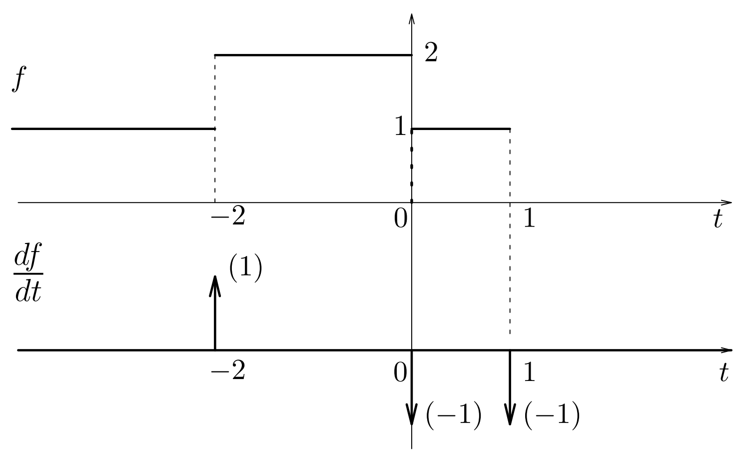
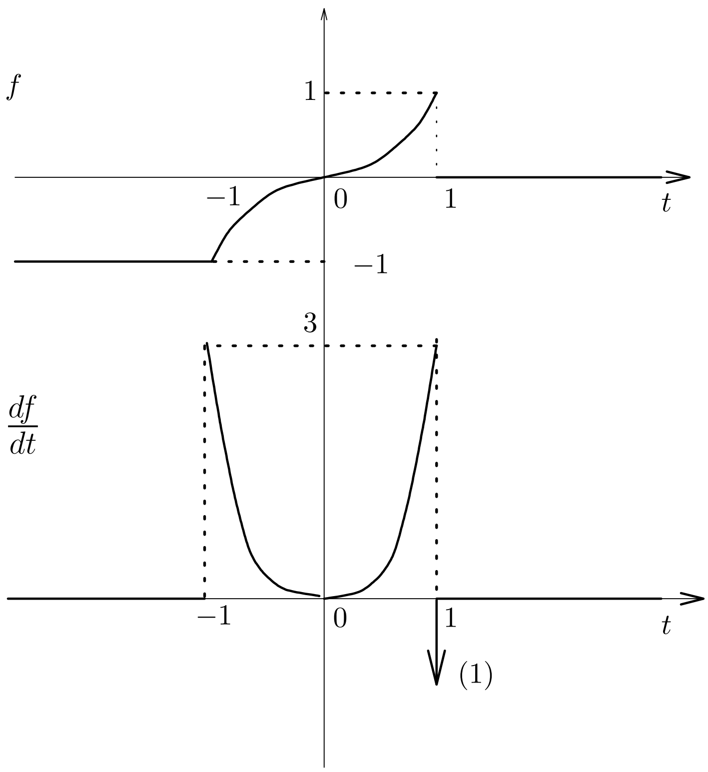
1. (a)

$$
\begin{aligned}
f(t) &= 1 - u(t) + u(t+2) - u(t-1) \\
\frac{df(t)}{dt} &= -\delta(t) + \delta(t+2) - \delta(t-1)
\end{aligned}
$$



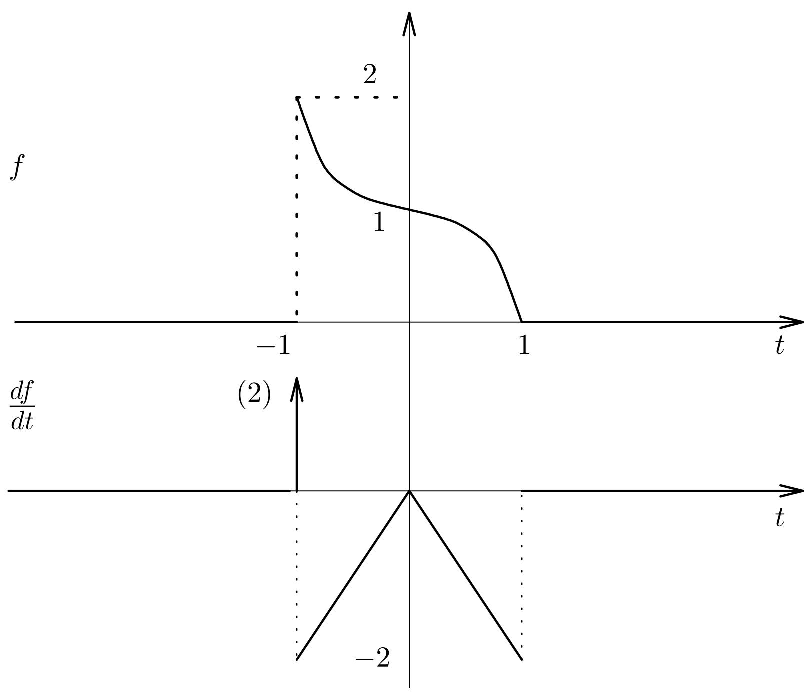
(b)

$$\begin{aligned}
f(t) &= -u(-t-1) + t^3[u(t+1) - u(t-1)] \\
\frac{df(t)}{dt} &= \delta(-t-1) + 3t^2[u(t+1) - u(t-1)] + t^3[\delta(t+1) - \delta(t-1)] \\
&= \delta(t+1) + 3t^2[u(t+1) - u(t-1)] - \delta(t+1) - \delta(t-1) \\
&= 3t^2[u(t+1) - u(t-1)] - \delta(t-1)
\end{aligned}$$



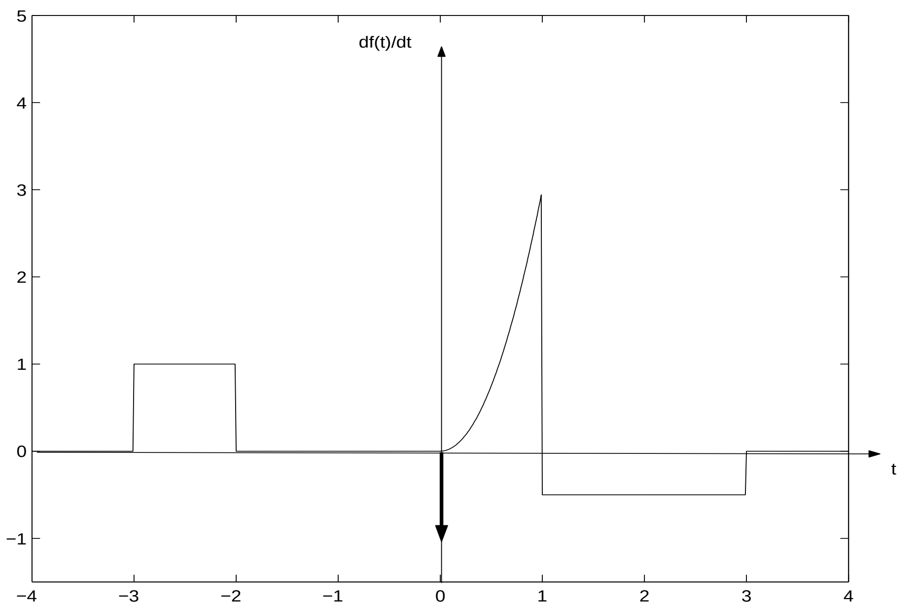
(c)

$$\begin{aligned}
f(t) &= (t^2+1)[u(t+1)-u(t)] + (1-t^2)[u(t)-u(t-1)] \\
\frac{df(t)}{dt} &= (t^2+1)[\delta(t+1)-\delta(t)] + 2t[u(t+1)-u(t)] \\
&\quad +(1-t^2)[\delta(t)-\delta(t-1)] - 2t[u(t)-u(t-1)] \\
&= 2\delta(t+1) - \delta(t) + 2t[u(t+1)-u(t)] + \delta(t) - 2t[u(t)-u(t-1)] \\
&= 2\delta(t+1) + 2t[u(t+1)-u(t)] - 2t[u(t)-u(t-1)]
\end{aligned}$$



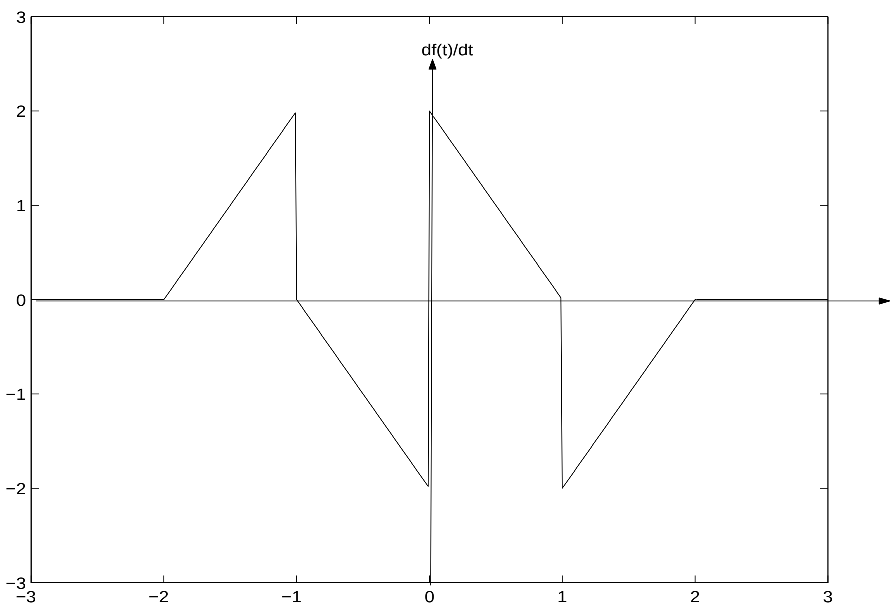
2. (a)

$$\begin{aligned} f(t) &= (t+3)[u(t+3)-u(t-2)]+[u(t+2)-u(t)]+t^3[u(t)-u(t-1)] \\ &\quad -\frac{1}{2}(t-3)[u(t-1)-u(t-3)] \\ \frac{df(t)}{dt} &= [u(t+3)-u(t-2)]+3t^2[u(t)-u(t-1)] \\ &\quad -\frac{1}{2}[u(t-1)-u(t-3)]-\delta(t) \end{aligned}$$



(b)

$$
\begin{aligned}
f(t) &= (t+2)^2[u(t+2)-u(t+1)] + (1-(t+1)^2)[u(t+1)-u(t)] \\
&\quad +(1-(t-1)^2)[u(t)-u(t-1)] + (t-2)^2[u(t-1)-u(t-2)] \\
\frac{df(t)}{dt} &= 2(t+2)[u(t+2)-u(t+1)] - 2(t+1)[u(t+1)-u(t)] \\
&\quad -2(t-1)[u(t)-u(t-1)] + 2(t-2)[u(t-1)-u(t-2)]
\end{aligned}
$$



3. (a) $\int_{-\infty}^{\infty} e^{\cos(\pi t)}\delta(t-1)dt = e^{\cos(\pi)} = e^{-1}$

   (b) $\int_{-\infty}^{0} \frac{\sin(t)}{t^4+1}\delta(t-1)dt = 0$ because $1 \notin (-\infty, 0)$.

   (c) $\int_{t^-}^{\infty} \sigma^5 \delta(t-\sigma)dt = t^5$

4. $y(t) = \int_{-\infty}^{t} \sin(t-\sigma)x(\sigma+1)d\sigma$

   (a)

$$\begin{aligned} h(t,\tau) &= \int_{-\infty}^{t} \sin(t-\sigma)\delta(\sigma+1-\tau)d\sigma \\ &= \sin(t-\tau+1)u(t-\tau+1) \end{aligned}$$

   (b) The system is time-invariant because $h(t,\tau)$ is only a function of $(t-\tau)$. The system is not causal because $h(t,\tau) \neq 0$ for $t < \tau$ (e.g., at $t-\tau = -\frac{1}{2}$).

5. (a)

$$\begin{aligned} \frac{dy(t)}{dt} &= u(t-1)x(t) \\ \int_0^t \frac{dy(t)}{dt} &= \int_0^t u(1-\sigma)x(\sigma)d\sigma \\ y(t) - 0 &= \int_0^t u(1-\sigma)x(\sigma)d\sigma \\ h(t,\tau) &= \int_0^t u(1-\sigma)\delta(\sigma-\tau)d\sigma \\ &= u(1-\tau)u(t-\tau) \end{aligned}$$

   (b) The system is time-variant because $h(t,\tau)$ is **not** only a function of $(t-\tau)$. The system is causal because $h(t,\tau) = 0$ for $t < \tau$.