EE 102 # Homework #1 **Fall 2001**

Professor Paganini **Due Wednesday 10/3/01**

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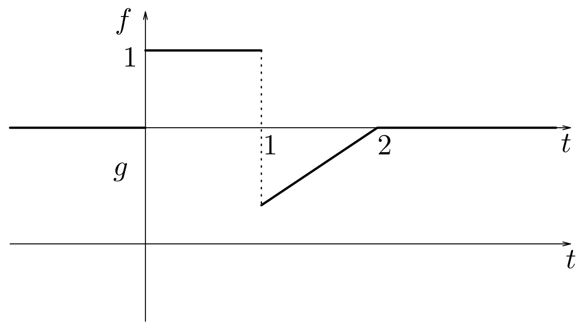
1. Review of integration.
   (a) Evaluate the integrals $\int_0^\pi t\cos(t)dt$ and $\int_0^\pi t^2\sin(t)dt$.
   (b) For a differentiable function f, derive the identity

$$\int_0^t f(t-\tau)d\tau = tf(t) - \int_0^t \tau f'(\tau)d\tau$$

   (c) The figure below contains a picture of a function $f(t)$. Find the function $g(t) = \int_{-\infty}^t f(\tau)d\tau$ and sketch it under $f(t)$.



2. Review of complex numbers
   (a) Find the following complex numbers (real and imaginary parts):

$$(1)\ e^{-\frac{27}{2}\pi i}, \quad (2)\ (i)^{i^6}$$

   (b) Change these complex numbers into exponential form:

$$(1)\ \alpha = \sqrt{3} - i, \quad (2)\ \beta = -i.$$

   (c) For the numbers in part (b), compute $\alpha^3/\bar{\beta}$, where $\bar{\beta}$ is the complex conjugate of β.
   (d) Find the complex roots to the polynomial equation $z^6 - 27 = 0$.

3. Given the differential equation for $t \geq 0$

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - 2x(t)$$

   - Let $x(0) = 0$ and $y(0) = 0$; solve for $y(t)$ in terms of $x(t)$.

4. For each of the following systems with input $x(t)$ and output $y(t)$, find out whether they are (i) linear, (ii) time invariant, (iii) causal. Justify your answer.

   (a) $y(t) = x(t+1) - 3$.

   (b) $y(t) = e^t x(t)$.

   (c) $y(t) = \int_t^\infty x(\tau) d\tau$.

   (d) The system where $y(t)$ is equal to $x(t)$ when $x(t) > 0$, and zero otherwise.

EE 102 # Homework #2 **Fall 2001**

Professor Paganini **Due Wednesday 10/10/01**

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1. Sketch $f(t)$ and $\frac{df}{dt}(t)$. State what $\frac{df}{dt}(t)$ is in the simplest form (e.g., $u(t-2)\delta(t-7)$ should be simplified to $\delta(t-7)$).

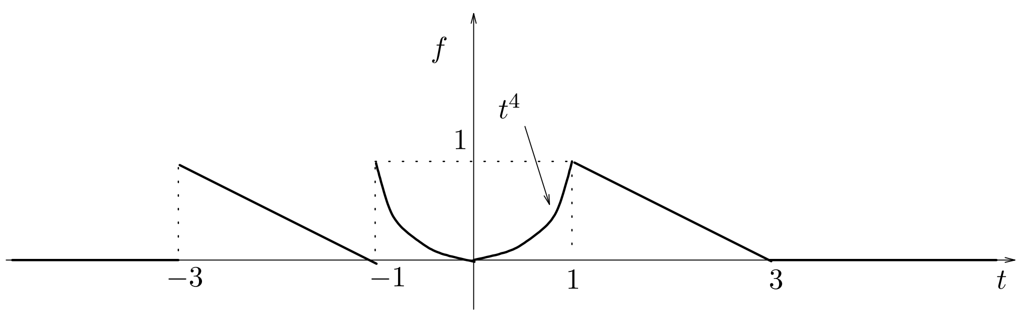
   (a) $f(t) = 1 - u(t+2) - u(t) + u(t-1)$.

   (b) $f(t) = \begin{cases} 2t+2 & \text{for } t \in (-1,0) \\ 2t-2 & \text{for } t \in (0,1) \\ 0 & \text{otherwise} \end{cases}$. Here you should first write an expression for $f(t)$.

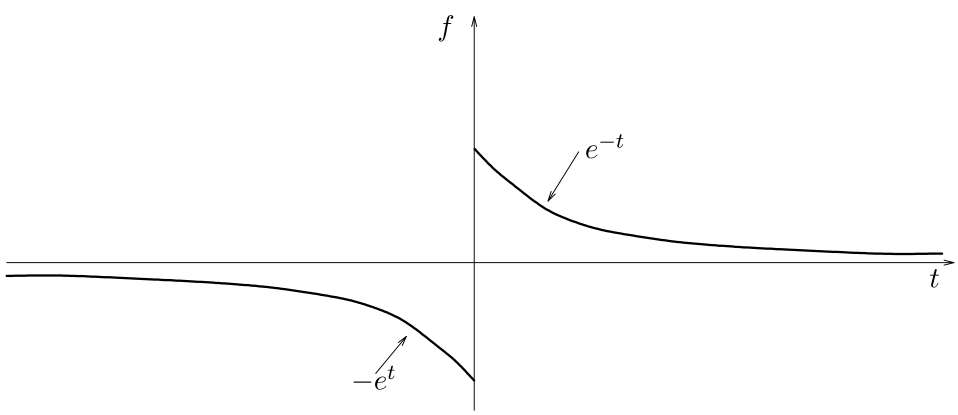
   (c) $f(t) = (t+1)^2[u(t+1) - u(t)] + (t-1)^2[u(t) - u(t-2)]$.

2. Sketch $\frac{df}{dt}(t)$, and find an expression for $f(t)$ and $\frac{df}{dt}(t)$.

   (a)



   (b)



3. Evaluate the following integrals.

$$\text{(a)} \quad \int_{-\infty}^{\infty} e^{\sin(\pi t)}\ \delta(t+\frac{1}{2})\ dt$$

$$\text{(b)} \quad \int_{-\infty}^{3} e^{t^2-3t-4}\delta(t-4)\ dt$$

$$\text{(c)} \quad \int_{a-}^{\infty} \cos(t)\delta(t-a)\ dt, \quad \text{where} \quad a \in \mathbb{R}.$$

4. Consider the system defined by the input-output relationship

$$y(t) = \int_{-\infty}^{t} \cos(t+\sigma)x(\sigma-1)d\sigma.$$

   (a) Find the system impulse response function $h(t,\tau)$.

   (b) Is the system time invariant? Causal?

5. Consider a system described by the differential equation

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - 2x(t),$$

studied in HW # 1. Signals are assumed to be zero for $t < 0$. i.e., the initial conditions are $y(0-) = x(0-) = 0$. Find the impulse response function $h(t)$.

EE 102

Homework #3

Fall 2001

Professor Paganini

Due Wednesday 10/17/01

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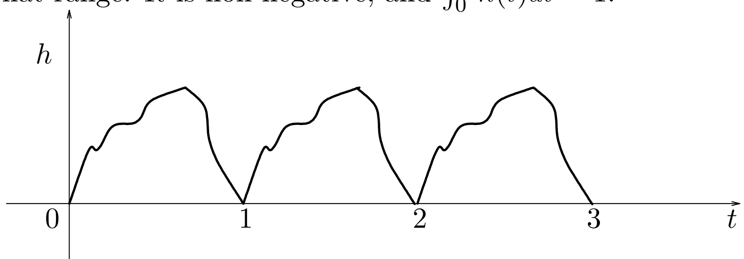
1. We consider a linear, time invariant system with impulse response $h(t)$ depicted in the figure. The function is made of three identical curves in the intervals $[0, 1]$, $[1, 2]$, $[2, 3]$, and is zero outside that range. It is non-negative, and $\int_0^1 h(t)dt = 1$.

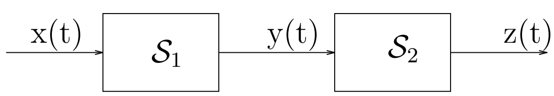


   Sketch the response of the system to the input $x(t) = u(t) - u(t-1)$.
   Your sketch cannot be exact since you don't know $h(t)$ exactly, but it must be consistent with the information given above.

2. Given the function $f(t) = e^{-t}u(t)$, where $u(t)$ is the step function, find the convolutions:

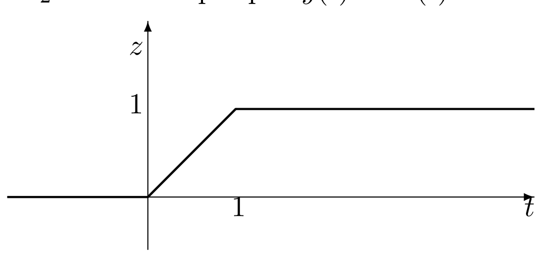
   (a) $u * f$;
   (b) $f * f$;
   (c) $u * u$.

3. Consider the cascade of linear, time-invariant systems $\mathcal{S}_1$ and $\mathcal{S}_2$.



   We know:

   - The impulse response function $h_1(t) = u(t) - u(t-2)$.
   - The response of system $\mathcal{S}_2$ to the ramp input $y(t) = tu(t)$ is the function $z(t)$ below.



   Find and sketch the impulse response of the cascade.

EE 102 # Homework #4 **Fall 2001**

Professor Paganini **Due Wednesday 10/24/01**

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1. Use the definition of the Laplace transform

$$F(s) = \int_0^\infty e^{-st} f(t) dt$$

to find the transform of the following functions. Do not invoke properties here; rather, perform the integration. In each case, specify the domain of convergence.

   (a) $u(t-2)e^{2t}$.

   (b) $u(t) - u(t-1) + u(t-2) - u(t-3)$.

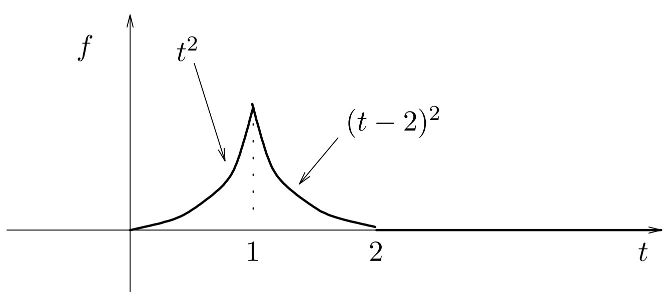
2. Find the Laplace transforms of the following functions using the properties of Laplace transform. Specify the properties being used, and the DOC.

   (a) $e^t u(t) + e^{-2t} u(t)$.

   (b) $u(t-\pi) e^{(t-\pi)} \cos(t)$.

   (c) $\int_0^t \sigma^2 e^{-\sigma} d\sigma$.

3. Consider the function $f(t)$ in the figure.



   (a) Find and sketch the derivatives $\frac{df}{dt}$, $\frac{d^2 f}{dt^2}$.

   (b) Find the Laplace transform of $\frac{d^2 f}{dt^2}$, and deduce the Laplace transforms of $\frac{df}{dt}$ and $f(t)$. Specify the domain of convergence.

4. Find $f(t)$ given $F(s)$.

$$\text{a) } F(s) = \frac{s+11}{s^2 - 3s - 4}; \quad \text{b) } F(s) = \frac{4s+10}{s^3 + 6s^2 + 10s}; \quad \text{c) } F(s) = \frac{2s^2 - s - 5}{(s-1)^2(s+3)}.$$

5. Consider the differential equation for $t \geq 0$:

$$\frac{d^2 f}{dt^2} + \alpha \frac{df}{dt} + f(t) = 1, \qquad f(0-) = \frac{df}{dt}(0-) = 0.$$

Here $\alpha \in \mathbb{R}$ is a parameter.

(a) Find the initial value $\lim_{t \to 0+} f(t)$; does your answer depend on α? Hint: you don't need to solve the differential equation.

(b) Repeat the above for the final value $\lim_{t \to +\infty} f(t)$.

(c) Now let $\alpha = 1$; solve the differential equation.