Homework 1 Solutions

Exercise 2.4

Input: X is a 4×4 matrix of elements $x_{ij} \in \{a, b, c, d\}$:

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix}$$

Output: $z \in \{0, 1\}$.

Function: The output is equal to 1 if one of x_{22}, x_{23}, x_{32}, or x_{33} is b and it is surrounded by eight a's. The conditional expression is

$$z = \begin{cases} 1 \textbf{ if } x_{i,j} = b \textbf{ and } (x_{k,m} = a \text{ for } k \in \{i-1, i, i+1\} \text{ and } m \in \{j-1, j, j+1\}) \\ \qquad \text{with } i, j \in \{2, 3\} \textit{ and } (k, m) \neq (i, j) \\ 0 \textbf{ otherwise} \end{cases}$$

A tabular representation would not be practical because there are $4 \times 4 = 16$ variables, each of which can have 4 values, resulting in a table of 4^{16} rows.

Exercise 2.8

(a) Month: $\lceil \log_2 12 \rceil = 4$ bits

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Code	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011

(b) Day: $\lceil \log_2 31 \rceil = 5$ bits
Year (assuming 0-2500): $\lceil \log_2 2501 \rceil = 12$ bits
Month: 4 bits
A total of 21 bits would be needed.

(c) Each decimal digit needs 4 bits. Two digits are necessary to represent the day, and 4 digits to represent the year. The number of bits to represent these fields would be $6 \times 4 = 24$ bits. Adding the bits used in the month field we get 28 bits total.
(d) If only one component is used, we need to specify a code for each particular day. This code may be calculated as:

$$(year - 1) * 12 * 31 + (month - 1) * 31 + day - 1$$

where *month*, *year* and *day* are the usual numbers associated to each field (Jan = 1, Feb = 2,...).
Maximum date is 12/31/2500, which corresponds to the value:
$2499 * 372 + 11 * 31 + 30 = 929999$
So $\lceil log_2 929999 \rceil = 20$ bits are needed to represent all possible codes. The disadvantage of this method is that many calculations must be done to convert the code to the usual representation of the date, and vice-versa.

Exercise 2.24

(a)

$$\begin{aligned} a'b' + ab + a'b &= a'(b+b') + ab \\ &= a'1 + ab \\ &= a' + b \end{aligned}$$

(b)

$$\begin{aligned} a' + a(a'b + b'c)' &= a' + (a'b + b'c)' \\ &= a' + (a+b')(b+c') \\ &= a' + ab + ac' + bb' + b'c' \\ &= a' + b + c' + b'c' \\ &= a' + b + c' \end{aligned}$$

(c)

$$\begin{aligned} (a'b' + c)(a+b)(b' + ac)' &= (a'b' + c)(a+b)b(a' + c') \\ &= (a'b' + c)b(a' + c') \\ &= (ba'b' + bc)(a' + c') \\ &= bc(a' + c') \\ &= a'bc \end{aligned}$$

(d)

$$\begin{aligned} ab' + b'c' + a'c' &= ab' + (a+a')b'c' + a'c' \\ &= ab' + ab'c' + a'b'c' + a'c' \\ &= ab'(1+c') + a'(1+b')c' \\ &= ab' + a'c' \end{aligned}$$

(e)

$$\begin{aligned} wxy + w'x(yz + yz') + x'(zw + zy') + z(x'w' + y'x) &= wxy + w'xy + z(x'(w+y') + x'w' + y'x) \\ &= (w+w')xy + z(x'(w+y'+w') + y'x) \\ &= xy + z(x' + xy') \\ &= xy + z(x' + y') \\ &= xy + z(xy)' \\ &= xy + z \end{aligned}$$

(f)

$$\begin{aligned} abc' + bc'd + a'bd &= abc' + (a+a')bc'd + a'bd \\ &= abc' + abc'd + a'bc'd + a'bd \\ &= abc'(1+d) + a'bd(1+c') \\ &= abc' + a'bd \end{aligned}$$

Exercise 2.26

$$
\begin{aligned}
a + a'b + a'b'c + a'b'c'd + a'b'c'd'e &= \\
&= a + a'b + a'b'c + a'b'c'(d + d'e) \\
&= a + a'b + a'b'(c + c'(d + e)) \\
&= a + a'b + a'b'(c + d + e) \\
&= a + a'(b + b'(c + d + e)) \\
&= a + a'(b + c + d + e) \\
&= a + b + c + d + e
\end{aligned}
$$

Exercise 2.38

(a) $E(a,b,c,d) = abc'd + ab'c + bc'd + ab'c' + acd + a'bcd$

$$\begin{aligned} E(a,b,c,d) &= c'd(ab+b) + ab'(c+c') + (a+a'b)cd \\ &= bc'd + ab' + (a+b)cd \\ &= bd(c+c') + ab' + acd \\ &= bd + ab' + a(b+b')cd \\ &= bd(1+ac) + ab'(1+cd) \\ &= ab' + bd \end{aligned}$$

(b) $E(a,b,c,d) = acb + ac'd + bc'd + a'b'c' + ab'c'd' + bc'd$

$$\begin{aligned} E(a,b,c,d) &= bc'(d+d') + b'c'(a' + ad') + acb + ac'd \\ &= bc' + b'c'a' + b'c'd' + acb + ac'd \\ &= c'(b + b'a' + b'd' + ad) + acb \\ &= c'(b + a' + d + d') + acb \\ &= c' + acb \\ &= ab + c' \end{aligned}$$

Exercise 2.40

(a)

$$\begin{aligned}
a'b + ac + bc &= a'b(c+c') + a(b+b')c + (a+a')bc \\
&= a'bc + a'bc' + abc + ab'c + abc + a'bc \\
&= m_3 + m_2 + m_7 + m_5 + m_7 + m_3 \\
&= \sum m(2,3,5,7) = \prod M(0,1,4,6)
\end{aligned}$$

(b) $g(a,b,c,d,e,f) = (ab+c)(d'e+f) = g_1(a,b,c)g_2(d,e,f)$

Since the g_1 and g_2 are disjoint, we may obtain their minterms separately and combine them later to get the minterms for $g(a,b,c,d,e,f)$.

$$g_1(a,b,c) = ab + c = abc + abc' + abc + a'bc + ab'c + a'b'c = \sum m(1,3,5,6,7) = \prod M(0,2,4)$$

$$g_2(d,e,f) = d'e + f = d'ef + d'ef' + def + d'ef + de'f + d'e'f = \sum m(1,2,3,5,7) = \prod M(0,4,6)$$

If g_1 has a minterm $m_i(a,b,c)$ and g_2 has a minterm $m_j(d,e,f)$, since the expression for g is the AND of both expressions, g has the minterms $m_{8i+j}(a,b,c,d,e,f) = m_i(a,b,c).m_j(d,e,f)$. Consequently, the minterms and maxterms of g are:

CSP: $\sum m(\ 9,10,11,13,15,25,26,27,29,31,41,42,43,45,47,49,50,51,53,55,57,58,59,61,63)$

CPS: $\prod M(\ 0,1,2,3,4,5,6,7,8,12,14,16,17,18,19,20,21,22,23,24,28,30,32,33,34,35,36,37,$
$38,39,40,44,46,48,52,54,56,60,62)$

(c)

$$\begin{aligned}
a'b(ab+c)(b+c'd) &= (a'bab + a'bc)(b+c'd) = a'bc(b+c'd) \\
&= a'bcb + a'bcc'd = a'bc(d+d') \\
&= a'bcd + a'bcd'
\end{aligned}$$

CSP: $\sum m(6,7)$

CPS: $\prod M(0,1,2,3,4,5,8,9,10,11,12,13,14,15)$

Exercise 2.42

a) We first convert to a sum of products

$$\begin{aligned}
&[((a+b+a'c')c+d)'+ab]' = \\
&\qquad = (((a+b+a'c')c+d)')'(ab)' \\
&\qquad = ((a+b+a'c')c+d)(a'+b') \\
&\qquad = (ac+bc+d)(a'+b') \\
&\qquad = a'bc+a'd+ab'c+b'd
\end{aligned}$$

To get the CSP we expand and eliminate repeated minterms

$$\begin{aligned}
&= a'bc(d+d')+a'd(b+b')(c+c')+ab'c(d+d')+b'd(a+a')(c+c') \\
&= a'bcd+a'bcd'+a'bc'd+a'b'cd+a'b'c'd+ab'cd+ab'cd'+ab'c'd
\end{aligned}$$

In m-notation,

$$E(a,b,c,d) = \sum m(1,3,5,6,7,9,10,11)$$

b) We first get a sum of products

$$\begin{aligned}
&[(w'+(xy+xyz'+(x+z)')')(z'+w')]' = \\
&= (w'+(xy+xyz'+x'z')')'+(z'+w')' \\
&= (w(xy+xyz'+x'z')+zw \\
&= wxy+wxyz'+wx'z'+wz
\end{aligned}$$

Now we expand and eliminate repeated minterms

$$\begin{aligned}
&= wxy(z+z')+wxyz'+wx'z'(y+y')wz(x+x')(y+y') \\
&= wxyz+wxyz'+wx'yz'+wx'y'z'+wxy'z+wx'yz+wx'y'z
\end{aligned}$$

In m-notation,

$$E(w,x,y,z) = \sum m(8,9,10,11,13,14,15)$$

Exercise 2.50 (a) A conditional expression is

$$L = \begin{cases} ON \text{ } \mathbf{if} \text{ an odd number of switches are ON} \\ OFF \text{ } \mathbf{otherwise} \end{cases}$$

The state of the light cannot be considered as an input because a feedback loop is formed and the circuit generated with this loop will not be stable (it will oscillate for some combinations of the switches).

(b) Calling w, x, y, z the four switches, the table for the switching functions is:

$wxyz$	L
0000	0
0001	1
0010	1
0011	0
0100	1
0101	0
0110	0
0111	1
1000	1
1001	0
1010	0
1011	1
1100	0
1101	1
1110	1
1111	0

(c) The minimal sum of products expression for this function is

$$L = w'x'y'z + w'x'yz' + w'xy'z' + w'xyz + wx'y'z' + wx'yz + wxy'z + wxyz'$$