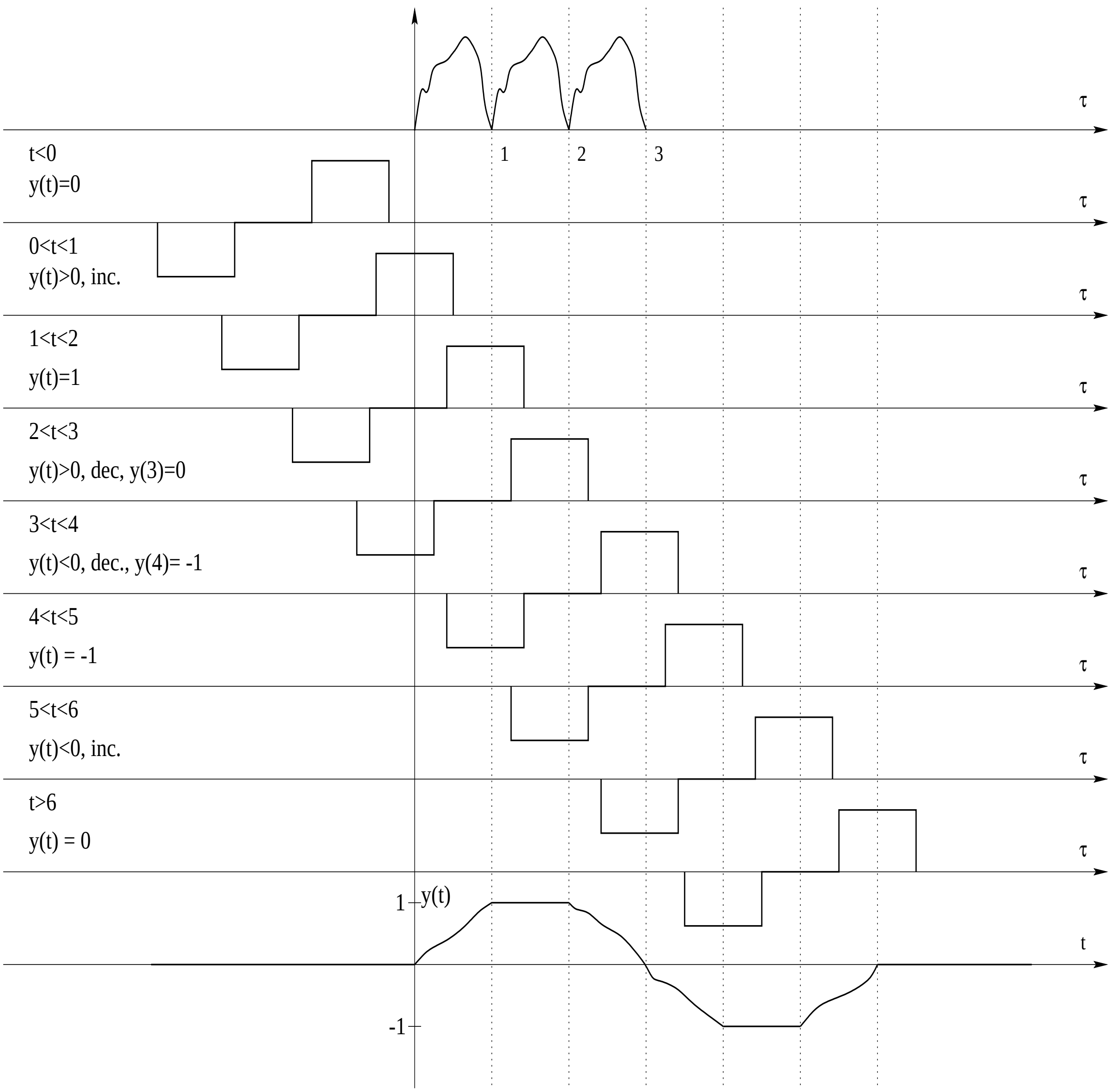
**EE 102** # Homework #3 Solutions **Winter 2004**

**Professor Paganini**

---

1. The step-by-step graphs corresponding to the convolution can be seen below:



2. (a)

$$
\begin{aligned}
(f*g)(t) &= \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} e^{-\tau}u(\tau)[u(t-\tau)-u(t-\tau-1)]d\tau \\
&= \begin{cases} 0 & \text{if } t \leq 0 \\ \int_0^t e^{-\tau}d\tau = [-e^{-\tau}]_0^t = 1-e^{-t} & \text{if } 0 < t \leq 1 \\ \int_{t-1}^t e^{-\tau}d\tau = [-e^{-\tau}]_{t-1}^t = e^{-t}(e-1) & \text{if } 1 < t \end{cases} \\
&= [u(t)-u(t-1)](1-e^{-t}) + u(t-1)e^{-t}(e-1)
\end{aligned}
$$

(b)

$$
\begin{aligned}
(f*h)(t) &= \int_{-\infty}^{\infty} e^{-(t-\tau)}u(t-\tau)u(\tau+1)e^{-\tau} \\
&= \begin{cases} 0 & \text{if } t < -1 \\ \int_{-1}^t e^{-(t-\tau)}e^{-\tau}d\tau = e^{-t}(t+1) & \text{otherwise} \end{cases} \\
&= e^{-t}(t+1)u(t+1)
\end{aligned}
$$

3. We know that for an LTI system, if applying input $x(t)$ generates output $y(t)$, then applying input $\frac{dx}{dt}$ generates output $\frac{dy}{dt}$. Using this fact for the system $\mathcal{S}_1$, we find the input-output pair $\frac{dx}{dt} = \delta(t)$ and $\frac{dy}{dt} = u(t) + t\delta(t) = u(t)$. So $h_1(t) = u(t)$ (this system is an integrator).

   Now we repeat the same argument for the second system $\mathcal{S}_2$, with input $y(t)$ and output $z(t)$. Taking derivatives of the given $y$ and $z$, we obtain the new input-output pair $y_1(t) = \frac{dy}{dt} = u(t)$, $z_1(t) = \frac{dz}{dt} = u(t) - u(t-1)$. Since we want to find an impulse response, we take another derivative and see that $\frac{dy_1}{dt} = \delta(t)$ will generate the output $\frac{dz_1}{dt} = \delta(t) - \delta(t-1)$. So $h_2(t) = \delta(t) - \delta(t-1)$. The following schematic summarizes our reasoning.

$$
\begin{array}{rcl}
y(t) = tu(t) & \rightsquigarrow & z(t) = t[u(t)-u(t-1)] + u(t-1) \\
& \downarrow & \\
\frac{dy}{dt} = u(t) = y_1(t) & \rightsquigarrow & z_1(t) = \frac{dz}{dt} = u(t) - u(t-1) \\
& \downarrow & \\
\frac{dy_1}{dt} = \delta(t) & \rightsquigarrow & \frac{dz_1}{dt} = \delta(t) - \delta(t-1)
\end{array}
$$

4. (a)

$$
\begin{aligned}
F(s) &= \int_{0-}^{\infty} e^{-st} u(t-1) e^{t} dt \\
&= \int_{1}^{\infty} e^{t(1-s)} dt \\
&= \frac{1}{1-s} \left[ e^{t(1-s)} \right]_1^{\infty} \\
&= \frac{1}{1-s} [0 - e^{1-s}] \quad (\text{if } Re[s] > 1) \\
&= \frac{e^{1-s}}{s-1}
\end{aligned}
$$

with DOC= $\{s | Re[s] > 1\}$.

(b)

$$
\begin{aligned}
F(s) &= \int_{0-}^{\infty} e^{-st} (u(t-a) - u(t-b)) dt \\
&= \int_{a}^{b} e^{-st} dt \\
&= \frac{1}{-s} \left[ e^{-st} \right]_a^{b} \\
&= \frac{e^{-as} - e^{-bs}}{s}
\end{aligned}
$$

with DOC=the whole complex plane.

**Note:** Superficially, it seems there is a pole at $s = 0$ which would limit the DOC. However, note that the numerator is also zero at that point.

(c)

$$
\begin{aligned}
F(s) &= \int_{0-}^{\infty} e^{-st} e^{t^3} u(t) dt \\
&= \int_{0-}^{\infty} e^{t(t^2 - s)} dt \\
&= \infty \text{ (no matter what } s \text{ is)}
\end{aligned}
$$

So the Laplace transform of $u(t)e^{t^3}$ is not defined for any $s$.