LECTURE 13

LECTURE NOTES: FEBRUARY 21, 2003

OUTLINE

REVIEW

- Inductor and Magnetic Induction
- Capacitor and Displacement Current
- Inductor and Capacitor Differential and Integral Voltage Relationships
- Power and Energy in Inductor and Capacitor Elements

HOMEWORK ASSIGNMENT 7

HOMEWORK DUE MARCH 3, 2003 BEFORE CLASS:

1. Problem 5.32: $i_a = 600\mu A$, $i_o = -760\mu A$, $v_x = -3.2V$
2. Problem 5.33: $v_{01} = 15.85V$, $v_{02} = 13.6V$
3. Problem 6.21 20H
4. Problem 6.24 $6\mu F$
5. Textbook Problem 7.5 : a) 0.2mA and 0.2mA, b) 0.2mA and –0.2mA, c) 0.2mAexp(-10^6t), d) -0.2mAexp(-10^6t)

6. PSPice Problem 3:
   a. Solve the Textbook Problem 5.40 parts a) and c) using PSpice. Provide hardcopy sheets for these two parts.
   b. For your Operational Amplifier, use one of the models below.
   c. Note that this operational amplifier contains an output resistance of Ro = 100 Ω and input resistance of R_{IN} = 400 kΩ
   d. Then use PSpice to determine the Thevenin equivalent output resistance. (Hint, review the Lecture notes on the operational current source). You may use a test voltage. Its value should be less than 15V. Why? Provide a hardcopy sheet showing current and voltage results that define the Thevenin Resistance and show a computation of the resistance value.

- We have two approaches for creating our non-ideal operational amplifier model. One with an "E" source, and one is implemented simply by the use of our ideal operational amplifier model. Both are shown below.

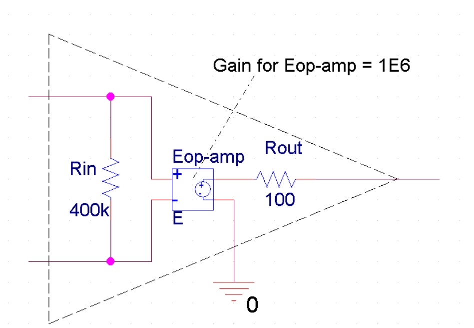


Figure 1. The Non-Ideal Operational Amplifier Model for PSpice Problem 3

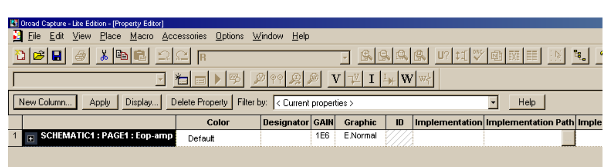


Figure 2. The Properties dialog box for the Eop-amp source. Please be sure to click on Apply before exiting

this dialog box.

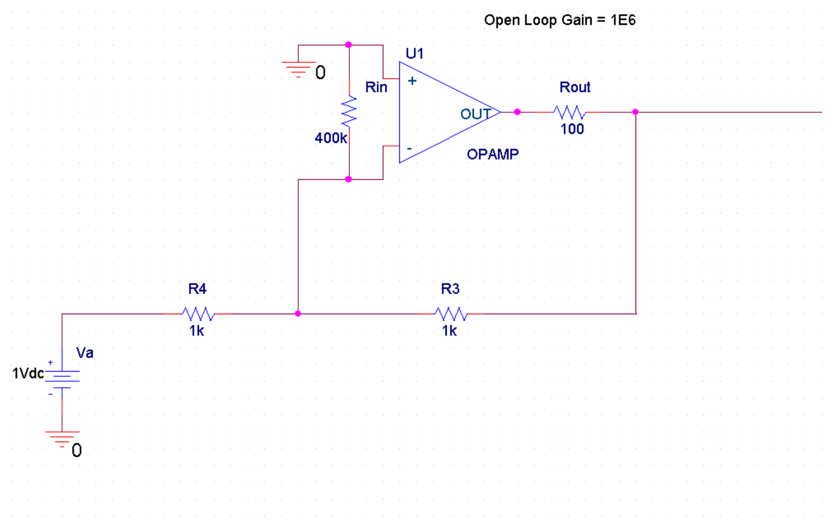


Figure 3. A circuit example where the operational amplifier is modified by the addition of the 100 ohm resistor, Rout, and 400k ohm resistor, R_{IN}

- Here is an example of the measurement of Thevenin Resistance for an operational amplifier circuit.

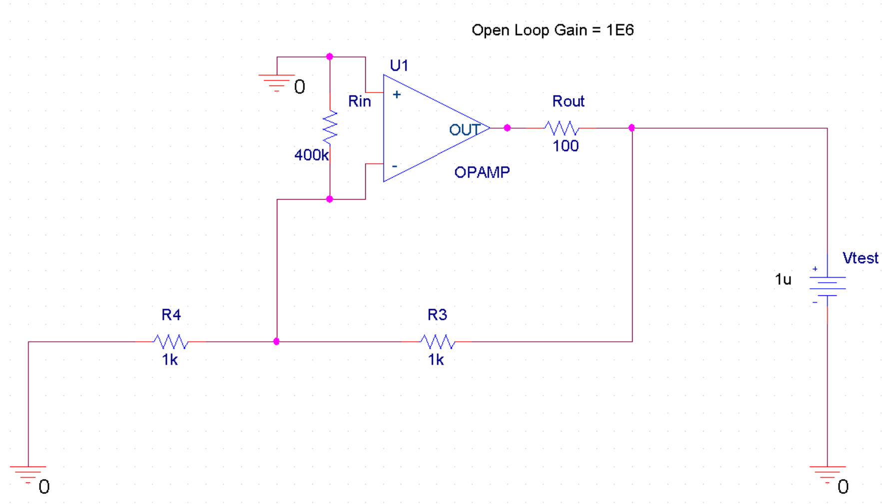


Figure 4. An example of a system used for computing output resistance. Note that V_{TEST} *is applied to the output. Its value is 1 microvolt (1u). (Recall from the PSpice tutorial that the unit for microvolt is lower case, "u") Now, we see that the Thevinin resistance is measured by replacing the independent source (*V_a*) by its zero element (a short) and providing* V_{TEST} *at the output. The current flowing is 5 mA with a 1 microvolt potential. There fore, the Thevenin resistance is* $2 \times 10^{-4}\,\Omega$

POWER DISSIPATION IN CAPACITOR CIRCUIT ELEMENTS

- Now, as for the inductor, the capacitor may be composed of conductors having essentially zero resistance. Thus, we would expect that for static voltage, that the inductor would support no voltage drop and the power dissipation in the capacitor, the current voltage product should always vanish.

- However, for time varying current and voltage, the electrtic field created by the capacitor is varying. Thus, the energy in the field is changing and this change in energy must be supplied by energy sources in circuits.

- The power dissipation in capacitors is via energy lost to fields. In contrast, the power dissipation in resistors is due to energy loss due to heating.

- Now, the we can compute power dissipation for the Capacitor using the Passive Sign Convention. Just as for the Resistor where we could compute power in terms of the resistance and voltage or current, here we can express power in terms of Capacitance, and current or voltage. We will encounter either a differential or integral expression.

$$p = iv = v \cdot C \frac{dv}{dt}$$

- Or,

$$p = iv = i \cdot \left(v(t_O) + \frac{1}{C} \int_{t_O}^{t} i(t')dt' \right)$$

ENERGY STORAGE IN CAPACITOR CIRCUIT ELEMENTS

- Now, when a voltage develops across a capacitor, and a field is created , a potential energy is associated with this field. The energy in this field may, in turn be absorbed by the capacitor and the collapse of this field induces a current.

- Frequently, it is useful to quote the energy, W(t), stored for a specified voltage. This is a useful quantity in the design of circuits associated with radio frequency, wireless systems, for example. We will also examine, in detail the specific case of power dissipation in digital circuits.

- We can compute this by integrating the inductor power dissipation over time since,

$$p \equiv \frac{dW}{dt}$$

- So, we will consider a capacitor system where we increase voltage from a zero value (where stored energy must be zero) to a finite value, I. Then we will compute energy at this value, V.

- First, we note that

$$p \equiv \frac{dW}{dt} = vC\frac{dv}{dt}$$

- So, we can multiply by differential time and get the differential equation:

$$dW = Cvdv$$

- Now, we can integrate from an operating point where $i = 0$ and $W=0$ to the value where $i = I$

$$\int_{W=0}^{W} dW' = 0 + C\int_{v'=0}^{v'=V} v'dv'$$

- And evaluating this is:

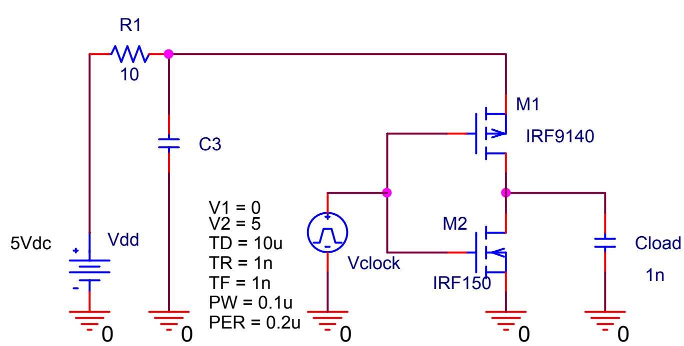
$$W = \frac{1}{2}CV^2$$

SPEED IN DIGITAL CIRCUITS

- Microprocessor speed has grown by a factor of over 1,000 in clock speed since their invention.
- Improvements in throughput are even greater with word lengths growing from 4 bits to 64 bits and with architectural improvements including on-board high speed cache, pipelining, vector processing, parallel execution, and branch prediction.
- But, what is the fundamental electrical engineering principle that limits clock speed – and fundamentally the rate at which instructions are executed?
- What also determines energy dissipation?
- We will address these issues next.

FUNDAMENTAL SWITCHING SPEED LIMITATIONS IN DIGITAL CIRCUITS

- At this point, we cannot examine the complete details of speed limitations for digital circuits, however, we can demonstrate primary problems.
- The circuit below is an example of one of the primary, primitive digital logic elements, the inverter.
- This circuit makes use of the EVAL MOSFET models included in PSpice 9.2. These are not typical of devices found in digital integrated circuits. However, they are completely applicable to this demonstration. You may compose your own circuits with these transistors.
- It accepts an input signal voltage where a positive voltage near Vdd implies a boolean TRUE or "1" state, and a voltage near ground, Vss implies a boolean FALSE or "0" state.
- Lets apply a signal to this system with PSpice and examine its response.



ENERGY DISSIPATION IN DIGITAL CIRCUITS

- Now, as we have seen, the energy stored in a capacitor is $\frac{1}{2}CV^2$ where V is the voltage drop across the capacitor terminals.
- Now, in a circuit like this inverter, for each cycle of the clock, we will alternately charge and discharge the capacitors in the circuit – to this energy where V is V_{DD}.
- The rate at which power flows will be $(\frac{1}{2}CV^2)f_{clock}$, where f_{clock} is the clock frequency.
- Lets check this behavior with PSpice. Is the average supply current proportional to f_{clock} ?
- Is the supply current proportional to V_{DD}^2
- What is maximum clock rate?
- How does this depend on Cload?
- How do we characterize this behavior in general?
- This is our next topic where we will begin studying R-C circuits.

INDUCTOR EQUIVALENT CIRCUITS

- As for resitor circuits, it is useful to replace inductor and capacitor circuit structures by simplified equivalents.
- Lets examine the series combination of inductors.

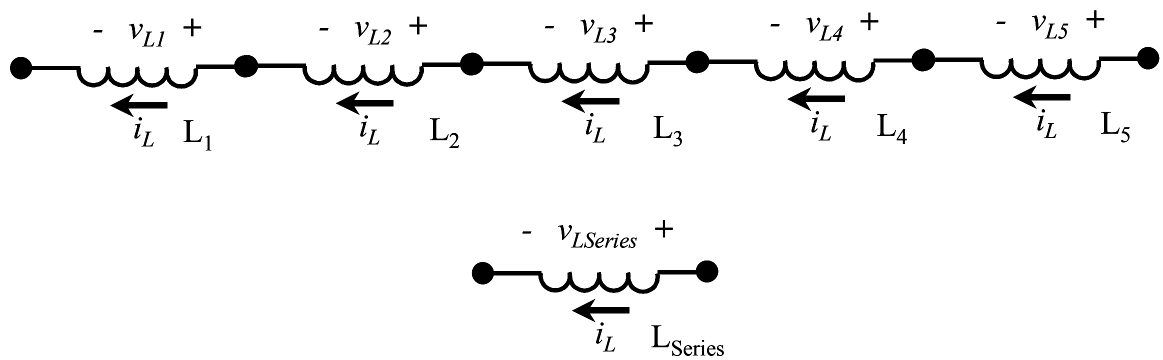


Figure 5. A series arrangement of inductors and their equivalent

- Now, if we examine the currents flowing in this series combination, we see that according to KCL,

$$i(t) = i_{L1}(t) = i_{L2}(t) = i_{L3}(t) = \ldots = i_{LN}(t)$$

- Also, the voltage across the series is just the sum of the individual voltage drops.

$$v(t) = v_{L1}(t) + v_{L2}(t) + v_{L3}(t) + \ldots + v_{LN}(t)$$

- We also see that the inductor voltages may be related to the currents as

$$v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + L_3 \frac{di(t)}{dt} + \ldots + L_N \frac{di(t)}{dt}$$

- And now, we can write the voltage across the series combination as

$$v(t) = \left(L_1 + L_2 + L_3 + \ldots + L_N\right) \frac{di(t)}{dt}$$

- Thus, the Equivalent Series Inductance is just

$$\boxed{L_{Series} = \sum_{n=1}^{N} L_n}$$

- Note here that the the largest inductance in the series may dominate the characteristics of the series equivalent.

- Lets further examine equivalent circuits, now with a parallel structure, as shown below.

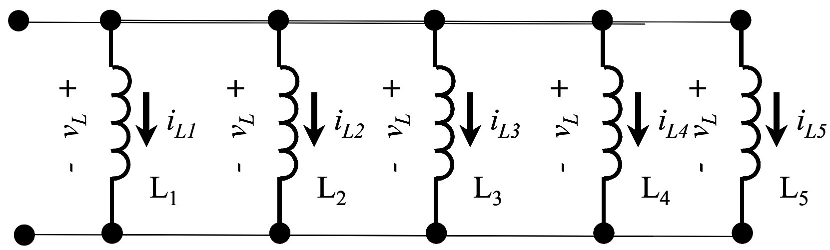


Figure 6. A Parallel combination of inductors

- Now, for this parallel structure, all inductors share the same node pair. So, all of their voltage drops are equal. However, their current values vary.

- We may write down the total current for a given applied voltage, using the expression for a single inductor

$$i(t) = i(t_O) + \frac{1}{L}\int_{t_O}^{t} v(t')dt'$$

- So,

$$i(t) = \sum_{n=1}^{N} i_n(t_o) + \sum_{n=1}^{N} \frac{1}{L_n}\int_{\tau=t_O}^{t} v(\tau)d\tau$$

- But,

$$i(t_O) = \sum_{n=1}^{N} i_n(t_o)$$

- So, we can rewrite the current expression now and define an equivalent inductance – then solve for this.

$$i(t) = i(t_o) + \sum_{n=1}^{N} \frac{1}{L_n}\int_{\tau=t_O}^{t} v(\tau)d\tau = i(t_o) + \frac{1}{L_{Parallel}}\int_{\tau=t_O}^{t} v(\tau)d\tau$$

- So, the equivalent inductance for Parallel Structures is

$$\boxed{\frac{1}{L_{Parallel}} = \sum_{n=1}^{N} \frac{1}{L_n}}$$

- Now, it is very important to note here that the parallel inductance is dominated by the least inductance value in the sum.

CAPACITOR EQUIVALENT CIRCUITS

- Now, we must also consider Capacitor equivalent circuits. Additional important intuition arises here.

- First, lets consider a parallel capacitor circuit.

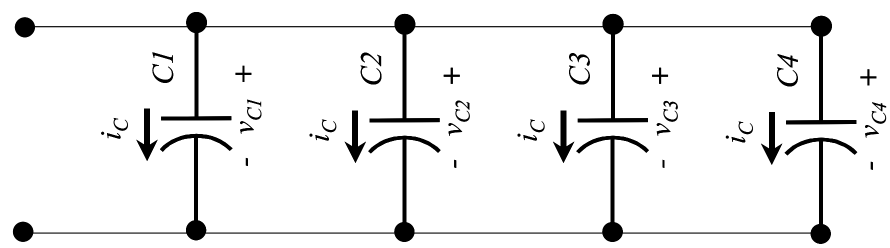


Figure 7. Parallel Capacitance strutcure

- As for the parallel inductor circuit, all voltages are equivalent and the total parallel structure current is the sum of the individual currents.

$$i(t) = i_{C1}(t) + i_{C2}(t) + i_{C3}(t) + \ldots + i_{CN}(t)$$

- But,

$$i_C = C\frac{dv_C}{dt}$$

- So,

$$i(t) = C_1\frac{dv_{C1}(t)}{dt} + C_2\frac{dv_{C2}(t)}{dt} + C_3\frac{dv_{C3}(t)}{dt} + \ldots + C_N\frac{dv_{CN}(t)}{dt}$$

- And, we can define

$$i(t) = (C_1 + C_2 + C_3 + \ldots + C_N)\frac{dv(t)}{dt} = C_{Parallel}\frac{dv(t)}{dt}$$

- So, we have an equivalent capacitance:

$$\boxed{C_{Parallel} = \sum_{n=1}^{N} C_n}$$

- Now, it is important to note here that the parallel capacitance is dominated by the greatest capacitance value in the sum.

- Finally, we must also consider the series capacitance structure.

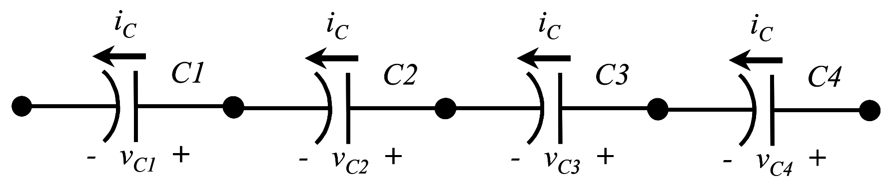


Figure 8. Series Capacitor Structure

- Now, the capacitor element currents are equal and the total series voltage is the sum of individual element voltage drops.

$$i(t) = i_{C1}(t) = i_{C2}(t) = i_{C3}(t) = ... = i_{CN}(t)$$

- And

$$v(t) = v_{C1}(t) + v_{C2}(t) + v_{C3}(t) + ... + v_{CN}(t)$$

- However, the individual element voltages are defined:

$$v(t) = v(t_O) + \frac{1}{C}\int_{\tau=t_O}^{t} i(\tau)d\tau$$

- So, the series sum will be

$$v(t) = \sum_{n=1}^{N}\left[v_n(t_O) + \frac{1}{C_n}\int_{\tau=t_O}^{t} i_n(\tau)d\tau\right]$$

- Also, lets consider $v(t_o)$. This will be described by the sum of the capacitor potentials evaluated at $t = t_O$.

- Also, we can introduce a Series Equivalent Capacitance and solve for this.

$$v(t) = \sum_{n=1}^{N} v_n(t_O) + \left[\sum_{n=1}^{N} \frac{1}{C_n}\right] \int_{\tau=t_O}^{t} i(\tau)d\tau = v(t_O) + \frac{1}{C_{Series}} \int_{\tau=t_O}^{t} i_n(\tau)d\tau$$

- And the Series Equivalent Capacitance is

$$\frac{1}{C_{Series}} = \sum_{n=1}^{N} \frac{1}{C_n}$$

- Now, it is very important to note here that the series capacitance is dominated by the least capacitance value in the sum.

RESPONSE OF R-C AND R-L CIRCUITS

- The response of circuit systems is often dominated by the effects of R-C and R-L networks.

- These account for the speed limitations of integrated circuit components in both the analog and digital domain as well as other large scale circuit systems.

- An example of this phenomena can be seen in the response of our example transistor circuit, above. The Figure below shows its complex time response.

- Lets discuss its significant limitations.

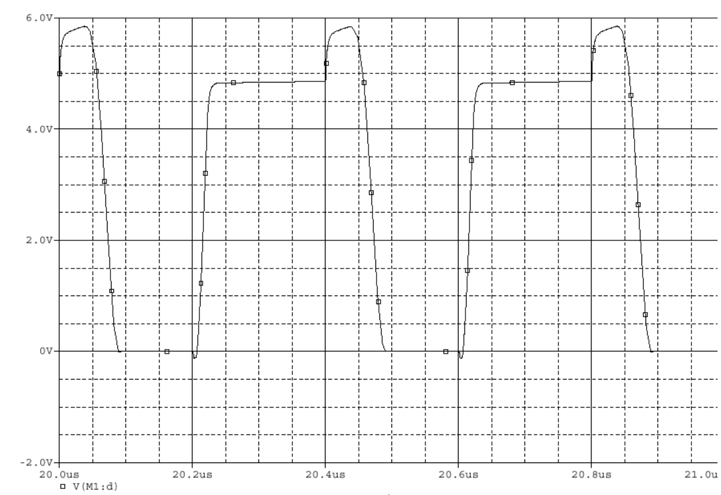


Figure 9. Simulation output for our Inverter circuit above.

- We will be investigating both the *Natural* and *Forced* response of R-C and R-L circuits. This will apply directly to the most important circuit performance issues.

NATURAL RESPONSE OF R-L CIRCUITS

- The natural response of a circuit is the behavior of a circuit occuring after it has first stabilized under the action of a source of energy and then after this source is suddenly removed.

- First, we will consider a circuit including a source equivalent, and an inductor and resistor. The response of this circuit will prepare us for understanding complex time-dependent R-L network systems.

- Consider the circuit below:

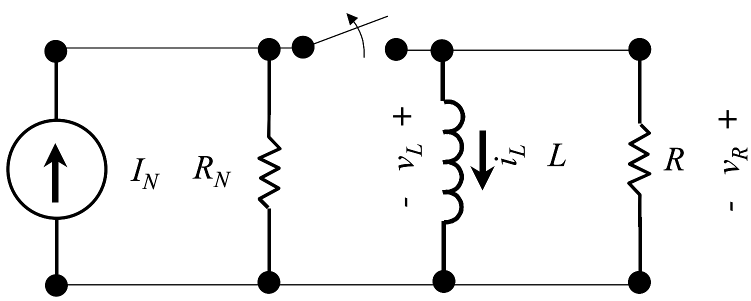


Figure 10. A circuit used for characterizing R-L system Natural Response

- This circuit contains a switch which will open at t = t_O.

- We will analyze the time response of this circuit (and others) after this time, t_O

- First, lets consider the inductor voltage and current before the switch is opened.

$$v_L(t < t_O) = 0$$

- Now, by definition

$$R_L = 0$$

- Now, current will be continuous at the time the switch opens.

$$i_L(t = t_O - t_\varepsilon) = i_L(t = t_O + t_\varepsilon) = i_L(t_O)$$

- We must carefully note polarities and definitions

- First,

$$v_R = i_R R$$

- And

$$v_L = L\frac{di_L}{dt}$$

- Note that the Passive Sign Convention applies to this definition.
- Completing a KVL sum around this loop, we have

$$v_L - v_R = 0$$

- Also, KCL at the node joining the switch, resistor, and inductor yields

$$i_L + i_R = 0$$

- So, we can now write

$$v_L = L\frac{di_L}{dt} = -i_L R$$

- Rearranging

$$\frac{di_L}{dt} = -\left(\frac{R}{L}\right)i_L$$

- We can manipulate this to develop the differential equation

$$\frac{di_L}{i_L} = -\left(\frac{R}{L}\right)dt$$

- Now, we can integrate to solve for current response

$$\int_{i'_L=i_L(t=t_O)}^{i_L(t)} \frac{di'_L}{i'_L} = -\left(\frac{R}{L}\right)\int_{\tau=t_O}^{t} d\tau$$

- And,

$$\ln\left(i_L(t)\right) - \ln\left(i_L(t_O)\right) = -\left(\frac{R}{L}\right)(t - t_O)$$

- Also, this is

$$\ln\left[\frac{i_L(t)}{i_L(t_O)}\right] = -\left(\frac{R}{L}\right)(t - t_O)$$

- Of couse, by definition, $\alpha = \exp(\ln(\alpha))$ and,

$$i_L(t) = i_L(t_O)e^{-\left(\frac{R}{L}\right)(t - t_O)}$$

- Now, lets consider the case where $t_O = 0$. Then, our time response for all time after the switch is thrown is,

$$i_L(t) = i_L(0)e^{-\left(\frac{R}{L}\right)t}$$

- Lets also compute voltage across the inductor:

$$v_L = L\frac{di_L(t)}{dt} = -Ri_L(0)e^{-\left(\frac{R}{L}\right)t}$$

- Note, as we stated before, $v_L - v_R = 0$

- So, voltage drop across resistor is

$$v_R = v_L = Ri_L(0)e^{-\left(\frac{R}{L}\right)t}$$

- And,

$$v_L(t < 0) = 0$$

- Further,

$$v_L(t = 0 + t_\varepsilon) = -i_L(0)R$$

- It is very important to note that inductor output voltage rises instantaneously at the point that the switch opens, whereas current remains at its initial value.

- Also, lets rewrite the current expression:

$$i_L(t) = i_L(0)e^{-\left(\frac{t}{\tau}\right)}$$

- We may define a Time Constant value, $\tau = L/R$

- Note that L/R will determine the properties of the Natural Response, as we shall see below.

- We know that for a time passage equal to one time constant, the inductor current will fall below its initial value by $1/e \approx 0.37$

- Lets examine this characteristic behavior with PSpice.

- Lets examine:

    - Current and Voltage Transients

    - Current and Voltage Natural Response

    - Dependence of response on circuit parameters

- Now, lets consider the inductor power dissipation here.

$$p = i_L v_L = i_L(0)e^{-\left(\frac{R}{L}\right)t}\left[-Ri_L(0)e^{-\left(\frac{R}{L}\right)t}\right] = -Ri_L{}^2(0)e^{-2\left(\frac{R}{L}\right)t}$$

- Note this is a negative definite quantity for all times. This means the inductor is delivering power.

- The power dissipated in the resistor is

$$p = \frac{v_R^2}{R} = \frac{R^2 i_L{}^2(0)e^{-2\left(\frac{R}{L}\right)t}}{R} = Ri_L{}^2(0)e^{-2\left(\frac{R}{L}\right)t}$$

- Note that this equal in magnitude and opposite to inductor power – energy is being delivered from inductor to resistor.

- Lets also examine power dissipation with PSpice

- Finally, lets also examine energy dissipation.
- First, initial energy is a result of the inductor current flow.

$$W(t=0) = \frac{1}{2} L i^2(0)$$

- And energy at all times is

$$W_L(t) = \int_{\tau=0}^{t} p(\tau)d\tau = -R\int_{\tau=0}^{t} i_L^2(0) e^{-2\left(\frac{R}{L}\right)\tau}$$

- Integrating, we find

$$W_L(t) = \frac{-1}{-2\left(\frac{R}{L}\right)} i_L^2(0)\, \mathrm{R} e^{-2\left(\frac{R}{L}\right)\tau} \Big|_o^t$$

- And

$$W_L(t) = \frac{L i_L^2(0)}{2}\left(e^{-2\left(\frac{R}{L}\right)t} - 1\right)$$

- Now, we can consider the limit where t $\rightarrow \infty$
- Then, the net energy absorbed by inductor is (please note error in previous notes version – "delivered" was written instead of "absorbed".)

$$W_L(t) = -\frac{L i_L^2(0)}{2}$$

- This is exactly equal in magnitude to the energy supplied to the inductor at t <0.

NATURAL RESPONSE OF R-C CIRCUITS

- Now, we will consider contrasting behavior of Resistor–Capacitor circuits, as for the circuit below.

- Again, we have a switch that opens at t = t_O.

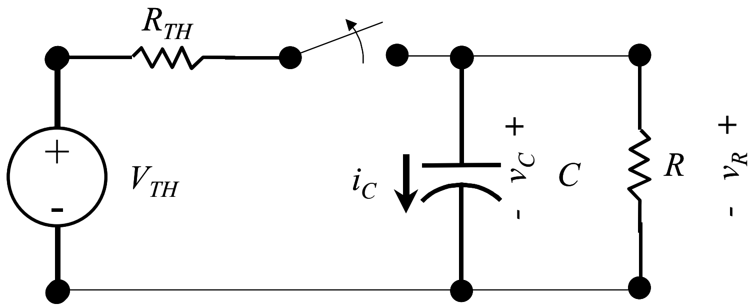


Figure 11. A circuit used for characterizing R-C system Natural Response

- Here we note that we may consider that any circuit may be implemented as a Thevenin equivalent, so this structure above is applicable to the analog and digital electronic systems we have just discussed.

- Lets begin analyzing this circuit by examining the circuit values prior to the time when our switch is opened.

$$v_C = v_{TH} \frac{R_{TH}}{R + R_{TH}} = V_R$$

$$i_R = \frac{v_R}{R}$$

- Now, lets open the switch.

- We can apply KCL at the node joining the switch, resistor, and capacitor. We see

$$i_C = -i_R$$

- But, by definition

$$i_C = C\frac{dv_C}{dt} = -i_R = -\frac{v_C}{R}$$

- Note that the Passive Sign Convention applies to the capacitor current expression above.

- Rearranging this:

$$\frac{dv_C}{dt} = -\frac{1}{RC}v_C$$

- And manipulating this to obtain a desired form for the differential equation. We have.

$$\frac{dv_C}{v_C} = -\frac{1}{RC}dt$$

- As for the previous problem, we may integrate to obtain

$$\int_{v'=v_C(t_O)}^{v_C(t)} dv' \frac{1}{v'} = -\int_{\tau=t_O}^{t} \frac{1}{RC} d\tau$$

$$\ln(v_C(t)) - \ln(v_C(t_O)) = -\frac{1}{RC}(t - t_O)$$

$$\ln\left(\frac{v_C(t)}{v_C(t_O)}\right) = -\frac{1}{RC}(t - t_O)$$

- Finally, we have the capacitor voltage.

$$v_C(t) = v_C(t_O)e^{-\frac{1}{RC}(t-t_O)}$$

- And, lets consider the special case of

$$t_o = 0$$

- Then,

$$v_C(t) = v_C(0)e^{-\frac{t}{RC}} = v_C(0)e^{-\frac{t}{\tau}}$$

- Note that another Time Constant appears, τ = RC
- Also, we must solve for capacitor voltage as well.
- This is

$$i_C(t) = C\frac{dv_C(t)}{dt} = -\frac{v_C(t)}{R}e^{-\frac{t}{RC}}$$

- Lets again examine the full set of behaviors with PSpice including:
  - Time Constant value
  - Transient Properties

        - Note that current changes instantaneously, whereas voltage remains fixed through the switch opening.

    - Natural Response

    - Lets also examine the role of the capacitor in power supply filter applications in digital systems.

- Lets also examine power and energy in the Natural Response of this system.

- Power for the capacitor is

$$p = v_C(t) i_C(t) = -\frac{{v_C}^2(t)}{R} e^{-2\frac{t}{RC}}$$

- Initial energy for the capacitor is

$$W(t=0) = \frac{1}{2} C v^2(0)$$

- And, after the switch opens,

$$W_C(t) = \int_{\tau=0}^{t} p(\tau) d\tau = -\frac{1}{R} \int_{\tau=0}^{t} v_C^2(\tau) e^{-2\left(\frac{\tau}{RC}\right)}$$

$$W_C(t) = \frac{RC}{2R} v_C^2(0) e^{-2\left(\frac{\tau}{RC}\right)} \Big|_0^t$$

$$W(t) = \frac{1}{2} C v_C^2(0) \left(e^{-2\left(\frac{t}{RC}\right)} - 1 \right) = -\frac{1}{2} C v_C^2(0) \left(1 - e^{-2\left(\frac{t}{RC}\right)} \right)$$

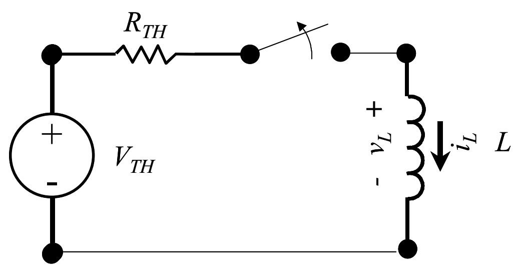
- Note that in the limit of infinite time, the total energy delivered to the resistor is equal to the energy initially supplied to the capacitor.

STEP RESPONSE OF R-L CIRCUITS

- We have examined the operation of circuits for systems in which energy is initially stored. Now, we must examine circuits which are initially operating without stored energy and then are suddenly subject to the action of a source. This is the circuit step response.

- Again, this understanding is directly applicable and required for understanding the digital system time response we have discussed.

- Our circuit is shown below:



- First, this circuit will have been operating with the switch open for a long period. The inductor current will be zero. Then, at t = 0, the switch in this circuit will close.

- So, for $t < 0$,

$$v_L = 0 \text{ and } i_L = 0$$

- However, at $t = 0 + t_\varepsilon$, we now may compute response.

- We will use $v_S = V_{TH}$

- We may apply KVL at the node at the switch and write

$$v_S - i_L R_{TH} - L\frac{di_L}{dt} = 0$$

- or

$$\frac{di_L}{dt} = \frac{v_S - i_L R_{TH}}{L} = -\frac{R}{L}\left(i_L - \frac{v_S}{R}\right)$$

- And this can be manipulated

$$\frac{di_L}{\left(i_L - \frac{v_S}{R}\right)} = -\frac{R}{L}dt$$

- Integrating from $i = 0$ to $i = i_L$ and from $\tau = 0$ to $\tau = t$

$$\int_{i'=0}^{i'=i_L(t)} \frac{di'}{\left(i' - \frac{v_S}{R}\right)} = -\int_o^t \frac{R}{L} d\tau$$

$$\ln\left(i_L(t) - \frac{v_S}{R}\right) - \ln\left(0 - \frac{v_S}{R}\right) = -\left(\frac{R}{L}\right)(t - t_O)$$

$$\ln\left(\frac{i_L(t) - \frac{v_S}{R}}{\frac{v_S}{R}}\right) = -\left(\frac{R}{L}\right)(t - t_O)$$

- And

$$i_L(t) = \frac{v_S}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)$$

- Finally, we again see the appearance of the time constant with

$$i_L(t) = \frac{v_S}{R}\left(1 - e^{-\left(\frac{t}{\tau}\right)}\right)$$

- And, = L/R
- We can examine this behavior
- Often we are particularly interested in the maximum time rate of change of current. This is
- at t = 0, $i_L = 0$
- but, the rate of change of i_L is

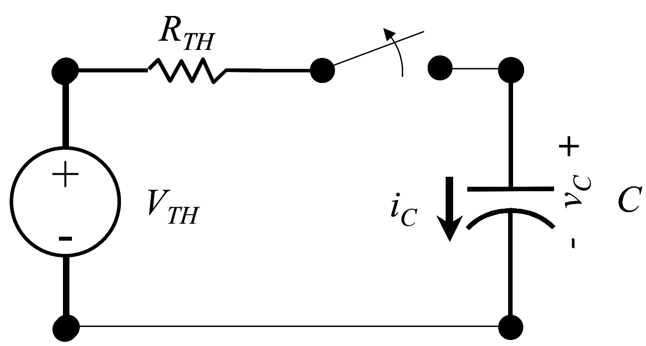
$$\frac{di_L(t)}{dt} = \frac{v_S}{L}$$

- And, we should note that at a time of t = τ = L/R, i(t) ≈ 0.63v_S/R

STEP RESPONSE OF R-C CIRCUITS

- The case of the Step Response of R-C circuits is also applicable to our digital system. Lets examine this first.

- Our circuit is shown below:



- First, this circuit will have been operating with the switch open for a long period. The inductor current will be zero. Then, at t = 0, the switch in this circuit will close.

- So, for $t < 0,$

$$v_C = 0 \text{ and } i_C = 0$$

- But at $t = 0 + t_\varepsilon$, our circuit responds to the step stimulus.

- Now we will use KVL for this loop and obtain

$$v_S - i_C R - v_C = 0$$

- Now, by definition.

$$i_C = C\frac{dv_C}{dt}$$

- So

$$v_S - RC\frac{dv_C}{dt} - v_C = 0$$

- Manipulating

$$\frac{dv_C(t)}{dt} = \frac{1}{RC}(v_s - v_c)$$

- And manipulating our differential equation

$$\frac{dv_C}{(v_C - v_S)} = -\frac{1}{RC}dt$$

- We may now integrate from $i = 0$ to $i = i_L$ and from $\tau = 0$ to $\tau = t$

$$\int_{v'=0}^{v'=v_C(t)} \frac{dv'}{(v' - v_S)} = -\int_{o}^{t} \frac{1}{RC} d\tau$$

- And, we use that

$$v_C(0) = 0$$

- So,

$$\ln(v_C(t) - v_S) - \ln(0 - v_S) = -\left(\frac{1}{RC}\right)t$$

- And

$$\ln\left(\frac{v_C(t) - v_S}{-v_S}\right) = -\left(\frac{1}{RC}\right)t$$

- Finally,

$$v_C(t) = v_S\left(1 - e^{-\left(\frac{t}{RC}\right)}\right)$$

- Again with a time constant appearing

$$v_C(t) = v_S\left(1 - e^{-\left(\frac{t}{\tau}\right)}\right)$$

- With

$$\tau = RC$$

- We can examine this behavior
- At t = 0, $v_C = 0$, but, the rate of change of v_C is

$$\frac{dv_C(t)}{dt} = \frac{v_S}{RC}$$

- We can examine this behavior in PSpice in detail.