SPECIAL ANNOUNCEMENTS

1. **Midterm Exam**
    a. Midterm Date Planned: Monday February 10
    b. Exam details as noted on the Web site and in First lecture.
    c. Material brought to exams may include one 81/2" x 11" sheet with notes, both sides
    d. You may bring a calculator to the exam
    e. Bring Engineering and Science exam books to Midterm and Final
2. **Graded Homework Assignment Distribution:**
    a. Graded Homework is placed on a table outside of 56-147L Engineering IV
3. There are separate folders with homework sorted alphabetically.
4. This class – review of Problem 4.6 at end of lecture.
5. Please Check Web Site for Tutorials on Problem Solving and Specific Problems

LECTURE NOTES: JANUARY 29, 2002

OUTLINE

REVIEW

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1. Principle of Superposition
2. Source Equivalents
3. Thevenin and Norton Equivalent Circuits
4. Computation of a Thevenin Equivalent for Circuits with Independent Souces

FOURTH HOMEWORK ASSIGNMENT DUE WEDNESDAY, FEBRUARY 5, 2003

These problems cover both our new skills from Mesh Current analysis as well as Review Material for the Midterm.

Homework Problems:

3.53 $R_{ab} = 90\Omega$

4.12 $P_{dissipated} = 306W$

4.16 $i_1 = 1mA$ $i_2 = -20mA$ $i_3 = 31mA$

4.24 $P_{dev} = 165W$

4.36 $i_a = 5.7A$, $i_b = 4.6A$, $i_c = 0.97A$, $i_d = -1.1A$, $i_e = 3.63A$

4.40 $P_{dev} = 1484W$

Optional Study and Review Problems:

2.27 $i_\Delta = 2A$, $v_o = 2V$, Total Power Absorbed = 356W

3.48 $v_1 = 23.2V$, $v_2 = 21V$ (this problem is complex, but, good practice)

4.23 Solution included in Tutorial section (This is a very good review problem – note that you can review the complete solution in the Tutorial section of the Web site.)

4.39 $v_o = 8V$, Absorbed power, $P_{4i\Delta} = 1W$ (this is a good review problem)

4.41 Solution included in Tutorial section (This is a very good review problem – note that you can review the complete solution in the Tutorial section of the Web site.)

SPECIAL REVIEW OF PROBLEM 4.6

We will solve this problem via two approaches now. The solution will also be posted.

REVIEW OF PROCEDURES FOR COMPUTING SOURCE EQUIVALENTS

PROCEDURES FOR THEVENIN EQUIVALENT CIRCUITS

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance R_{AB}. This resistance will equal R_{TH}

4) Return all Independent Sources to their original values and compute the voltage value corresponding to the voltage drop from A to B, V_{AB} (with Nodes A and B open-circuited). This voltage, $V_{AB} = V_{Th}$

5) You may compute this voltage using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

6) Draw the new Thevenin Equivalent Circuit.

PROCEDURES FOR NORTON EQUIVALENT CIRCUITS

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance R_{AB}. This resistance will equal R_N

4) Return all Independent Sources to their original values and then apply a short circuit at the terminals A and B. Compute the current value from A to B, I_{AB} (with Nodes A and B short-circuited). This current is the Norton Equivalent Current, $I_{AB} = I_N$

5) You may compute this current using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

6) Draw the new Norton Equivalent Circuit using this I_N and $R_{N.}$ Note polarities for I_N.

NORTON EQUIVALENT CIRCUITS

- The dual of the Thevenin Source Equivalent is the Norton Source Equivalent.

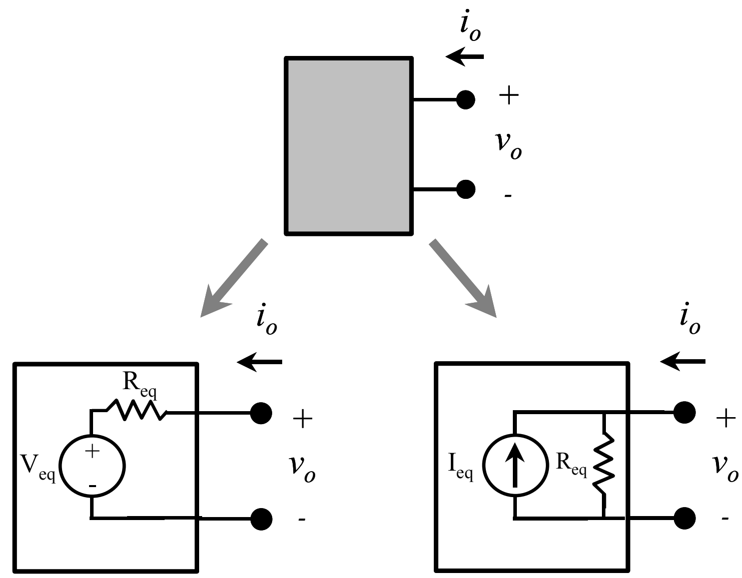


Figure 1. Replacement of a Linear Circuit System with either a Thevenin Source Equivalent or Norton Source Equivalent Circuit containing an Equivalent Resistor. Note that R_{EQ} *is the same for both circuits.*

- Lets compute a Thevinin and Norton Source Equivalent for a circuit.

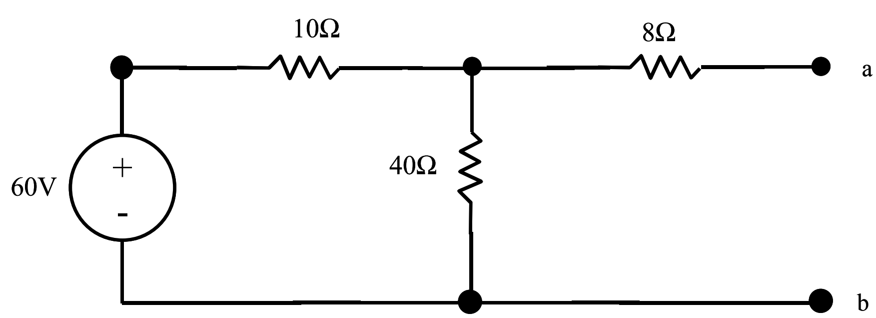


Figure 2. The Example Circuit

- Here is the procedure:

PROCEDURES FOR THEVENIN EQUIVALENT CIRCUITS

1)	Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.
2)	Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.
3)	Compute the resistance R_{AB}. This resistance will equal R_{TH}
4)	Return all Independent Sources to their original values and compute the voltage value corresponding to the voltage drop from A to B, V_{AB} (with Nodes A and B open-circuited). This voltage, $V_{AB} = V_{Th}$
5)	You may compute this voltage using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.
6)	Draw the new Thevenin Equivalent Circuit.

- Lets follow this. First, we see that the terminals are labeled.
- Lets remove the voltage source and replace it with its "zero" element – a short circuit.
- Then, lets compute R_{AB}.
- This is simple, R_{AB} will be a resistance consisting of an 8Ω resistor in series with a parallel combination of a 10Ω and 40Ω resistor. This parallel combination is itself, 8Ω, so $R_{AB} = R_{TH} = 16\Omega$
- Now, lets compute $V_{TH.}$
- We return the voltage source to its original value, and compute the voltage at the terminals, a and b.
- Here, we can just use the Voltage Divider equation. Why?
- Now, $V_{AB} = V_{TH} = 48V$. Thus, our equivalent circuit is:

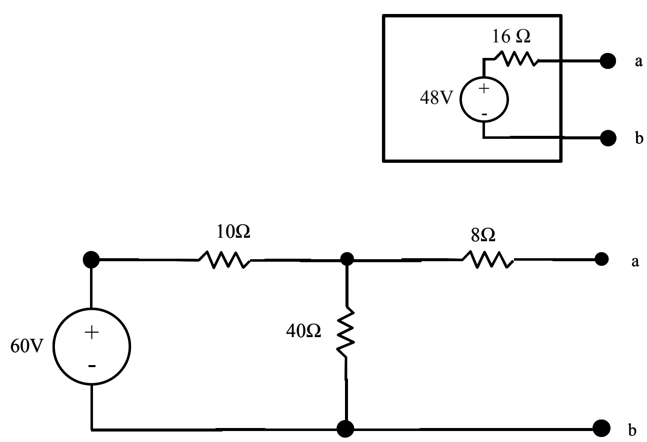


Figure 3. The Example Circuit and its Thevenin Equivalent

- Now, lets compute the Norton Equivalent

PROCEDURES FOR NORTON EQUIVALENT CIRCUITS

1)	Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.
2)	Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.
3)	Compute the resistance R_{AB}. This resistance will equal R_N
4)	Return all Independent Sources to their original values and then apply a short circuit at the terminals A and B. Compute the current value from A to B, I_{AB} (with Nodes A and B short-circuited). This current is the Norton Equivalent Current, $I_{AB} = I_N$
5)	You may compute this current using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.
6)	Draw the new Norton Equivalent Circuit using this I_N and R_N. Note polarities for I_N.

- Lets follow this. First, the terminals are labeled.
- We remove the voltage source and replace it with its "zero" element – a short circuit.
- Then, we compute R_{AB}.
- We just found this to be $R_{AB} = R_N = 16\Omega$
- Now, lets compute I_N.
- We return the voltage source to its original value, and compute the short circuit current value at the terminals, a and b. Lets examine the circuit that results from this:

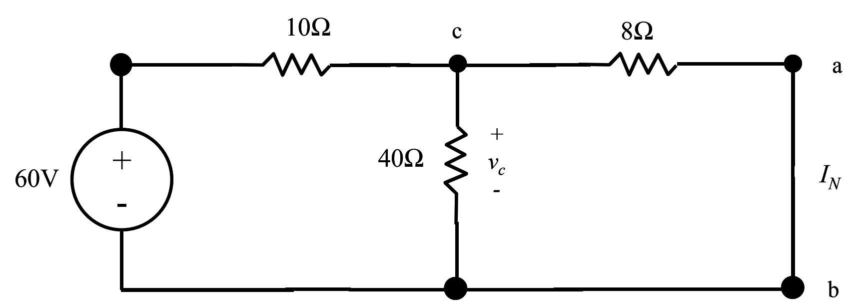


Figure 4. Circuit construction for computing the Norton Equivalent Current

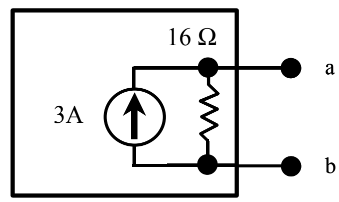
- Lets solve for the current flowing through the 8Ω resistor. This is a familiar problem. Lets just compute the voltage drop across the 8Ω resistor. This is just equal to v_c.
- First, we can compute the voltage at the node joining the 40Ω and 8Ω resistors. We start by replacing them with their parallel equivalent. This parallel equivalent is just

6.66Ω

- Now, then we can use the Voltage Divider Equation again. The voltage, v_c ,from this is,

$$v_c = 60V(6.66\Omega/16.66\Omega) = 24V$$

- Then, the current through the 8Ω resistor is just 24V/8Ω = 3A
- Thus, I_N = 3A. Thus, our equivalent circuit is:



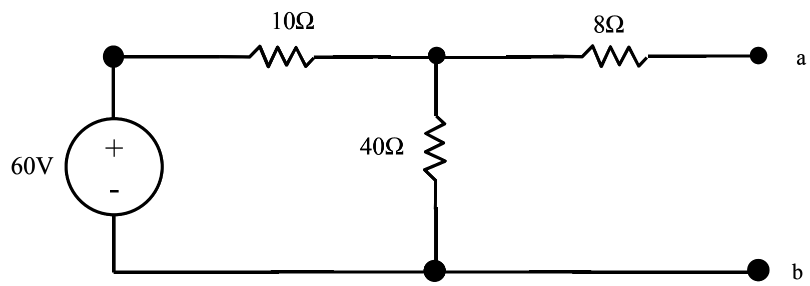


Figure 5. The Norton Source Equivalent for our Example Circuit

- Lets check this result and our understanding.
- First, these two circuits should show the same open circuit output voltage:
  - The Thevenin Source Equivalent output voltage is 48V – this is easily seen.
  - The Norton Source Equivalent is also 48V, just the voltage drop due to the 3A current flow through the 16Ω resistor.
- Also, these circuits should show the same short circuit output current.
  - The Thevenin Source Equivalent short circuit current is 3A
  - The Norton Source Equivalent short circuit current is 3A
- These three circuits, the original circuit and the two Thevenin and Norton Equivalents all show exactly the same terminal characteristics.

SOURCE EQUIVALENT CIRCUITS WITH DEPENDENT SOURCES

- In the development of amplifier circuits, we will frequently encounter the need for developing Source Equivalent circuits involving dependent sources.
- All of our techniques will apply.
- However, we must introduce one new method for computing resistance. This will be useful to you later, for example in EE115B
- Here is our circuit (a textbook Drill Exercise):

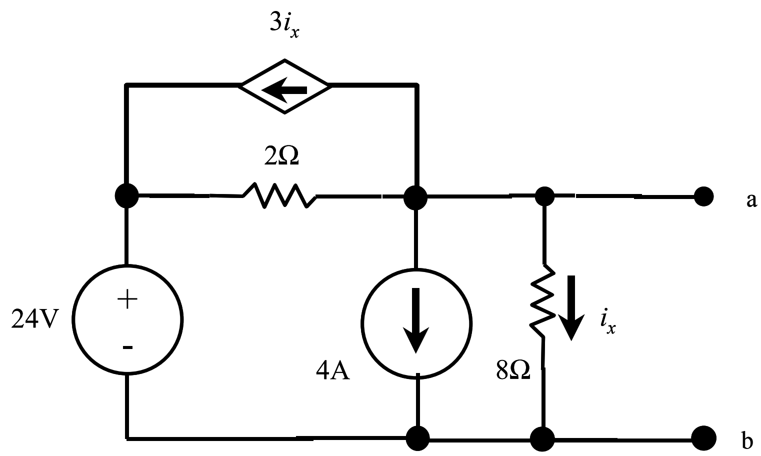


Figure 6. An Example Circuit for computation of Source Equivalents

- First, lets use the general procedures to compute the Thevenin Source Equivalent.
- We will begin by computing V_{TH}
- Lets use the Node Voltage method with a Reference Node where the two Independent Sources and the 8Ω resistor meet.
- Now, note that there is only one Node Equation required. Why?
- At terminal a, an Essential Node, we have the KCL equation:

$$(24V - v_a)/2 + (-3i_x) - (4A) - v_a/8 = 0$$

- Also, the Dependent Source constraint equation is:

$$i_x = v_a/8$$

- This is now easy to solve. Substituting for i_x.

$$(24V - v_a)/2 + (-3\,v_a/8) - (4A) - v_a/8 = 0$$

- and

$$v_a = v_{TH} = 8V$$

- Now, we must compute R_{AB}
- There are two approaches:
  - 1)
    - One may construct a method where we short circuit the output terminals and compute current in the short circuit, i_{SC}
    - Then, the resistance, R_{AB}, is just v_{TH}/i_{SC}
    - Lets return to our definitions of the Thevenin Equivalent to see this.
  - 2)
    - An alternative (and our recommend) approach for problems we will encounter, is to compute R_{AB} in a fundamental method:
      - Replace all Independent Sources with their "zero" elements. Note, Dependent Sources remain.
      - Apply a test voltage, V_T to the output terminals
      - This will yield a test current, I_T
      - Compute $R_{AB} = V_T/I_T$
      - This is a powerful method that will never let you down.
- Let us proceed with the second case
- Let us replace the all Independent Sources with their "zero" elements. Also, lets apply the voltage, V_T.

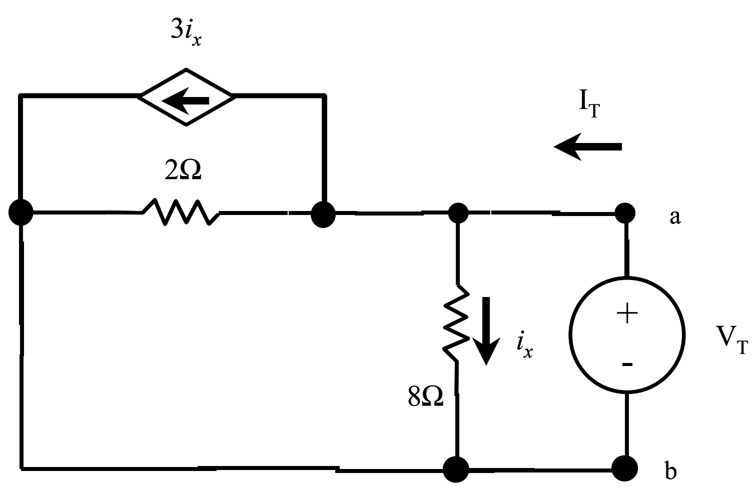


Figure 7. Our Example Cicuit placed in a form required to compute R_{AB}.

- Let us again use a Node Voltage method to compute I_T.
- Again., we will place our Reference at the lower node on this diagram where the 8Ω, 2Ω, and the two sources meet.
- Then, the Node Voltage Equation is:

$$(-V_T)/2\Omega + (-3i_x) - V_T/8\Omega + I_T = 0$$

- but,

$$i_x = V_T/8$$

- So, our equation becomes:

$$(-V_T)/2\Omega + (-3V_T/8\Omega) - V_T/8\Omega + I_T = 0$$

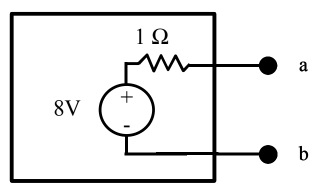
- Rearranging,

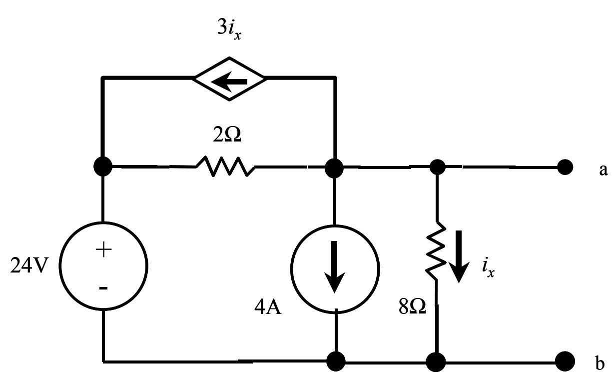
$$-V_T/1\Omega + I_T = 0$$

- But, by definition:

$$R_{AB} = V_T/I_T = 1\Omega$$

- Thus, our Thevenin Source Equivalent circuit is





- Point for discussion: note that the Thevenin resistance is a very low value. Why? This has application in low input impedance amplifiers.

APPLICATIONS OF EQUIVALENT CIRCUITS

- Frequently, in design or design analysis, we must meet a design specification for a value for "input" or "output" resistance (or impedance) for our circuit system.

- This results from the need to enable specifications for *interfaces* between components. Interface specifications include both physical node connections as well as electrical specifications.

- Specification of electronic characteristics is best accomplished via source equivalents.

- An example may be the following:

    - We are asked to supply a voltage of 10V-12V to a resistive load of that will be no less than 1kΩ. (This resistiave load may appear at the "input" terminals of a second, large circuit system.) We may also be limited to implementing our circuit with supply potentials no greater than 12V.

    - Thus, we would specify that our circuit Thevenin equivalent that would drive this load will be 12V, and the Thevinin resistance, would be no greater than R_{TH} = 200Ω.

- For many examples, we wish to select a circuit resistive load value such that our circuit system transfers the largest power to the load.

- To address this problem, we will use a Thevinin Equivalent.

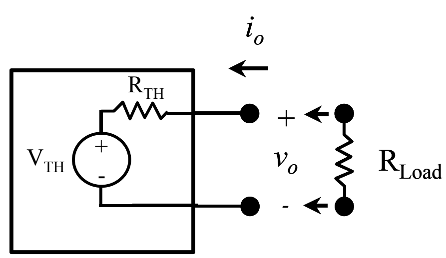


Figure 8 A Thevenin equivalent source and load resistor.

- Consider the circuit in Figure 2. Here, a load resistor, R_{Load} is applied to a circuit structure arranged as a Thevenin equivalent, voltage source and series resistor pair.

- We may compute the power dissipated in R_{Load}. This will be

$$P_{\text{Absorbed}} = v_O^2/R_{Load.}$$

- However, we may compute v_O directly with the voltage divider equation. So,

$$P_{\text{Absorbed}} = v_O^2/R_{\text{Load.}} = V_{TH}^2[R_{\text{Load}}/(R_{\text{Load}} + R_{TH})]^2$$

- So, to compute the maximum value, lets begin by computing the derivative of power, P_{Absorbed} with respect to R_{Load}. This is

$$dP/dR_{\text{Load}} = V_{TH}^2\,[1/(R_{\text{Load}} + R_{TH})^2 - 2R_{TH}(R_{\text{Load}} + R_{TH})/(R_{\text{Load}} + R_{TH})^4\,]$$

- Now, for non-zero V_{TH}, this derivative is zero at a value of R_{Load} where,

$$1/(R_{\text{Load}} + R_{TH})^2 - 2R_{TH}(R_{\text{Load}} + R_{TH})/(R_{\text{Load}} + R_{TH})^4 = 0$$

- or

$$[(R_{\text{Load}} + R_{TH})^2 - 2R_{TH}(R_{\text{Load}} + R_{TH})]/(R_{\text{Load}} + R_{TH})^4 = 0$$

- or

$$(R_{\text{Load}} + R_{TH})^2 - 2R_{TH}(R_{\text{Load}} + R_{TH}) = 0$$

- Finally, this is just

$$R_{\text{Load Max}} = R_{TH}$$

- So, this frequently used result states that the maximum power transfer to a load from a *fixed* Thevenin circuit equivalent, occurs when the load resistor value equals the Thevenin resistance value.

- Also, the specific value of the maximum power can be computed from the above results, with

$$P_{\text{Absorbed Max}} = v_O^2/\,R_{\text{Load Max.}} = V_{TH}^2[R_{TH}/(R_{TH} + R_{TH})]^2$$

- or

$$P_{\text{Absorbed Max}} = V_{TH}^2/4R_{TH} = V_{TH}^2/4R_{\text{Load}}\,\text{Max}$$

PROBLEM 4.6: NODE VOLTAGE METHOD DEMONSTRATION WITH TWO CHOICES FOR THE REFERENCE POTENTIAL.

- The results of the Node Voltage Method for solving circuit problems is, as we know, independent of the choice of which Essential Node is selected as the Reference Node.

- We will now develop the Node Voltage equations and solve the problem 4.6 for two choices of Reference Node.

- First, consider the Problem 4.6 with the choice of Reference Node as shown and with Non-Reference Nodes labled. Note that since we have 4 Essential Nodes, we should have (4-1) = 3 independent equations.

- First, this is the circuit:

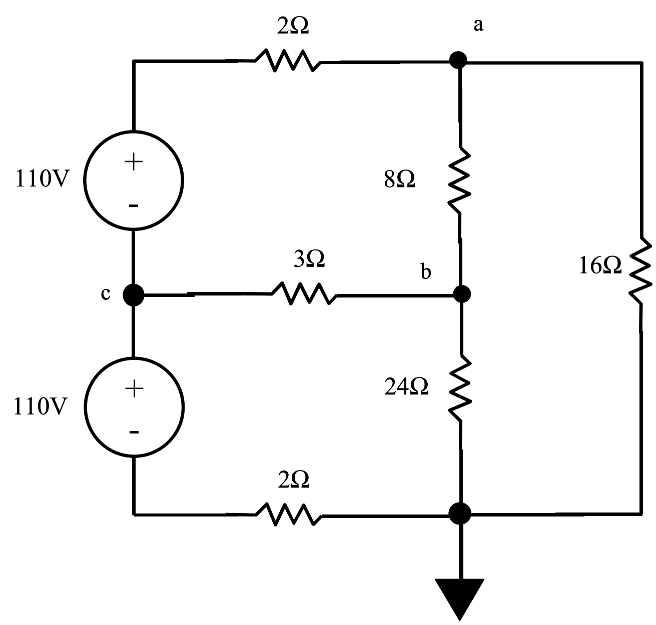


Figure 9. Circuit of Problem 4.6. Note the choice of Reference Node.

- Let us write down the Node Voltage Equations. ***For a reminder on the procedures, please refer to the Tutorial*** EE10 April 24, 2002: Solving for Voltage Differences in Node Voltage Equations

- Node a: $-(v_a - v_c - 110\text{V})/2 - v_a/16 - (v_a - v_b)/8 = 0$

- Node b: $(v_a - v_b)/8 + (v_c - v_b)/3 - v_b/24 = 0$

- Node b: $-(v_c - 110)/2 - (v_c - v_b)/3 + (v_a - v_c - 110\text{V})/2 = 0$

- These equations are solved by the following steps:

- Multiply Node a Equation by 16
- Multiply Node b Equation by 24
- Multiply Node c Equation by 6
- Combine the a and c equations to eliminate v_c , combine the b and c equations to eliminate v_c , and combine these resulting equations to eliminate v_b
- Solving, we find:
- v_a = 157.14V, v_b = 94.29V, and v_c = 82.5
- Now, let us compute this circuit solution with a new location for the Reference Node, as shown below.

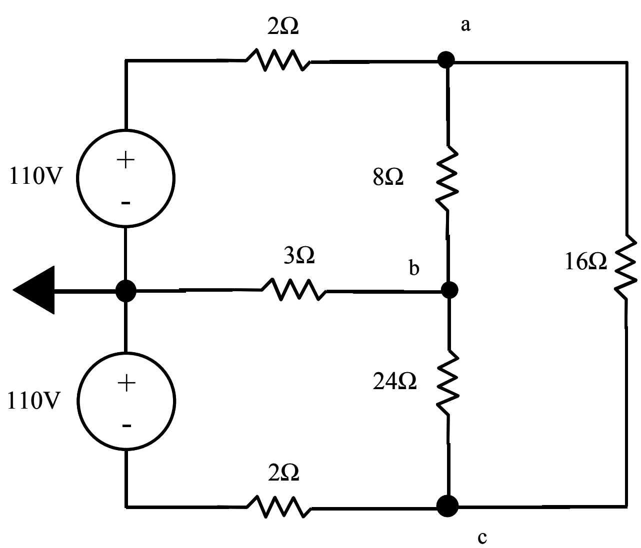


*Figure 10. Circuit of Problem 4.6. Note the **New** choice of Reference Node, compare to Figure 1.*

This yields a set of three new Node Voltage equations:

- Node a: $-(v_a - 110V)/2 - (v_a - v_c)/16 - (v_a - v_b)/8 = 0$
- Node b: $(v_a - v_b)/8 - v_b/3 - (v_b - v_cI)/24 = 0$
- Node b: $-(v_c + 110)/2 + (v_b - v_c)/24 + (v_a - v_c)/16 = 0$
- Solving, we find:
- v_a = 74.64V, v_b = 11.79V, and v_c = 82.5

COMPARISON

- We can directly compare these two results:

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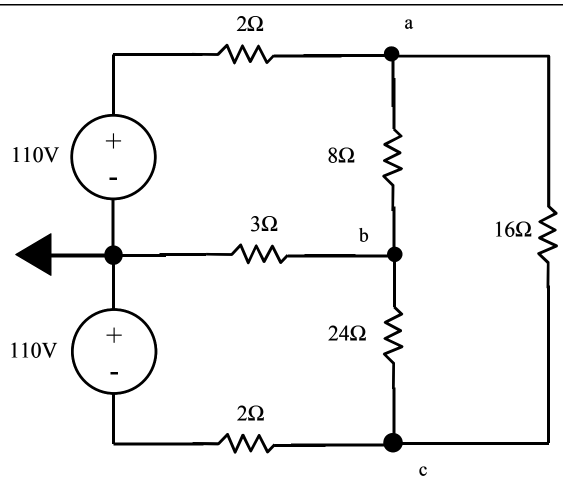
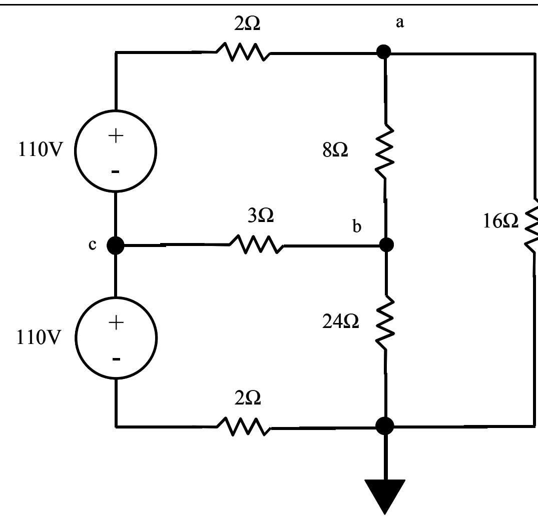
<tr><th>Solution I</th><th>Solution II</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>$v_a = 74.64$V, $v_b = 11.79$V, and $v_c = -82.5$</td><td>$v_a = 157.14$V, $v_b = 94.29$V, and $v_c = 82.5$</td></tr>
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- Now, the Reference Node voltages are defined differently between these two circuits. Therefore, the Node Voltage values must differ. However, the actual variable values for current and voltage across any element or between any two nodes must be the same in the two circuit solutions.

- Let us check a few results:

Solution I	Solution II
$v_{8\Omega} = v_a - v_b = 74.64 - 11.79 = 62.86$	$v_{8\Omega} = v_a - v_b = 157.14 - 82.5 = 62.86$
$v_{24\Omega} = v_b - v_c = 11.79 - (-82.5) = 94.29$	$v_{24\Omega} = v_a = 94.29$
$v_{16\Omega} = v_a - v_c = 157.14$	$v_{16\Omega} = v_a = 157.14$

- Note the agreement between the results of these methods.