LECTURE 15

LECTURE NOTES: MARCH 5, 2003

OUTLINE

PRINCIPLES AND PROBLEM SOLVING PROCEDURES FOR RC AND RL CIRCUITS

RL NATURAL RESPONSE

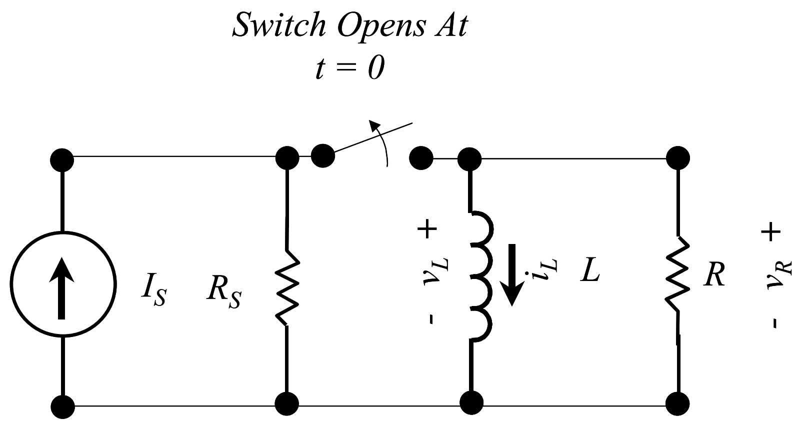


Figure 1. RL Natural Response Circuit

TIME CONSTANT

$\tau = L/R$

CONDITIONS PRIOR TO T = 0

$i_L(t < 0) = I_S$

$v_L(t < 0) = v_R(t < 0) = 0$

- Note $v_L(0)$ will vary with each circuit. (For the particular circuit above $v_L(0) = 0$)

$i_R(t < 0) = 0$

CONDITIONS AT T = 0^+ (AN INFITESIMAL TIME AFTER T = 0)

$i_L(t = 0^+) = I_S$

CURRENT AND VOLTAGE AT T > 0

$$i_L(t) = i_L(0)e^{-\left(\frac{t}{\tau}\right)}$$

$$v_L(t) = -i_L(0)\mathrm{Re}^{-\left(\frac{t}{\tau}\right)}$$

$$v_L(t) = v_R(t)$$

$$i_L(t) = -i_R(t)$$

RC NATURAL RESPONSE

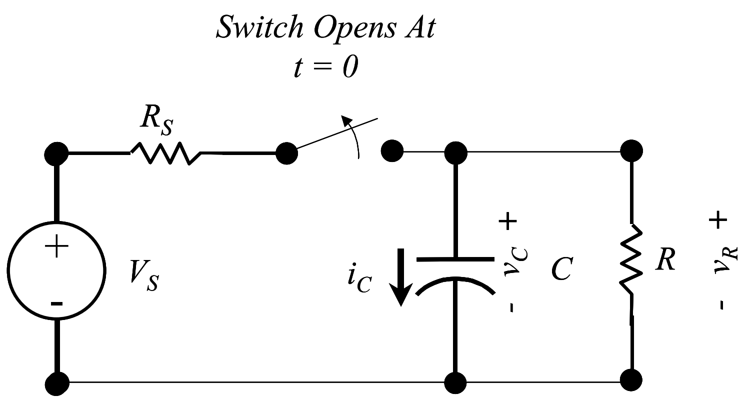


Figure 2. RC Natural Response Circuit

TIME CONSTANT

$\tau = RC$

CONDITIONS PRIOR TO T = 0

$i_C(t < 0) = 0$

$v_C(t < 0) = v_C(0)$

- Note $v_C(0)$ will vary with each circuit. (For the particular circuit above $v_C(0) = V_S[(R/(R+R_S)]$)

$i_R(t < 0) = v_C(0)/R$

CONDITIONS AT T = 0^+ (AN INFITESIMAL TIME AFTER T = 0)

$v_C(t = 0^+) = v_C(0)$

CURRENT AND VOLTAGE AT T > 0

$$v_C(t) = v_C(0)e^{-\left(\frac{t}{\tau}\right)}$$

$$i_C(t) = -\frac{v_C(0)}{R}e^{-\left(\frac{t}{\tau}\right)}$$

$$v_C(t) = v_R(t)$$

$$i_C(t) = -i_R(t)$$

RL STEP RESPONSE

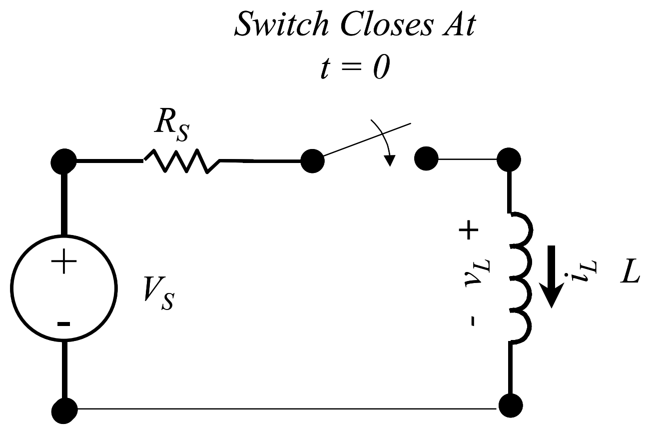


Figure 3. RL Natural Response Circuit

TIME CONSTANT

$\tau = L/R_S$

CONDITIONS PRIOR TO T = 0

v_L (t < 0) = 0

i_L(t < 0) = i_L (0)

- Note i_L (0) will vary with each circuit. (For the particular circuit above i_L (0) = 0. This will not always be true. For some circuits, i_L (0) $\neq$ 0)

i_R(t < 0) = V_S /R_S

CONDITIONS AT T = 0^+ (AN INFITESIMAL TIME AFTER T = 0)

i_L(t =0^+) = i_L (0)

CURRENT AND VOLTAGE AT T > 0

$$i_L(t) = \frac{V_S}{R_S} + \left(i_L(0) - \frac{V_S}{R_S} \right) e^{-\left(\frac{t}{\tau}\right)}$$

$$v_L(t) = \left(V_S - i_L(0) R_S\right) e^{-\left(\frac{t}{\tau}\right)}$$

$$V_S = v_R(t) + v_L(t)$$

$$i_L(t) = i_R(t)$$

RC STEP RESPONSE

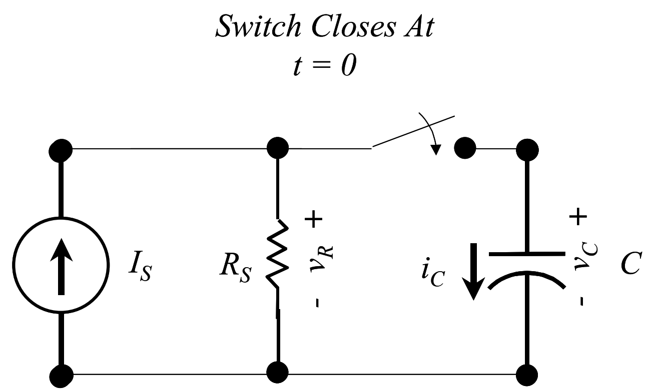


Figure 4. RC Natural Response Circuit

TIME CONSTANT

$\tau = R_S C$

CONDITIONS PRIOR TO T = 0

$v_C(t < 0) = v_C(0)$

$i_C(t < 0) = 0$

- Note $v_c(0)$ will vary with each circuit. (For the particular circuit above $v_c(0) = 0$. This will not always be true. For some circuits, $v_C(0) \neq 0$

CONDITIONS AT T = 0^+ (AN INFITESIMAL TIME AFTER T = 0)

$v_C(t = 0^+) = v_C(0)$

CURRENT AND VOLTAGE AT T > 0

$$v_C(t) = I_S R_S + \left(v_c(0) - I_S R_S\right)e^{-\left(\frac{t}{\tau}\right)}$$

$$i_C(t) = \left(I_S - \frac{v_c(0)}{R_S}\right)e^{-\left(\frac{t}{\tau}\right)}$$

$$v_R(t) = v_C(t)$$

$$i_C(t) = -i_R(t)$$

HINT FOR PROBLEM 7.22

As always for our problems, we wish to manipulate and simplify the circuit problem to place it in the form of the rules we have for RL and RC step and natural response problems.

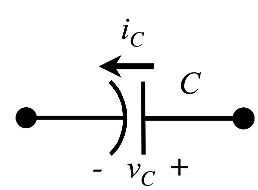
It is important to note that for this problem, after switch closure, the 2μF and 8μF capacitors are combined. They form a single 1.6uF capacitor. The voltage drop across one is 75V, and across the other is 0V at the time of switch closure.

So, the circuit appears as a 1.6uF and 5k resistor with an initial condition of 75V across the 1.6uF.

This combined circuit allows us to set the value of the time constant.

Now, note that the current flows through both C_1 and C_2.

We must always use the Displacement Current Law with the proper sign convention.



$$i_C = C\frac{dv_C}{dt}$$

According to the assigned current in the problem, you will observe that the current flows through both capacitors and the 5 kΩ resistor. But, it flows according to the Passive Sign Convention through one element, but, not the other since their assigned voltage variables correspond to Active and Passive. You must make sure to use the proper sign for current in your equations.

EXAMPLE PROBLEM 7.27

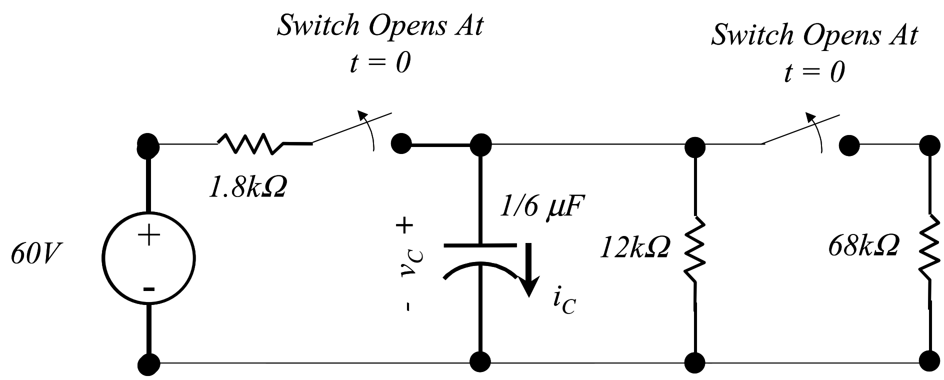


Figure 5. Circuit of Problem 7.27

Problem Statement:

a) Compute v_O and i_O for t<0, t=0⁺, and t >0.

b) Compute initial energy stored in the capacitor

c) Compute power dissipated in the 12k resistor for t >0

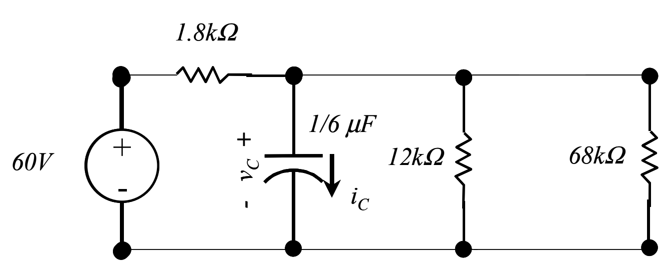
d) Compute energy absorbed in the 12k resistor for the period up to t = 2ms

e) Compute the time required to dissipate 95% of the initial capacitor energy

Step 1) **First, for this problem, as always, we must arrange the circuit into a form that will allow us to use either or both of the results from natural and step response equations (see the Appendix). We must simplify the circuit by creating just a source and resistor for each of the conditions with the switch open and closed.**

Step 2) **Compute Initial Conditions:**

a. Lets draw the circuit for t < 0 with the switches closed.



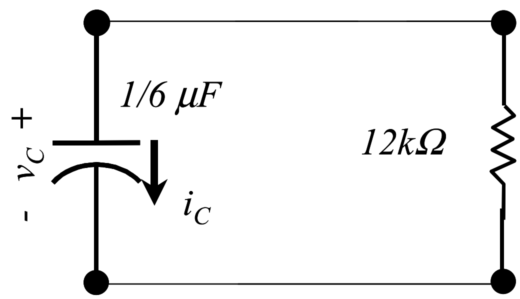
b. Now, we know that for after the circuit has stabilized at t <0, there will be no capacitor current.

c. So, the voltage, v_C is just determined by a voltage divider involving the 60V source and the 1.8k resistor in series with the parallel combination of the 12k and 68k resistors.

d. So, the initial voltage v_C (t < 0) = 51V

e. This provides the critical initial condition that we need.

Step 3) Now, lets draw the circuit after the switches have opened.



a. Now, we have the circuit in the form that allow us to use our standard circuits in the Appendix.

Step 4) First, we can compute the time constant.

a. $\tau = RC = 1/6\mu F * 12k\Omega = 2$ msec.

Step 5) Now, we apply the RC Natural Response expressions for voltage:

$$v_C(t) = v_C(0)e^{-\left(\frac{t}{\tau}\right)}$$

$$i_C(t) = -\frac{v_C(0)}{R}e^{-\left(\frac{t}{\tau}\right)}$$

$$v_C(t) = v_R(t)$$

$$i_C(t) = -i_R(t)$$

a. And,

$$v_C(t) = 51Ve^{-\left(\frac{t}{2mS}\right)} = 51Ve^{-(500t)}$$

Step 6) Energy stored in the capacitor at t < 0 is

$$W(t) = C\frac{V^2}{2} = 0.217mJ$$

Step 7) Now, we may compute power,

$$P_R(t) = \frac{{v_R}^2}{R} = \left(\frac{\left(51Ve^{-\left(\frac{t}{2mS}\right)}\right)^2}{12k\Omega}\right) = 0.217We^{-(1000t)}$$

a. And, absorbed in the resistor at time t, is

$$W(t) = \int_o^t d\tau P_R(\tau) = \int_o^t d\tau 0.217We^{-(1000t)}$$

$$W(t) = \frac{0.217W}{-1000}e^{-(1000t)}\Big|_0^t = 0.217mJ(1-e^{-(1000t)})$$

b. Note for infinite time, the total energy absorbed is equal to the total energy stored in the capacitor at t <0.

Step 8) We can now compute the energy transferred from capacitor to resistor in 2 ms.

a. This is

$$W(2m\sec) = 0.217mJ(1-e^{-(2t)}) = 0.187mJ$$

Step 9) Lets compute the time required for the capacitor to lose 95% of its initial energy:

$$W(t) = 0.217mJ(1-e^{-(1000t)}) = 0.95 * 0.217mJ$$

$$(1-e^{-(1000t)}) = 0.95$$

a. This time is t = 3msec

EXAMPLE PROBLEM 7.45

(This problem will prepare you for problems 7.8 and 7.22. It is more complex than the homework problems, thus this provides some useful background. Please read this carefully. We will also discuss this problem in Lecture on June 3.)

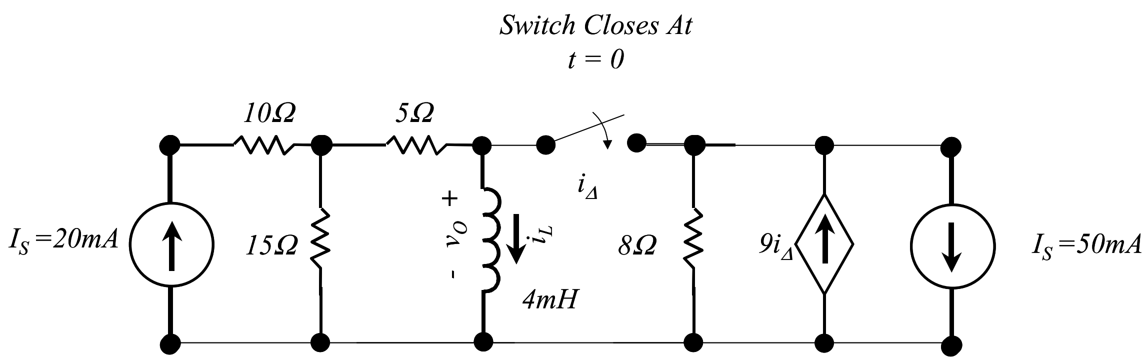


Figure 6. Circuit of Problem 7.45

Problem Statement:

Step 1) Compute v_O and i_O for t<0, t=0$^+$, and t >0.

Step 2) Compute energy storage in the inductor for t < 0.

This includes more questions than are requested in the textbook problem. Here we will be seeking more detail to illustrate additional fundamentals.

Step 10) **First, for this problem, we must arrange the circuit into a form that will allow us to use either or both of the results from natural and step response equations (see the Appendix). We must simplify the circuit by creating just a source and resistor for each of the conditions with the switch open and closed.**

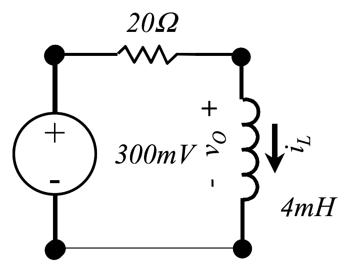
Step 11) Thus, lets compute Thevenin equivalents for the source and resistors that are connected to the inductor prior to the switch closure at t = 0.

a. The Thevenin resistance across the terminals of the inductor is computed with the current source replaced by its zero element (open circuit). This resistance is 5Ω + 15Ω = 20Ω

b. The Thevenin voltage is the open circuit voltage across the terminals attached to the inductor - with the inductor terminals open circuited (inductor removed).

c. This voltage is generated by the 20mA source supplying current to the 10Ω and 15Ω series resistors. Now, the voltage drop across the inductor terminals with the inductor terminals open circuited (inductor removed) is just 20mA x 15Ω = 300mV.

Step 12) Lets use this result to determine the initial current and energy. Our circuit will appear as below:



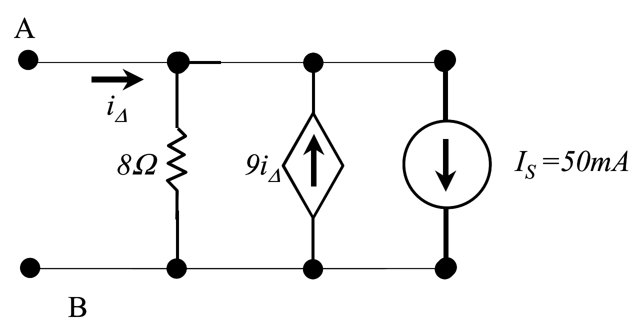
Step 13) The first part of our problem is solved: We can see immediately that for t < 0,

$$i_L(t < 0) = 300mV/20\Omega = 15mA$$

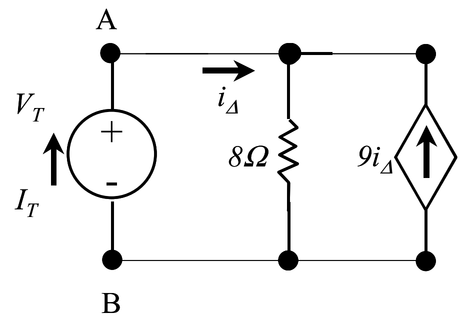
$$W(t < 0) = LI^2/2 = 4mH \times (15mA)^2/2 = 0.45\mu J$$

Step 14) Then, lets compute the Thevenin equivalent for the sources and resistors that are connected to the inductor to the right of the switch. This will simplify our circuit as well.

a. Lets draw the circuit along with the Thevenin equivalent we just used. Again, the inductor will not be present in the problem since we want to compute the Thevenin equivalent across *its terminals*. We will label the Terminals A and B and compute R_{AB}



b. Now, to compute R_{AB} , the Thevenin equivalent resistance, we replace the independent sources with zero elements and apply a test voltage, V_T to the terminals A and B and compute I_T. Our new circuit becomes:



c. We can use node voltage analysis here. We have only two Essential Nodes, and therefore one equation. We wish to solve for V_T/I_T. Our node voltage equation at Node A with Node B as the Reference Node is:

$$I_T - \frac{V_T}{8\Omega} + 9i_\Delta = 0$$

d. Also, we can write an equation for i_Δ in terms of I_T = we see this is simple.

$$i_\Delta = I_T$$

e. Manipulating

$$I_T = \frac{V_T}{80\Omega}$$

f. Finally, this means that $R_{AB} = V_T/I_T = 80\Omega$

Step 15) Now, we also need the Thevenin voltage.

a. This is computed by returning to the circuit of Step 5 (a) and computing the voltage at the terminals, A and B.

b. Again, this is a node voltage problem.

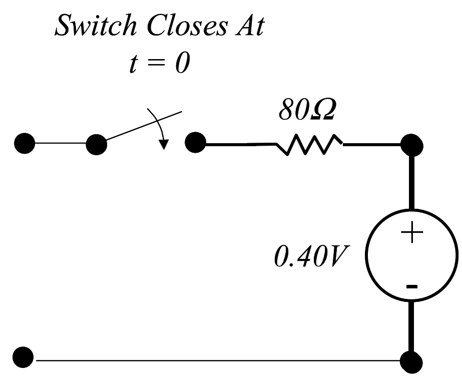
c. We return to the circuit of Figure 5a.

d. There are two Essential Nodes, and therefore one Node Voltage equation. This is:

$$\frac{v_A}{8\Omega} + 9i_\Delta - 0.05A = 0$$

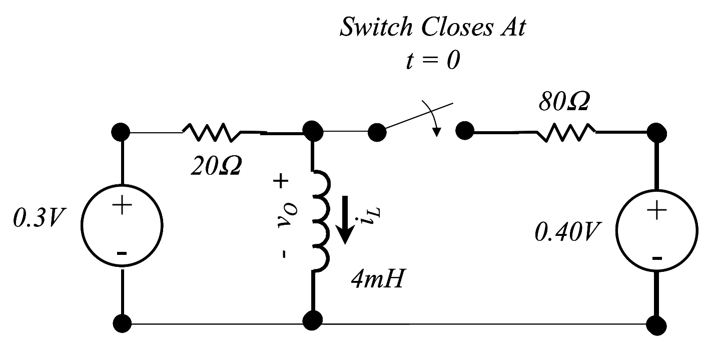
$$i_\Delta = 0$$

e. So, v_A = 0.40V and our Thevenin equivalent circuit is



Step 16) Finally, our circuit is in a form that we can use.

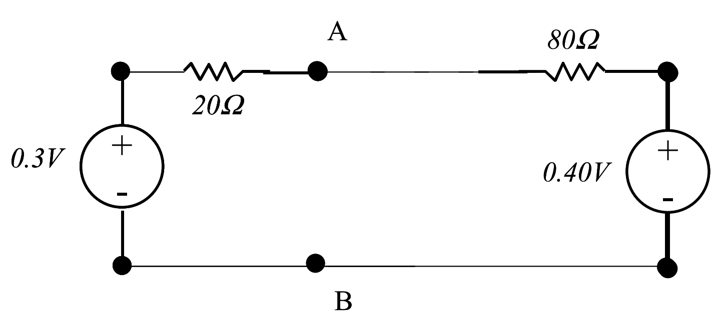
a. We can now combine the circuits of Step 3 and Step 6e to obtain an equivalent circuit.



b. Recall, now that the inductor current at t = 0+ is 15 mA as can be seen from the above.

Step 17) Now, the last step is to draw the Thevenin equivalent circuit for the system with the switch closed.

a. We need to compute the Thevenin equivalent across the terminals A, and B, for the circuit below.

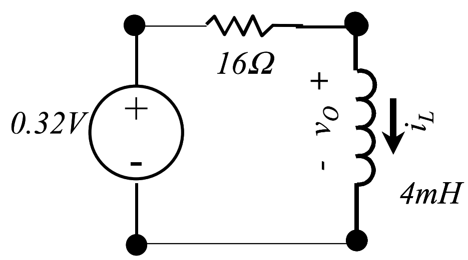


b. Fortunately, this is straightforward.

c. First, we see that the Thevenin equivalent resistance is just computed with the voltage sources replaced by their zero element short circuits. And the resistance is just the parallel combination of the 20 and 80 Ω resistors – or 16Ω.

d. The open circuit voltage is easy to compute using the voltage divider equation. We have a 0.1 V voltage drop across a the 20 and 80Ω resistor series combination. Thus, the voltage drop across the 20Ω resistor is 0.02V. This means that the voltage at Node a is 0.3 + 0.02 = 0.32V

e. Thus, our Thevenin equivalent for the condition after switch closure is:



f. Also, the time constant $\tau = L/R = 250\mu$sec

Step 18) **This step is our important goal. So, the current dependence on time for t > 0 can be computed from our results in the Appendix. Note the equation from the Appendix:**

$$i_L(t) = \frac{V_S}{R} + \left(i_L(0) - \frac{V_S}{R}\right)e^{-\left(\frac{t}{\tau}\right)}$$

a. Thus,

$$i_L(t) = \frac{0.32}{16} + \left(15mA - \frac{0.32}{16}\right)e^{-\left(\frac{t}{250E-6}\right)}$$

b. Finally, performing the arithmetic:

c. $i_L(t) = 20mA - (5mA)e^{-(4000t)}$

d. We can also compute the voltage dependence from the equation in the Appendix

$$v_L(t) = (V_S - i_L(0)R)e^{-\left(\frac{t}{\tau}\right)}$$

e. We can see that $V_S = 0.32$ and the current at t=0 is 15mA, so

$$v_L(t) = (0.32V - 15mA \bullet 16\Omega)e^{(-4000t)}$$

f. Or

$$v_L(t) = (80mV)e^{(-4000t)}$$

EXAMPLE PROBLEM 7.87

This problem will provide background for the solution of Problem 7.85

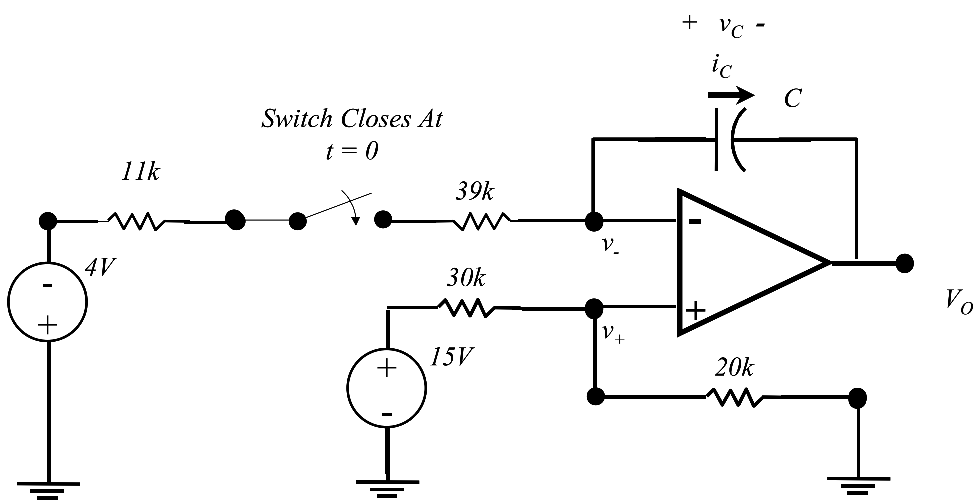


Figure 7. Circuit for Solution of Problem 7.87

Problem Statement:

a) The capacitor carries an initial voltage of v_C = 16V at t < 0.

b) Compute v_O for t<0, t=0$^+$, and t >0.

c) Compute the time after t = 0 at which the output voltage reaches zero.

Step 3) First, we note that this is an ideal operational amplifier. Thus, we may solve this with the assumption in place that:

$$v_+ = v_-$$

Step 4) Now, our operational amplifier circuit procedure calls for us to compute the voltages at the inverting and non-inverting terminals.

Step 5) Since this is ideal and there is no input current at these terminals, we may use the voltage divider expression to compute $v+$

$$v_+ = 15V \frac{20k}{20k + 30k} = 6V$$

Step 6) Now, we can write a node voltage equation for the node at the inverting input for the t >0

$$\frac{-4V - v_-}{11k + 39k} - i_C = \frac{-4V - v_-}{50k} - i_C = 0$$

Step 7) Also, we can write an equation for i_C

$$i_C = 0.5\mu F \frac{d}{dt} v_C = 0.5\mu F \frac{d}{dt}(v_- - v_O)$$

Step 8) Now, with

$$v_+ = v_- = 6V$$

Step 9) We can substitute this result into the above equations and differentiate

$$\frac{-4V - 6V}{50k} - 0.5\mu F \frac{d}{dt}(6V - v_O) = 0$$

$$\frac{d}{dt}(v_O) = 400V/\sec$$

Step 10) Now, we may integrate this.

$$\int_0^t d\tau \frac{d}{d\tau}(v_O) = \int_0^t d\tau 400V/\sec$$

$$v_O(t) - v_O(0) = t \bullet 400V$$

Step 11) Now, we can compute v_O(t=0).

Step 12) We know that v_C(0)=16V. Also, at all times, since this is an ideal operational amplifier, it holds that

$$v_+ = v_- = 6V$$

Step 13) Therefore, we may use KVL around a loop including the reference, the inverting input and output nodes we find.

$$v_-(0) - v_C(0) - v_O(0) = 0$$

$$6V - 16V - v_O(0) = 0$$

$$v_O(0) = -10V$$

Step 14) So, our voltage expression for t > 0 is,

$$v_O(t) = (400t - 10V)$$

Step 15) Finally, solving this we see that the time at which the output voltage reaches zero is, t = (1/400)sec = 25msec.