LECTURE 16

LECTURE NOTES: MARCH 10, 2003

OUTLINE

GENERAL PROBLEM SOLVING STEPS FOR PARALLEL RLC CIRCUITS

Step 1) For RC, RL, and RLC circuits we must examine first whether our circuit corresponds to Natural or Step Response circuit. As we had discussed in Lecture, if a circuit has been supplied with and stabilized with an initial energy storage in L and/or C elements, and then all sources are suddenly removed, this corresponds to a Natural Response. However, if after a switching event, there are new or different source values than had previously existed, this corresponds to Step Response. We will examine some examples in this Tutorial.

Step 2) For RLC circuits we must then also determine whether our circuit is a parallel or series structure.

Step 3) We must simplify the circuit and provide appropriate conversions of resistive elements and sources to produce Thevenin or Norton equivalents so that we can use the standard structures to determine circuit response. Whether Norton or Thevenin depends on the type of circuit.

Step 4) For Parallel RLC circuits we must use Norton Equivalents

Step 5) For Series RLC circuits we must use Thevenin Equivalents.

Step 6) For RLC circuits we must then calculate the damping coefficient (neper frequency) and the resonance frequency.

Step 7) We must then determine from these results whether the circuit is overdamped, critically damped, or underdamped.

Step 8) We then must compute the initial conditions for voltage and current.

Step 9) Finally, we can use our standard calculation procedures provided in this Appendix for computing response.

First, we will provide the key results for parallel RLC circuits.

PARALLEL RLC NATURAL RESPONSE

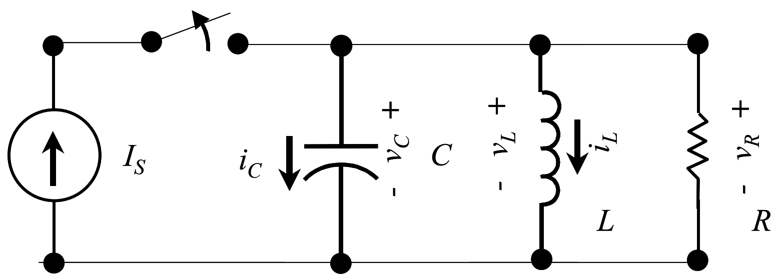


Figure 1. Parallel RLC Natural Response Circuit

CIRCUIT CHARACTERISTICS

CIRCUIT VOLTAGE

$$v_L = v_C = v_R = v$$

DAMPING COEFFICIENT

$$\alpha \equiv \frac{1}{2RC}$$

RESONANCE FREQUENCY IN RADIANS/SEC

$$\omega_O \equiv \frac{1}{\sqrt{LC}}$$

RESONANCE FREQUENCY IN HZ.

$$\omega_O \equiv 2\pi f_O$$

DETERMINING OVERDAMPED, CRITICALLY DAMPED, OR UNDERDAMPED RESPONSE CLASS

OVERDAMPED

$$\omega_O < \alpha$$

CRITICALLY DAMPED

$$\omega_O = \alpha$$

UNDERDAMPED

$$\omega_O > \alpha$$

PARALLEL RLC NATURAL OVERDAMPED RESPONSE

OVERDAMPED

$$\omega_O < \alpha$$

- We can solve for voltage and current in circuits of this type by first solving the Node Voltage equation and determining the initial conditions for voltage and current.

VOLTAGE RESPONSE

$$v(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$
$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_O{}^2}$$
$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_O{}^2}$$

CURRENT RESPONSE

$$i_L(t) = i_L(t_O) + \frac{1}{L}\int_{t_O}^{t} v(\tau)d\tau$$
$$i_C = C\frac{dv}{dt}$$
$$i_R = \frac{v}{R}$$

KCL CURRENT RELATIONSHIP FOR T >0

$$0 = i_L + i_R + i_C$$

$$i_C = -i_L - \frac{v}{R}$$

INITIAL CONDITIONS

$$i_L(t<0) = i_L(t=0) = i_L(t=0^+) = i_S$$
$$v(t<0) = v(t=0) = v(t=0^+)$$
$$i_C(t=0) = C\frac{d}{dt}v(t=0) = C(S_1A_1 + S_2A_2) = -i_L(t=0) - \frac{v(t=0)}{R}$$
$$C(S_1A_1 + S_2A_2) = -i_L(t<0) - \frac{v(t<0)}{R}$$
$$A_1 + A_2 = v(t<0)$$

PARALLEL RLC CRITICALLY DAMPED RESPONSE

CRITICALLY DAMPED

$\omega_O = \alpha$

- We can solve for voltage and current in circuits of this type by first solving the Node Voltage equation and determining the initial conditions for voltage and current.

VOLTAGE RESPONSE

$$v(t) = (D_1 + D_2 t)e^{-\alpha t}$$

CURRENT RESPONSE

$$i_L(t) = i_L(t_O) + \frac{1}{L}\int_{t_O}^{t} v(\tau)d\tau$$

$$i_C = C\frac{dv}{dt}$$

$$i_R = \frac{v}{R}$$

KCL CURRENT RELATIONSHIP FOR T >0

$$0 = i_L + i_R + i_C$$

$$i_C = -i_L - \frac{v}{R}$$

INITIAL CONDITIONS

$$i_L(t<0) = i_L(t=0) = i_L(t=0^+) = i_S$$

$$v(t<0) = v(t=0) = v(t=0^+)$$

$$i_C(t=0) = C\frac{d}{dt}v(t=0) = C\left(-\alpha D_1 + D_2\right) = -i_L(t=0) - \frac{v(t=0)}{R}$$

$$C\left(-\alpha D_1 + D_2\right) = -i_L(t<0) - \frac{v(t<0)}{R}$$

$$D_1 = v(t<0)$$

PARALLEL RLC NATURAL UNDERDAMPED RESPONSE

UNDERDAMPED

$$\omega_O > \alpha$$

- We can solve for voltage and current in circuits of this type by first solving the Node Voltage equation and determining the initial conditions for voltage and current.

VOLTAGE RESPONSE

$$v(t) = e^{-\alpha t}\left[B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$$

$$\omega_d \equiv \sqrt{{\omega_O}^2 - \alpha^2}$$

CURRENT RESPONSE

$$i_L(t) = i_L(t_O) + \frac{1}{L}\int_{t_O}^{t} v(\tau)d\tau$$

$$i_C = C\frac{dv}{dt}$$

$$i_R = \frac{v}{R}$$

KCL CURRENT RELATIONSHIP FOR T >0

$$0 = i_L + i_R + i_C$$

$$i_C = -i_L - \frac{v}{R}$$

INITIAL CONDITIONS

$$i_L(t<0) = i_L(t=0) = i_L(t=0^+) = i_S$$

$$v(t<0) = v(t=0) = v(t=0^+)$$

$$i_C(t=0) = C\frac{d}{dt}v(t=0) = C\left(-\alpha B_1 + \omega_d B_2\right) = -i_L(t=0) - \frac{v(t=0)}{R}$$

$$C\left(-\alpha B_1 + \omega_d B_2\right) = -i_L(t<0) - \frac{v(t<0)}{R}$$

$$B_1 = v(t<0)$$

PARALLEL RLC STEP RESPONSE

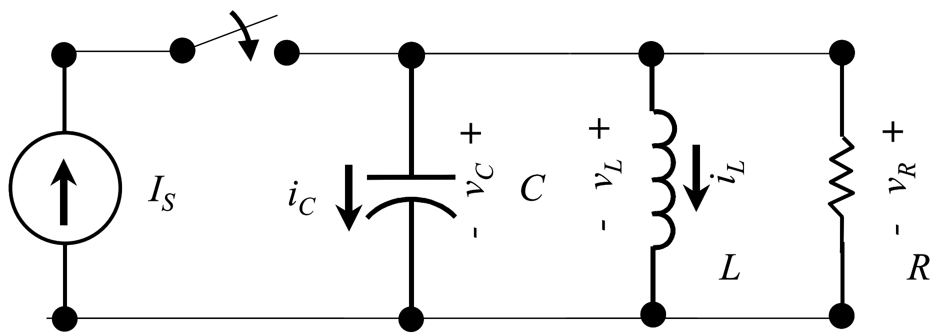


Figure 2. Parallel RLC Natural Response Circuit. Note that this is drawn with a Norton Current Source.

PARALLEL RLC OVERDAMPED STEP RESPONSE

OVERDAMPED

$\omega_O < \alpha$

- We can solve for voltage and current in circuits of this type by first solving the Node Voltage equation and determining the initial conditions for voltage and current.

VOLTAGE RESPONSE

$$v(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - {\omega_O}^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - {\omega_O}^2}$$

CURRENT RESPONSE

$$i_L(t) = i_L(t_O) + \frac{1}{L}\int_{t_O}^{t} v(\tau)d\tau$$

$$i_C = C\frac{dv}{dt}$$

$$i_R = \frac{v}{R}$$

KCL CURRENT RELATIONSHIP FOR T >0

$$I_S = i_L + i_R + i_C$$

$$I_S = i_L + \frac{v}{R} + i_C$$

$$i_C = I_S - i_L - \frac{v}{R}$$

INITIAL CONDITIONS

$$i_L(t<0) = i_L(t=0) = i_L(t=0^+) = i_O$$

$$i_C(t=0) = C\frac{d}{dt}v(t=0) = C\left(S_1 A_1 + S_2 A_2\right)$$

$$C\left(S_1 A_1 + S_2 A_2\right) = I_S - i_L(t<0) - \frac{v(t<0)}{R}$$

$$A_1 + A_2 = v(t<0)$$

PARALLEL RLC CRITICALLY DAMPED STEP RESPONSE

CRITICALLY DAMPED

$$\omega_O = \alpha$$

- We can solve for voltage and current in circuits of this type by first solving the Node Voltage equation and determining the initial conditions for voltage and current.

VOLTAGE RESPONSE

$$v(t) = (D_1 + D_2 t)e^{-\alpha t}$$

CURRENT RESPONSE

$$i_L(t) = i_L(t_O) + \frac{1}{L}\int_{t_O}^{t} v(\tau)d\tau$$

$$i_C = C\frac{dv}{dt}$$

$$i_R = \frac{v}{R}$$

KCL CURRENT RELATIONSHIP FOR T >0

$$I_S = i_L + i_R + i_C$$

$$I_S = i_L + \frac{v}{R} + i_C$$

$$i_C = I_S - i_L - \frac{v}{R}$$

INITIAL CONDITIONS

$$i_L(t<0) = i_L(t=0) = i_L(t=0^+) = i_O$$

$$i_C(t=0) = C\frac{d}{dt}v(t=0) = C\left(-\alpha D_1 + D_2\right)$$

$$C\left(-\alpha D_1 + D_2\right) = I_S - i_L(t<0) - \frac{v(t<0)}{R}$$

$$D_1 = v(t<0)$$

PARALLEL RLC UNDER DAMPED STEP RESPONSE

UNDER DAMPED

$\omega_O > \alpha$

- We can solve for voltage and current in circuits of this type by first solving the Node Voltage equation and determining the initial conditions for voltage and current.

VOLTAGE RESPONSE

$$v(t) = e^{-\alpha t}\left[B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$$

$$\omega_d \equiv \sqrt{\omega_O{}^2 - \alpha^2}$$

CURRENT RESPONSE

$$i_L(t) = i_L(t_O) + \frac{1}{L}\int_{t_O}^{t} v(\tau)d\tau$$

$$i_C = C\frac{dv}{dt}$$

$$i_R = \frac{v}{R}$$

KCL CURRENT RELATIONSHIP FOR T >0

$$I_S = i_L + i_R + i_C$$

$$I_S = i_L + \frac{v}{R} + i_C$$

$$i_C = I_S - i_L - \frac{v}{R}$$

INITIAL CONDITIONS

$$i_L(t<0) = i_L(t=0) = i_L(t=0^+) = i_O$$

$$i_C(t=0) = C\frac{d}{dt}v(t=0) = C\left(-\alpha B_1 + \omega_d B_2\right) = I_S - i_L(t=0) - \frac{v(t=0)}{R}$$

$$C\left(-\alpha B_1 + \omega_d B_2\right) = I_S - i_L(t<0) - \frac{v(t<0)}{R}$$

$$B_1 = v(t<0)$$

PROBLEM 8.34

Please Read Through the Appendix First Before proceeding.

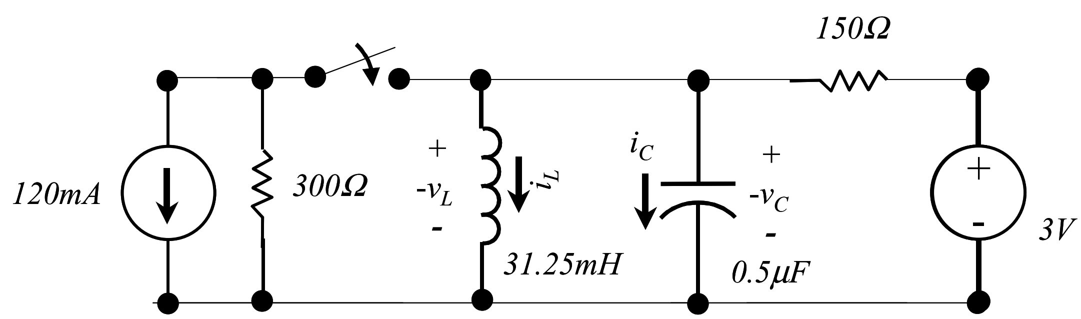


Figure 3. Circuit of Problem 8.34

Problem Statement:

a) Compute v_L and i_L for t<0, t=0$^+$, and t >0.

Lets proceed to solve this problem using the methods of the Appendix.

Step 1) *For RC, RL, and RLC circuits we must examine first whether our circuit corresponds to Natural or Step Response circuit. As we had discussed in Lecture, if a circuit has been supplied with and stabilized with an initial energy storage in L and/or C elements, and then all sources are suddenly removed, this corresponds to a Natural Response. However, if after a switching event, there are new or different source values than had previously existed, this corresponds to Step Response. We will examine some examples in this Tutorial.*

This is clearly a Step Response circuit.

Step 2) *For RLC circuits we must then also determine whether our circuit is a parallel or series structure.*

This is a parallel structure

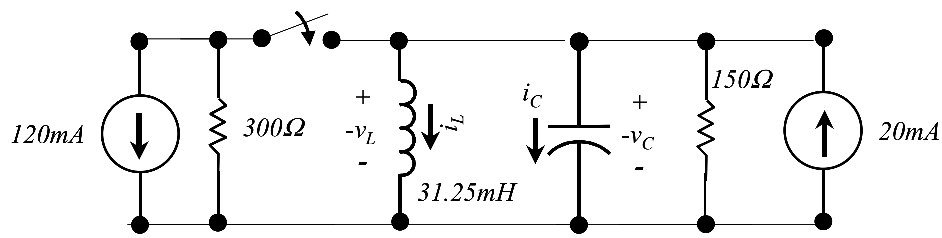
Step 3) *We must simplify the circuit and provide appropriate conversions of resistive elements and sources to produce Thevenin or Norton equivalents so that we can use the standard structures to determine circuit response. Whether Norton or Thevenin depends on the type of circuit.*

Step 4) *For Parallel RLC circuits we must use Norton Equivalents*

Lets place the circuit into the form of two Norton Equivalents.

The circuit to the right of the switch shows a 3V source and 150Ω resistor. This must be converted to a Norton Equivalent.

First, we may compute its short circuit current. This is I_{SC} = 3V/150Ω = 20mA. Also, the Norton resistance computed by replacing the Voltage Source by its zero element short circuit is just 150Ω. So, we can redraw this circuit



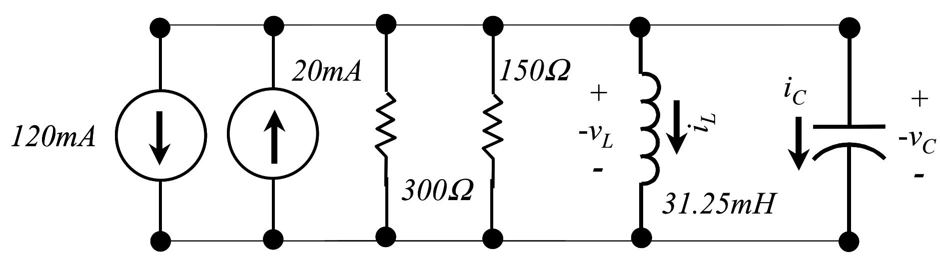
Now, we will use this circuit to determine our initial conditions.

Step 5) *For Series RLC circuits we must use Thevenin Equivalents.*

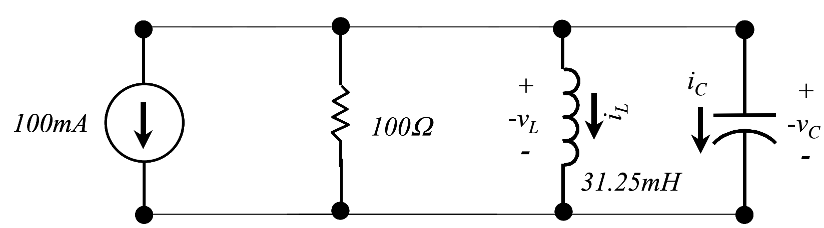
This step is not needed for our circuit.

Step 6) For RLC circuits we must then calculate the damping coefficient (neper frequency) and the resonance frequency.

To accomplish this, we must first consider the circuit ***after the switch closure.*** Note that now the two sources are in parallel. We will draw them together to make this clear.



Now, we can simplify this to obtain our required Norton Equivalent Source.



Finally, this is our circuit after switch closure.

Step 7) *We must then determine from these results whether the circuit is overdamped, critically damped, or underdamped.*

Lets compute the damping coefficient and resonance frequency.

$$\alpha \equiv \frac{1}{2RC} = \frac{1}{2(100)(0.5\mu F)} = 10^4$$

$$\omega_O \equiv \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(31.25mH)(0.5\mu F)}} = 8\times 10^3$$

Therefore since

$$\omega_O < \alpha$$

This is overdamped.

Step 8) *We then must compute the initial conditions for voltage and current.*

This is straightforward. We proceed to the simplified circuit of Step 4.

Here we can see that

$$i_L(t < 0) = 20\text{mA}.$$

Also,

$$v(t < 0) = v(t = 0) = v(t = 0^+) = 0$$

Step 9) *Finally, we can use our standard calculation procedures provided in this Appendix for computing response.*

We must compare our circuit with the Step Response RLC Circuit.

We see that by definition

$$I_S = -100\text{mA}$$

Also , we can now write.

$$v(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

where

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_O{}^2} = -4\times 10^3$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_O{}^2} = -16\times 10^3$$

Now, we can use our initial conditions.

$$A_1 + A_2 = v(t < 0) = 0$$

Also,

$$C\left(S_1 A_1 + S_2 A_2\right) = I_S - i_L(t < 0) - \frac{v(t < 0)}{R}$$

We have I_S and the initial conditions from above.

$$C\left(S_1 A_1 + S_2 A_2\right) = -100mA - 20mA - 0$$

And,

So, we have two equations in A_1 and A_2 These are:

$$A_1 + A_2 = 0$$

And

$$S_1 A_1 + S_2 A_2 = -\frac{120mA}{C}$$

Solving we have,

$$A_1 = 20V, A_2 = -20V$$

$$v(t) = 20Ve^{-4000t} - 20e^{-16000t}$$

Also, we have the equations from the Appendix

$$I_S = i_L + i_R + i_C$$

$$i_C = C\frac{dv}{dt}$$

$$i_R = \frac{v}{R}$$

So,

$$i_L = I_S - i_R - i_C = I_S - \frac{v}{R} - C\frac{d}{dt}v$$

We can proceed to substitute

$$i_L = -100mA - \frac{1}{R}\left(20Ve^{-4000t} - 20e^{-16000t}\right) - C\frac{d}{dt}\left(20Ve^{-4000t} - 20e^{-16000t}\right)$$

Computing the derivative

$$i_L = -100mA - 200mAe^{-4000t} + 200mAe^{-16000t} + 40mAe^{-4000t} - 160e^{-16000t}$$

Finally, combining terms, we have the current

$$i_L = -100mA - 160mAe^{-4000t} + 40mAe^{-16000t}$$