Order-driven markets

Econophysics

A. Djermani A. El Aamrani W. Lhourti

CentraleSupelec

June 4th 2018

Sommaire

Introduction

Modelling financial markets :

- A good characterization of financial markets has important practical consequences for risk control and option pricing.

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Econophysics models :

- 3 Simple order books models inspired from Physics.
- Analogy : order $\equiv$ particle

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Analogy with a diffusion model

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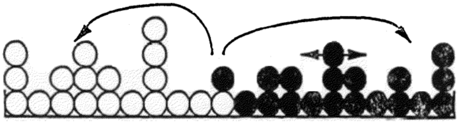

Figure – Illustration of the Bak, Paczuski and Shubik model

Physics	Bak et al. (1997)
Particles	Orders
Finite pipe	Order book
Collision	Transaction

Market mechanism

Market rules

There is **one type of stock**

Each agent can own at most **one share**

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Types of agents

N **noise traders** :

- $N/2$ potential **sellers**
- $N/2$ potential **buyers**

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Prices

$$Prices \in [\![0, P_{max}]\!]$$

Market initialization

$$P_b^0 \hookrightarrow U_{[\![0,\frac{P_{max}}{2}]\!]}$$
$$P_s^0 \hookrightarrow U_{[\![\frac{P_{max}}{2},P_{max}]\!]}$$

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Price evolution

At each time step t, with **equal probability** :

$$P_b^{t+1} = P_b^t \pm 1$$
$$P_s^{t+1} = P_s^t \pm 1$$

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    - If there is a transaction, the **buyer becomes a seller**, with a random price in $[\![P^t, P_{max}]\!]$, and the **seller becomes a buyer**, with a random price in $[\![0, P^t]\!]$

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Setting Diffusion Simulation

```
# Defining the model parameters
N = 500
price_max = 250
T = 100000
# Running the simulation
prices, agents= diffusion_Model(N, price_max, T)
```

Figure – Setting Diffusion parameters

Transaction price evolution

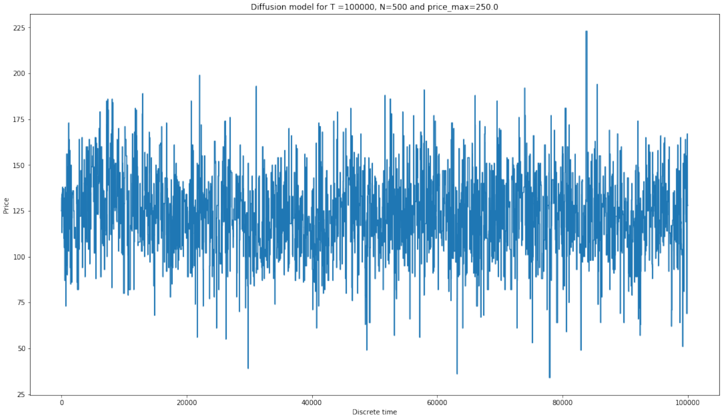
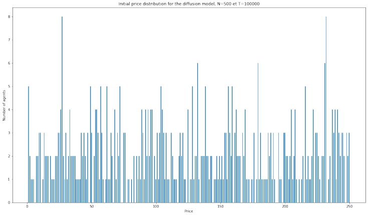
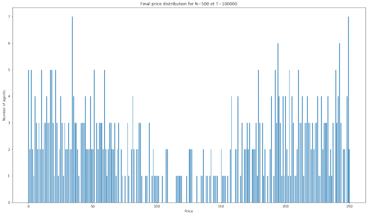


Figure – Evolution of the transaction price in a diffusion model

Agents price distribution



(a) Initial distribution



(b) Final distribution

Physics theory of the diffusion model

Some authors showed that the **position of annihilation** in diffusion models varies with time intervals Δt following :

$$P_{\Delta t} \sim \Delta t^{\frac{1}{4}} ln(\Delta t)^{\frac{1}{2}}$$

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Rescaled range

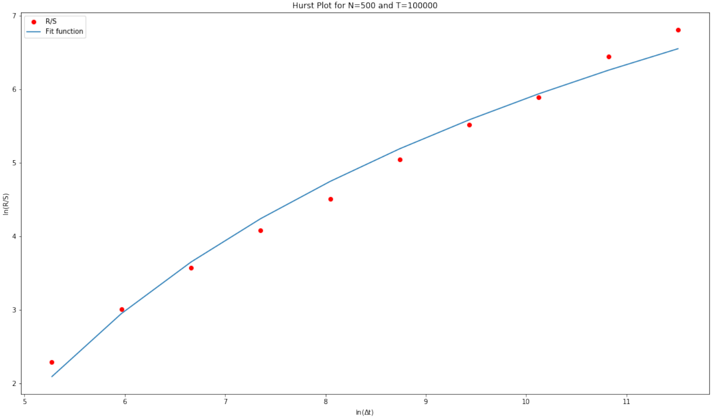

Figure – The Hurst plot

Models conclusions

Stylized fact

- $H > 0.5 \quad (H \approx 0.7)$ in real markets

Model's weakness

- "Moving" orders are very **unrealistic**

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- Infer qualitative behaviors of the distribution of price fluctuations and long range correlations from simulations

The Maslov model is a theoretical model, which assumes very simple hypothesis upon agents behavior and limit order market structure.

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Best ask $p_a(t)$ and Best bid $p_b(t)$

The best ask is the **lowest** price among the ask limit orders
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Binary behavior of agents

Two possibilities : make an order at the best price or store a limit order

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Unique size for all orders

Unitary lot of stock in each order

Sequences

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2. Either he is buyer, or he is a seller
3. With probability q_{lo}, he places a limit order, with probability 1 - q_{lo}, he trades at the market price ...

Sequences

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- If a market order is executed, then the corresponding best limit order is removed and its price becomes the new market price
- Else, the price of the new limit order is fixed : $p = p(t) \pm \Delta$ and a new expiration time is given
- For each limit order, if its expiration time is reached, then it disappears from the order book

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Setting Maslov Simulation

Before the evolution of prices, we generate

- an array of bid limit orders {990, 991, ..., 999}
- an array of ask limit orders {1001, 1002, ..., 1010}

At each simulation step t, two random numbers u and v are generated :

- If $u \leq 0.5$, the agent is a buyer.
- Else, he is a seller.
- For each :
    - If $v > q_{lo}$, the order is an effective market order.
    - The agent places a limit order.

Setting Maslov Simulation

```
N = 100000
buy_sell_param = 0.5
qlo = 0.5
bids = np.linspace(990,999,10).tolist()
asks = np.linspace(1001,1010,10).tolist()
ask_limit_orders = {i: asks[i] for i in range(10)}
bid_limit_orders = {i: bids[i] for i in range(10)}
initialPrice = 1000
expiration_time = 1000
prices = Maslov_expiration_model(N,buy_sell_param,qlo,bid_limit_orders, ask_limit_orders, initialPrice, expiration_time
```

Figure – Setting Maslov parameters

We obtain an array of N prices (current price for each step t)

Market price evolution

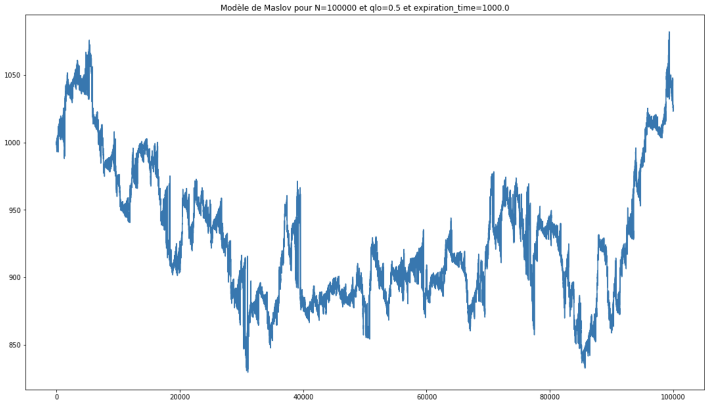

Figure – Evolution of the market price for $N = 100000$ steps

Market price increments

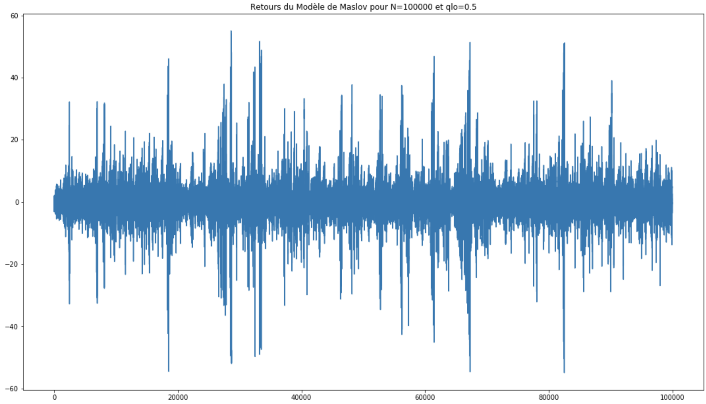

Figure – Evolution of the market price increments

Heavy tails of the log-increments

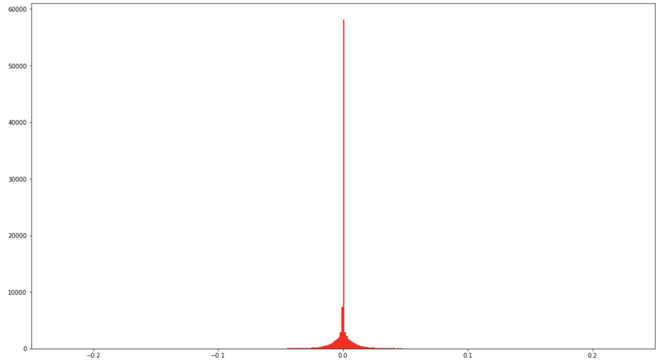

Figure – The distribution of the log-increments of market prices

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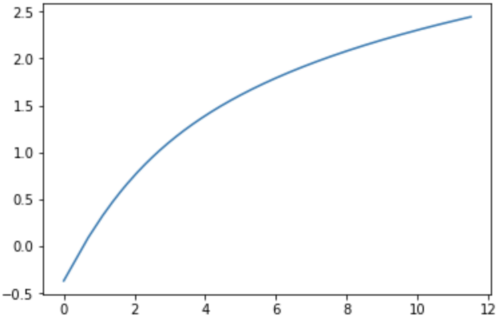

Figure – The distribution of the log-increments of market prices

Observations

- High volatility of prices is observed
- Phenomenon of clustering : price increments are grouped and separated by "quiet" zones of fluctuation
- Long-range auto correlation of price
- Shows a non trivial, unexpected Hirst Exponent $H = 1/4$

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Logarithmic returns |r(t)|

$$r(t) = \pi_m(t) - \pi_m(t-1)$$

Order placement

The model for order placement is developed in the same style as that of Challet and Stinchcombe (2001).

Physics	Challet and Stinchcombe (2001)
Particles	Orders
Infinite lattice	Order book
Deposition	Limit orders submission
Evaporation	Limit orders cancelation
Annihilation	Transaction

Table – Analogy between the deposition-evaporation process and the order book of Challet and Stinchcombe (2001)

Cancellation model

Introducing an empirical cancellation probability

- The position in the order book $y(t) = \frac{\Delta(t)}{\Delta(0)}$
- The total number $N(t) = N_a(t) + N_b(t)$ of orders in the book.
- The order imbalance $N_{imb}(t) = \frac{N_i(t)}{N(t)}$

$$P\left(C|y(t), N_{imb}(t), N_t(t)\right) = A(1 - e^{-y(t)})(N_{imb}(t) + B)\frac{1}{N_t(t)}$$

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Describing the simulation

Before the evolution of prices, we generate

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Check if orders should be canceled

Setting Mike-Farmer Simulation

```
# Defining hyperparameters
T = 100000
alpha_x = 1.3
sigma_x = 0.0024
Hs = 0.75
A = 1.12
B = 0.2
ask_lo = np.log(np.linspace(101,110,10)).tolist()
bid_lo = np.log(np.linspace(90,99,10)).tolist()

# Simulating the model
log_prices, log_returns, spreads = Mike_and_Farmer(T, Hs, A, B, alpha_x, sigma_x, ask_lo, bid_lo)
```

Figure – Setting Mike-Farmer parameters

Transaction price evolution

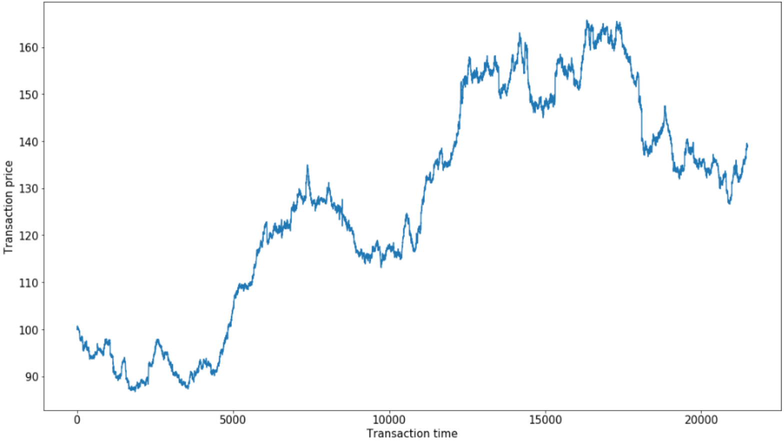

Figure – Evolution of the transaction price of a random run

Stylized facts

Distribution of the log-returns :

- "The returns are distributed according to the cubic law"
- The power law exponent for the log returns is $\alpha = 3.37$

Heavy tails of the log-returns

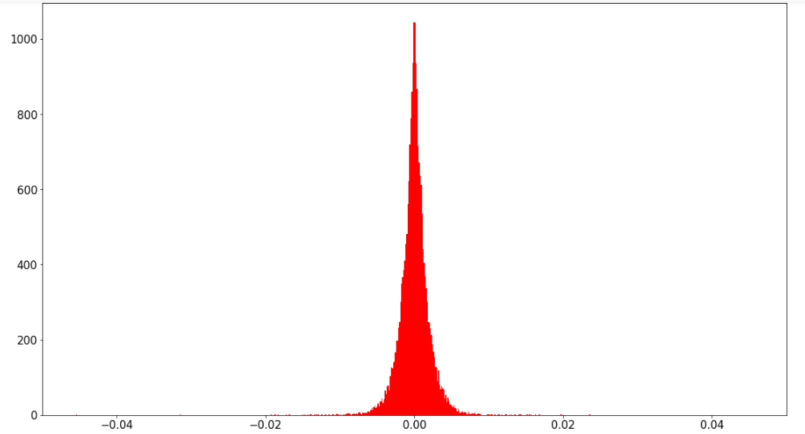

Figure – The distribution of the log-returns of transaction prices

Stylized facts

Hurst exponent of the log transaction prices

- $H_p \sim 0.6 \in]0.5, 1[$
- "The DFA scaling exponent of returns is close to 0.5"
- Stylized fact absent in the previous models

Prediction

Stock sizer	H_s	α_x	$\sigma_x \times 10^{-3}$	A	B
AZN	0.77	1.31	2.4	1.12	0.2

Figure – The measured parameters of our order flow models

Stock sizer	$\mathbb{E}(\lvert r\rvert) \times 10^{-4}$	$\mathbb{E}(s) \times 10^{-4}$	$\sigma(\lvert r\rvert) \times 10^{-4}$	$\sigma(s) \times 10^{-4}$	$\alpha(\lvert r\rvert)$	$\alpha(s)$
AZN	5.4	13.9	7.2	12.1	2.4	3.3
Predicted	3.94	40.21	9.83	57.41	3.4	3.1

Figure – The statistical properties of our order flow models

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"Effect of tick size on model stability"

Mike and Farmer model

Equation of state

Mike and Farmer model

Equation of state

The properties of order flow

Mike and Farmer model

Equation of state

The properties of order flow

$\updownarrow$

The properties of prices

Conclusion

Comparing models

	Heavy tails	Long-memory	Empirical
Bak (1997)	-	-	-
Maslov (2000)	+	-	-
M&F (2008)	+	+	+

Table – Models comparison