Lab #3

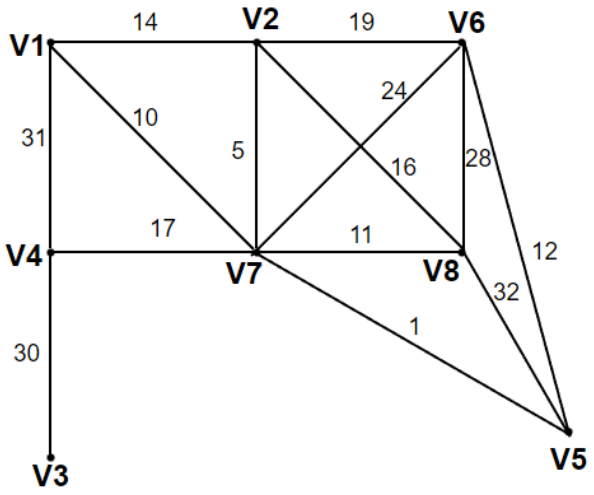
Graph Algorithm Tasks

Input data:

Var 33 : v3 ⟶ v5
w({v1,v2})=14, w({v1,v4})=31, w({v1,v7})=10, w({v2,v6})=19, w({v2,v7})=5, w({v2,v8})=16, w({v3,v4})=30, w({v4,v7})=17, w({v5,v6})=12, w({v5,v7})=1, w({v5,v8})=32, w({v6,v7})=24, w({v6,v8})=28, w({v7,v8})=11.

(1) Using the Floyd-Warshall algorithm, find the weight matrix of the paths and the matrix of the first vertices' indices for these paths.

Graph diagram:



ADJACENCY MATRIX:

Vertices: V = {a, b, c, d, e, f, g, o}

v1=a, v2=b, v3=c, v4=d, v5=e, v6=f, v7=g, v8=o

Edge Set: : E = ({v1,v2})=14, w({v1,v4})=31, w({v1,v7})=10, w({v2,v6})=19, w({v2,v7})=5, w({v2,v8})=16, w({v3,v4})=30, w({v4,v7})=17, w({v5,v6})=12, w({v5,v7})=1, w({v5,v8})=32, w({v6,v7})=24, w({v6,v8})=28, w({v7,v8})=11.

	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	1	0	1	0	0	1
b V2	1	0	0	0	0	1	1
c V3	0	0	**0**	1	0	0	0
d V4	1	0	1	0	0	0	1
e V5	0	0	0	0	0	1	1
f V6	0	1	0	0	1	**0**	1
g V7	1	1	0	1	1	1	**0**

o V8	0	1	0	0	1	1	1

EDGE WEIGHT MATRIX W

W	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	∞	31	∞	∞	10
b V2	14	0	∞	∞	∞	19	5
c V3	∞	∞	**0**	30	∞	∞	∞
d V4	31	∞	30	0	∞	∞	17
e V5	∞	∞	∞	∞	0	12	1
f V6	∞	19	∞	∞	12	**0**	24
g V7	10	5	∞	17	1	24	**0**
o V8	∞	16	∞	∞	32	28	11

Floyd-Warshall Algorithm:

Initial conditions::

0) (L(0), P(0))

L(0) = W

L(0) =

L(0)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	∞	31	∞	∞	10
b V2	14	0	∞	∞	∞	19	5
c V3	∞	∞	**0**	30	∞	∞	∞
d V4	31	∞	30	0	∞	∞	17
e V5	∞	∞	∞	∞	0	12	1
f V6	∞	19	∞	∞	12	**0**	24
g V7	10	5	∞	17	1	24	**0**
o V8	∞	16	∞	∞	32	28	11

P(0) =

P(0)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	1	2	0	4	0	0	7
b V2	1	2	0	0	0	6	7
c V3	0	0	3	4	0	0	0
d V4	1	0	3	4	0	0	7
e V5	0	0	0	0	5	6	7
f V6	0	2	0	0	5	6	7
g V7	1	2	0	4	5	6	7
o V8	0	2	0	0	5	6	7

1) (L1, P1)

L(1) =

L(1)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	∞	31	∞	∞	10
b V2	14	0	∞	[45]	∞	19	5
c V3	∞	∞	**0**	30	∞	∞	∞

d V4	31	[45]	30	0	∞	∞	17
e V5	∞	∞	∞	∞	0	12	1
f V6	∞	19	∞	∞	12	**0**	24
g V7	10	5	∞	17	1	24	**0**
o V8	∞	16	∞	∞	32	28	11

P(1) =

P(1)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	1	2	0	4	0	0	7
b V2	1	2	0	[1]	0	6	7
c V3	0	0	3	4	0	0	0
d V4	1	[1]	3	4	0	0	7
e V5	0	0	0	0	5	6	7
f V6	0	2	0	0	5	6	7
g V7	1	2	0	4	5	6	7
o V8	0	2	0	0	5	6	7

2) **(L2, P2)**

L(2) =

L(2)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	∞	31	∞	[33]	10
b V2	14	0	∞	45	∞	19	5
c V3	∞	∞	**0**	30	∞	∞	∞
d V4	31	45	30	0	∞	[64]	17
e V5	∞	∞	∞	∞	0	12	1
f V6	[33]	19	∞	[64]	12	**0**	24
g V7	10	5	∞	17	1	24	**0**
o V8	[30]	16	∞	[61]	32	28	11

P(2) =

P(2)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	1	2	0	4	0	[2]	7
b V2	1	2	0	1	0	6	7
c V3	0	0	3	4	0	0	0
d V4	1	1	3	4	0	[1]	7
e V5	0	0	0	0	5	6	7
f V6	[2]	2	0	[2]	5	6	7
g V7	1	2	0	4	5	6	7
o V8	[2]	2	0	[2]	5	6	7

3) **(L3, P3)**

L(3) = Nothing has changed

L(3)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	∞	31	∞	33	10

b V2	14	0	∞	45	∞	19	5
c V3	∞	∞	**0**	30	∞	∞	∞
d V4	31	45	30	0	∞	64	17
e V5	∞	∞	∞	∞	0	12	1
f V6	33	19	∞	64	12	**0**	24
g V7	10	5	∞	17	1	24	**0**
o V8	30	16	∞	61	32	28	11

P(3) =

P(3)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	1	2	0	4	0	2	7
b V2	1	2	0	1	0	6	7
c V3	0	0	3	4	0	0	0
d V4	1	1	3	4	0	1	7
e V5	0	0	0	0	5	6	7
f V6	2	2	0	2	5	6	7
g V7	1	2	0	4	5	6	7
o V8	2	2	0	2	5	6	7

4) **(L4, P4)**

L(4) =

L(4)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	[61]	31	∞	33	10
b V2	14	0	[75]	45	∞	19	5
c V3	[61]	[75]	**0**	30	∞	[94]	[47]
d V4	31	45	30	0	∞	64	17
e V5	∞	∞	∞	∞	0	12	1
f V6	33	19	[94]	64	12	**0**	24
g V7	10	5	[47]	17	1	24	**0**
o V8	30	16	[91]	61	32	28	11

P(4) =

P(4)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	1	2	[4]	4	0	2	7
b V2	1	2	[1]	1	0	6	7
c V3	[4]	[4]	3	4	0	[4]	[4]
d V4	1	1	3	4	0	1	7
e V5	0	0	0	0	5	6	7
f V6	2	2	[2]	2	5	6	7
g V7	1	2	[4]	4	5	6	7
o V8	2	2	[2]	2	5	6	7

5) **(L5, P5)**

L(5) =

L(5)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	61	31	∞	33	10

b V2	14	0	75	45	∞	19	5
c V3	61	75	**0**	30	∞	94	47
d V4	31	45	30	0	∞	64	17
e V5	∞	∞	∞	∞	0	12	1
f V6	33	19	94	64	12	**0**	[13]
g V7	10	5	47	17	1	[13]	**0**
o V8	30	16	91	61	32	28	11

P(5) =

P(5)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	1	2	4	4	0	2	7
b V2	1	2	1	1	0	6	7
c V3	4	4	3	4	0	4	4
d V4	1	1	3	4	0	1	7
e V5	0	0	0	0	5	6	7
f V6	2	2	2	2	5	6	[5]
g V7	1	2	4	4	5	[5]	7
o V8	2	2	2	2	5	6	7

6) **(L6, P6)**

L(6) =

L(6)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	61	31	[45]	33	10
b V2	14	0	75	45	[31]	19	5
c V3	61	75	**0**	30	[106]	94	47
d V4	31	45	30	0	[76]	64	17
e V5	[45]	[31]	[106]	[76]	0	12	1
f V6	33	19	94	64	12	**0**	13
g V7	10	5	47	17	1	13	**0**
o V8	30	16	91	61	32	28	11

P(6) =

P(6)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	1	2	4	4	[2]	2	7
b V2	1	2	1	1	[6]	6	7
c V3	4	4	3	4	[4]	4	4
d V4	1	1	3	4	[1]	1	7
e V5	[6]	[6]	[6]	[6]	5	6	7
f V6	2	2	2	2	5	6	5
g V7	1	2	4	4	5	5	7
o V8	2	2	2	2	5	6	7

7) **(L7, P7)**

L(7) =

L(7)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	[57]	[27]	[11]	[23]	10

b V2	14	0	[52]	[22]	[6]	[18]	5
c V3	[57]	[52]	**0**	30	[48]	[60]	47
d V4	[27]	[22]	30	0	[18]	[30]	17
e V5	[11]	[6]	[48]	[18]	0	12	1
f V6	[23]	[18]	[60]	[30]	12	**0**	13
g V7	10	5	47	17	1	13	**0**
o V8	[21]	16	[58]	[28]	[12]	[24]	11

P(7) =

P(7)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	1	2	[7]	[7]	[7]	[7]	7
b V2	1	2	[7]	[7]	[7]	[7]	7
c V3	4	4	3	4	[4]	[4]	4
d V4	[7]	[7]	3	4	[7]	[7]	7
e V5	[7]	[7]	[7]	[7]	5	6	7
f V6	[5]	[5]	[5]	[5]	5	6	5
g V7	1	2	4	4	5	5	7
o V8	[7]	2	[7]	[7]	[7]	[7]	7

8) **(L8, P8)**

L(8) = nothing has changed and we get an answer !

ANSWER: The matrix of route weights

L(8)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	57	27	11	23	10
b V2	14	0	52	22	6	18	5
c V3	57	52	**0**	30	48	60	47
d V4	27	22	30	0	18	30	17
e V5	11	6	48	18	0	12	1
f V6	23	18	60	30	12	**0**	13
g V7	10	5	47	17	1	13	**0**
o V8	21	16	58	28	12	24	11

P(8) = nothing has changed and we get an answer!

ANSWER: The matrix of the numbers of the first vertices of these routes:

P(8)	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	1	2	7	7	7	7	7
b V2	1	2	7	7	7	7	7
c V3	4	4	3	4	4	4	4
d V4	7	7	3	4	7	7	7
e V5	7	7	7	7	5	6	7
f V6	5	5	5	5	5	6	5
g V7	1	2	4	4	5	5	7
o V8	7	2	7	7	7	7	7

(2) For the given pair of vertices vb→v , extract the path from vertex vb to vertex

vt from the path matrix in the form of a sequence of vertex indices.

The path from v3 → v5.

Route weight L35 = 48

The route from the vertex V3 → V5 in the form of a sequence of vertex numbers

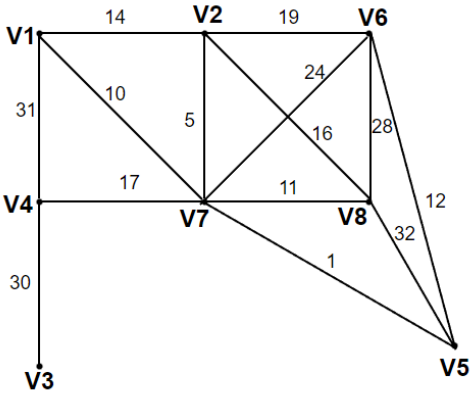
s0 = v3, k1 = P35 = 4

s1 = v4, k2 = P45 = 7

s2 = v7, k3 = P75 = 5

s3 = v5, k4 = P55 = 5

Answer: The route- v3 v4 v7 v5



(3) Using Dijkstra's algorithm for the same pair of vertices, find the weight and the path of the minimum weight.

Path from v3 → v5.

EDGE WEIGHT MATRIX W

W	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	**∞**	31	**∞**	**∞**	10
b V2	14	0	**∞**	**∞**	**∞**	19	5
c V3	**∞**	**∞**	**0**	30	**∞**	**∞**	**∞**
d V4	31	**∞**	30	0	**∞**	**∞**	17
e V5	**∞**	**∞**	**∞**	**∞**	0	12	1
f V6	**∞**	19	**∞**	**∞**	12	**0**	24
g V7	10	5	**∞**	17	1	24	**0**
o V8	**∞**	16	**∞**	**∞**	32	28	11

b = 3, t = 5

k	**T(k)**	**1**	**2**	**3**	**4**	5	6
0	L	**∞**	**∞**	0	**∞**	**∞**	**∞**
	P	0	0	0	0	0	0
	C	0	0	1	0	0	0

1	L	∞	∞	0	30	∞	∞
	P	0	0	0	3	0	0
	C	0	0	1	1	0	0
2	L	31	∞	0	30	∞	∞
	P	4	0	0	3	0	0
	C	0	0	1	1	0	0
3	L	10	5	0	30	1	24
	P	7	7	0	3	7	7
	C	0	0	1	1	1	0

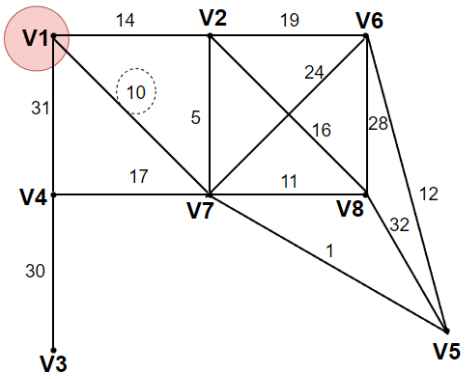
The route : 3 ← 4 ← 7← 5

Weight - 30 + 17 + 1 = 48

(4) (4) Using Prim's algorithm, find the minimum spanning tree for the given graph in the form of a list of the spanning tree's edges.

EDGE WEIGHT MATRIX W

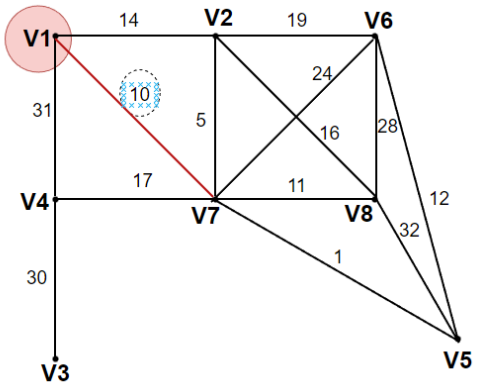
W	a v1	b V2	c V3	d V4	e V5	f V6	g V7
a V1	**0**	14	∞	31	∞	∞	10
b V2	14	0	∞	∞	∞	19	5
c V3	∞	∞	**0**	30	∞	∞	∞
d V4	31	∞	30	0	∞	∞	17
e V5	∞	∞	∞	∞	0	12	1
f V6	∞	19	∞	∞	12	**0**	24
g V7	10	5	∞	17	1	24	**0**
o V8	∞	16	∞	∞	32	28	11



1 iteration:

W	v1	V2	V3	V4	V5	V6	V7
V1	**0**	14	0	31	0	0	10 - min

V2	14	0	0	0	0	19	5
V3	0	0	**0**	30	0	0	0
V4	31	0	30	0	0	0	17
V5	0	0	0	0	0	12	1
V6	0	19	0	0	12	**0**	24
V7	10	5	0	17	1	24	**0**
V8	0	16	0	0	32	28	11
-							

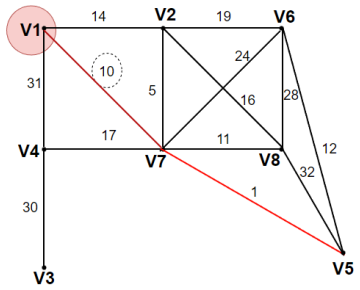


10 = min({14, 31, 10})

Edge 1 - 7 are connected.

2 iteration:

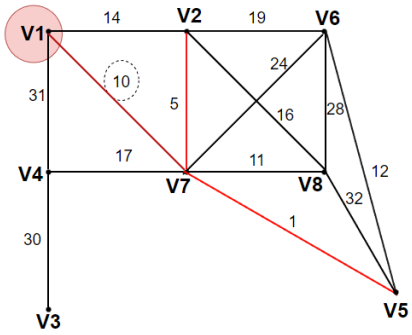
	v1	V2	V3	V4	V5	V6	V7
V1		14	0	31	0	0	
V2		0	0	0	0	19	
V3		0	**0**	30	0	0	
V4		0	30	0	0	0	
V5		0	0	0	0	12	
V6		19	0	0	12	**0**	
V7		5	0	17	1 - min	24	
V8		16	0	0	32	28	
-							



Edge 7 - 5 are connected.

3 iteration:

	v1	V2	V3	V4	V5	V6	V7
V1		14	0	31		0	
V2		0	0	0		19	
V3		0	**0**	30		0	
V4		0	30	0		0	
V5		0	0	0		12	
V6		19	0	0		**0**	
V7		5 - min	0	17		24	
V8		16	0	0		28	
-							

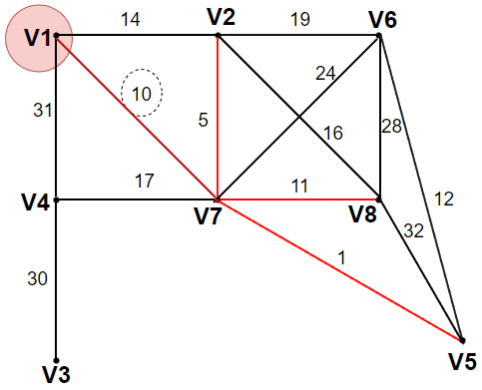


7-2

4 iteration:

	v1	V2	V3	V4	V5	V6	V7
V1			0	31		0	
V2			0	0		19	
V3			**0**	30		0	
V4			30	0		0	
V5			0	0		12	
V6			0	0		0	

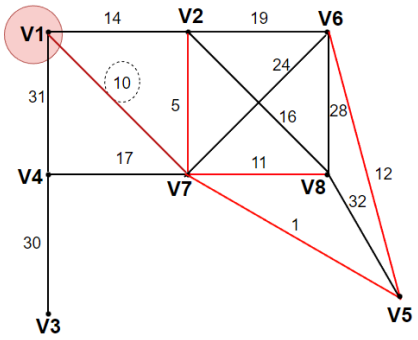
V7			0	17		24	
V8			0	0		28	
-							



7 - 8

5 - iteration:

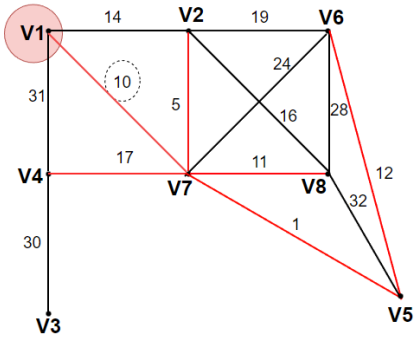
	v1	V2	V3	V4	V5	V6	V7
V1			0	31		0	
V2			0	0		19	
V3			**0**	30		0	
V4			30	0		0	
V5			0	0		12 - min	
V6			0	0		0	
V7			0	17		24	
V8			0	0		28	
-							



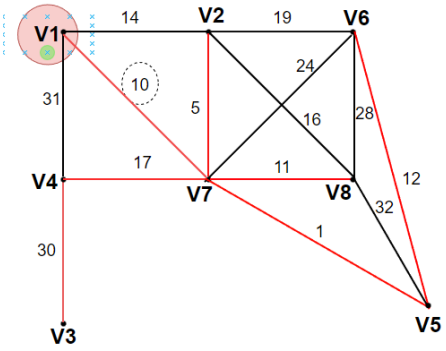
6 iteration:

	v1	V2	V3	V4	V5	V6	V7
V1			0				
V2			0				
V3			**0**				

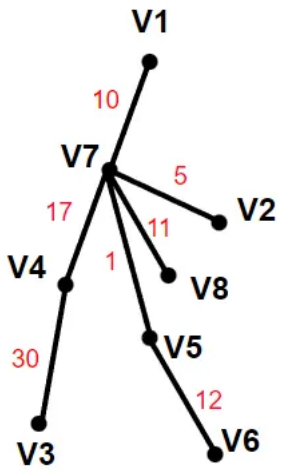
V4			30				
V5			0				
V6			0				
V7			0				
V8			0				
-							



7 iteration:



Минимальный остов по сумме весов :



w = 10 + 17 + 30 + 1 + 11 + 5+ 12 = 86