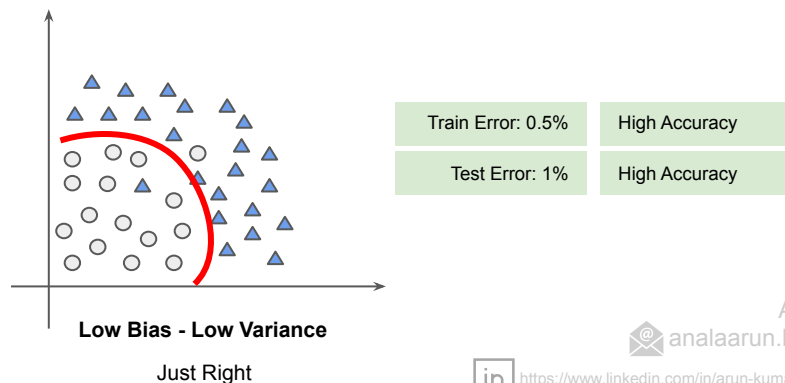
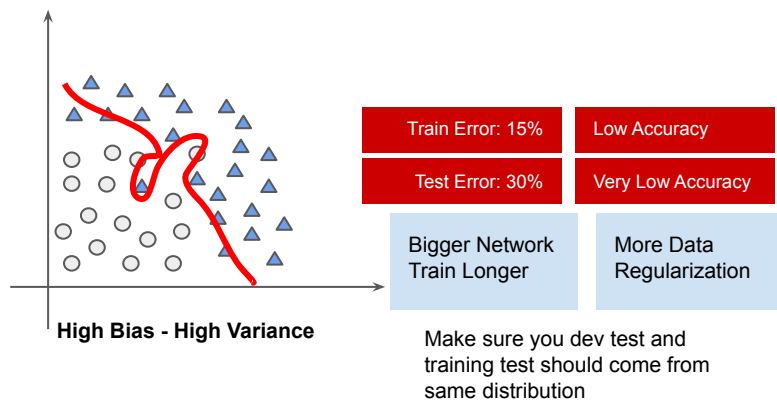
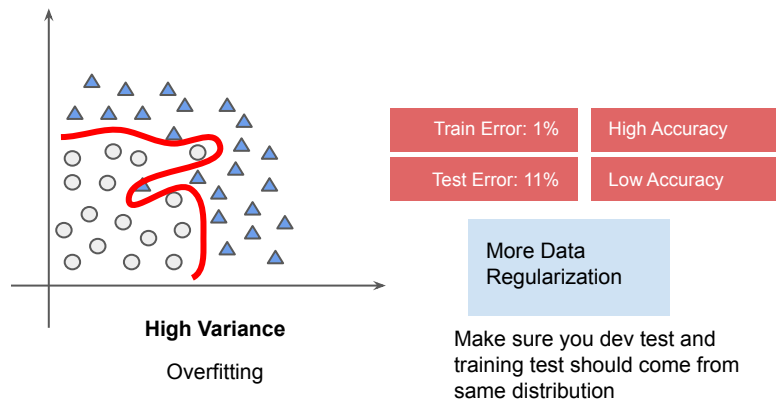
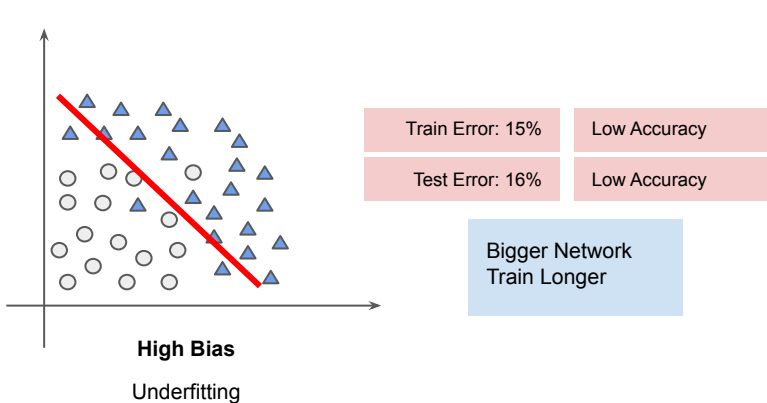


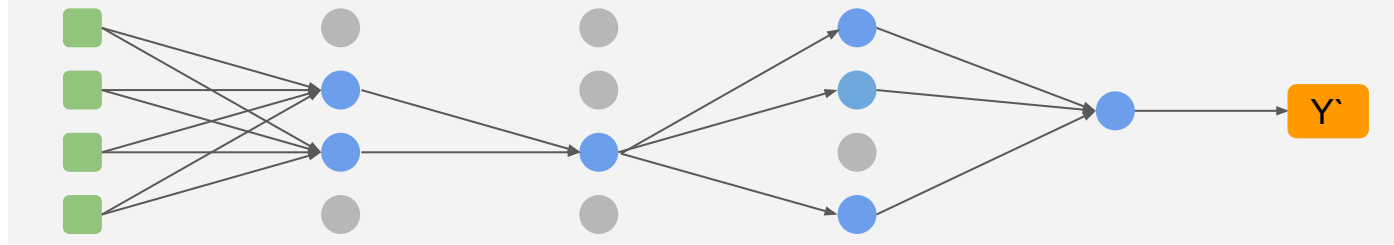
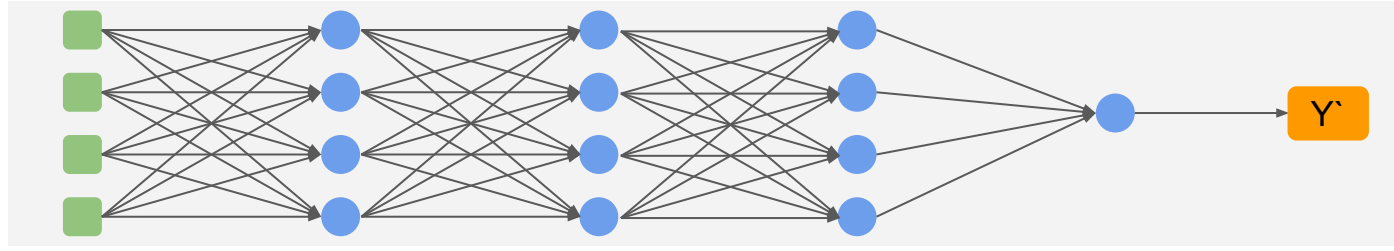
Deep Learning

Week 3

Bias and Variance



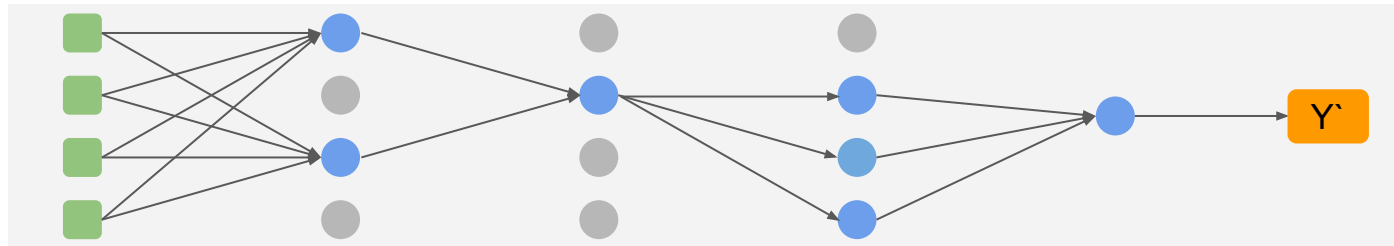
Regularization Drop-Out



drop_prob = 0.5

drop_prob = 0.75

drop_prob = 0.25



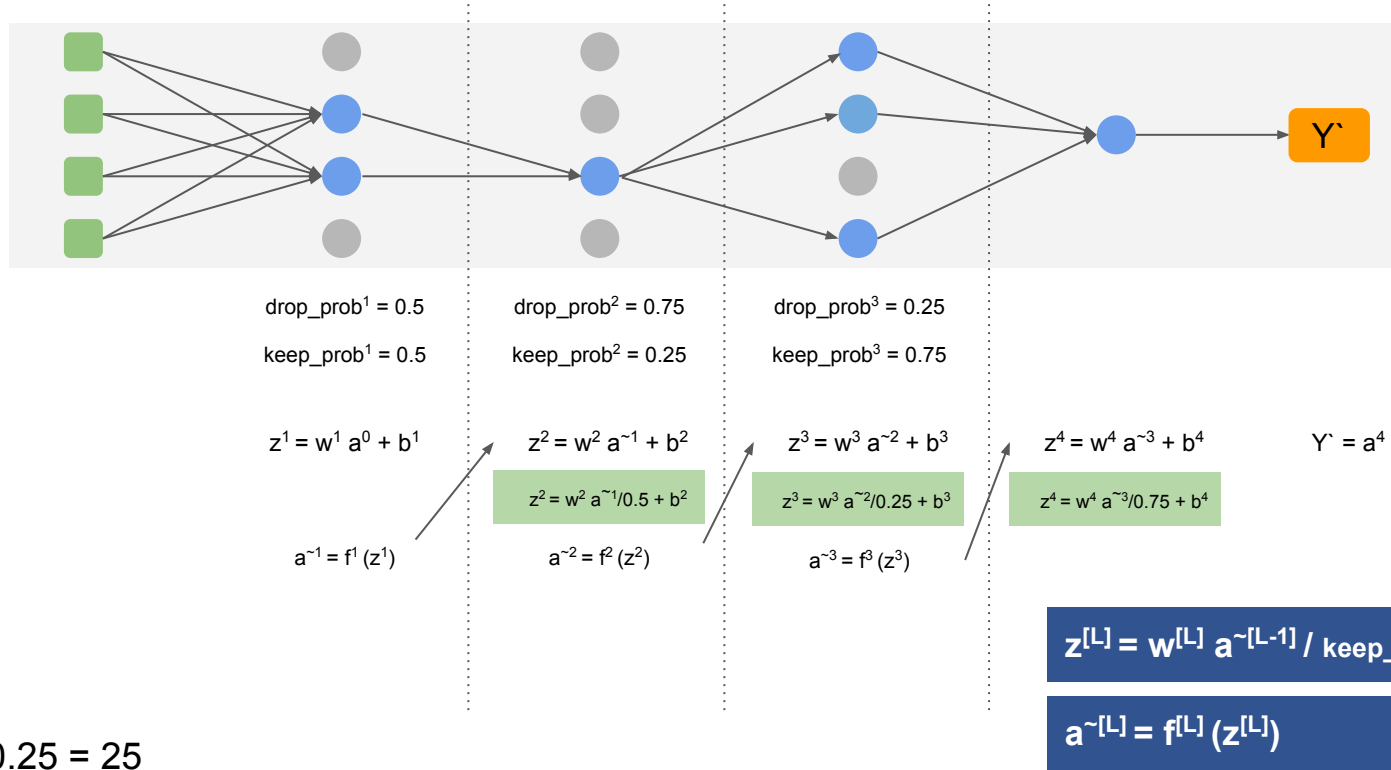
drop_prob = 0.5

drop_prob = 0.75

drop_prob = 0.25



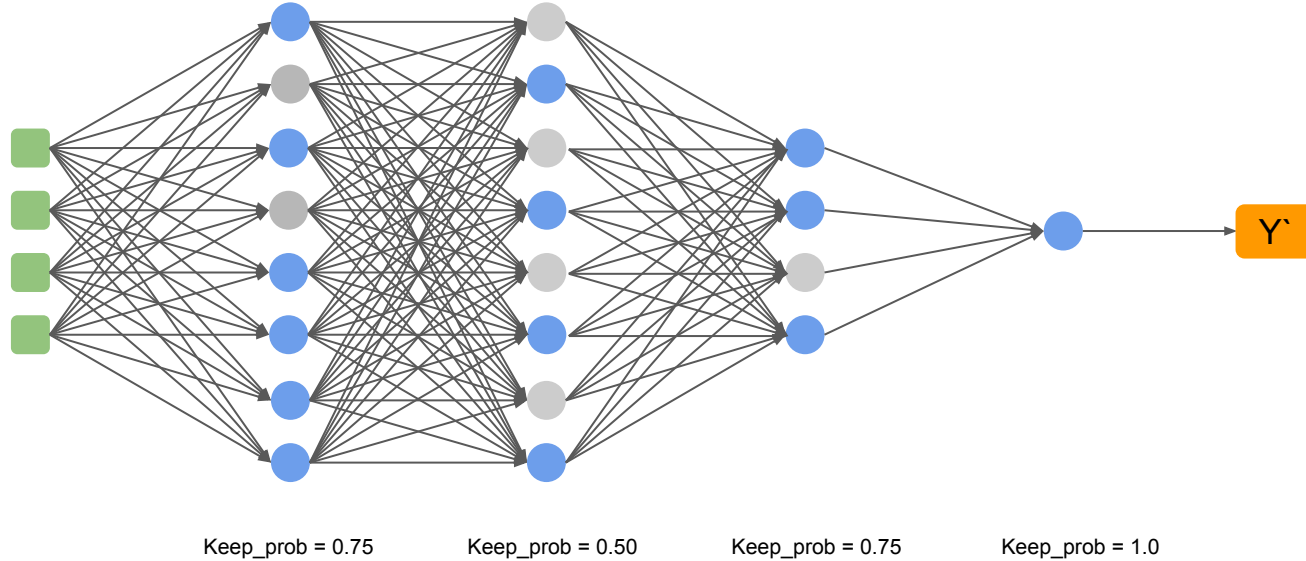
Regularization Drop-Out (Inverted dropout)



$$100 * 0.25 = 25$$

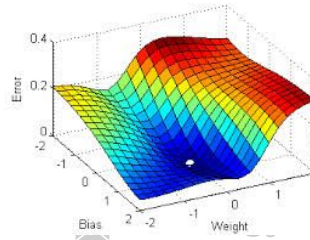
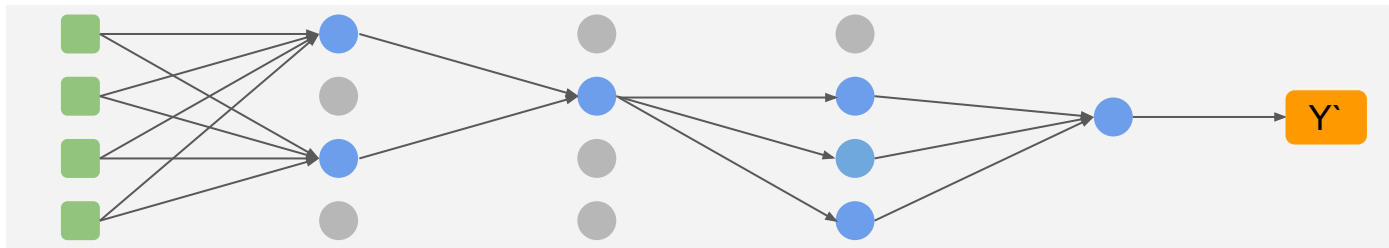
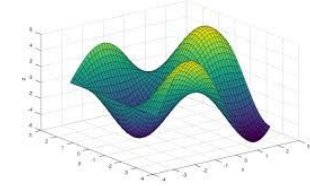
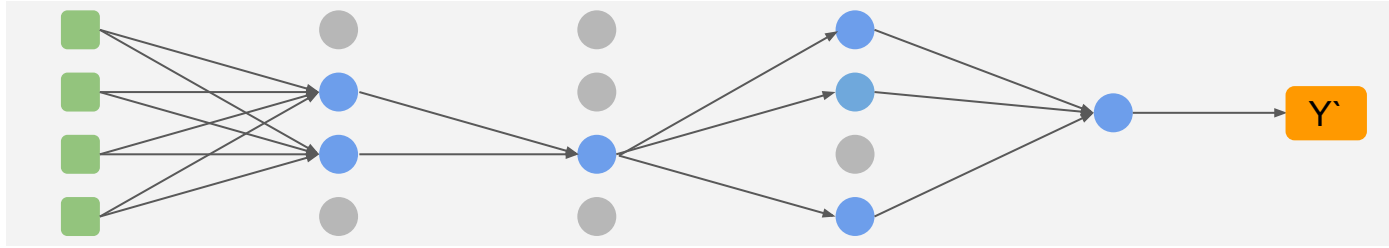
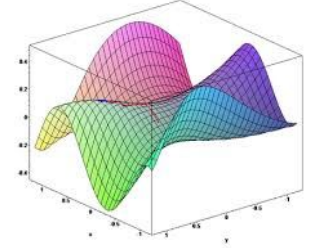
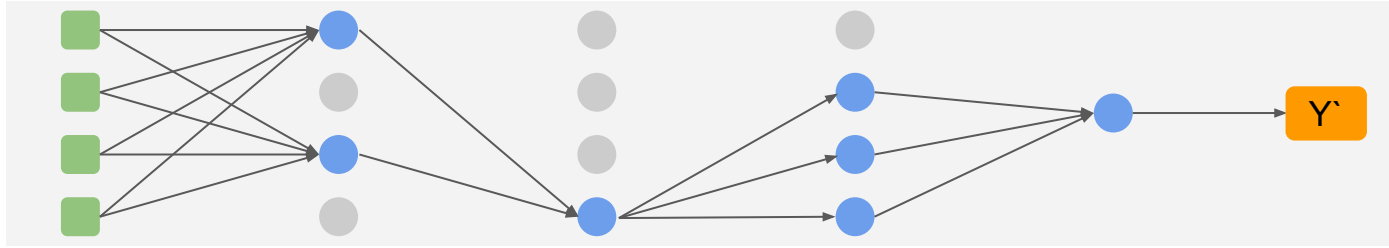
$$25 / 0.25 = 100$$

Regularization Drop-Out (Why it works?)



Can't rely on single feature, so spread out weights.
Reduces overfitting and high variance, since more weights
between layers can cause more learning and so overfitting.

Regularization Drop-Out (Downside - Gradient Descent)



drop_prob = 0.5

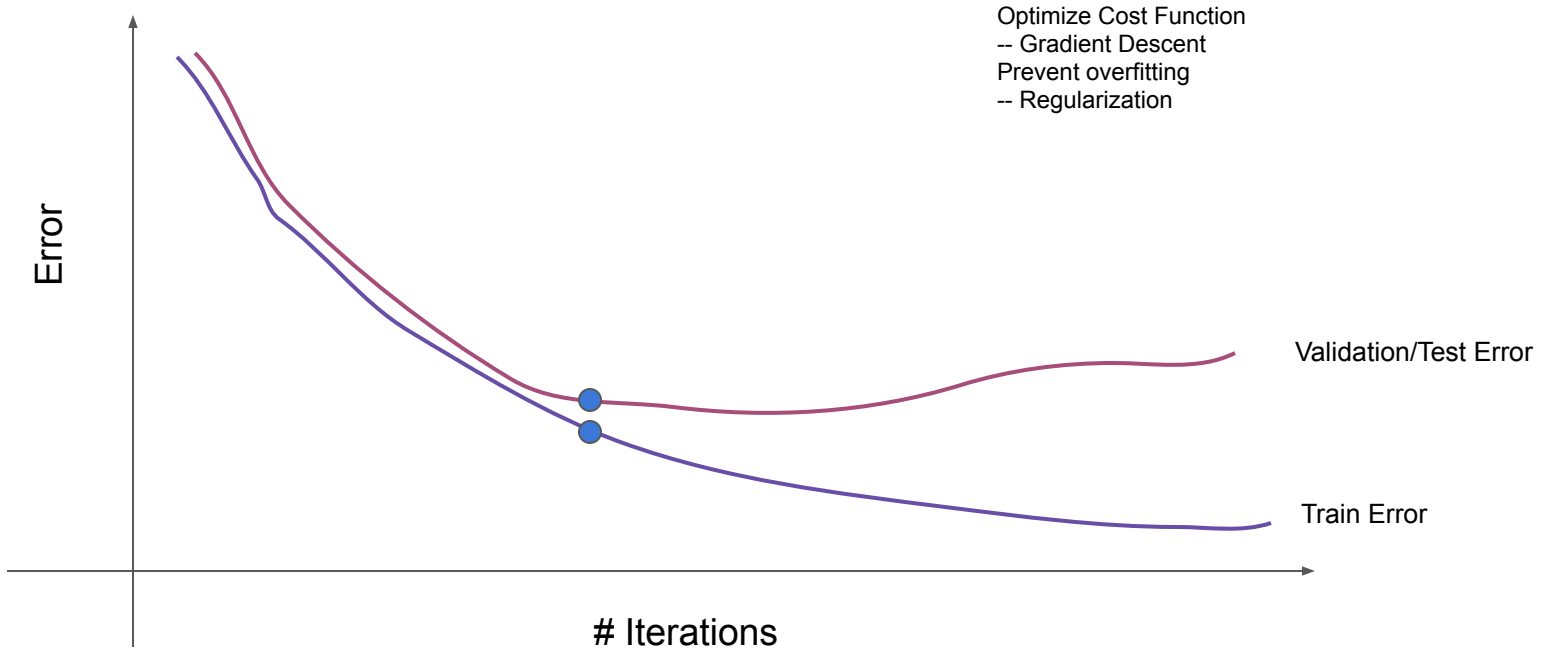
drop_prob = 0.75

drop_prob = 0.25

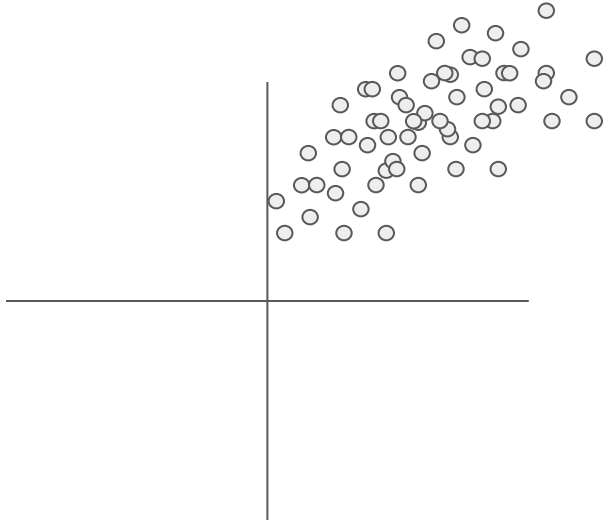


<https://www.linkedin.com/in/arun-kumar-anala-35760523/>

Early Stopping

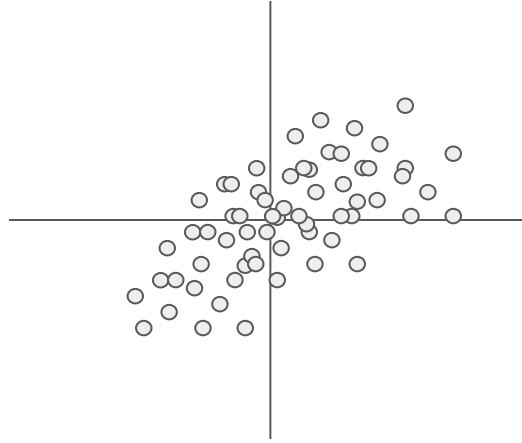


Normalizing a distribution



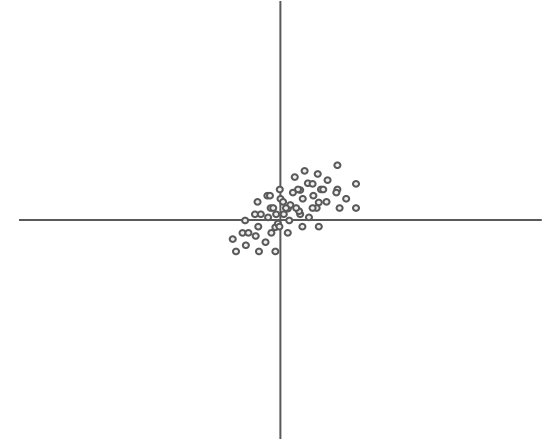
Original Distribution

$$z^1$$



Zero Centered Distribution

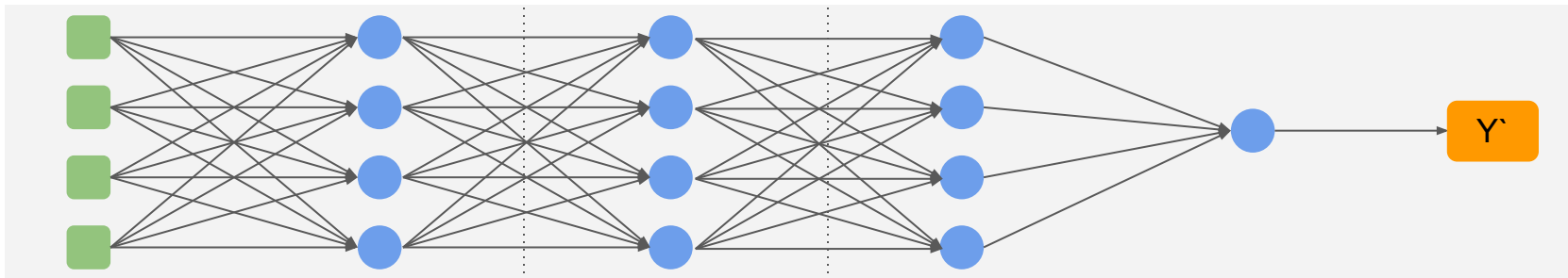
$$(z^1 - \mu)$$



Normalized Distribution

$$(z^1 - \mu^1) / \sqrt{(\sigma^2 + \epsilon)}$$

Batch Normalization



z value

$$z^1 = w^1 a^0 + b^1$$

$$z^2 = w^2 a^1 + b^2$$

$$z^3 = w^3 a^2 + b^3$$

$$z^4 = w^4 a^3 + b^4$$

$$Y^* = a^4$$

Batch
Statistics

mean

$$\mu^1 = 1/m \sum z^1$$

$$\mu^2 = 1/m \sum z^2$$

$$\mu^3 = 1/m \sum z^3$$

Non-Trainable
Parameters

variance

$$\sigma^{2[1]} = 1/m \sum (z^1 - \mu^1)^2$$

$$\sigma^{2[2]} = 1/m \sum (z^2 - \mu^2)^2$$

$$\sigma^{2[3]} = 1/m \sum (z^3 - \mu^3)^2$$

z norm value

$$z^1_{\text{norm}} = (z^1 - \mu^1) / \sqrt{(\sigma^{2[1]} + \epsilon)}$$

$$z^2_{\text{norm}} = (z^2 - \mu^2) / \sqrt{(\sigma^{2[2]} + \epsilon)}$$

$$z^3_{\text{norm}} = (z^3 - \mu^3) / \sqrt{(\sigma^{2[3]} + \epsilon)}$$

z with gamma & beta

Trainable Parameters

$$z^1 = \gamma^1 z^1_{\text{norm}} + \beta^1$$

$$z^2 = \gamma^2 z^2_{\text{norm}} + \beta^2$$

$$z^3 = \gamma^3 z^3_{\text{norm}} + \beta^3$$

Activation value

$$a^1 = f^1(z^1)$$

$$a^2 = f^2(z^2)$$

$$a^3 = f^3(z^3)$$



$$z^1_{\text{norm}} = (z^1 - \mu^1) / \sqrt{(\sigma^{2[1]} + \epsilon)}$$

$$z^1_{\text{norm}} \sqrt{(\sigma^{2[1]} + \epsilon)} = (z^1 - \mu^1)$$

$$z^1_{\text{norm}} \sqrt{(\sigma^{2[1]} + \epsilon)} + \mu^1 = z^1$$

$$z^1 = \sqrt{(\sigma^{2[1]} + \epsilon)} * z^1_{\text{norm}} + \mu^1$$

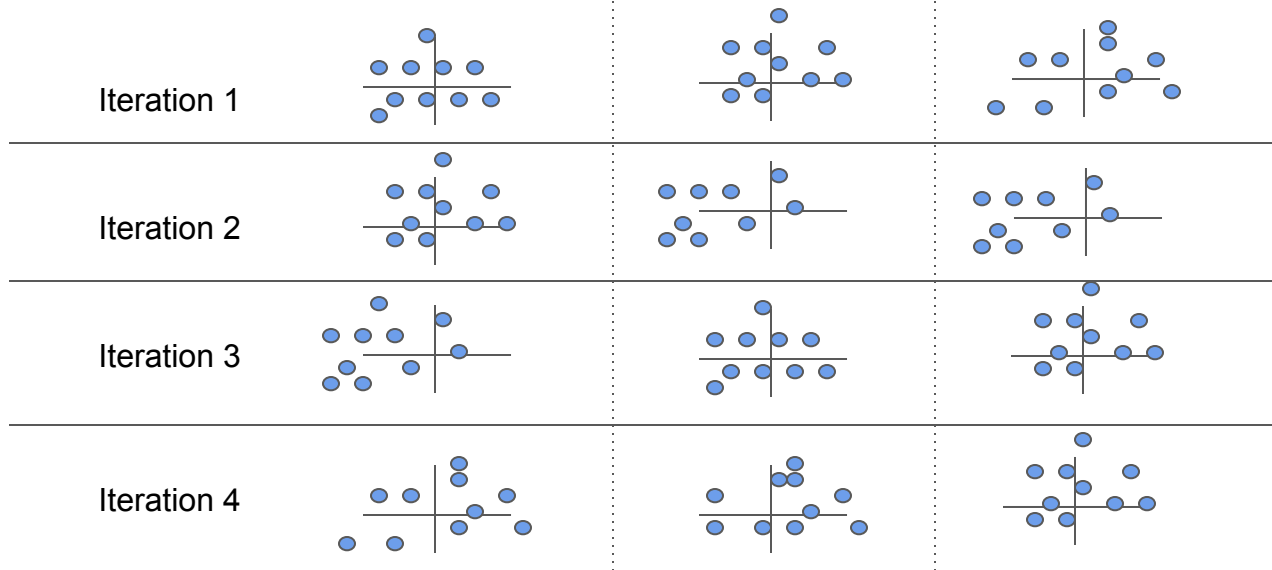
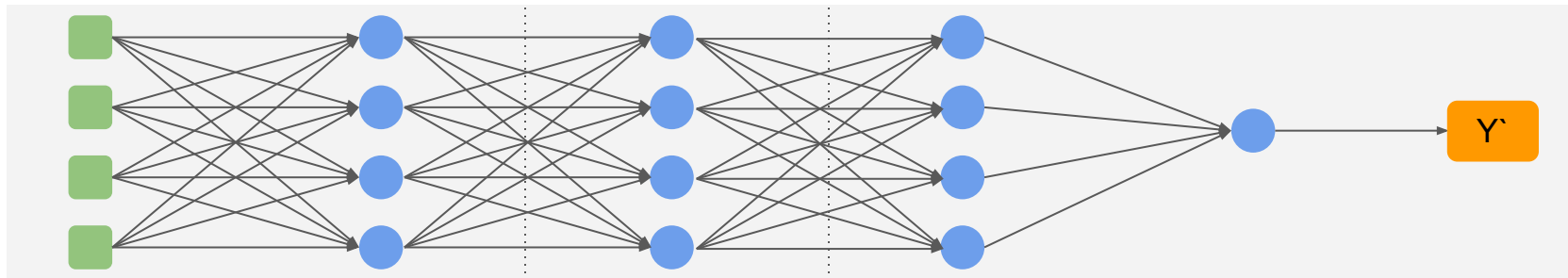
$$z^1 = \gamma^1 z^1_{\text{norm}} + \beta^1$$

$$\gamma^1 = \sqrt{(\sigma^{2[1]} + \epsilon)} \quad \beta^1 = \mu^1$$

$$z^1 = z^1$$

Without Batch Normalization - Covariate Shift

Learning of Shifting Input Distribution



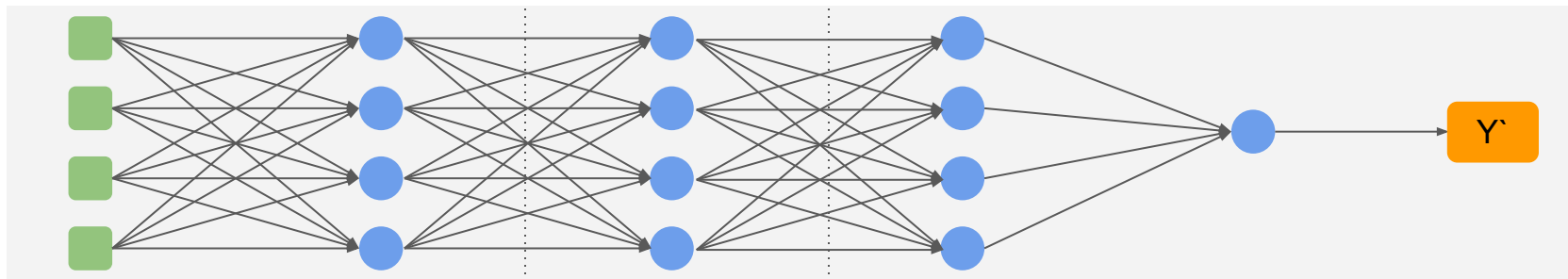
Arun Kumar A
analaarun.k@gmail.com



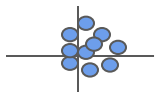
<https://www.linkedin.com/in/arun-kumar-anala-35760523/>

Batch Normalization - Covariate Shift

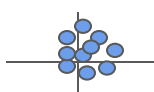
Learning of Shifting Input Distribution



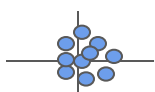
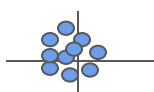
Iteration 1



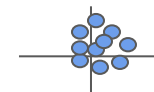
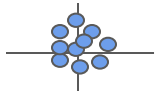
Iteration 2



Iteration 3



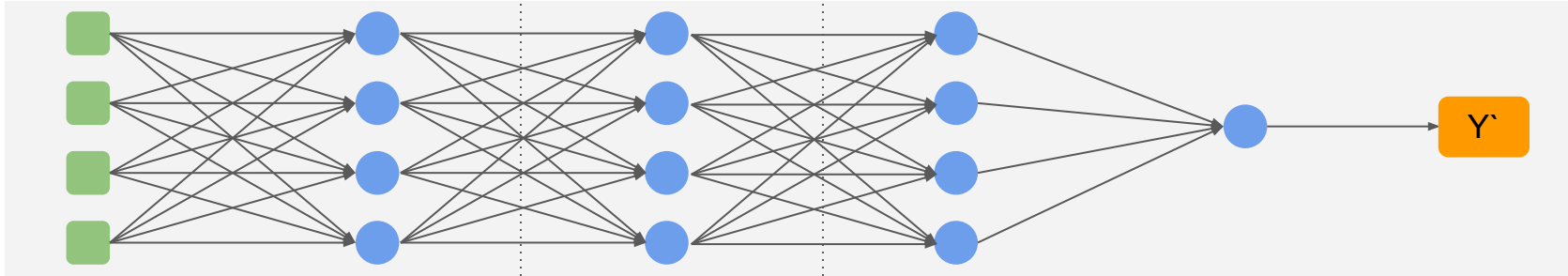
Iteration 4



Less prone to noise of the input.
This makes the learning faster and efficient
Improves gradient flow through the network.

Batch Normalization - Mini batch

Train set : 1000
Mini Batch : 200



1 Epoch or Iteration

| Mini Batch 1 | $\mu^{1[1]}, \sigma^{2[1][1]}$ | $\mu^{2[1]}, \sigma^{2[2][1]}$ | $\mu^{3[1]}, \sigma^{2[3][1]}$ |
|--------------|--------------------------------|--------------------------------|--------------------------------|
| Mini Batch 2 | $\mu^{1[2]}, \sigma^{2[1][2]}$ | $\mu^{2[2]}, \sigma^{2[2][2]}$ | $\mu^{3[2]}, \sigma^{2[3][2]}$ |
| Mini Batch 3 | $\mu^{1[3]}, \sigma^{2[1][3]}$ | $\mu^{2[3]}, \sigma^{2[2][3]}$ | $\mu^{3[3]}, \sigma^{2[3][3]}$ |
| Mini Batch 4 | $\mu^{1[4]}, \sigma^{2[1][4]}$ | $\mu^{2[4]}, \sigma^{2[2][4]}$ | $\mu^{3[4]}, \sigma^{2[3][4]}$ |
| Mini Batch 5 | $\mu^{1[5]}, \sigma^{2[1][5]}$ | $\mu^{2[5]}, \sigma^{2[2][5]}$ | $\mu^{3[5]}, \sigma^{2[3][5]}$ |

Estimate using Exponential
Weighted Average across
mini-batch

$$\mu^1, \sigma^{2[1]}$$

$$\mu^2, \sigma^{2[2]}$$

$$\mu^3, \sigma^{2[3]}$$

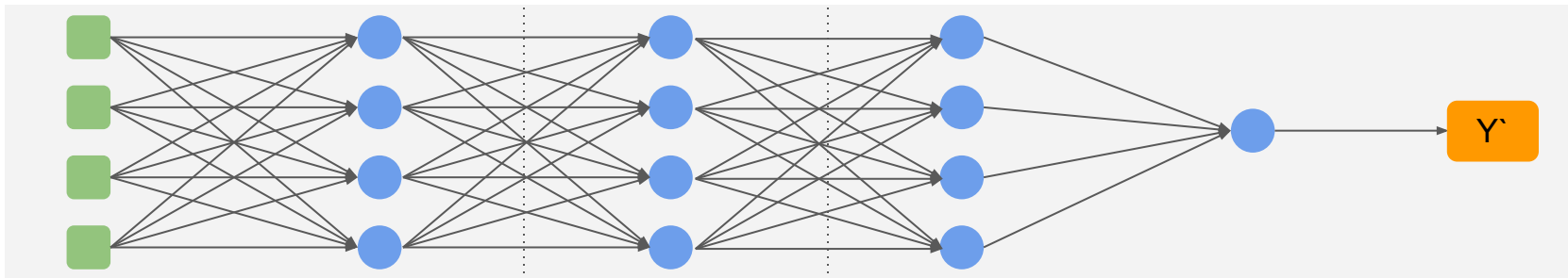
Arun Kumar A
analaarun.k@gmail.com



<https://www.linkedin.com/in/arun-kumar-anala-35760523/>

Batch Normalization - Mini batch

Train set : 1000
Mini Batch : 200



1 Epoch or Iteration

| | | | |
|--------------|--------------------------------|--------------------------------|--------------------------------|
| Mini Batch 1 | $\mu^{1[1]}, \sigma^{2[1][1]}$ | $\mu^{2[1]}, \sigma^{2[2][1]}$ | $\mu^{3[1]}, \sigma^{2[3][1]}$ |
| Mini Batch 2 | $\mu^{1[2]}, \sigma^{2[1][2]}$ | $\mu^{2[2]}, \sigma^{2[2][2]}$ | $\mu^{3[2]}, \sigma^{2[3][2]}$ |
| Mini Batch 3 | $\mu^{1[3]}, \sigma^{2[1][3]}$ | $\mu^{2[3]}, \sigma^{2[2][3]}$ | $\mu^{3[3]}, \sigma^{2[3][3]}$ |
| Mini Batch 4 | $\mu^{1[4]}, \sigma^{2[1][4]}$ | $\mu^{2[4]}, \sigma^{2[2][4]}$ | $\mu^{3[4]}, \sigma^{2[3][4]}$ |
| Mini Batch 5 | $\mu^{1[5]}, \sigma^{2[1][5]}$ | $\mu^{2[5]}, \sigma^{2[2][5]}$ | $\mu^{3[5]}, \sigma^{2[3][5]}$ |

Compute Moving Average

Estimate using Exponential
Weighted Average across
mini-batch

$$\mu^1, \sigma^{2[1]}$$

$$\mu^2, \sigma^{2[2]}$$

$$\mu^3, \sigma^{2[3]}$$



analaarun.k@gmail.com

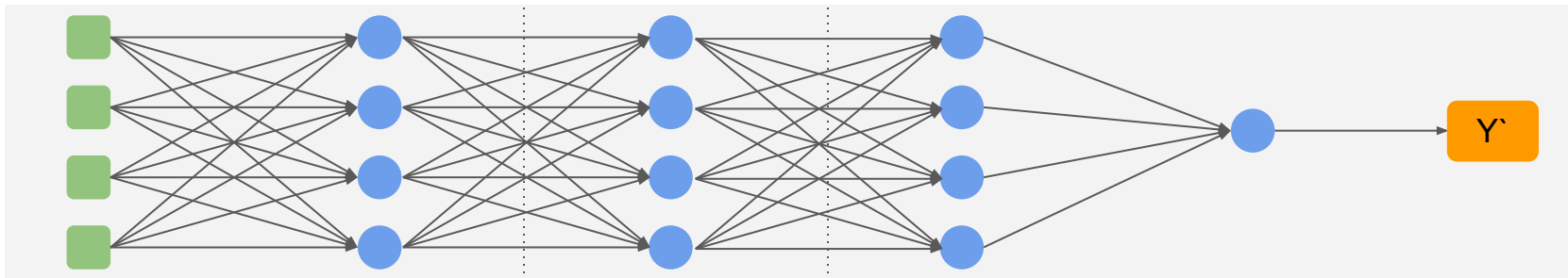


<https://www.linkedin.com/in/arun-kumar-anala-35760523/>

Arun Kumar A

Batch Normalization - Mini batch

Train set : 1000
Mini Batch : 200



1 Epoch or Iteration

| Mini Batch 1 | $\mu^{1[1]}, \sigma^{2[1][1]}$ | $\mu^{2[1]}, \sigma^{2[2][1]}$ | $\mu^{3[1]}, \sigma^{2[3][1]}$ |
|--------------|--------------------------------|--------------------------------|--------------------------------|
| Mini Batch 2 | $\mu^{1[2]}, \sigma^{2[1][2]}$ | $\mu^{2[2]}, \sigma^{2[2][2]}$ | $\mu^{3[2]}, \sigma^{2[3][2]}$ |
| Mini Batch 3 | $\mu^{1[3]}, \sigma^{2[1][3]}$ | $\mu^{2[3]}, \sigma^{2[2][3]}$ | $\mu^{3[3]}, \sigma^{2[3][3]}$ |
| Mini Batch 4 | $\mu^{1[4]}, \sigma^{2[1][4]}$ | $\mu^{2[4]}, \sigma^{2[2][4]}$ | $\mu^{3[4]}, \sigma^{2[3][4]}$ |
| Mini Batch 5 | $\mu^{1[5]}, \sigma^{2[1][5]}$ | $\mu^{2[5]}, \sigma^{2[2][5]}$ | $\mu^{3[5]}, \sigma^{2[3][5]}$ |

Compute Moving Average

Estimate using Exponential
Weighted Average across
mini-batch

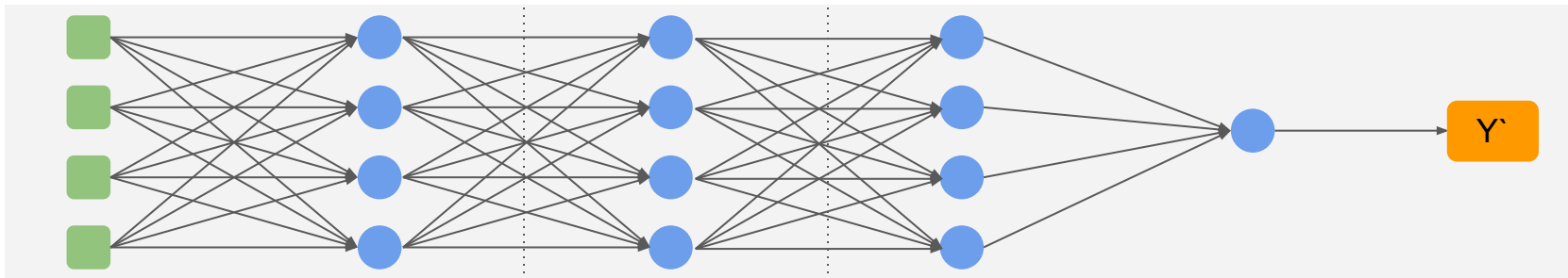
$\mu^1, \sigma^{2[1]}$

$\mu^2, \sigma^{2[2]}$

$\mu^3, \sigma^{2[3]}$

Batch Normalization - Mini batch

Train set : 1000
Mini Batch : 200



1 Epoch or Iteration

| Mini Batch 1 | $\mu^1[1], \sigma^2[1][1]$ | $\mu^2[1], \sigma^2[2][1]$ | $\mu^3[1], \sigma^2[3][1]$ |
|--------------|----------------------------|----------------------------|----------------------------|
| Mini Batch 2 | $\mu^1[2], \sigma^2[1][2]$ | $\mu^2[2], \sigma^2[2][2]$ | $\mu^3[2], \sigma^2[3][2]$ |
| Mini Batch 3 | $\mu^1[3], \sigma^2[1][3]$ | $\mu^2[3], \sigma^2[2][3]$ | $\mu^3[3], \sigma^2[3][3]$ |
| Mini Batch 4 | $\mu^1[4], \sigma^2[1][4]$ | $\mu^2[4], \sigma^2[2][4]$ | $\mu^3[4], \sigma^2[3][4]$ |
| Mini Batch 5 | $\mu^1[5], \sigma^2[1][5]$ | $\mu^2[5], \sigma^2[2][5]$ | $\mu^3[5], \sigma^2[3][5]$ |

Compute Moving Average

Estimate using Exponential
Weighted Average across
mini-batch

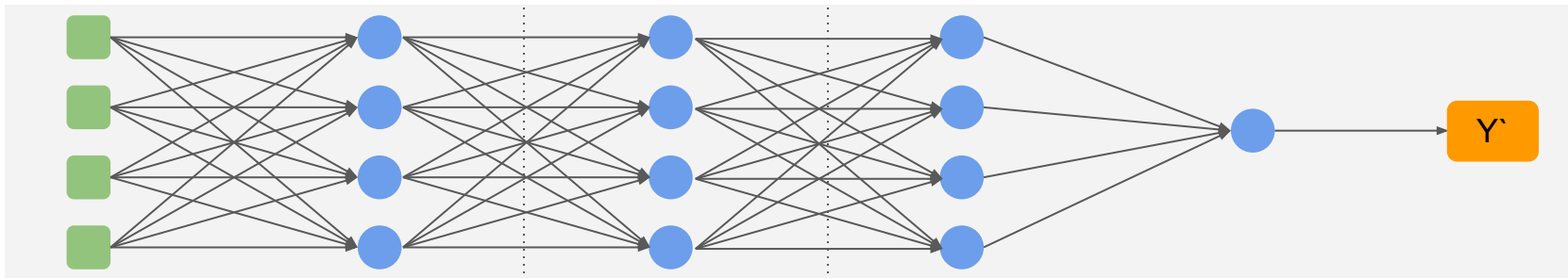
$\mu^1, \sigma^2[1]$

$\mu^2, \sigma^2[2]$

$\mu^3, \sigma^2[3]$

Batch Normalization - Mini batch

Train set : 1000
Mini Batch : 200



1 Epoch or Iteration

| Mini Batch 1 | $\mu^1[1], \sigma^2[1][1]$ | $\mu^2[1], \sigma^2[2][1]$ | $\mu^3[1], \sigma^2[3][1]$ |
|--------------|----------------------------|----------------------------|----------------------------|
| Mini Batch 2 | $\mu^1[2], \sigma^2[1][2]$ | $\mu^2[2], \sigma^2[2][2]$ | $\mu^3[2], \sigma^2[3][2]$ |
| Mini Batch 3 | $\mu^1[3], \sigma^2[1][3]$ | $\mu^2[3], \sigma^2[2][3]$ | $\mu^3[3], \sigma^2[3][3]$ |
| Mini Batch 4 | $\mu^1[4], \sigma^2[1][4]$ | $\mu^2[4], \sigma^2[2][4]$ | $\mu^3[4], \sigma^2[3][4]$ |
| Mini Batch 5 | $\mu^1[5], \sigma^2[1][5]$ | $\mu^2[5], \sigma^2[2][5]$ | $\mu^3[5], \sigma^2[3][5]$ |

In Exponential Weighted Average, high weightage is given to new values

Compute Moving Average

Estimate using Exponential Weighted Average across mini-batch

$\mu^1, \sigma^2[1]$

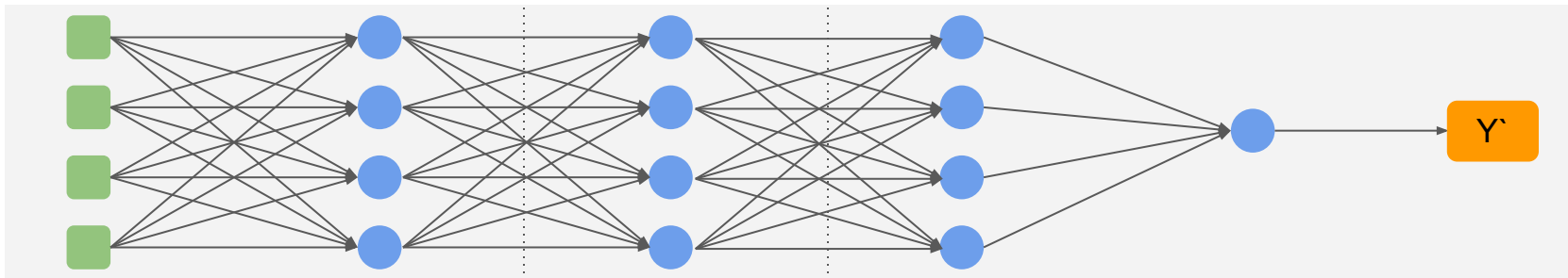
$\mu^2, \sigma^2[2]$

$\mu^3, \sigma^2[3]$

These values would be finally used for testing to compute the z_{norm} value

Batch Normalization - Mini batch

Train set : 1000
Mini Batch : 200



1 Epoch or Iteration

| Mini Batch 1 | $\mu^{1[1]}, \sigma^{2[1][1]}$ | $\mu^{2[1]}, \sigma^{2[2][1]}$ | $\mu^{3[1]}, \sigma^{2[3][1]}$ |
|--------------|--------------------------------|--------------------------------|--------------------------------|
| Mini Batch 2 | $\mu^{1[2]}, \sigma^{2[1][2]}$ | $\mu^{2[2]}, \sigma^{2[2][2]}$ | $\mu^{3[2]}, \sigma^{2[3][2]}$ |
| Mini Batch 3 | $\mu^{1[3]}, \sigma^{2[1][3]}$ | $\mu^{2[3]}, \sigma^{2[2][3]}$ | $\mu^{3[3]}, \sigma^{2[3][3]}$ |
| Mini Batch 4 | $\mu^{1[4]}, \sigma^{2[1][4]}$ | $\mu^{2[4]}, \sigma^{2[2][4]}$ | $\mu^{3[4]}, \sigma^{2[3][4]}$ |
| Mini Batch 5 | $\mu^{1[5]}, \sigma^{2[1][5]}$ | $\mu^{2[5]}, \sigma^{2[2][5]}$ | $\mu^{3[5]}, \sigma^{2[3][5]}$ |

Estimate using Exponential
Weighted Average across
mini-batch

$$\mu^1, \sigma^{2[1]}$$

$$\mu^2, \sigma^{2[2]}$$

$$\mu^3, \sigma^{2[3]}$$

The Batch Normalization in Mini batch generates noise which propagates to next hidden layers, so similar to drop out it has slight regularization effect.

So less records in mini-batch, more noise it adds to the next layer.