

# Convolutional Neural Network - 1

CNN or ConvNet

# Computer Vision Challenges

## Image Classification



Is it Cat?  
[0/1]

## Object Detection



## Neural Style Transfer

Content Image +  
Style Image =  
Generated Image



# Image - RGB vs B&W



3	4	6	2	1	7
3	5	4	9	8	8
1	7	5	4	6	6
7	2	6	5	4	5
6	1	7	7	2	4
5	3	3	1	4	3

6 x 6 x 1



		3	4	6	2	1	7	
	3	4	6	2	1	7		8
3	4	6	2	1	7	8		6
3	5	4	9	8	8	6		5
1	7	5	4	6	6	5		4
7	2	6	5	4	5	4		3
6	1	7	7	2	4	3		
5	3	3	1	4	3			

6 x 6 x 3

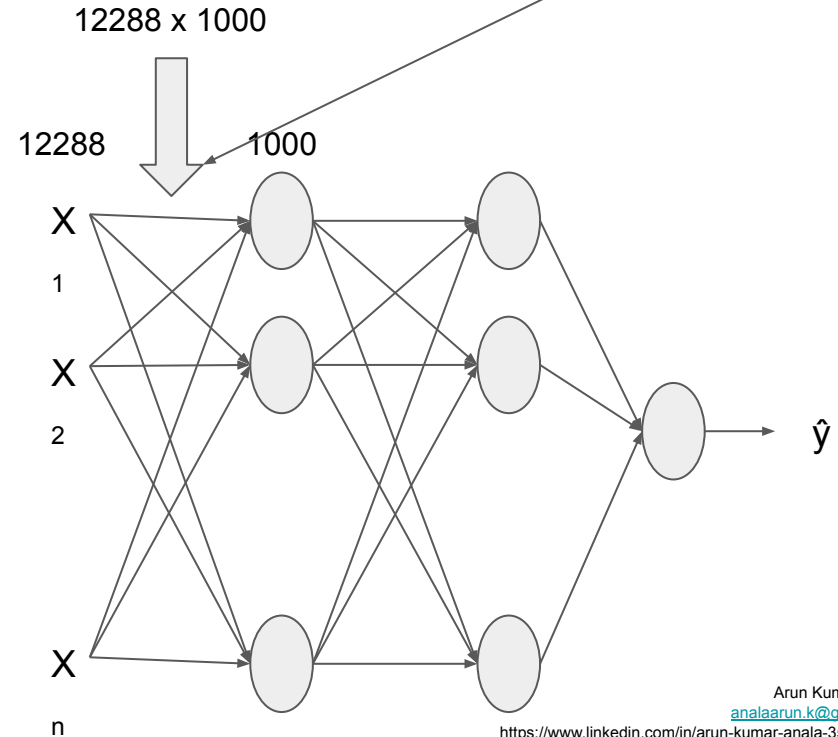
# Fully Connected Network on Images

## Image Classification



Is it Cat?  
[0/1]

$$64 \times 64 \times 3 \\ = 12288$$



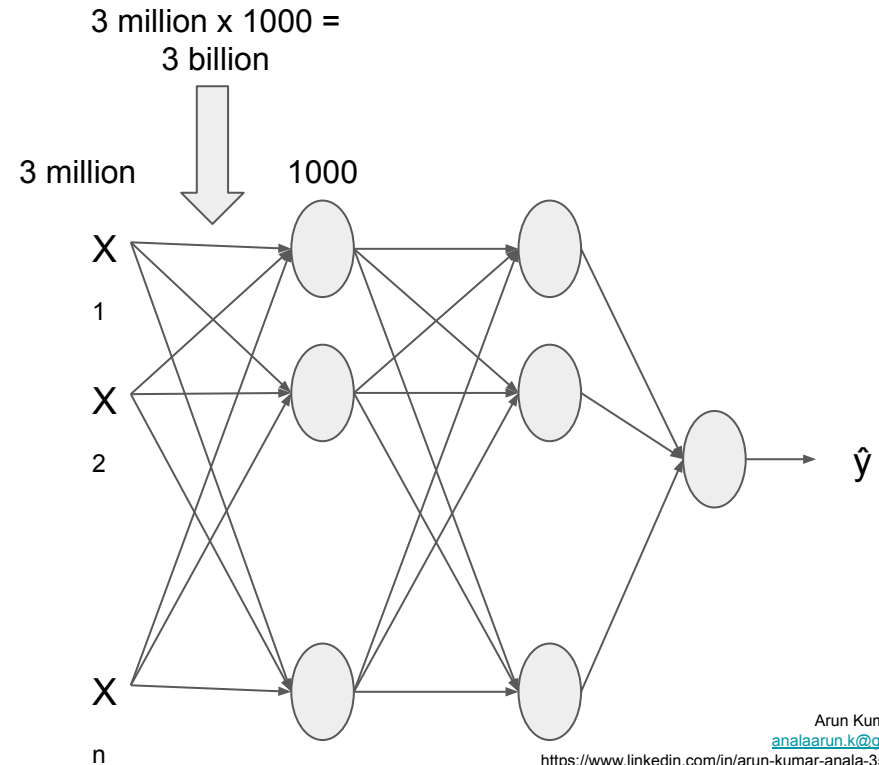
# Fully Connected Network on Images

## Image Classification



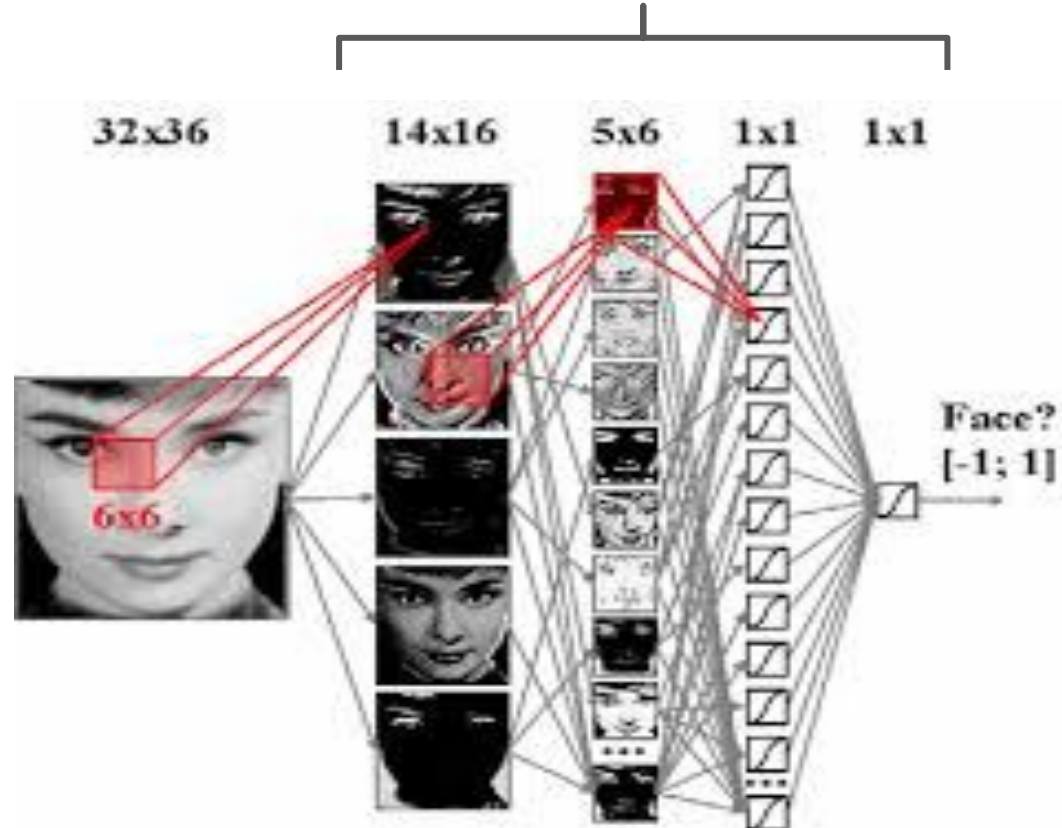
Is it Cat?  
[0/1]

$$1000 \times 1000 \times 3 \\ = 3 \text{ million}$$



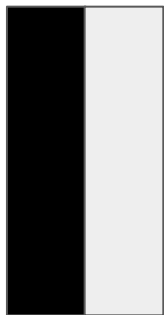
# CNN - High Level

Filter helps in detecting patterns in images



# Filter - I know it

## Detect Vertical Edge



10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

\*

1	0	-1
1	0	-1
1	0	-1

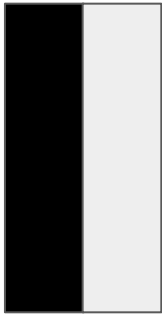
=

0			

$$[10 \times 1 + 10 \times 0 + 10 \times (-1)] + [10 \times 1 + 10 \times 0 + 10 \times (-1)] + [10 \times 1 + 10 \times 0 + 10 \times (-1)] = 0$$

# Filter - I know it

## Detect Vertical Edge



10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

\*

1	0	-1
1	0	-1
1	0	-1

=

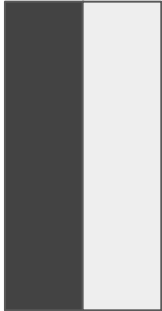
0	30		

$$[10 \times 1 + 10 \times 0 + 0 \times (-1)] + [10 \times 1 + 10 \times 0 + 0 \times (-1)] + [10 \times 1 + 10 \times 0 + 0 \times (-1)] = 30$$



# Filter - I know it

## Detect Vertical Edge



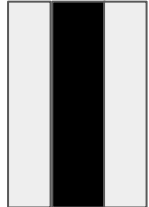
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0
10	10	10	0	0	0

\*

1	0	-1
1	0	-1
1	0	-1

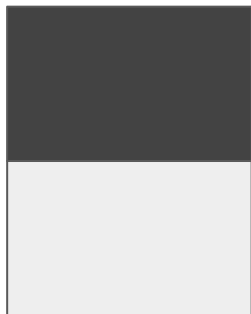
=

0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0



# Filter - I know it

## Detect Horizontal Edge



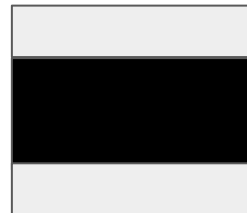
10	10	10	10	10	10
10	10	10	10	10	10
10	10	10	10	10	10
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

\*

1	1	1
0	0	0
-1	-1	-1

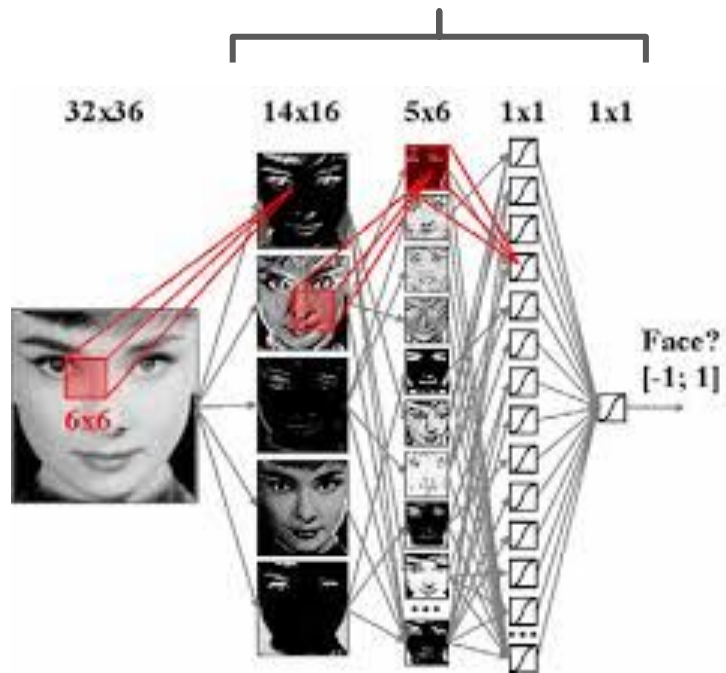
=

0	0	0	0
30	30	30	30
30	30	30	30
0	0	0	0



# Filter - I don't know, but let CNN learn it

Filter helps in detecting patterns in images



## Detect Pattern in Images

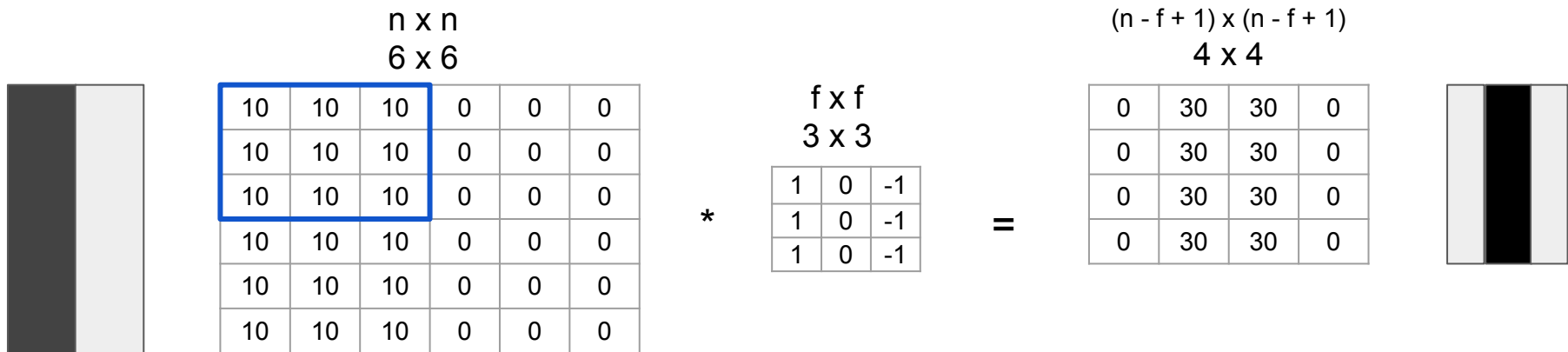
112	76	144	55	43	23
164	43	43	34	25	13
12	13	30	67	44	14
6	45	24	77	87	89
64	64	60	67	65	67
34	35	54	60	44	56



$$* \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} = \text{[Filtered Image]}$$

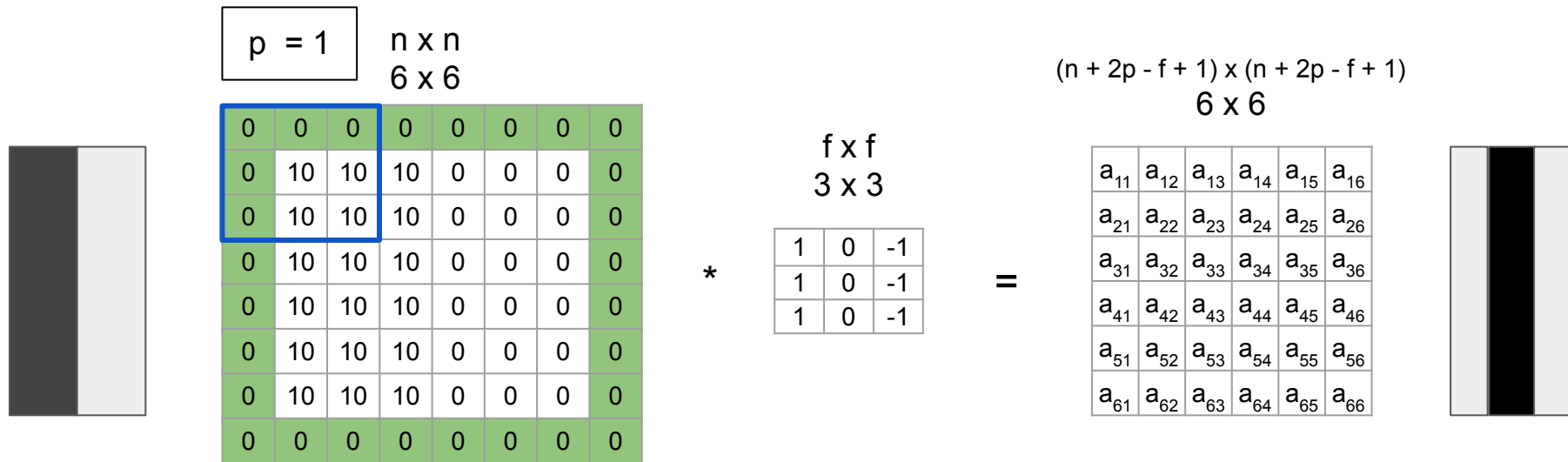
?

# Valid Convolution



Output size is less than the input size

# Same Convolution - Padding



Pad so that the output size is same as the input size

# Strided Convolution

s = 2		n x n 7 x 7				
10	10	10	0	0	0	0
10	10	10	0	0	0	0
10	10	10	0	0	0	0
10	10	10	0	0	0	0
10	10	10	0	0	0	0
10	10	10	0	0	0	0

$$\left[ \frac{(n + 2p - f)}{s} + 1 \right] \times \left[ \frac{(n + 2p - f)}{s} + 1 \right]$$

3 x 3

a <sub>11</sub>	a <sub>12</sub>	a <sub>13</sub>
a <sub>21</sub>	a <sub>22</sub>	a <sub>23</sub>
a <sub>31</sub>	a <sub>32</sub>	a <sub>33</sub>

f x f 3 x 3		
1	0	-1
1	0	-1
1	0	-1

\*

=

# Filter for RGB

		3	4	6	2	1	7
	3	4	6	2	1	7	8
3	4	6	2	1	7	8	6
3	5	4	9	8	8	6	5
1	7	5	4	6	6	5	4
7	2	6	5	4	5	4	3
6	1	7	7	2	4	3	
5	3	3	1	4	3		

6 x 6 x 3

\*

		1	0	-1
	1	0	-1	-1
1	0	-1	-1	-1
1	0	-1	-1	
1	0	-1		

3 x 3 x 3

---

0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0

4 x 4

# Filter for RGB

		1	0	-1
	1	0	-1	-1
1	0	-1	-1	-1
1	0	-1	-1	
1	0	-1		

0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0

0	30	30	0
0	30	30	0
0	30	30	0
0	30	30	0

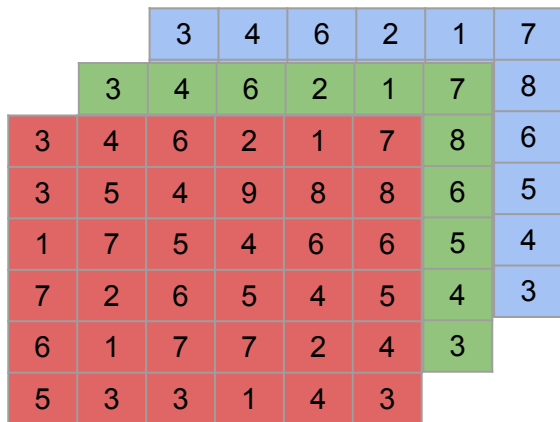
						3	4	6	2	1	7
						3	5	4	9	8	8
						1	7	5	4	6	6
			3	4	6	2	1	7	5	4	5
			3	5	4	9	8	8	7	2	4
			1	7	5	4	6	6	1	4	3
3	4	6	2	1	7	5	4	5			
3	5	4	9	8	8	7	2	4			
1	7	5	4	6	6	1	4	3			
7	2	6	5	4	5						
6	1	7	7	2	4						
5	3	3	1	4	3						

						3	4	6	2	1	7
						3	5	4	9	8	8
						1	7	5	4	6	6
			3	4	6	2	1	7	5	4	5
			3	5	4	9	8	8	7	2	4
			1	7	5	4	6	6	1	4	3
3	4	6	2	1	7	5	4	5			
3	5	4	9	8	8	7	2	4			
1	7	5	4	6	6	1	4	3			
7	2	6	5	4	5						
6	1	7	7	2	4						
5	3	3	1	4	3						

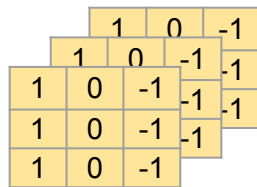
Arun Kumar Ar



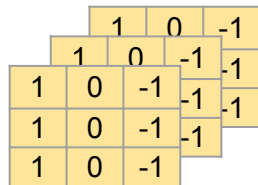
# Multiple Filters for RGB


$$6 \times 6 \times 3$$

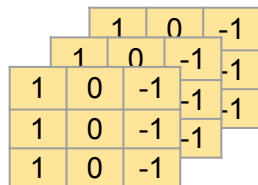
\*



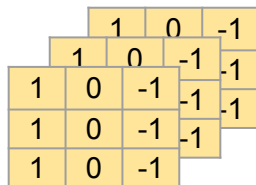
3 x 3 x 3



3 x 3 x 3

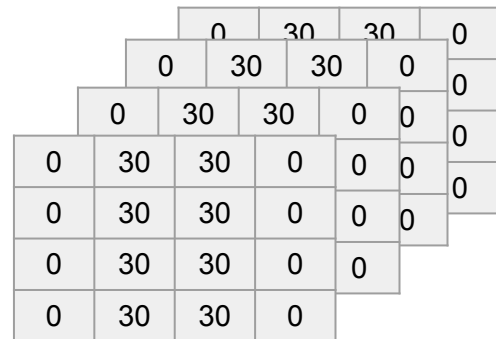


3 x 3 x 3



3 x 3 x 3

---



4 x 4 x 4

# Question

In a layer, if you have 20 filters of dimension 3x3x3 each, then how many parameter does that layer have?

For 1 filter

$3 \times 3 \times 3 = 27$  weights

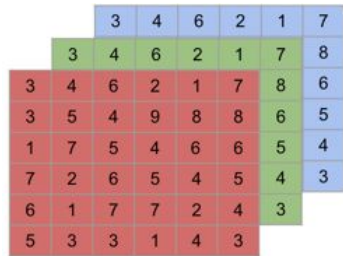
1 = bias

So 28 parameter for 1 layer

For 20 filters in a layer

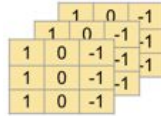
$28 \times 20 = \mathbf{560}$  parameters

# Example of Convolutional layer

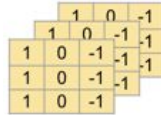


6 x 6 x 3

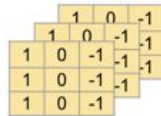
108



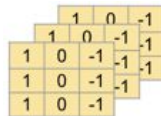
3 x 3 x 3



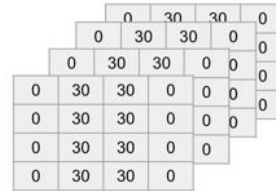
3 x 3 x 3



3 x 3 x 3

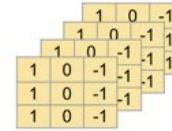


3 x 3 x 3

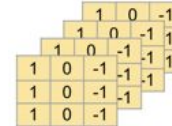


4 x 4 x 4

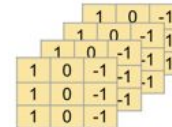
64



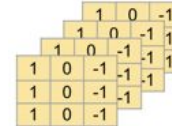
3 x 3 x 4



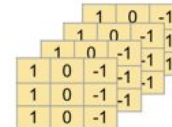
3 x 3 x 4



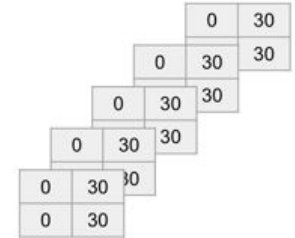
3 x 3 x 4



3 x 3 x 4



3 x 3 x 4

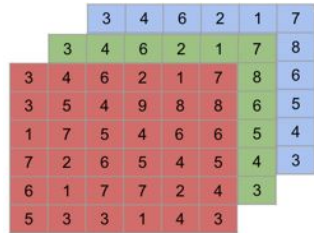


2 x 2 x 5

20

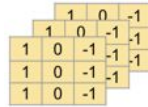
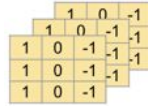
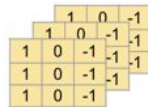
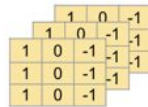
Length Decreases  
Width Decreases  
Depth **Increases**

Input : (L-1)

 $6 \times 6 \times 3$ 

$$n^{L-2}_H \times n^{L-2}_W \times n^{L-2}_C$$

Layer : (L-1)

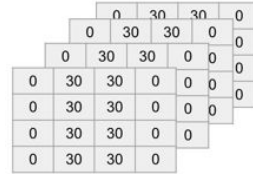
 $3 \times 3 \times 3$  $3 \times 3 \times 3$  $3 \times 3 \times 3$  $3 \times 3 \times 3$ 

$$f^{L-1}_H \times f^{L-1}_W \times n^{L-2}_C \times n^{L-1}_C$$

 $s^{L-1}$  $p^{L-1}$ 

Output : (L-1)

Input : (L)

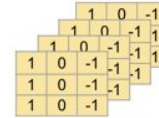
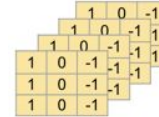
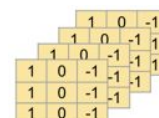
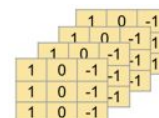
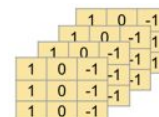
 $4 \times 4 \times 4$ 

$$n^{L-1}_H \times n^{L-1}_W \times n^{L-1}_C$$

$$n^{L-1}_H = n^{L-2}_H - f^{L-1}_H + 1$$

$$n^{L-1}_H = (n^{L-2}_H + 2p^{L-1} - f^{L-1}_H) / s^{L-1} + 1$$

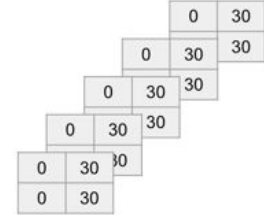
Layer : (L)

 $3 \times 3 \times 4$  $3 \times 3 \times 4$  $3 \times 3 \times 4$  $3 \times 3 \times 4$  $3 \times 3 \times 4$ 

$$f^L_H \times f^L_W \times n^{L-1}_C \times n^L_C$$

 $s^L$  $p^L$ 

Output : (L)

 $2 \times 2 \times 5$ 

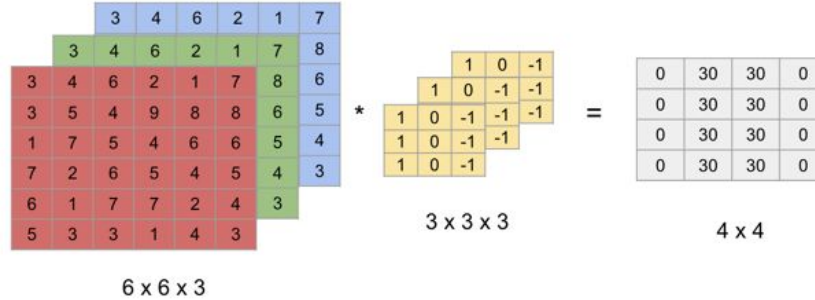
$$n^L_H \times n^L_W \times n^L_C$$

$$n^L_H = n^{L-1}_H - f^L_H + 1$$

$$n^L_H = (n^{L-1}_H + 2p^L - f^L_H) / s^L + 1$$

# Types of layer in Convolutional Network

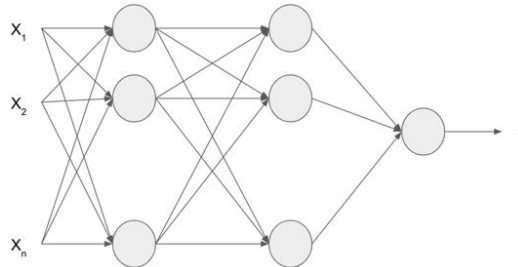
Convolution



Pool



Fully  
Connected



# Max Pooling Layer

3	4	6	2	1	7
3	5	4	9	8	8
1	7	5	4	6	6
7	2	6	5	4	5
6	1	7	7	2	4
5	3	3	1	4	3

6 x 6 x 1

f = 3



7	9
7	7

2 x 2 x 1

# Max Pooling Layer

			3	4	6	2	1	7	
		3	4	6	2	1	7	8	
	3	4	6	2	1	7	8	6	
3	5	4	9	8	8	6	5		
1	7	5	4	6	6	5	4		
7	2	6	5	4	5	4	3		
6	1	7	7	2	4	3			
5	3	3	1	4	3				

$6 \times 6 \times 3$

$f = 3$



			7		9	
		7		9		
	7		9			
7		9				
7		7				

$2 \times 2 \times 3$

# Average Pooling Layer

3	4	6	2	1	7
3	5	4	9	8	8
1	7	5	4	6	6
7	2	6	5	4	5
6	1	7	7	2	4
5	3	3	1	4	3

6 x 6 x 1

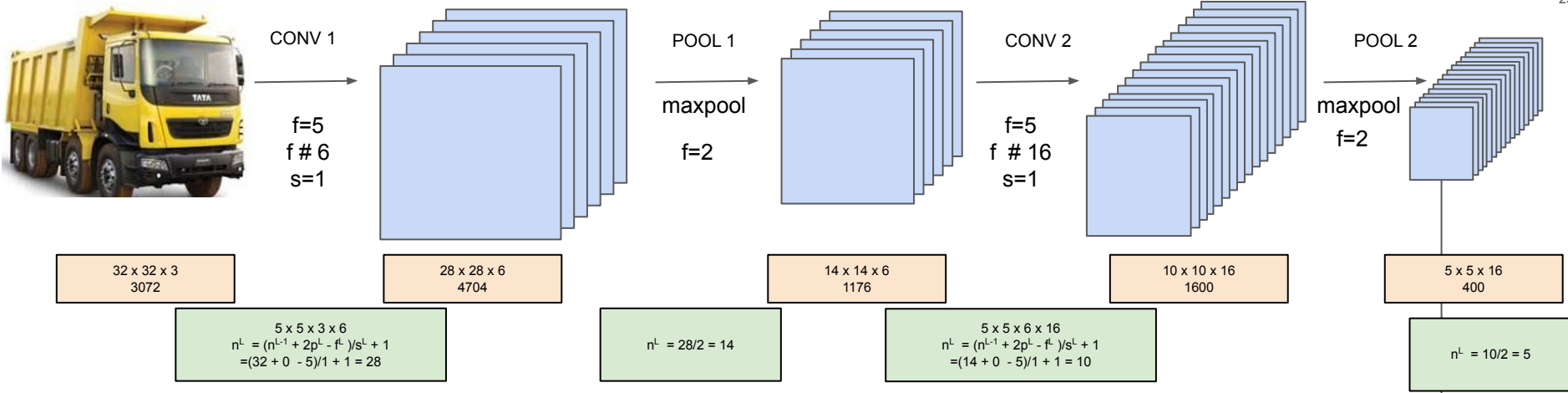
f = 3



4.22	5.66
4.44	3.89

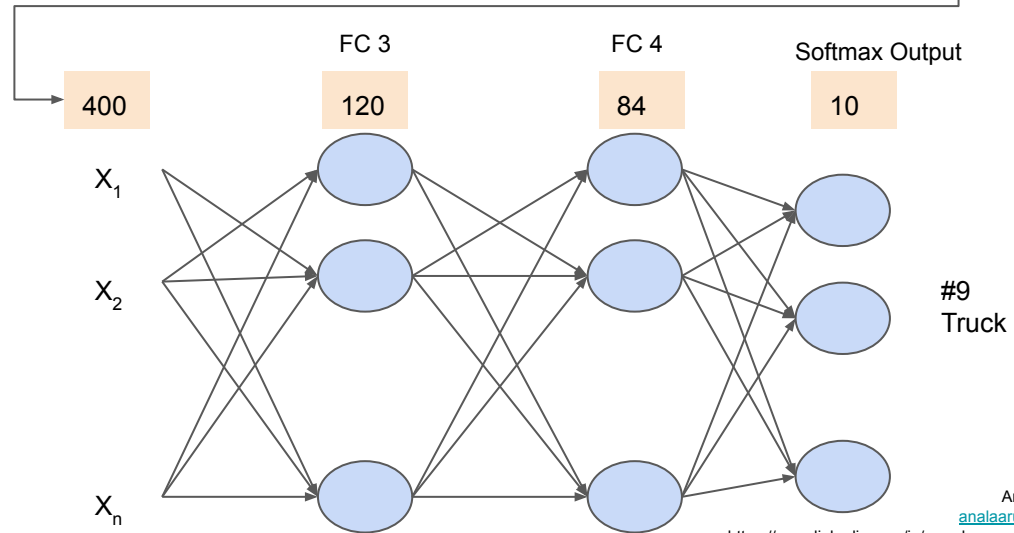
2 x 2 x 1





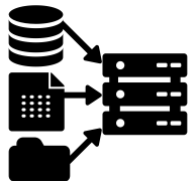
# CNN Example

## Image Classification using CIFAR10 dataset



# Exercise Image Classification

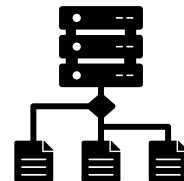
## Load Dataset



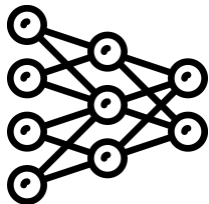
## Analyze Dataset



## Prepare Dataset



## Define Model



## Train & Evaluate Model



## Summarize Results



## Predict New Image



# Model Summary

Layer (type)	Filter/Kernel for CNN Neurons for FCN	Output Shape	Params #	Comments
Input		(32, 32, 3)		
CONV 1	$5 \times 5 \times 3 \times 6$	(28, 28, 6)	456	$(5 \times 5 \times 3 + 1) \times 6$
POOL 1	$2 \times 2$	(14, 14, 6)	0	
CONV 2	$5 \times 5 \times 6 \times 16$	(10, 10, 16)	2416	$(5 \times 5 \times 6 + 1) \times 16$
POOL 2	$2 \times 2$	(5, 5, 16)	0	
FC 1	$5 \times 5 \times 16 = 400$	120	48120	$(400 + 1) \times 120$
FC 2	120	84	10164	$(120 + 1) \times 84$
Softmax Output	84	10	850	$(84 + 1) \times 10$
Total			62,006	

# CNN vs Fully Connected Network on Images

## Image Classification



$64 \times 64 \times 3$   
 $= 12288$

Is it Cat?  
[0/1]



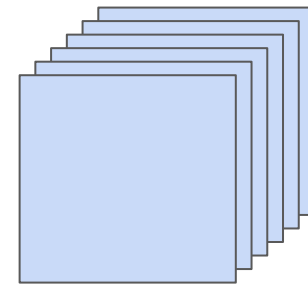
$64 \times 64 \times 3 =$   
 $12288$

Filters# 6  
 $5 \times 5 \times 3$

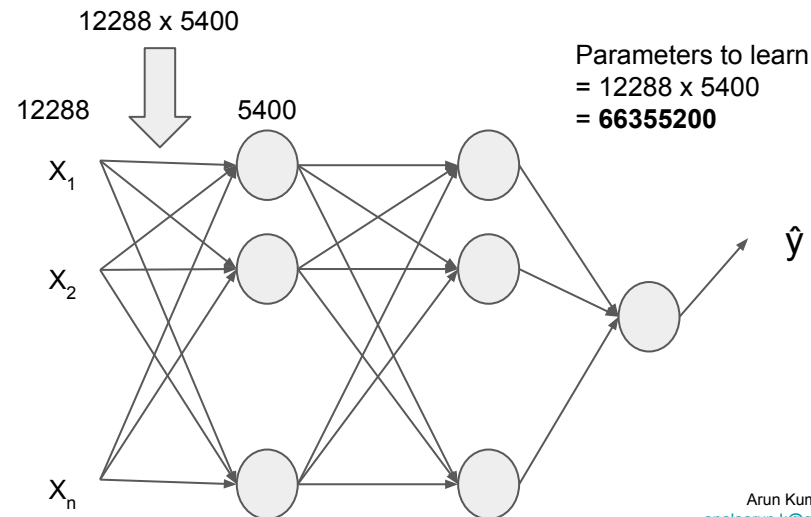
Strides = 2

Parameters to  
learn

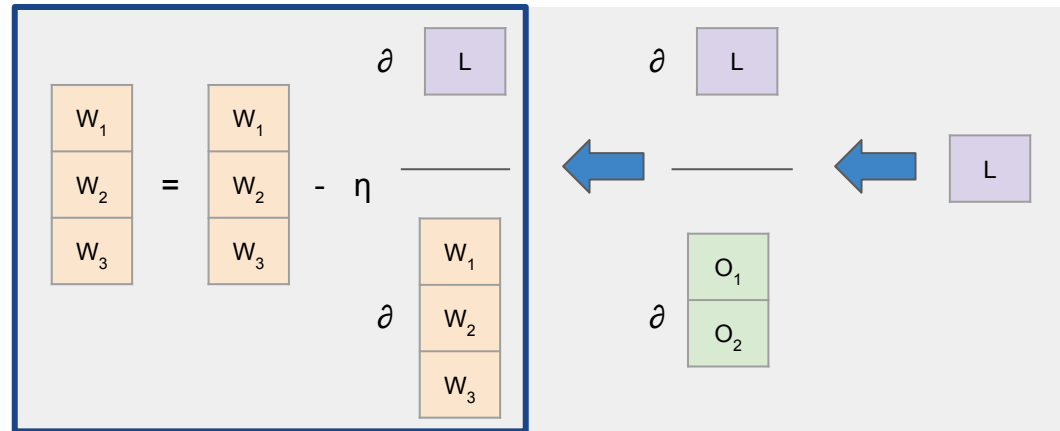
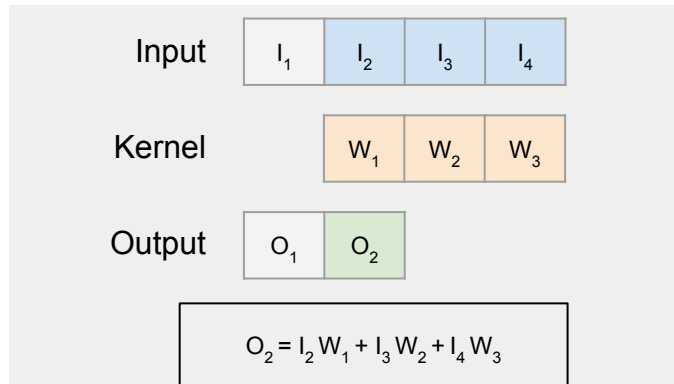
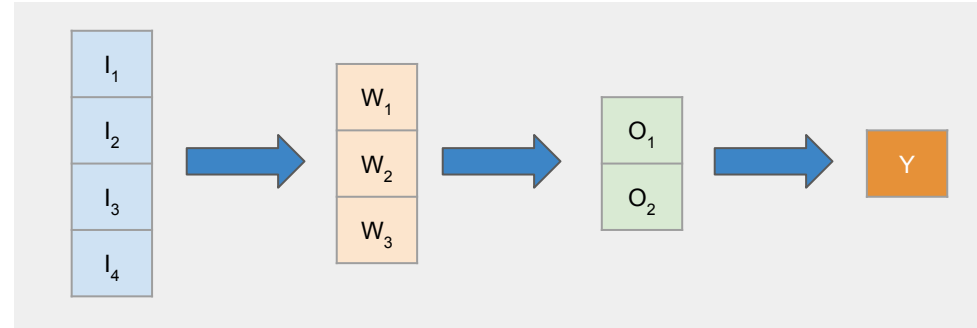
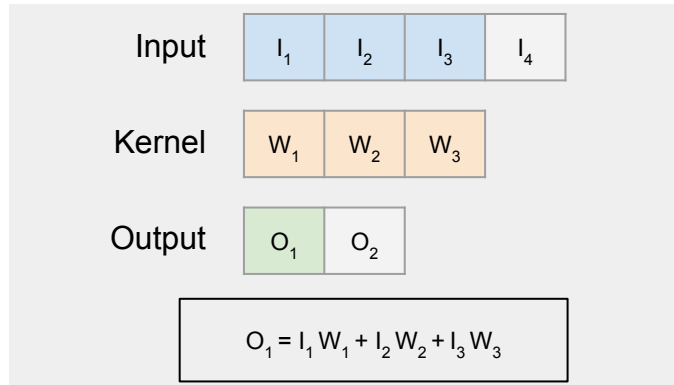
$= (5 \times 5 \times 3 + 1) \times 6$   
 $= 456$



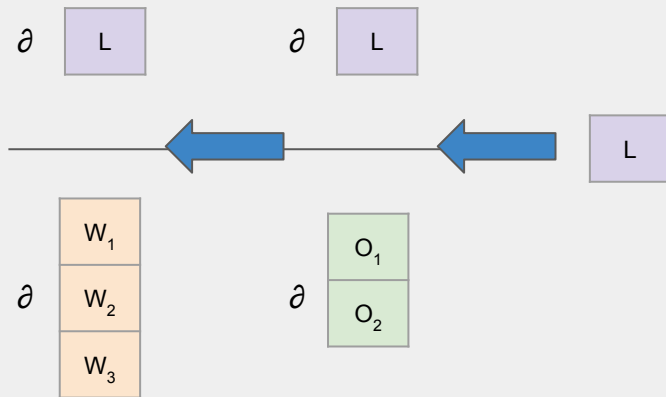
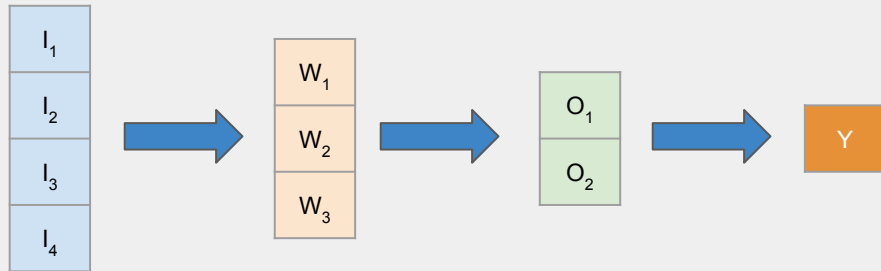
$30 \times 30 \times 6 =$   
 $5400$



# CNN Back Prop



# CNN Back Prop



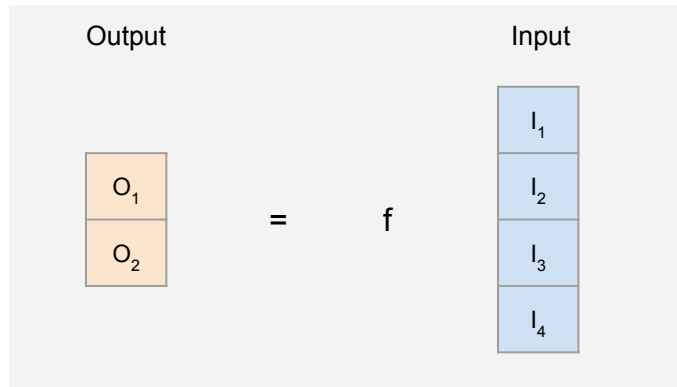
Chain Rule of Partial Derivative

$$\frac{\partial L}{\partial W_1} = \frac{\partial L}{\partial O_1} \times \frac{\partial O_1}{\partial W_1}$$

$$\frac{\partial L}{\partial W_2} = \frac{\partial L}{\partial O_1} \times \frac{\partial O_1}{\partial W_2} + \frac{\partial L}{\partial O_2} \times \frac{\partial O_2}{\partial W_2}$$

$$\frac{\partial L}{\partial W_3} = \frac{\partial L}{\partial O_2} \times \frac{\partial O_2}{\partial W_3}$$

# CNN Back Prop - Just Input & Output



$$y = f(x)$$

$$\frac{\partial y}{\partial x} = f'(x)$$

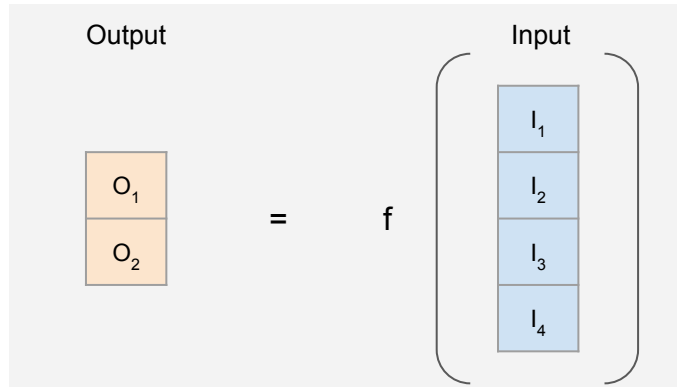
$$O_1 = I_1 W_1 + I_2 W_2 + I_3 W_3$$

$$O_2 = I_2 W_1 + I_3 W_2 + I_4 W_3$$

Diagram illustrating the backward pass (backpropagation) of a CNN layer. The partial derivatives of the output with respect to the input are calculated using the chain rule. The input vector  $\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$  is shown on the left, and the output vector  $\begin{bmatrix} O_1 \\ O_2 \end{bmatrix}$  is shown on the right. The partial derivatives are arranged in a grid:

$$\frac{\partial}{\partial} \begin{bmatrix} O_1 \\ O_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial O_1}{\partial I_1} & \frac{\partial O_1}{\partial I_2} & \frac{\partial O_1}{\partial I_3} & \frac{\partial O_1}{\partial I_4} \\ \frac{\partial O_2}{\partial I_1} & \frac{\partial O_2}{\partial I_2} & \frac{\partial O_2}{\partial I_3} & \frac{\partial O_2}{\partial I_4} \end{bmatrix} = \begin{bmatrix} & & & \\ & & & \end{bmatrix}$$

# CNN Back Prop - Just Input & Output



$$y = f(x)$$

$$\partial y / \partial x = f'(x)$$

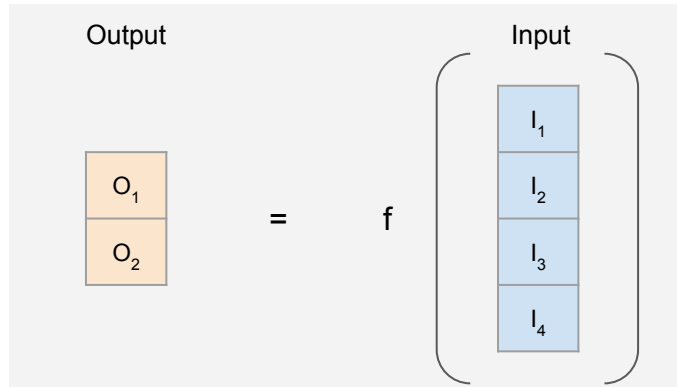
$$O_1 = I_1 W_1 + I_2 W_2 + I_3 W_3$$

$$O_2 = I_2 W_1 + I_3 W_2 + I_4 W_3$$

$$\frac{\partial \begin{bmatrix} O_1 \\ O_2 \end{bmatrix}}{\partial \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}} = \begin{bmatrix} \partial O_1 / \partial I_1 & \partial O_1 / \partial I_2 & \partial O_1 / \partial I_3 & \partial O_1 / \partial I_4 \\ \partial O_2 / \partial I_1 & \partial O_2 / \partial I_2 & \partial O_2 / \partial I_3 & \partial O_2 / \partial I_4 \end{bmatrix} = \begin{bmatrix} W_1 & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$



# CNN Back Prop - Just Input & Output



$$y = f(x)$$

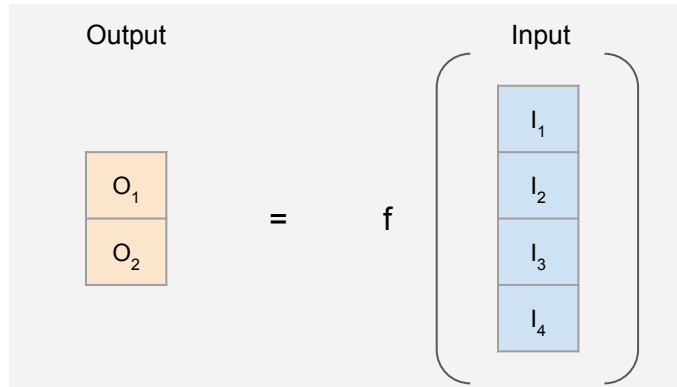
$$\frac{\partial y}{\partial x} = f'(x)$$

$$O_1 = I_1 W_1 + I_2 W_2 + I_3 W_3$$

$$O_2 = I_2 W_1 + I_3 W_2 + I_4 W_3$$

$$\frac{\partial \begin{bmatrix} O_1 \\ O_2 \end{bmatrix}}{\partial \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}} = \begin{bmatrix} \frac{\partial O_1}{\partial I_1} & \frac{\partial O_1}{\partial I_2} & \frac{\partial O_1}{\partial I_3} & \frac{\partial O_1}{\partial I_4} \\ \frac{\partial O_2}{\partial I_1} & \frac{\partial O_2}{\partial I_2} & \frac{\partial O_2}{\partial I_3} & \frac{\partial O_2}{\partial I_4} \end{bmatrix} = \begin{bmatrix} W_1 & W_2 & & \\ & & & \end{bmatrix}$$

# CNN Back Prop - Just Input & Output



$$y = f(x)$$

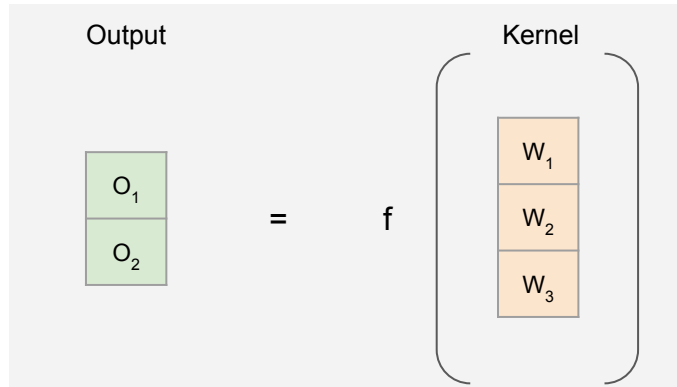
$$\partial y / \partial x = f'(x)$$

$$O_1 = I_1 W_1 + I_2 W_2 + I_3 W_3$$

$$O_2 = I_2 W_1 + I_3 W_2 + I_4 W_3$$

$$\frac{\partial \begin{bmatrix} O_1 \\ O_2 \end{bmatrix}}{\partial \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}} = \begin{bmatrix} \partial O_1 / \partial I_1 & \partial O_1 / \partial I_2 & \partial O_1 / \partial I_3 & \partial O_1 / \partial I_4 \\ \partial O_2 / \partial I_1 & \partial O_2 / \partial I_2 & \partial O_2 / \partial I_3 & \partial O_2 / \partial I_4 \end{bmatrix} = \begin{bmatrix} W_1 & W_2 & W_3 & 0 \\ 0 & W_1 & W_2 & W_3 \end{bmatrix}$$

# CNN Back Prop - Just Kernel & Output



$$y = f \left[ x \right]$$

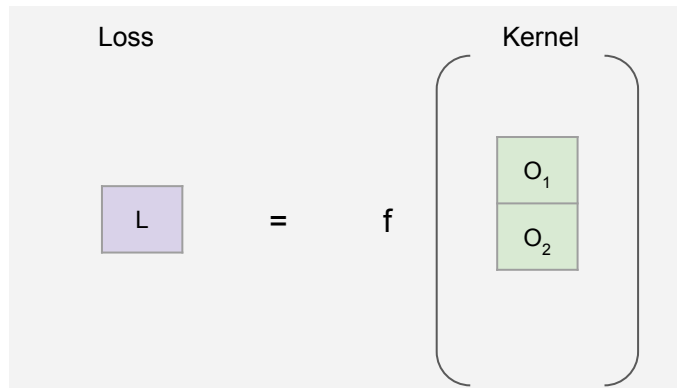
$$\partial y / \partial x = f' \left[ x \right]$$

$$O_1 = I_1 W_1 + I_2 W_2 + I_3 W_3$$

$$O_2 = I_2 W_1 + I_3 W_2 + I_4 W_3$$

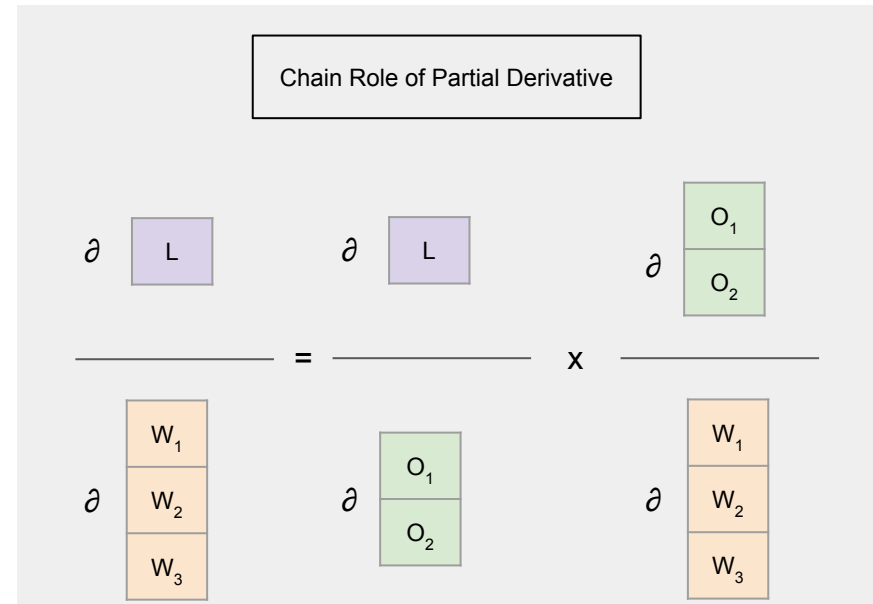
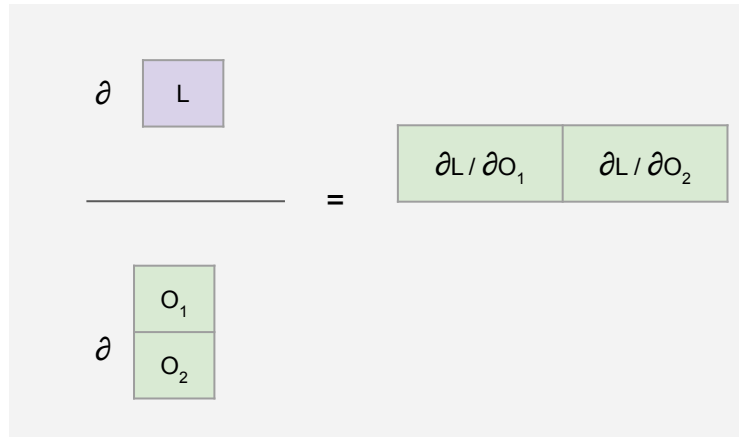
$$\frac{\partial \begin{bmatrix} O_1 \\ O_2 \end{bmatrix}}{\partial \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}} = \begin{bmatrix} \partial O_1 / \partial W_1 & \partial O_1 / \partial W_2 & \partial O_1 / \partial W_3 \\ \partial O_2 / \partial W_1 & \partial O_2 / \partial W_2 & \partial O_2 / \partial W_3 \end{bmatrix} = \begin{bmatrix} I_1 & I_2 & I_3 \\ I_2 & I_3 & I_4 \end{bmatrix}$$

# CNN Back Prop - Consider Loss and Output



$$y = f(x)$$

$$\partial y / \partial x = f'(x)$$



# CNN Back Prop - Consider Loss and Output

$$\frac{\partial L}{\partial \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}} = \frac{\partial L}{\partial \begin{bmatrix} O_1 \\ O_2 \end{bmatrix}} \times \frac{\partial \begin{bmatrix} O_1 \\ O_2 \end{bmatrix}}{\partial \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}} = \begin{bmatrix} \partial L / \partial O_1 & \partial L / \partial O_2 \end{bmatrix}_{1 \times 2} \times \begin{bmatrix} I_1 & I_2 & I_3 \\ I_2 & I_3 & I_4 \end{bmatrix}_{2 \times 3}$$

$$\frac{\partial L}{\partial w} = \begin{bmatrix} \partial L / \partial O_1 \times I_1 + \partial L / \partial O_2 \times I_2 & \partial L / \partial O_1 \times I_2 + \partial L / \partial O_2 \times I_3 & \partial L / \partial O_1 \times I_3 + \partial L / \partial O_2 \times I_4 \end{bmatrix}$$

# CNN Back Prop - Consider Loss and Output

$$\frac{\partial L}{\partial w} = \left( \frac{\partial L}{\partial O_1} \times I_1 + \frac{\partial L}{\partial O_2} \times I_2 \right) \frac{\partial L}{\partial O_1} \times I_2 + \frac{\partial L}{\partial O_2} \times I_3 \quad \frac{\partial L}{\partial O_1} \times I_3 + \frac{\partial L}{\partial O_2} \times I_4$$

Input	$I_1$	$I_2$	$I_3$	$I_4$
Loss	$\frac{\partial L}{\partial O_1}$	$\frac{\partial L}{\partial O_2}$		

# CNN Back Prop - Consider Loss and Output

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial O_1} \times I_1 + \frac{\partial L}{\partial O_2} \times I_2 \quad \boxed{\frac{\partial L}{\partial O_1} \times I_2 + \frac{\partial L}{\partial O_2} \times I_3} \quad \frac{\partial L}{\partial O_1} \times I_3 + \frac{\partial L}{\partial O_2} \times I_4$$

Input 

$I_1$	$I_2$	$I_3$	$I_4$
-------	-------	-------	-------

Loss 

$\frac{\partial L}{\partial O_1}$	$\frac{\partial L}{\partial O_2}$
-----------------------------------	-----------------------------------

Input 

$I_1$	$I_2$	$I_3$	$I_4$
-------	-------	-------	-------

Loss 

$\frac{\partial L}{\partial O_1}$	$\frac{\partial L}{\partial O_2}$
-----------------------------------	-----------------------------------

# CNN Back Prop - Consider Loss and Output

$$\frac{\partial L}{\partial w} = \left[ \frac{\partial L}{\partial O_1} \times I_1 + \frac{\partial L}{\partial O_2} \times I_2 \right] \left[ \frac{\partial L}{\partial O_1} \times I_2 + \frac{\partial L}{\partial O_2} \times I_3 \right] \boxed{\frac{\partial L}{\partial O_1} \times I_3 + \frac{\partial L}{\partial O_2} \times I_4}$$

Input	$I_1$	$I_2$	$I_3$	$I_4$
Loss	$\frac{\partial L}{\partial O_1}$	$\frac{\partial L}{\partial O_2}$		

Input	$I_1$	$I_2$	$I_3$	$I_4$
Loss		$\frac{\partial L}{\partial O_1}$	$\frac{\partial L}{\partial O_2}$	

Hence, backward propagation of a convolution is also as convolution

Input	$I_1$	$I_2$	$I_3$	$I_4$
Loss			$\frac{\partial L}{\partial O_1}$	$\frac{\partial L}{\partial O_2}$



# CNN - Translation Invariance

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