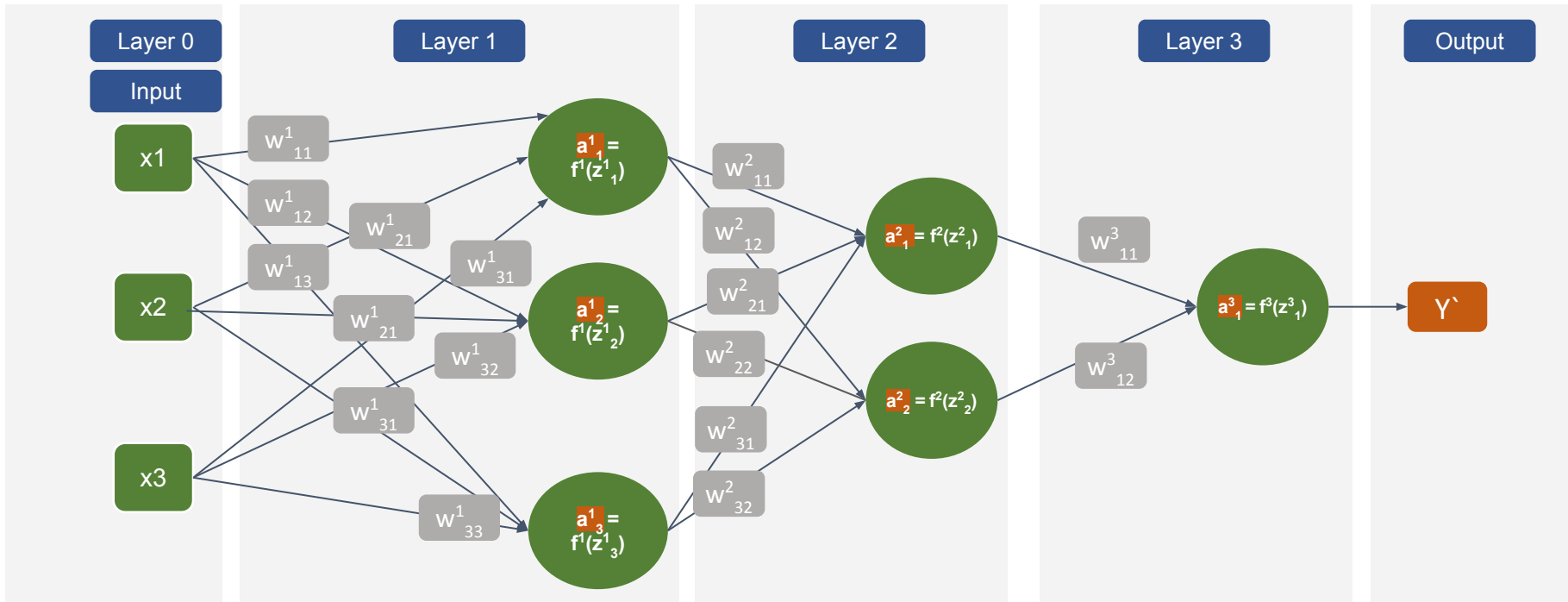


Deep Learning



Layers $L = 3$

$n^{[L]} = \# \text{ units in layer } L$ $n^{[1]} = 3, n^{[2]} = 2, n^{[3]} = 1$

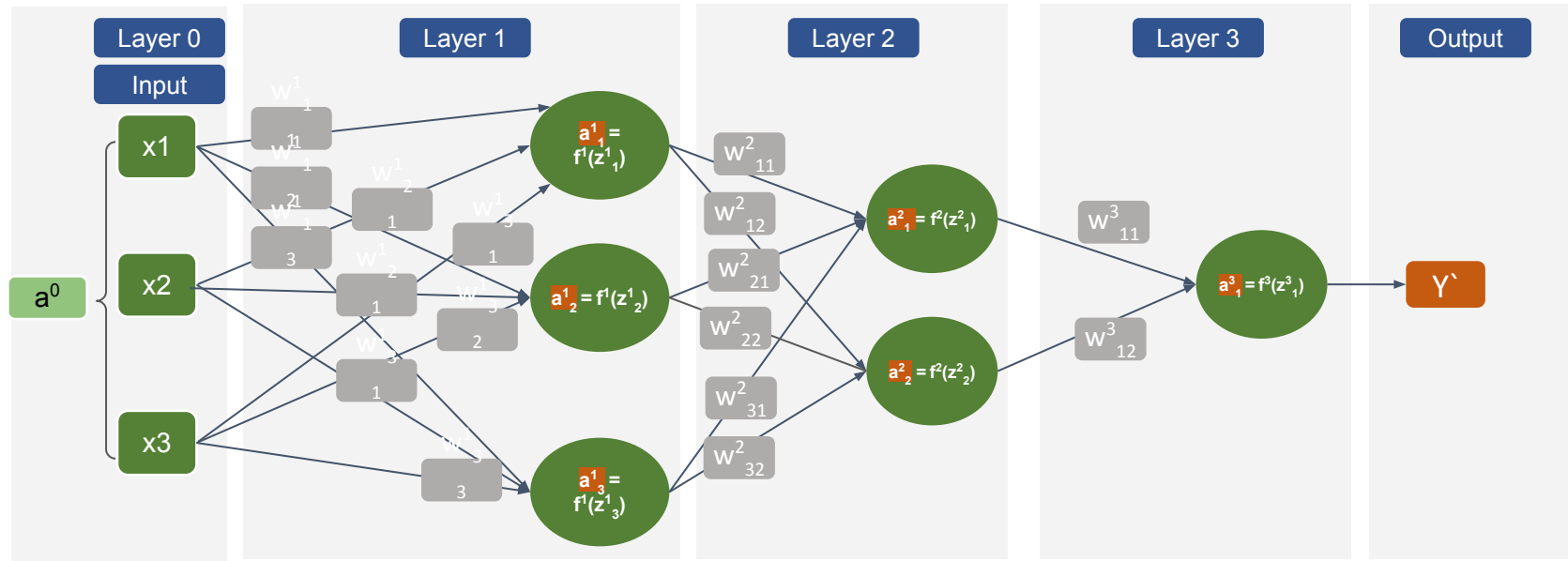
$a^{[L]} = \# \text{ activations in layer } L = f^{[L]}(z^{[L]})$

$w^{[L]} = \text{weights for layer } L$

$b^{[L]} = \text{biases for layer } L$



For 1 Training Sample : Forward Propagation



Layers L = 3

$n^{[L]}$ = # units in layer L

$a^{[L]}$ = # activations in layer L = $f^{[L]}(z^{[L]})$

$n^{[1]} = 3$, $n^{[2]} = 2$, $n^{[3]} = 1$

$w^{[L]}$ = weights for layer L

$b^{[L]}$ = biases for layer L

$$z^1 = w^1 x + b^1$$

$$a^1 = f^1(z^1)$$

$$z^2 = w^2 a^1 + b^2$$

$$a^2 = f^2(z^2)$$

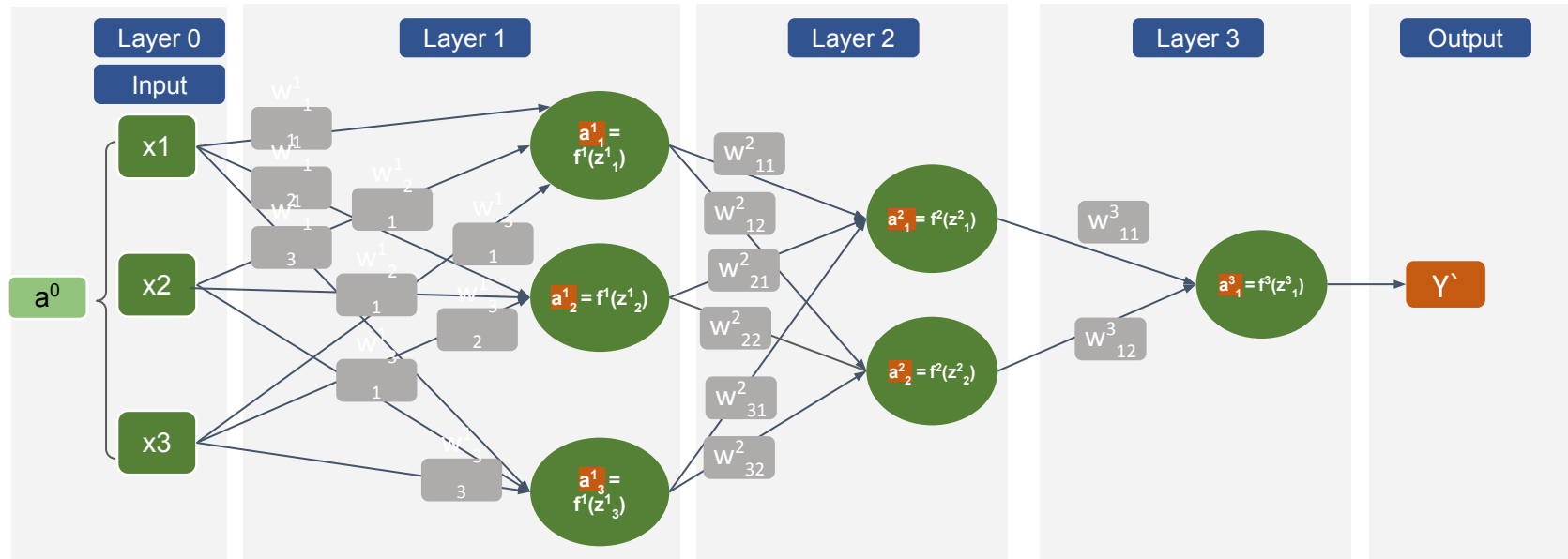
$$z^3 = w^3 a^2 + b^3$$

$$a^3 = f^3(z^3)$$

$$Y^* = a^3$$

$$z^{[L]} = w^{[L]} a^{[L-1]} + b^{[L]}$$

$$a^{[L]} = f^{[L]}(z^{[L]})$$



Layers $L = 3$

$n^{[L]} = \#$ units in layer L

$a^{[L]} = \#$ activations in layer $L = f^{[L]}(z^{[L]})$

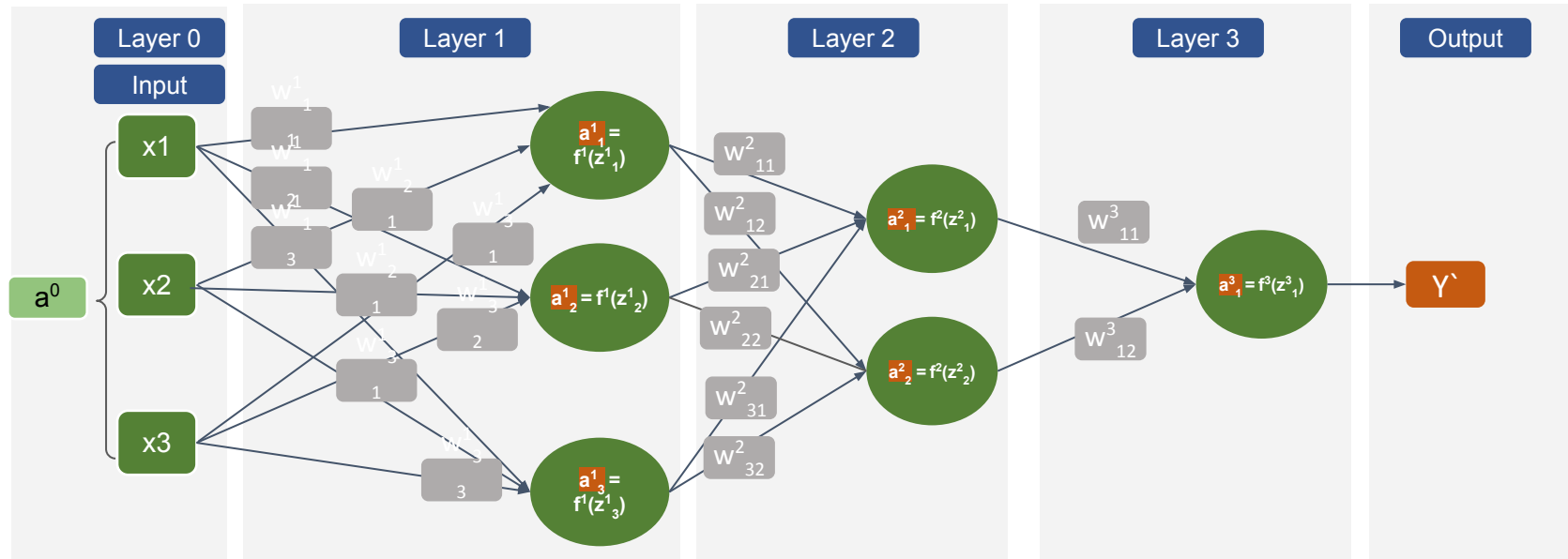
$n^{[1]} = 3, n^{[2]} = 2, n^{[3]} = 1$

$w^{[L]} =$ weights for layer L

$b^{[L]} =$ biases for layer L

$$z^1 = w^1 x + b^1$$

$$\begin{bmatrix} z^1_1 \\ z^1_2 \\ z^1_3 \end{bmatrix} = \begin{bmatrix} w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} \end{bmatrix} \times \begin{bmatrix} a^0_1 \\ a^0_2 \\ a^0_3 \end{bmatrix} + \begin{bmatrix} b^1_1 \\ b^1_2 \\ b^1_3 \end{bmatrix}$$



Layers L = 3

$n^{[L]}$ = # units in layer L

$a^{[L]}$ = # activations in layer L = $f^{[L]}(z^{[L]})$

$n^{[1]} = 3$, $n^{[2]} = 2$, $n^{[3]} = 1$

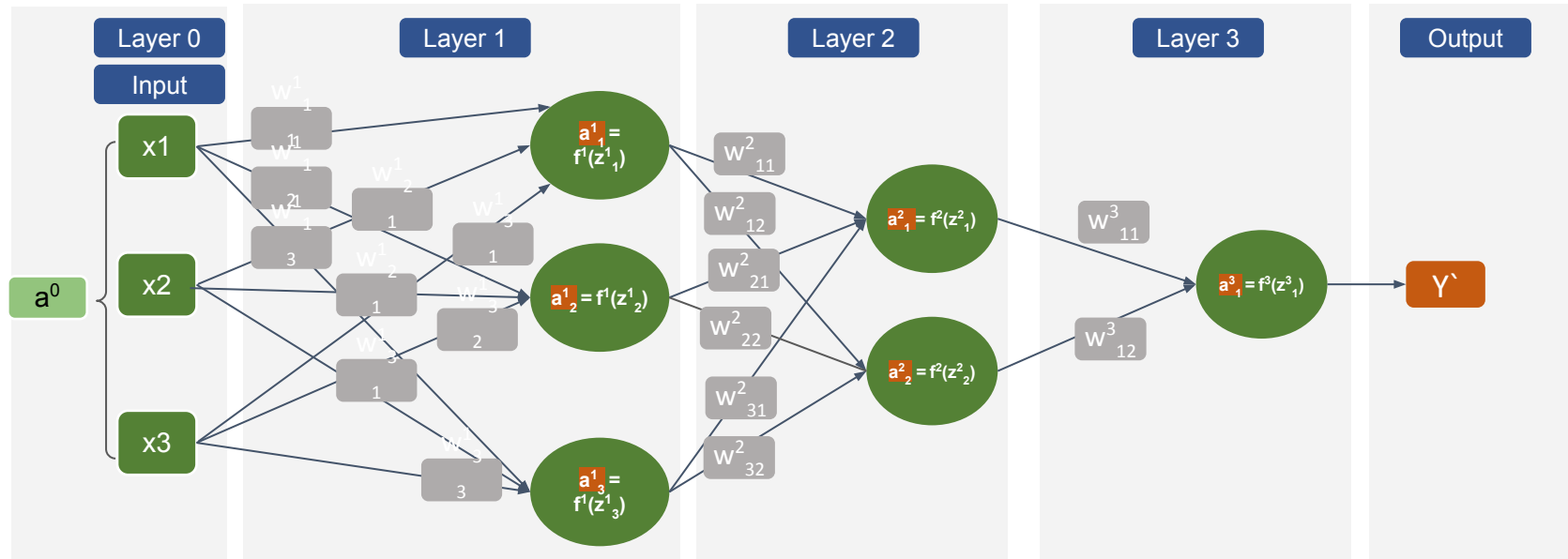
$w^{[L]}$ = weights for layer L

$b^{[L]}$ = biases for layer L

$$z^1 = w^1 x + b^1 \quad z^1 = w^1 a^0 + b^1$$

$$\begin{bmatrix} z^1_1 \\ z^1_2 \\ z^1_3 \end{bmatrix} = \begin{bmatrix} w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} \end{bmatrix} \times \begin{bmatrix} a^0_1 \\ a^0_2 \\ a^0_3 \end{bmatrix} + \begin{bmatrix} b^1_1 \\ b^1_2 \\ b^1_3 \end{bmatrix}$$

$$z^1_1 = w^1_{11} a^0_1 + w^1_{21} a^0_2 + w^1_{31} a^0_3 + b^1_1$$



Layers L = 3

$n^{[L]}$ = # units in layer L

$a^{[L]}$ = # activations in layer L = $f(z^{[L]})$

$n^{[1]} = 3$, $n^{[2]} = 2$, $n^{[3]} = 1$

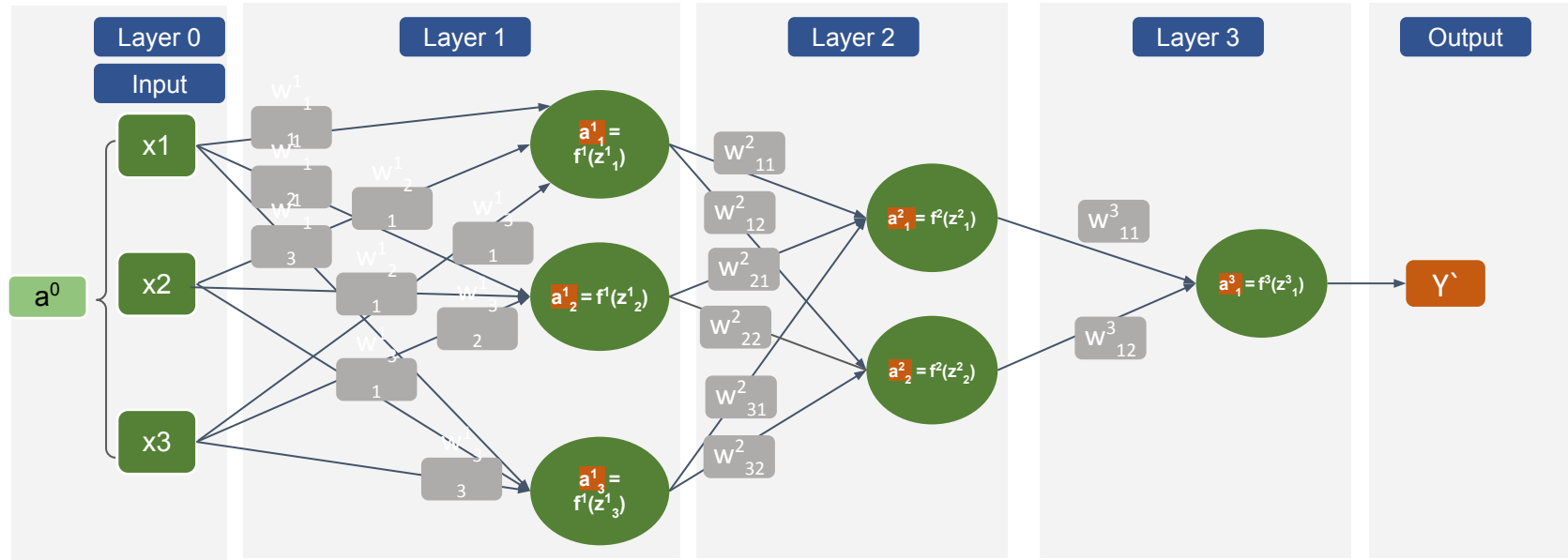
$w^{[L]}$ = weights for layer L

$b^{[L]}$ = biases for layer L

$$z^1 = w^1 x + b^1 \quad z^1 = w^1 a^0 + b^1$$

$$\begin{bmatrix} z^1_1 \\ z^1_2 \\ z^1_3 \end{bmatrix} = \begin{bmatrix} w^1_{11} & w^1_{12} & w^1_{13} \\ w^1_{21} & w^1_{22} & w^1_{23} \\ w^1_{31} & w^1_{32} & w^1_{33} \end{bmatrix} \times \begin{bmatrix} a^0_1 \\ a^0_2 \\ a^0_3 \end{bmatrix} + \begin{bmatrix} b^1_1 \\ b^1_2 \\ b^1_3 \end{bmatrix}$$

$$z^1_2 = w^1_{12} a^0_1 + w^1_{22} a^0_2 + w^1_{32} a^0_3 + b^1_2$$



Layers $L = 3$

$n^{[L]} = \#$ units in layer L

$a^{[L]} = \#$ activations in layer $L = f(z^{[L]})$

$n^{[1]} = 3, n^{[2]} = 2, n^{[3]} = 1$

$w^{[L]} =$ weights for layer L

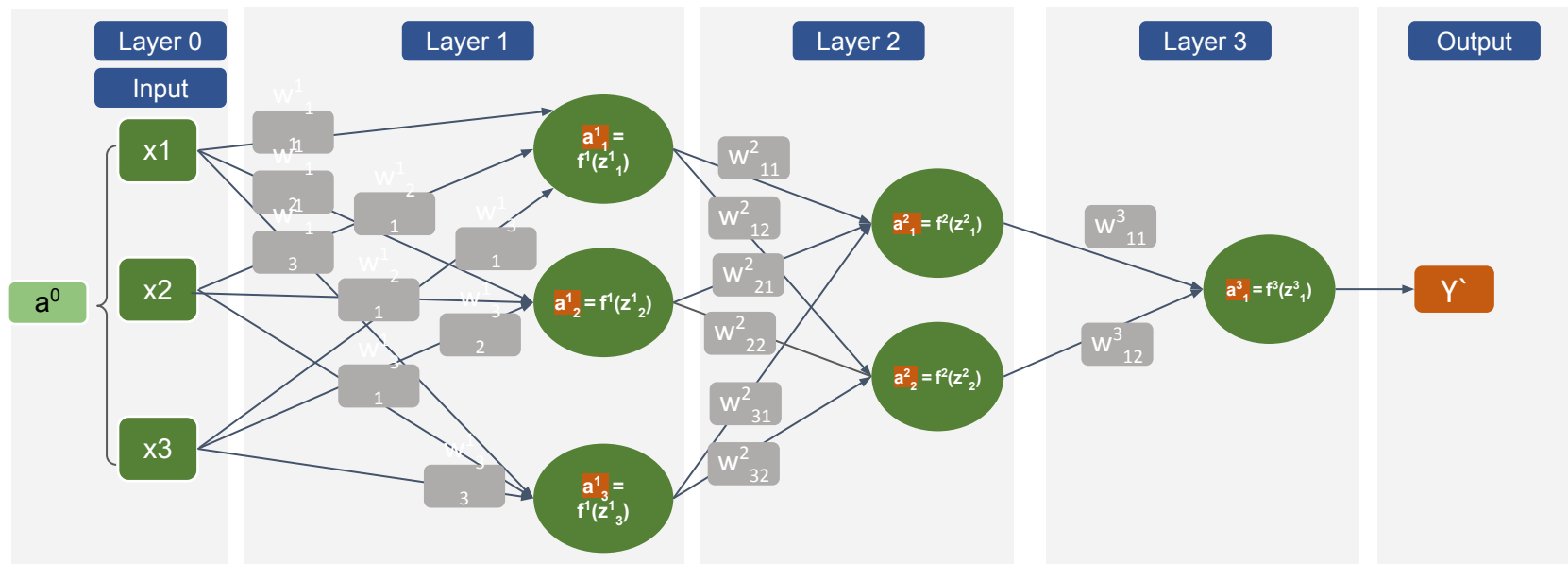
$b^{[L]} =$ biases for layer L

$$z^1 = w^1 x + b^1 \quad z^1 = w^1 a^0 + b^1$$

$$\begin{bmatrix} z^1_1 \\ z^1_2 \\ z^1_3 \end{bmatrix} = \begin{bmatrix} w^1_{11} & w^1_{12} & w^1_{13} \\ w^1_{21} & w^1_{22} & w^1_{23} \\ w^1_{31} & w^1_{32} & w^1_{33} \end{bmatrix} \times \begin{bmatrix} a^0_1 \\ a^0_2 \\ a^0_3 \end{bmatrix} + \begin{bmatrix} b^1_1 \\ b^1_2 \\ b^1_3 \end{bmatrix}$$

$$z^1_3 = w^1_{13} a^0_1 + w^1_{23} a^0_2 + w^1_{33} a^0_3 + b^1_3$$

For 1 Training Sample : Forward Propagation



Layers $L = 3$

$n^{[L]} = \# \text{ units in layer } L$

$a^{[L]} = \# \text{ activations in layer } L = f^{[L]}(z^{[L]})$

$n^{[1]} = 3, n^{[2]} = 2, n^{[3]} = 1$

$w^{[L]} = \text{weights for layer } L$

$b^{[L]} = \text{biases for layer } L$

$$z^1 = w^1 x + b^1 \quad z^1 = w^1 a^0 + b^1$$

$$\begin{bmatrix} z^1_1 \\ z^1_2 \\ z^1_3 \end{bmatrix} = \begin{bmatrix} w^1_{11} & w^1_{12} & w^1_{13} \\ w^1_{21} & w^1_{22} & w^1_{23} \\ w^1_{31} & w^1_{32} & w^1_{33} \end{bmatrix} \times \begin{bmatrix} a^0_1 \\ a^0_2 \\ a^0_3 \end{bmatrix} + \begin{bmatrix} b^1_1 \\ b^1_2 \\ b^1_3 \end{bmatrix}$$

$$\begin{bmatrix} a^1_1 \\ a^1_2 \\ a^1_3 \end{bmatrix} = f^1 \left[\begin{bmatrix} z^1_1 \\ z^1_2 \\ z^1_3 \end{bmatrix} \right]$$

$$z^2 = w^2 a^1 + b^2$$

$$\begin{bmatrix} z^2_1 \\ z^2_2 \end{bmatrix} = \begin{bmatrix} w^2_{11} & w^2_{12} & w^2_{13} \\ w^2_{21} & w^2_{22} & w^2_{23} \end{bmatrix} \times \begin{bmatrix} a^1_1 \\ a^1_2 \\ a^1_3 \end{bmatrix} + \begin{bmatrix} b^2_1 \\ b^2_2 \end{bmatrix}$$

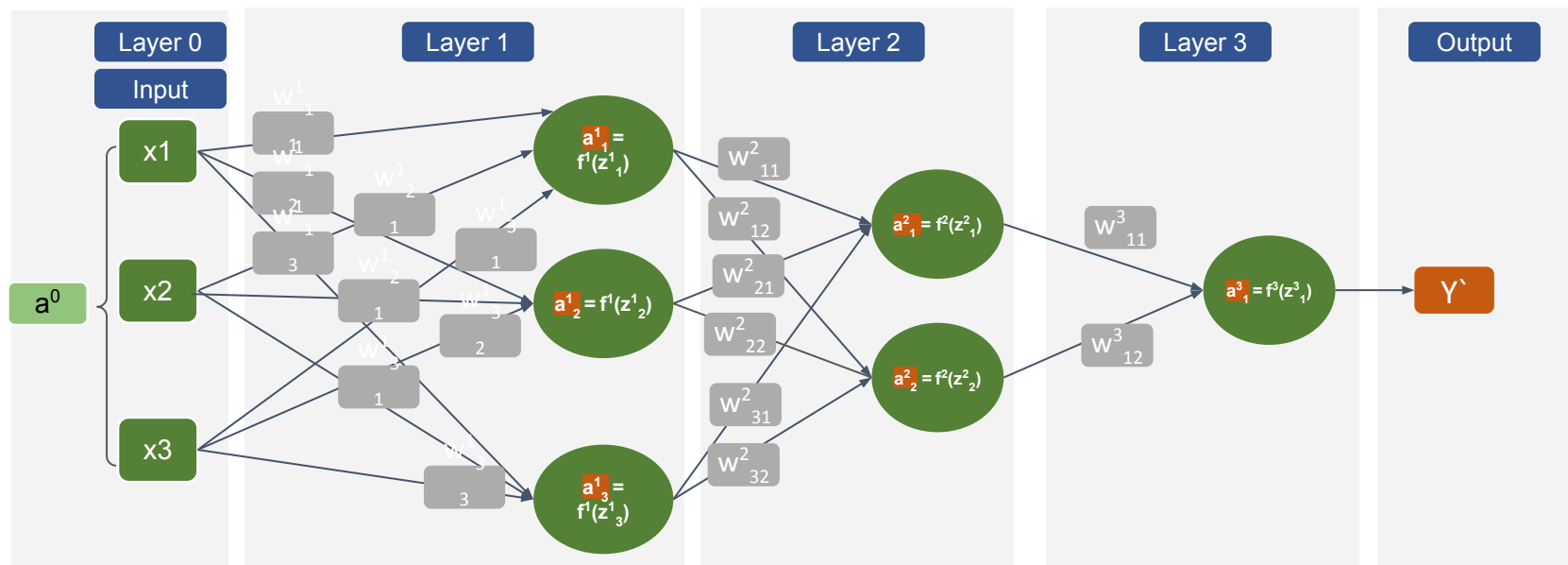
$$z^2_1 = w^2_{11} a^1_1 + w^2_{21} a^1_2 + w^2_{31} a^1_3 + b^2_1$$

$$z^2_2 = w^2_{12} a^1_1 + w^2_{22} a^1_2 + w^2_{32} a^1_3 + b^2_2$$

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<https://www.linkedin.com/company/analaarun-kumar-anala-35760523/>

For 1 Training Sample : Forward Propagation



Layers $L = 3$

$n^{[L]} = \# \text{ units in layer } L$

$a^{[L]} = \# \text{ activations in layer } L = f^{[L]}(z^{[L]})$

$n^{[1]} = 3, n^{[2]} = 2, n^{[3]} = 1$

$w^{[L]} = \text{weights for layer } L$

$b^{[L]} = \text{biases for layer } L$

$$z^1 = w^1 x + b^1 \quad z^1 = w^1 a^0 + b^1$$

$$\begin{pmatrix} z^1_1 \\ z^1_2 \\ z^1_3 \end{pmatrix} = \begin{pmatrix} w^1_{11} & w^1_{12} & w^1_{13} \\ w^1_{21} & w^1_{22} & w^1_{23} \\ w^1_{31} & w^1_{32} & w^1_{33} \end{pmatrix} \times \begin{pmatrix} a^0_1 \\ a^0_2 \\ a^0_3 \end{pmatrix} + \begin{pmatrix} b^1_1 \\ b^1_2 \\ b^1_3 \end{pmatrix}$$

(3,1) (3,3) (3,1) (3,1)

$$z^2 = w^2 a^1 + b^2$$

$$\begin{pmatrix} z^2_1 \\ z^2_2 \end{pmatrix} = \begin{pmatrix} w^2_{11} & w^2_{12} \\ w^2_{21} & w^2_{22} \end{pmatrix} \times \begin{pmatrix} a^1_1 \\ a^1_2 \\ a^1_3 \end{pmatrix} + \begin{pmatrix} b^2_1 \\ b^2_2 \end{pmatrix}$$

(2,1) (2,3) (3,1) (2,1)

$$w^{[L]} : (n^{[L]}, n^{[L-1]})$$

$$b^{[L]} : (n^{[L]}, 1)$$

$$w^{[1]} : (3, 3) \quad b^{[1]} : (3, 1)$$

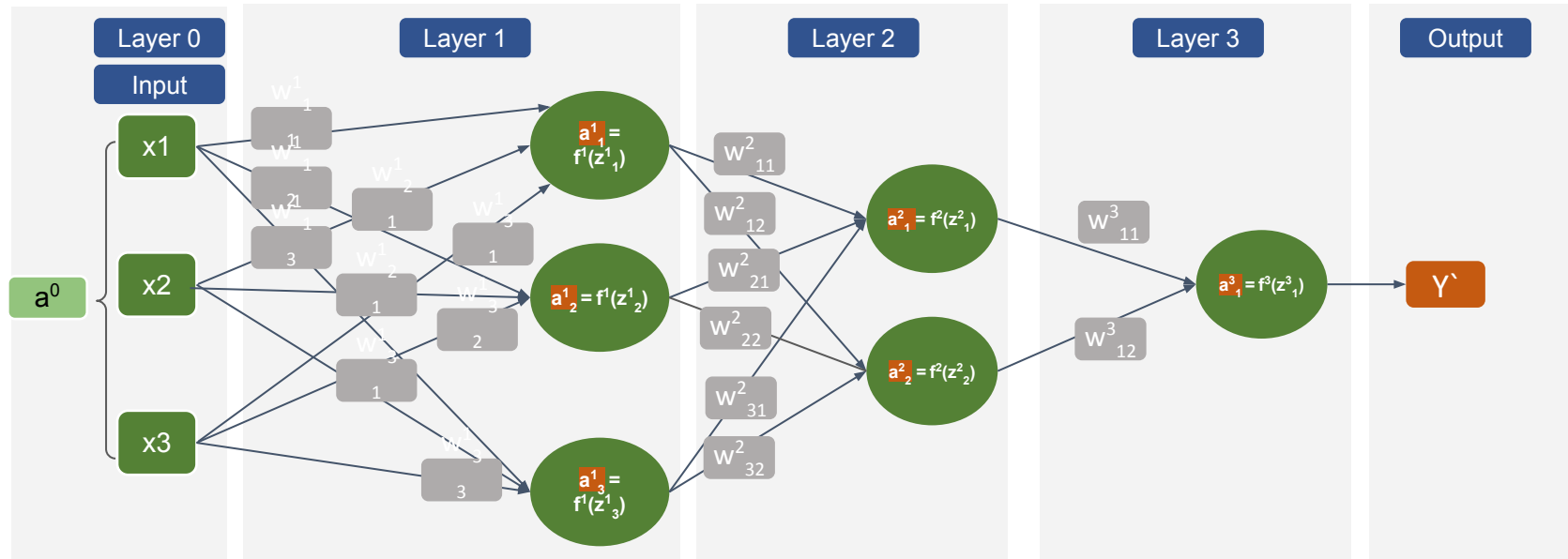
$$w^{[2]} : (2, 3) \quad b^{[2]} : (2, 1)$$

$$w^{[3]} : (1, 2) \quad b^{[3]} : (1, 1)$$

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For 1 Training Sample : Forward Propagation



Layers $L = 3$

$n^{[L]} = \#$ units in layer L

$a^{[L]} = \#$ activations in layer $L = f^{[L]}(z^{[L]})$

$n^{[1]} = 3$, $n^{[2]} = 2$, $n^{[3]} = 1$

$w^{[L]} =$ weights for layer L

$b^{[L]} =$ biases for layer L

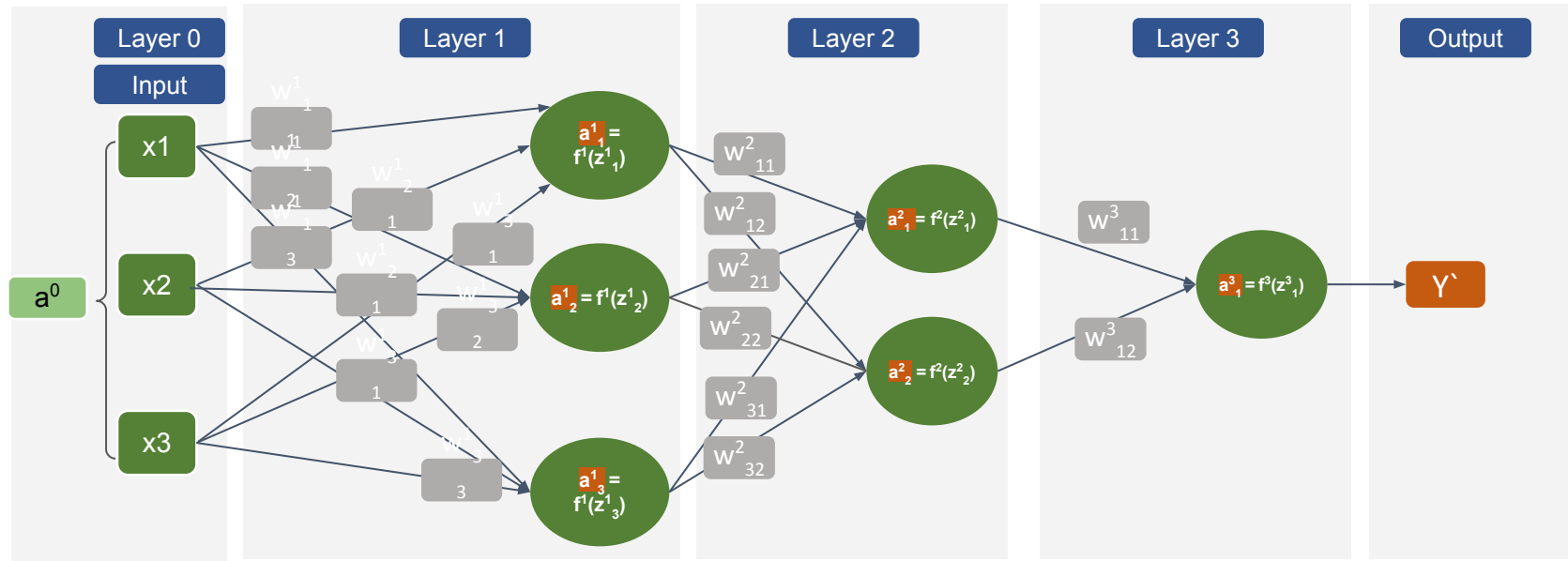
$$z^{[L]} = w^{[L]} a^{[L-1]} + b^{[L]}$$

$$a^{[L]} = f^{[L]}(z^{[L]})$$

$$w^{[L]} : (n^{[L]}, n^{[L-1]})$$

$$b^{[L]} : (n^{[L]}, 1)$$

For 1 Training Sample : Forward Propagation



Layers $L = 3$

$n^{[L]} = \# \text{ units in layer } L$

$a^{[L]} = \# \text{ activations in layer } L = f^{[L]}(z^{[L]})$

$n^{[1]} = 3, n^{[2]} = 2, n^{[3]} = 1$

$w^{[L]} = \text{weights for layer } L$

$b^{[L]} = \text{biases for layer } L$

$$z^{[L]} = w^{[L]} a^{[L-1]} + b^{[L]}$$

$$a^{[L]} = f^{[L]}(z^{[L]})$$

$$z^{[L]} : (n^{[L]}, 1)$$

$$a^{[L]} : (n^{[L]}, 1)$$

$$w^{[L]} : (n^{[L]}, n^{[L-1]})$$

$$b^{[L]} : (n^{[L]}, 1)$$

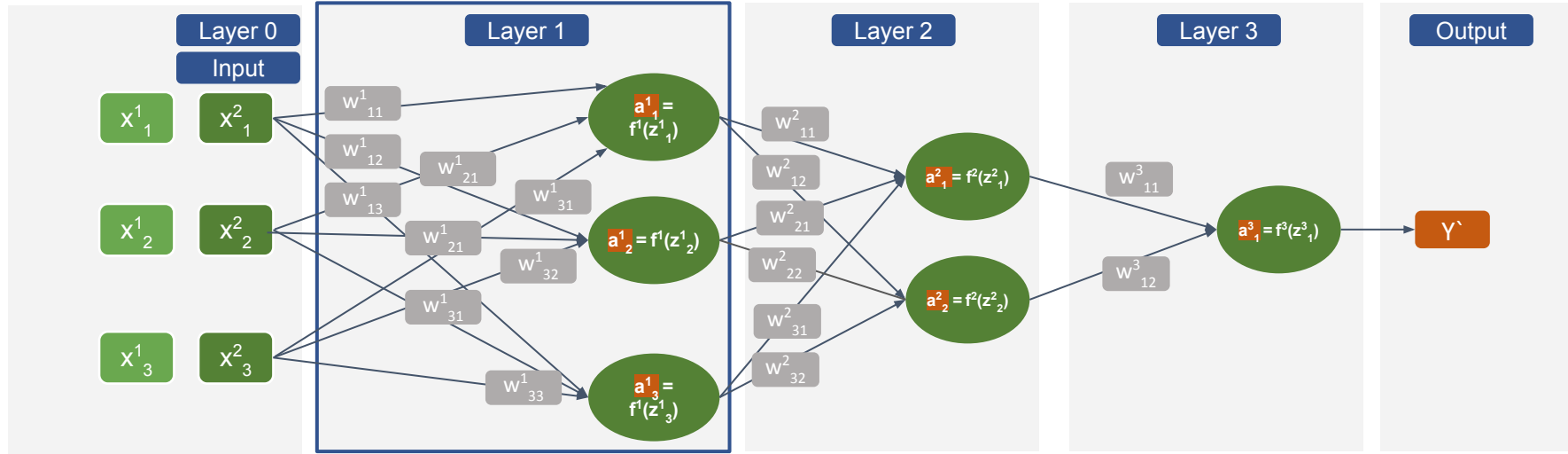
$$\partial z^{[L]} : (n^{[L]}, 1)$$

$$\partial a^{[L]} : (n^{[L]}, 1)$$

$$\partial w^{[L]} : (n^{[L]}, n^{[L-1]})$$

$$\partial b^{[L]} : (n^{[L]}, 1)$$

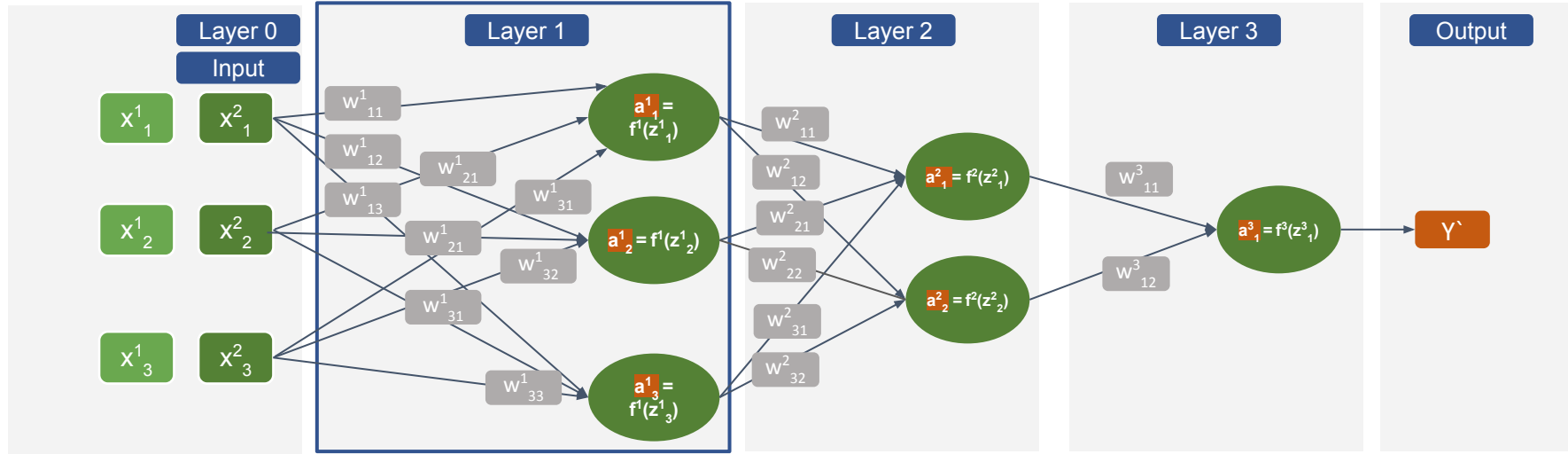
For 2 Training Sample : Forward Propagation



$$\begin{matrix} z^1_1 & z^1_1 \\ z^1_2 & z^1_2 \\ z^1_3 & z^1_3 \end{matrix} = \begin{matrix} w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} \end{matrix} \begin{matrix} w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} \end{matrix} \times \begin{matrix} a^0_1 & a^0_1 \\ a^0_2 & a^0_2 \\ a^0_3 & a^0_3 \end{matrix} + \begin{matrix} b^1_1 & b^1_1 \\ b^1_2 & b^1_2 \\ b^1_3 & b^1_3 \end{matrix}$$

(3,2)
((3, 3) , 2)
(3,2)
(3,2)

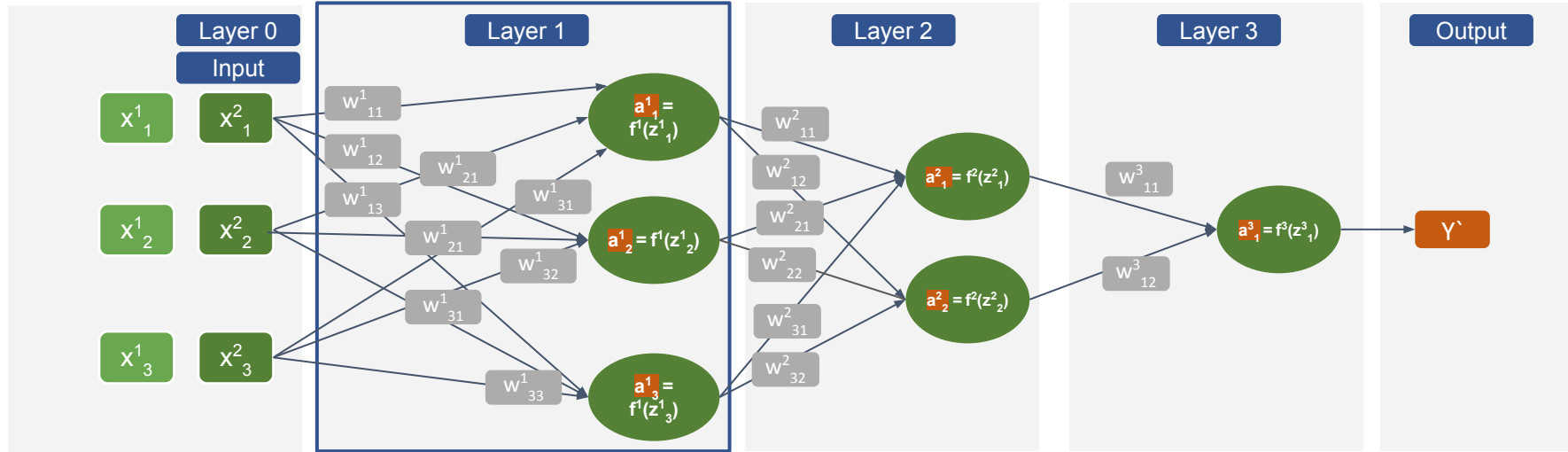
For 2 Training Sample : Forward Propagation



$$\begin{bmatrix} z^1_1 & z^1_1 \\ z^1_2 & z^1_2 \\ z^1_3 & z^1_3 \end{bmatrix} = \begin{bmatrix} w^1_{11} & w^1_{21} & w^1_{31} & w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} & w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} & w^1_{13} & w^1_{23} & w^1_{33} \end{bmatrix} \times \begin{bmatrix} a^0_1 & a^0_1 \\ a^0_2 & a^0_2 \\ a^0_3 & a^0_3 \end{bmatrix} + \begin{bmatrix} b^1_1 & b^1_1 \\ b^1_2 & b^1_2 \\ b^1_3 & b^1_3 \end{bmatrix}$$

$(3,2) \qquad ((3,3), 2) \qquad (3,2) \qquad (3,2)$

For 2 Training Sample : Forward Propagation



$$\begin{bmatrix} z_1^1 & z_1^1 \\ z_1^2 & z_1^2 \\ z_1^3 & z_1^3 \end{bmatrix}$$

(3,2)

=

$$\begin{bmatrix} w_{11}^1 & w_{21}^1 & w_{31}^1 \\ w_{12}^1 & w_{22}^1 & w_{32}^1 \\ w_{13}^1 & w_{23}^1 & w_{33}^1 \end{bmatrix}$$

(3, 3)

X

$$\begin{bmatrix} a_1^0 & a_1^0 \\ a_2^0 & a_2^0 \\ a_3^0 & a_3^0 \end{bmatrix}$$

(3,2)

+

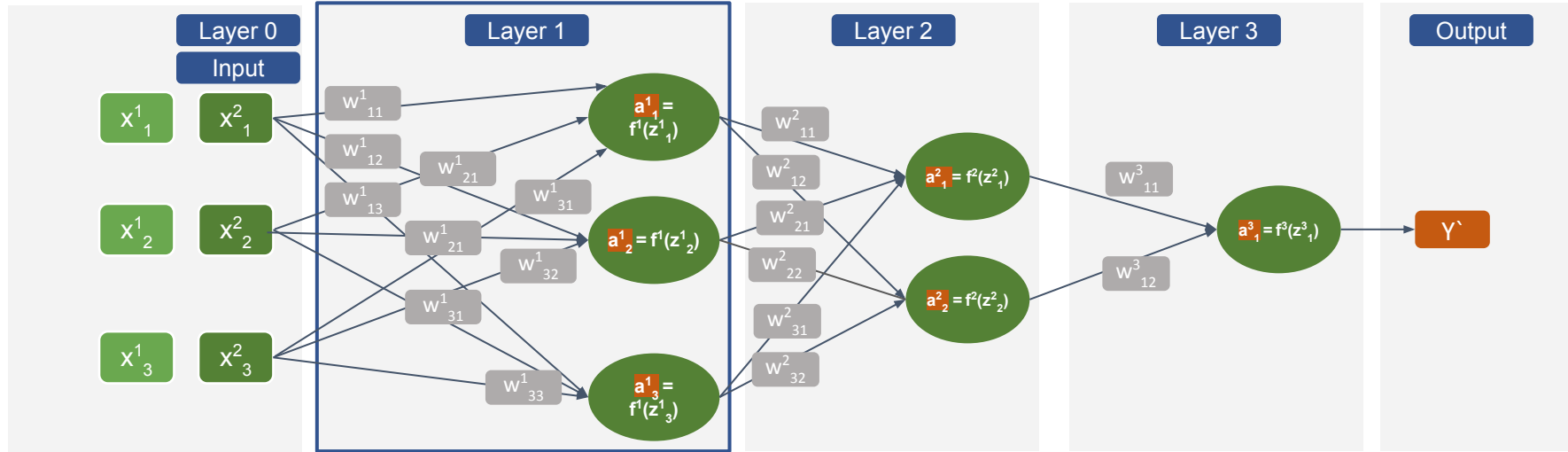
$$\begin{bmatrix} b_1^1 & b_1^1 \\ b_2^1 & b_2^1 \\ b_3^1 & b_3^1 \end{bmatrix}$$

(3,2)

$$\begin{bmatrix} w_{11}^1 & w_{21}^1 & w_{31}^1 \\ w_{12}^1 & w_{22}^1 & w_{32}^1 \\ w_{13}^1 & w_{23}^1 & w_{33}^1 \end{bmatrix}$$

(2, (3, 3))

For 2 Training Sample : Forward Propagation

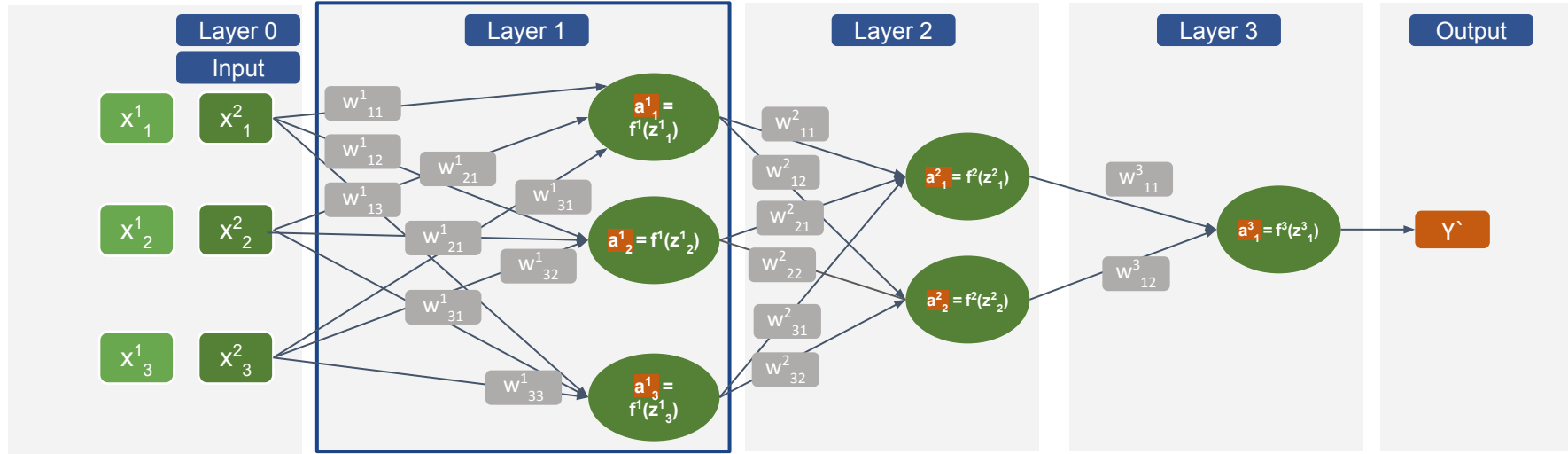


$$\begin{matrix} z_1^1 & z_1^1 \\ z_2^1 & z_2^1 \\ z_3^1 & z_3^1 \end{matrix} = \begin{matrix} w_{11}^1 & w_{21}^1 & w_{31}^1 \\ w_{12}^1 & w_{22}^1 & w_{32}^1 \\ w_{13}^1 & w_{23}^1 & w_{33}^1 \end{matrix} \times \begin{matrix} a_1^0 & a_1^0 \\ a_2^0 & a_2^0 \\ a_3^0 & a_3^0 \end{matrix} + \begin{matrix} b_1^1 & b_1^1 \\ b_2^1 & b_2^1 \\ b_3^1 & b_3^1 \end{matrix}$$

(3,2) (3,3) (3,2) (3,2)

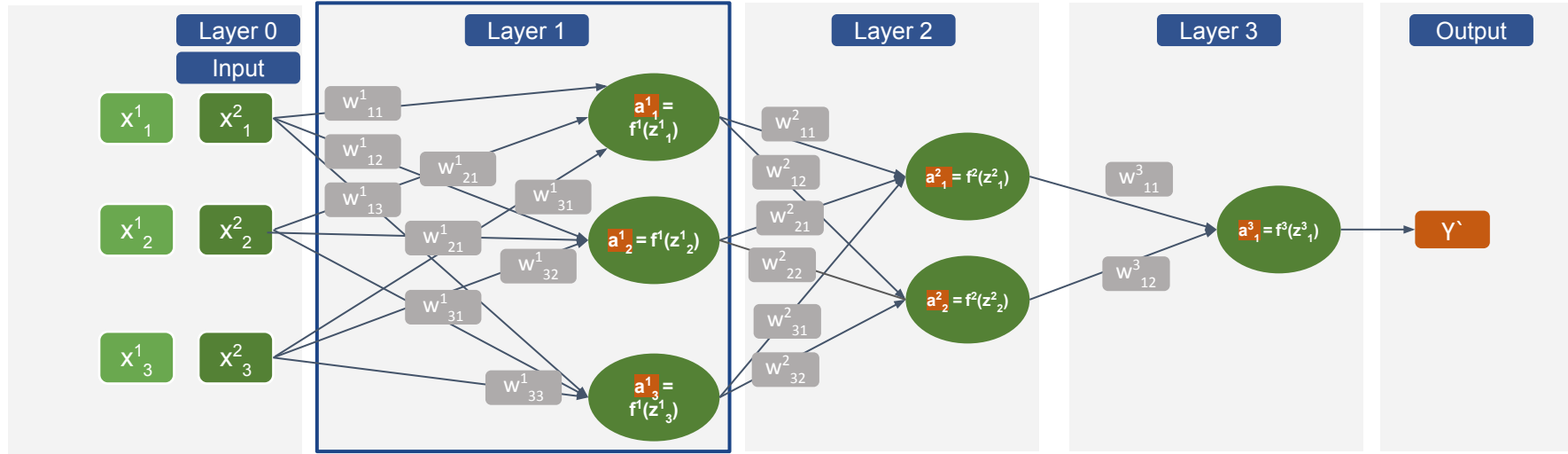
(2, (3, 3))

For 2 Training Sample : Forward Propagation



$$\begin{bmatrix} z^1_1 & z^1_1 \\ z^1_2 & z^1_2 \\ z^1_3 & z^1_3 \end{bmatrix}_{(3,2)} = \begin{bmatrix} w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} \end{bmatrix}_{(3,3)} \times \begin{bmatrix} a^0_1 & a^0_1 \\ a^0_2 & a^0_2 \\ a^0_3 & a^0_3 \end{bmatrix}_{(3,2)} + \begin{bmatrix} w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} \end{bmatrix}_{(3,3)} \times \begin{bmatrix} a^0_1 & a^0_1 \\ a^0_2 & a^0_2 \\ a^0_3 & a^0_3 \end{bmatrix}_{(3,2)} + \begin{bmatrix} b^1_1 & b^1_1 \\ b^1_2 & b^1_2 \\ b^1_3 & b^1_3 \end{bmatrix}_{(3,2)}$$

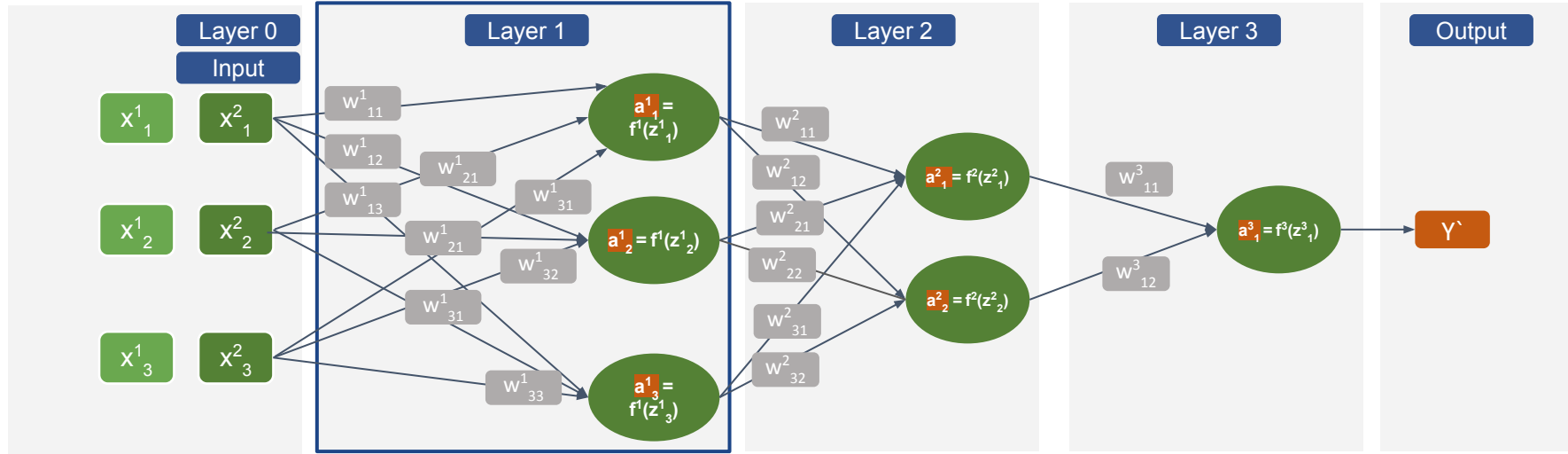
For 2 Training Sample : Forward Propagation



$$\begin{matrix} z^1_1 & z^1_1 \\ z^1_2 & z^1_2 \\ z^1_3 & z^1_3 \end{matrix} = \begin{matrix} w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} \end{matrix} \times \begin{matrix} a^0_1 & a^0_1 \\ a^0_2 & a^0_2 \\ a^0_3 & a^0_3 \end{matrix} + \begin{matrix} w^2_{11} & w^2_{21} & w^2_{31} \\ w^2_{12} & w^2_{22} & w^2_{32} \\ w^2_{13} & w^2_{23} & w^2_{33} \end{matrix} \times \begin{matrix} a^0_1 & a^0_1 \\ a^0_2 & a^0_2 \\ a^0_3 & a^0_3 \end{matrix} + \begin{matrix} b^1_1 & b^1_1 \\ b^1_2 & b^1_2 \\ b^1_3 & b^1_3 \end{matrix}$$

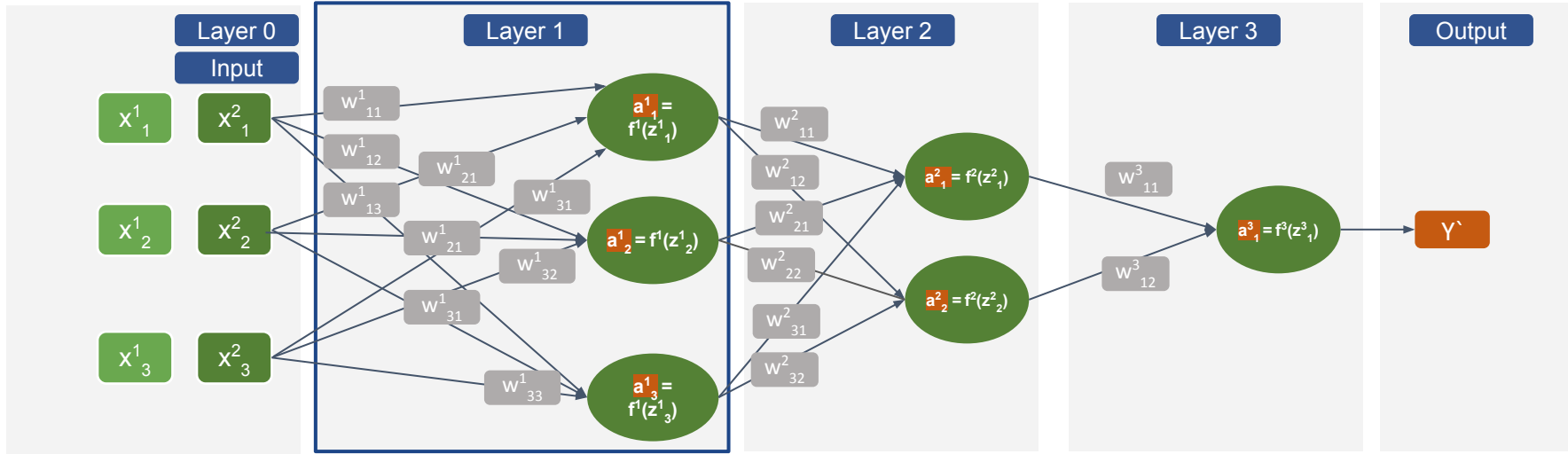
(3,2) (3, 3) (3,2) (3, 3) (3,2) (3,2)

For 2 Training Sample : Forward Propagation



$$\begin{bmatrix} z^1_1 & z^1_1 \\ z^1_2 & z^1_2 \\ z^1_3 & z^1_3 \end{bmatrix}_{(3,2)} = \begin{bmatrix} w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} \end{bmatrix}_{(3,3)} \times \begin{bmatrix} a^0_1 & a^0_1 \\ a^0_2 & a^0_2 \\ a^0_3 & a^0_3 \end{bmatrix}_{(3,2)} + \begin{bmatrix} w^1_{11} & w^1_{21} & w^1_{31} \\ w^1_{12} & w^1_{22} & w^1_{32} \\ w^1_{13} & w^1_{23} & w^1_{33} \end{bmatrix}_{(3,3)} \times \begin{bmatrix} a^0_1 & a^0_1 \\ a^0_2 & a^0_2 \\ a^0_3 & a^0_3 \end{bmatrix}_{(3,2)} + \begin{bmatrix} b^1_1 & b^1_1 \\ b^1_2 & b^1_2 \\ b^1_3 & b^1_3 \end{bmatrix}_{(3,2)}$$

For 2 Training Sample : Forward Propagation



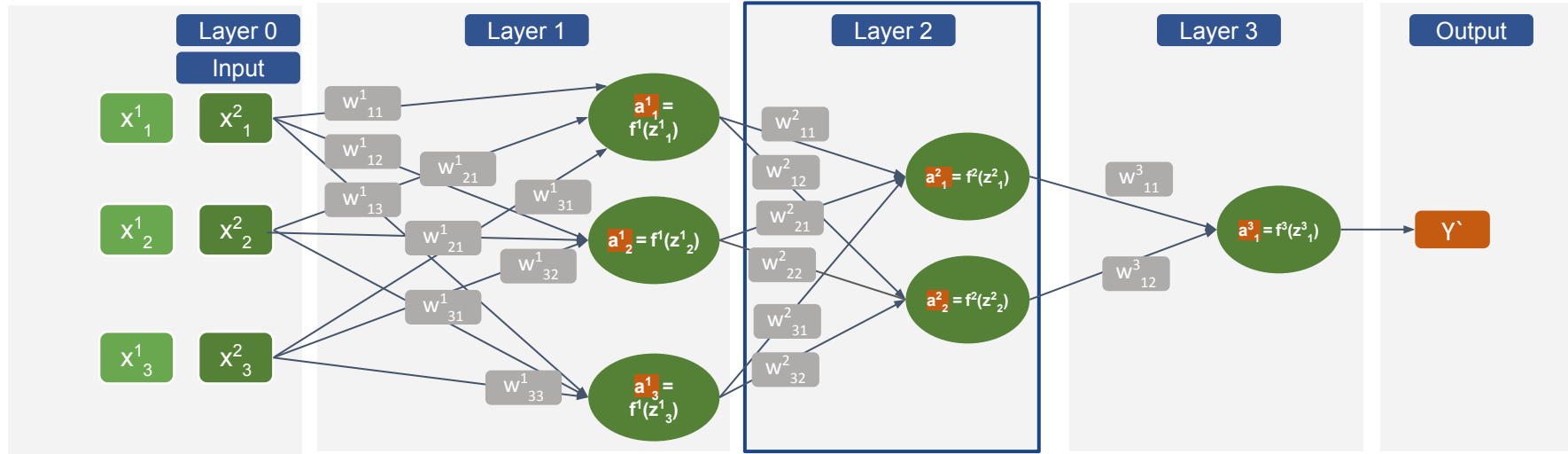
$$\begin{pmatrix} z_1^1 & z_1^1 \\ z_2^1 & z_2^1 \\ z_3^1 & z_3^1 \end{pmatrix} = \begin{pmatrix} w_{11}^1 & w_{21}^1 & w_{31}^1 \\ w_{12}^1 & w_{22}^1 & w_{32}^1 \\ w_{13}^1 & w_{23}^1 & w_{33}^1 \end{pmatrix} \times \begin{pmatrix} a_1^0 & a_1^0 \\ a_2^0 & a_2^0 \\ a_3^0 & a_3^0 \end{pmatrix} + \begin{pmatrix} w_{11}^1 & w_{21}^1 & w_{31}^1 \\ w_{12}^1 & w_{22}^1 & w_{32}^1 \\ w_{13}^1 & w_{23}^1 & w_{33}^1 \end{pmatrix} \times \begin{pmatrix} a_1^0 & a_1^0 \\ a_2^0 & a_2^0 \\ a_3^0 & a_3^0 \end{pmatrix} + \begin{pmatrix} b_1^1 & b_1^1 \\ b_2^1 & b_2^1 \\ b_3^1 & b_3^1 \end{pmatrix}$$

(3,2) (3,3) (3,2) (3,3) (3,2) (3,2)

$$\begin{pmatrix} a_1^1 & a_1^1 \\ a_2^1 & a_2^1 \\ a_3^1 & a_3^1 \end{pmatrix} = \mathbf{f} \left(\begin{pmatrix} z_1^1 & z_1^1 \\ z_2^1 & z_2^1 \\ z_3^1 & z_3^1 \end{pmatrix} \right)$$

(3,2) (3,2)

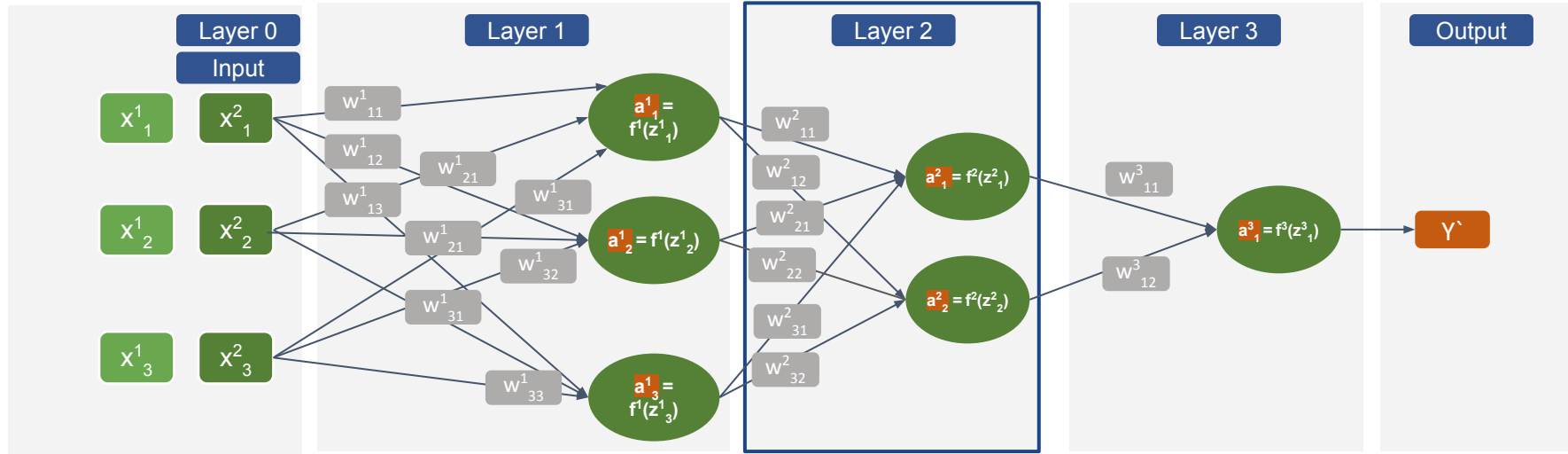
For 2 Training Sample : Forward Propagation



$$\begin{matrix} z^2_1 & z^2_1 \\ z^2_2 & z^2_2 \end{matrix} = \begin{matrix} w^2_{11} & w^2_{21} & w^2_{31} \\ w^2_{12} & w^2_{22} & w^2_{32} \end{matrix} \begin{matrix} w^2_{11} & w^2_{21} & w^2_{31} \\ w^2_{12} & w^2_{22} & w^2_{32} \end{matrix} \times \begin{matrix} a^1_1 & a^1_1 \\ a^1_2 & a^1_2 \\ a^1_3 & a^1_3 \end{matrix} + \begin{matrix} b^2_1 & b^2_1 \\ b^2_2 & b^2_2 \end{matrix}$$

(2,2)
((2, 3) , 2)
(3,2)
(3,2)

For 2 Training Sample : Forward Propagation



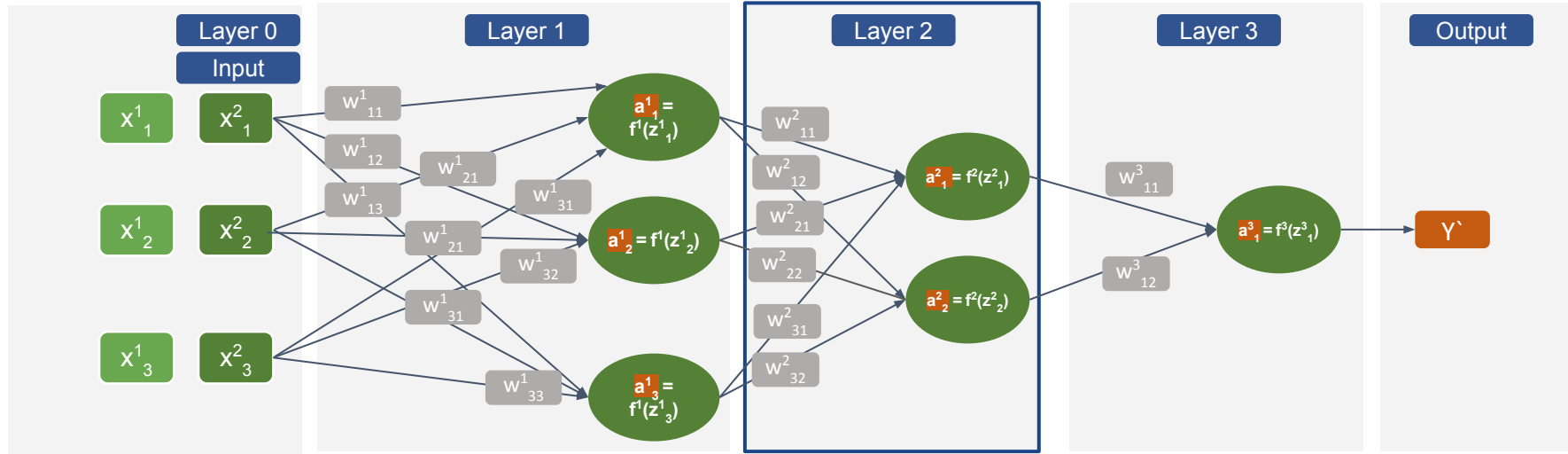
$$\begin{matrix} z_1^2 & z_2^2 \\ z_2^2 & z_2^2 \end{matrix} = \begin{matrix} w_{11}^2 & w_{21}^2 & w_{31}^2 \\ w_{12}^2 & w_{22}^2 & w_{32}^2 \end{matrix} \times \begin{matrix} a_1^1 & a_1^1 \\ a_1^2 & a_1^2 \\ a_1^3 & a_1^3 \end{matrix} + \begin{matrix} b_1^2 & b_1^2 \\ b_2^2 & b_2^2 \end{matrix}$$

(2,2) (2,3) (3,2) (2,2)

$$\begin{matrix} w_{11}^2 & w_{21}^2 & w_{31}^2 \\ w_{12}^2 & w_{22}^2 & w_{32}^2 \end{matrix}$$

(2, (2, 3))

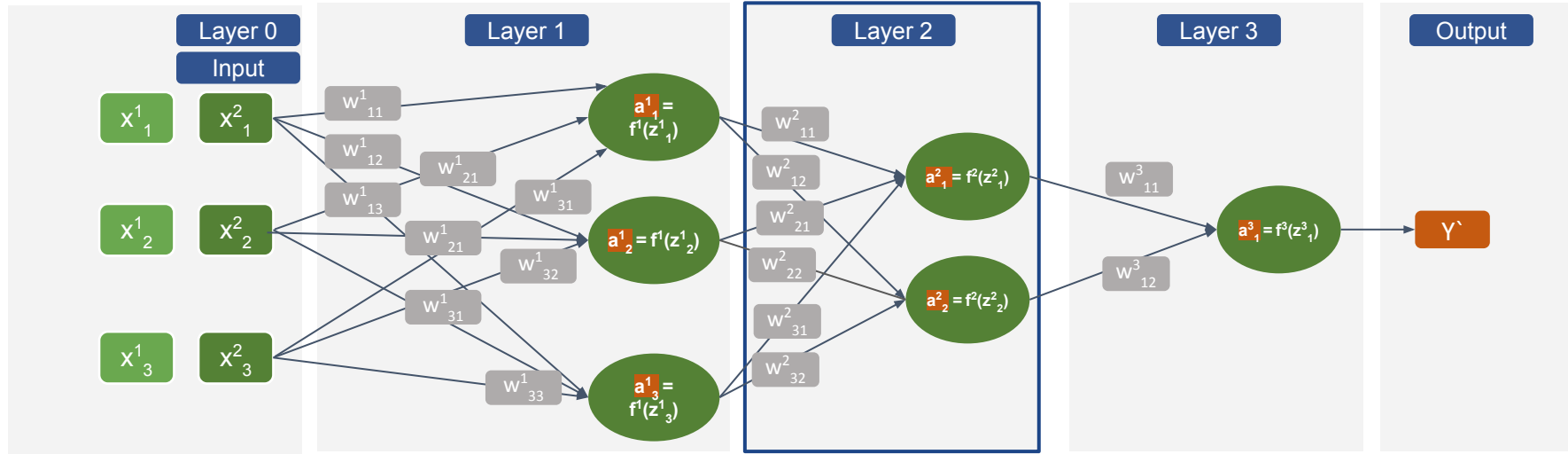
For 2 Training Sample : Forward Propagation



$$\begin{bmatrix} z^2_1 & z^2_1 \\ z^2_2 & z^2_2 \end{bmatrix} = \begin{bmatrix} w^2_{11} & w^2_{21} & w^2_{31} \\ w^2_{12} & w^2_{22} & w^2_{32} \end{bmatrix} \times \begin{bmatrix} a^1_1 & a^1_1 \\ a^1_2 & a^1_2 \\ a^1_3 & a^1_3 \end{bmatrix} + \begin{bmatrix} w^2_{11} & w^2_{21} & w^2_{31} \\ w^2_{12} & w^2_{22} & w^2_{32} \end{bmatrix} \times \begin{bmatrix} a^1_1 & a^1_1 \\ a^1_2 & a^1_2 \\ a^1_3 & a^1_3 \end{bmatrix} + \begin{bmatrix} b^2_1 & b^2_1 \\ b^2_2 & b^2_2 \end{bmatrix}$$

(2,2) (2,3) (3,2) (2,3) (3,2) (2,2)

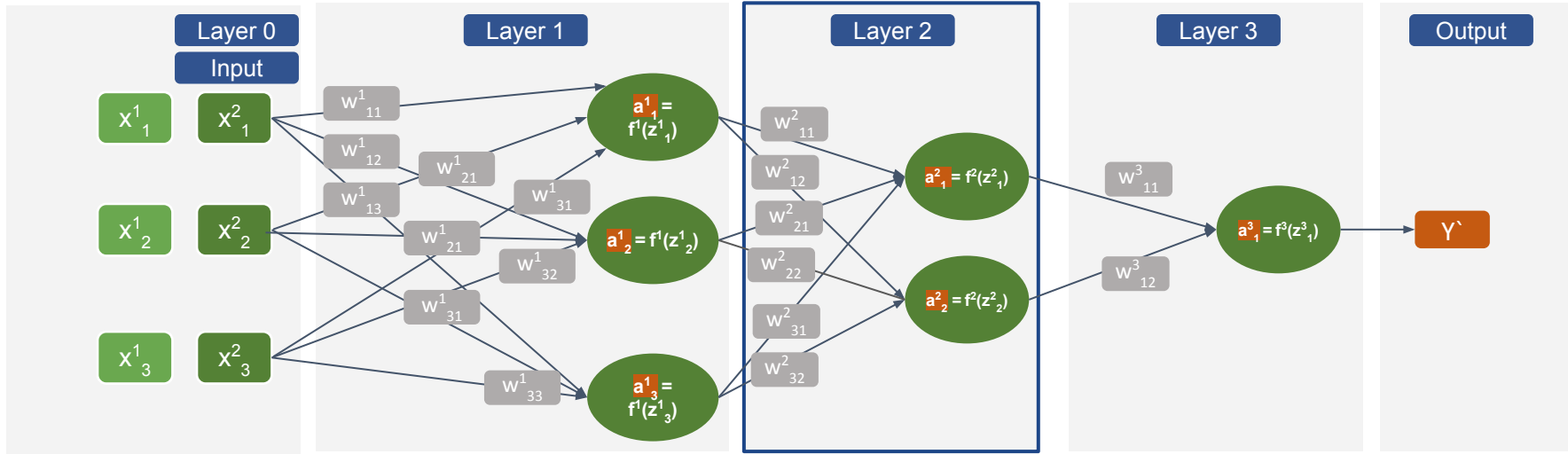
For 2 Training Sample : Forward Propagation



$$\begin{matrix} z^2_1 & z^2_1 \\ z^2_2 & z^2_2 \end{matrix} = \begin{matrix} w^2_{11} & w^2_{21} & w^2_{31} \\ w^2_{12} & w^2_{22} & w^2_{32} \end{matrix} \times \begin{matrix} a^1_1 & a^1_1 \\ a^1_2 & a^1_2 \\ a^1_3 & a^1_3 \end{matrix} + \begin{matrix} w^2_{11} & w^2_{21} & w^2_{31} \\ w^2_{12} & w^2_{22} & w^2_{32} \end{matrix} \times \begin{matrix} a^1_1 & a^1_1 \\ a^1_2 & a^1_2 \\ a^1_3 & a^1_3 \end{matrix} + \begin{matrix} b^2_1 & b^2_1 \\ b^2_2 & b^2_2 \end{matrix}$$

(2,2) (2,3) (3,2) (2,3) (3,2) (2,2)

For 2 Training Sample : Forward Propagation



$$\begin{matrix} z^2_1 & z^2_1 \\ z^2_2 & z^2_2 \end{matrix} = \begin{matrix} w^2_{11} & w^2_{21} & w^2_{31} \\ w^2_{12} & w^2_{22} & w^2_{32} \end{matrix} \times \begin{matrix} a^1_1 & a^1_1 \\ a^1_2 & a^1_2 \\ a^1_3 & a^1_3 \end{matrix} + \begin{matrix} w^2_{11} & w^2_{21} & w^2_{31} \\ w^2_{12} & w^2_{22} & w^2_{32} \end{matrix} \times \begin{matrix} a^1_1 & a^1_1 \\ a^1_2 & a^1_2 \\ a^1_3 & a^1_3 \end{matrix} + \begin{matrix} b^2_1 & b^2_1 \\ b^2_2 & b^2_2 \end{matrix}$$

(2,2) (2,3) (3,2) (2,3) (3,2) (2,2)

$$\begin{matrix} a^1_1 & a^1_1 \\ a^1_2 & a^1_2 \end{matrix} = f \left(\begin{matrix} z^2_1 & z^2_1 \\ z^2_2 & z^2_2 \end{matrix} \right)$$

(2,2) (2,2)

$$\begin{array}{c} \begin{matrix} z_1^1 & z_2^1 \\ z_2^1 & z_2^1 \\ z_3^1 & z_3^1 \end{matrix} \\ (3,2) \\ Z^1 \end{array} = \begin{array}{c} \begin{matrix} w_{11}^1 & w_{21}^1 & w_{31}^1 \\ w_{12}^1 & w_{22}^1 & w_{32}^1 \\ w_{13}^1 & w_{23}^1 & w_{33}^1 \end{matrix} \\ ((3,3), 2) \\ W^1 \end{array} \times \begin{array}{c} \begin{matrix} a_1^0 & a_1^0 \\ a_2^0 & a_2^0 \\ a_3^0 & a_3^0 \end{matrix} \\ (3,2) \\ A^0 \end{array} + \begin{array}{c} \begin{matrix} b_1^1 & b_1^1 \\ b_2^1 & b_2^1 \\ b_3^1 & b_3^1 \end{matrix} \\ (3,2) \\ B^1 \end{array} \Bigg| \begin{array}{c} \begin{matrix} a_1^1 & a_1^1 \\ a_2^1 & a_2^1 \\ a_3^1 & a_3^1 \end{matrix} \\ (3,2) \\ A^1 \end{array} = f \left(\begin{array}{c} \begin{matrix} z_1^1 & z_1^1 \\ z_2^1 & z_2^1 \\ z_3^1 & z_3^1 \end{matrix} \\ (3,2) \\ Z^1 \end{array} \right) \quad Z^1 = (W^1)^T A^0 + B^1$$

$$\begin{array}{c} \begin{matrix} z_1^2 & z_2^2 \\ z_2^2 & z_2^2 \\ z_2^2 & z_2^2 \end{matrix} \\ (2,2) \\ Z^2 \end{array} = \begin{array}{c} \begin{matrix} w_{11}^2 & w_{21}^2 & w_{31}^2 \\ w_{12}^2 & w_{22}^2 & w_{32}^2 \\ w_{12}^2 & w_{22}^2 & w_{32}^2 \end{matrix} \\ ((2,3), 2) \\ W^2 \end{array} \times \begin{array}{c} \begin{matrix} a_1^1 & a_1^1 \\ a_2^1 & a_2^1 \\ a_3^1 & a_3^1 \end{matrix} \\ (3,2) \\ A^1 \end{array} + \begin{array}{c} \begin{matrix} b_1^2 & b_2^2 \\ b_2^2 & b_2^2 \\ b_2^2 & b_2^2 \end{matrix} \\ (2,2) \\ B^2 \end{array} \Bigg| \begin{array}{c} \begin{matrix} a_1^1 & a_1^1 \\ a_2^1 & a_2^1 \end{matrix} \\ (2,2) \\ A^2 \end{array} = f \left(\begin{array}{c} \begin{matrix} z_1^2 & z_2^2 \\ z_2^2 & z_2^2 \end{matrix} \\ (2,2) \\ Z^2 \end{array} \right) \quad Z^2 = (W^2)^T A^1 + B^2$$

$$Z^{[L]} = (W^{[L]})^T A^{[L-1]} + B^{[L]}$$

$$A^{[L]} = f^{[L]}(Z^{[L]})$$

$$Z^{[L]} : (n^{[L]}, m)$$

$$A^{[L]} : (n^{[L]}, m)$$

$$\partial Z^{[L]} : (n^{[L]}, m)$$

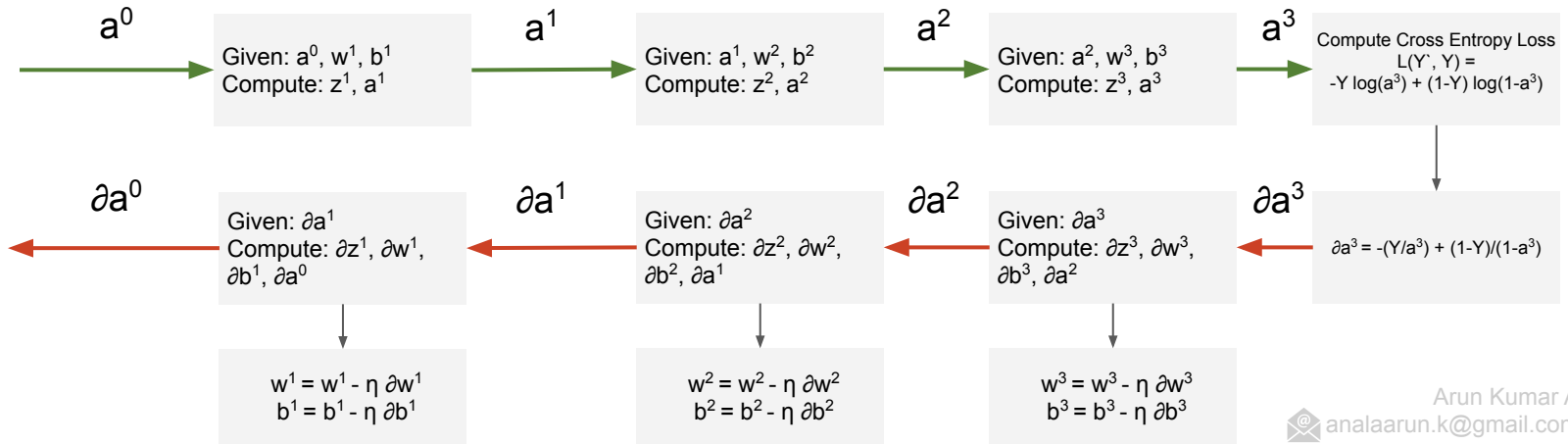
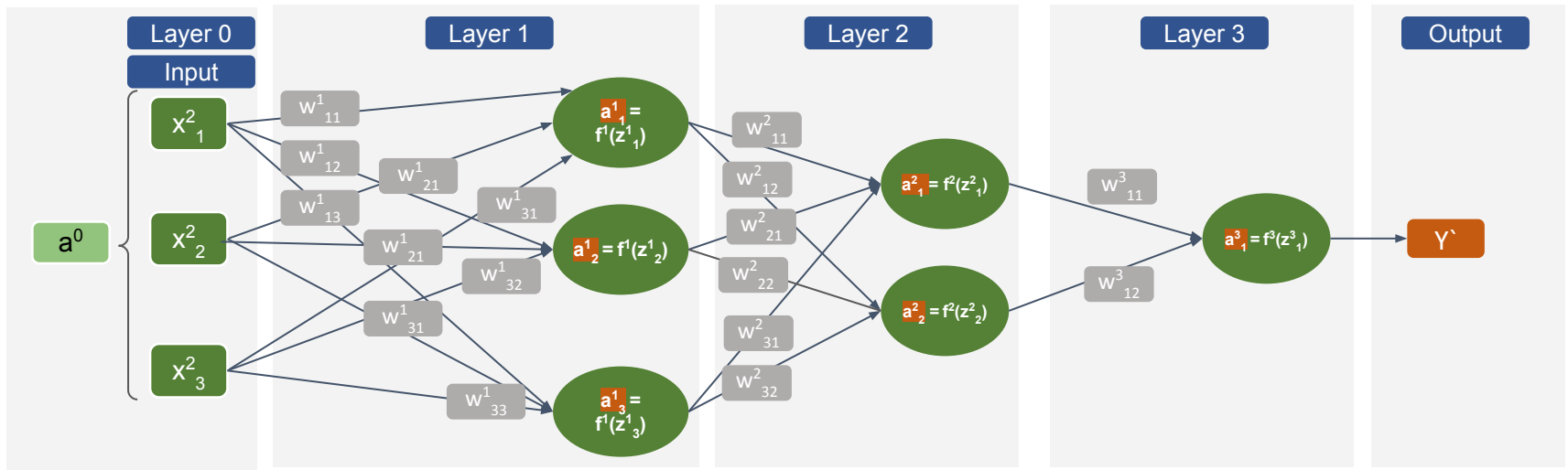
$$\partial A^{[L]} : (n^{[L]}, m)$$

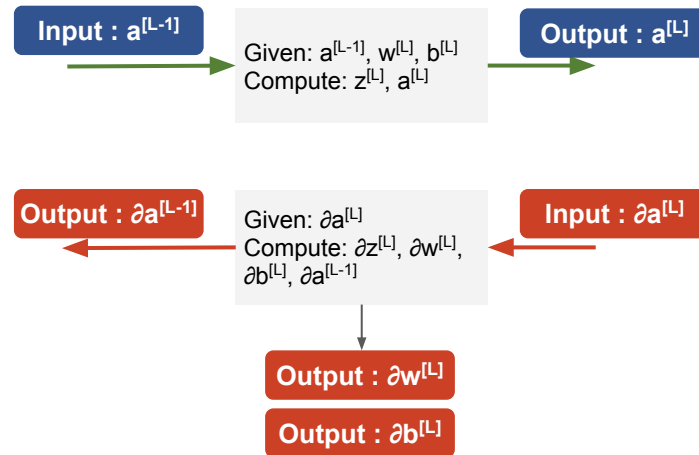
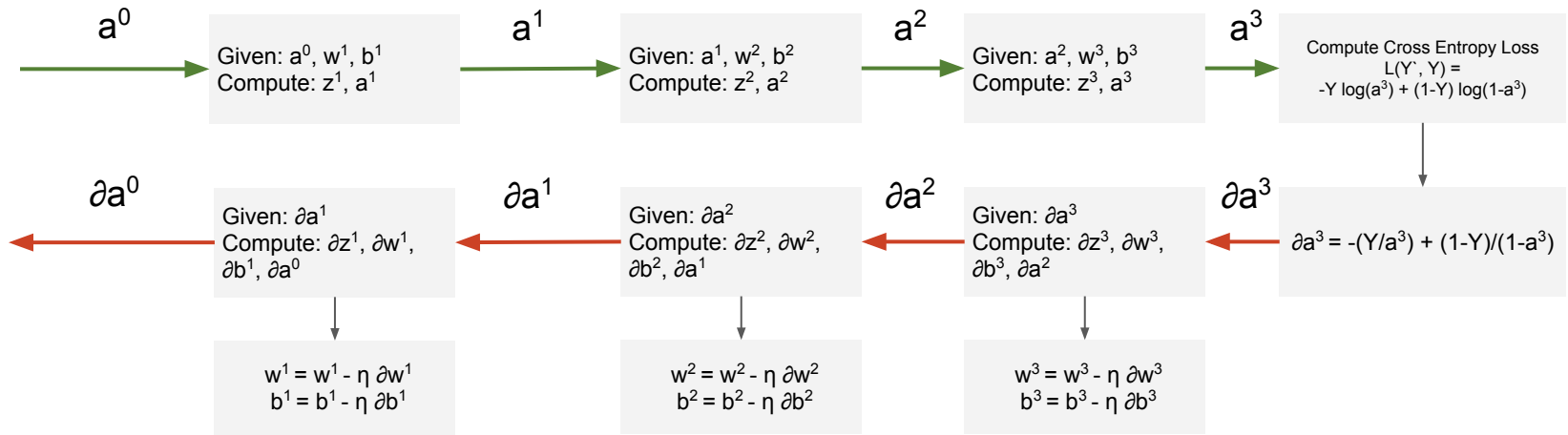
$$W^{[L]} : (n^{[L]}, n^{[L-1]}, m)$$

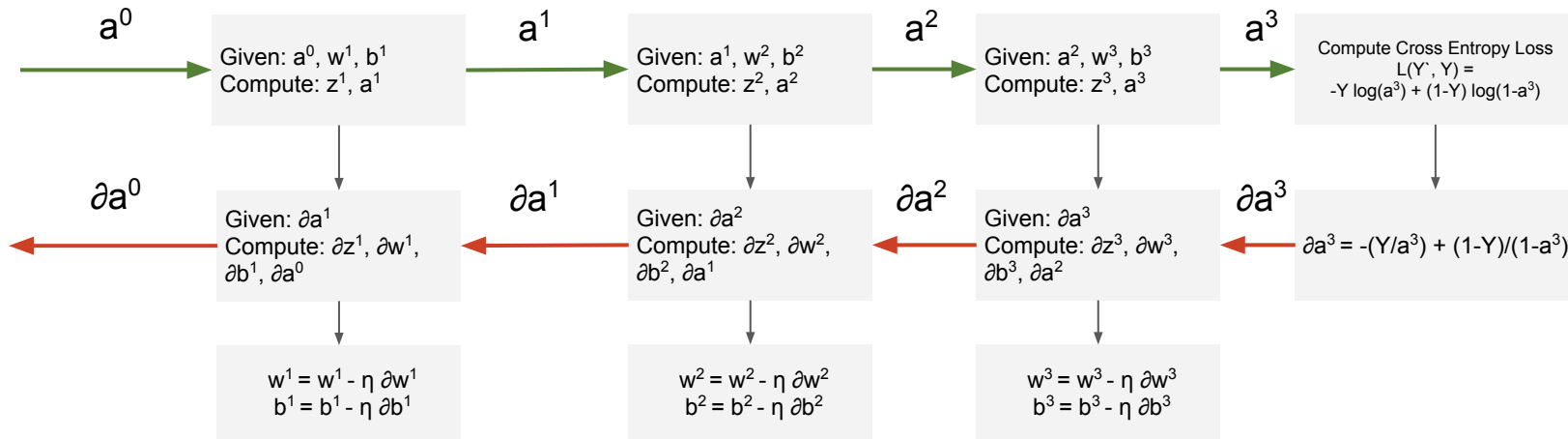
$$B^{[L]} : (n^{[L]}, m)$$

$$\partial W^{[L]} : (n^{[L]}, n^{[L-1]}, m)$$

$$\partial B^{[L]} : (n^{[L]}, m)$$







$$a^{[L]} = f^{[L]}(z^{[L]})$$

$$z^{[L]} = w^{[L]} a^{[L-1]} + b^{[L]}$$

$$\partial z^{[L]} = \partial a^{[L]} * f^{[L]'}(z^{[L]})$$

$$\partial w^{[L]} = \partial z^{[L]} * a^{[L-1]}$$

$$\partial b^{[L]} = \partial z^{[L]}$$

$$\partial a^{[L-1]} = w^{[L]T} * \partial z^{[L]}$$

$$\partial z^{[L]} = w^{[L+1]T} * \partial z^{[L+1]} * f^{[L]'}(z^{[L]})$$

$$\partial Z^{[L]} = \partial A^{[L]} * f^{[L]'}(Z^{[L]})$$

$$\partial w^{[L]} = 1/m * \partial Z^{[L]} * A^{[L-1]T}$$

$$\partial b^{[L]} = 1/m * \Sigma(\partial z^{[L]})$$

$$\partial A^{[L-1]} = W^{[L]T} * \partial Z^{[L]}$$

For 1 Training Sample

Forward Prop

Input : $a^{[L-1]}$

Output : $a^{[L]}$

Backward Prop

Input : $\partial a^{[L]}$

Output : $\partial a^{[L-1]}$

$$z^{[L]} = w^{[L]} a^{[L-1]} + b^{[L]}$$

$$\partial z^{[L]} = \partial a^{[L]} * f^{[L]'}(z^{[L]})$$

$$a^{[L]} = f^{[L]}(z^{[L]})$$

$$\partial w^{[L]} = \partial z^{[L]} * a^{[L-1]}$$

$$\partial b^{[L]} = \partial z^{[L]}$$

$$\partial a^{[L-1]} = w^{[L]T} * \partial z^{[L]}$$

$$z^{[L]} : (n^{[L]}, 1)$$

$$\partial z^{[L]} : (n^{[L]}, 1)$$

$$a^{[L]} : (n^{[L]}, 1)$$

$$\partial a^{[L]} : (n^{[L]}, 1)$$

$$w^{[L]} : (n^{[L]}, n^{[L-1]})$$

$$\partial w^{[L]} : (n^{[L]}, n^{[L-1]})$$

$$b^{[L]} : (n^{[L]}, 1)$$

$$\partial b^{[L]} : (n^{[L]}, 1)$$

For m Training Samples

Also called Vectorized Implementation

Forward Prop

Input : $a^{[L-1]}$

Output : $A^{[L]}$

Backward Prop

Input : $\partial A^{[L]}$

Output : $\partial A^{[L-1]}$

$$Z^{[L]} = (W^{[L]})^T A^{[L-1]} + B^{[L]}$$

$$\partial Z^{[L]} = \partial A^{[L]} * f^{[L]'}(Z^{[L]})$$

$$A^{[L]} = f^{[L]}(Z^{[L]})$$

$$\partial w^{[L]} = 1/m * \partial Z^{[L]} * A^{[L-1]T}$$

$$\partial b^{[L]} = 1/m * \Sigma(\partial z^{[L]})$$

$$\partial A^{[L-1]} = W^{[L]T} * \partial Z^{[L]}$$

$$Z^{[L]} : (n^{[L]}, m)$$

$$\partial Z^{[L]} : (n^{[L]}, m)$$

$$A^{[L]} : (n^{[L]}, m)$$










$$\partial A^{[L]} : (n^{[L]}, m)$$

$$W^{[L]} : (n^{[L]}, n^{[L-1]}, m)$$

$$\partial W^{[L]} : (n^{[L]}, n^{[L-1]}, m)$$

$$B^{[L]} : (n^{[L]}, m)$$

$$\partial B^{[L]} : (n^{[L]}, m)$$

Name	Plot	Equation	Derivative
Identity		$f(x) = x$	$f'(x) = 1$
Binary step		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1 + e^{-x}}$	$f'(x) = f(x)(1 - f(x))$
TanH		$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2 + 1}$
Rectified Linear Unit (ReLU)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU) [2]		$f(x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Exponential Linear Unit (ELU) [3]		$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
SoftPlus		$f(x) = \log_e(1 + e^x)$	$f'(x) = \frac{1}{1 + e^{-x}}$