Variance Ratio Tests for Random Walk Hypothesis Using NIFTY100.

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INTRODUCTION

Markets are said to be efficient when current stock prices fully reflect available information about the value of the firm. There is no way to earn excess profits by using this information. In an efficient market, competition causes the full effects of new information to be reflected instantaneously in actual prices. Variance Ratio is used to test this efficient market hypothesis to check if the stocks follow a random walk. We look at the weak form of market efficiency which is the least restrictive form, where current prices fully incorporate information in past history of prices only, i.e. nobody can beat the market by analyzing past prices.

Although, the markets may not always be efficient in nature. Certain kinds of deviations can occur in case of inefficiency: mean reversion and persistence. Mean Reversion is the situation where any information shock to the price series is temporary in nature and the effects dies down quickly. It is often attributed to overreaction by investors, leading to temporary deviations from long term mean and is subsequently corrected. On the other hand, persistence refers to a long-term memory in prices due to prolonged effect of any shock which is attributed to slow reaction by investors to the current information as compared to prior information. The existence of mean reversion and persistence imply presence of predictable information in prices. This however, does not lead to profitable trading strategies due to transaction costs.

METHODOLY TO TEST HYPOTHESIS

Lo and Mac (1988) suggest the use of a Variance Ratio (VR) statistic to test the returns in the random walk hypothesis. The idea behind Variance Ratio test is that if the natural logarithm series Y_t is a pure random walk, the variance of its q-differences grows proportionally with difference q. This implies, the variance of the increments in a random walk is linear in the sampling interval. Therefore, if a time series follows a random walk process, the variance of its q-differences should be q times the variance of its first differences. The variance ratio, VR(q), is defined as:

$$\boldsymbol{VR(q) = \frac{\sigma^2(q)}{\sigma^2(1)}}$$

Where, $\sigma^2(q)$ is 1/q the variance of the q-differences and $\sigma^2(1)$ is the variance of the first differences. According to Lo and MacKinlay (1988), formulas for the calculation of $\sigma^2(q)$ and $\sigma^2(1)$ are as follows:

$$\boldsymbol{\sigma^2(q) = \frac{1}{q(nq - q + 1)(1 - \frac{q}{nq})} \sum_{t=q}^{nq} (Y_t - Y_{t-q} - q\hat{\mu})^2}$$

and

$$\boldsymbol{\sigma^2(1) = \frac{1}{(nq-1)}\sum_{t=1}^{nq}(Y_t - Y_{t-1} - \hat{\mu})^2}$$

where,

$$\boldsymbol{\hat{\mu} = \frac{1}{nq}(Y_{nq} - Y_0)}$$

Y_0 and Y_{nq} are the first and last observations of the time series. The distribution of the statistic is as follows:

$$\boldsymbol{\psi(q) = \sqrt{nq}(VR(q) - 1)\left(\frac{2(2q-1)(q-1)}{3q}\right)^{-\frac{1}{2}} \sim N(0,1)}$$

The Variance Ratio is not unbiased and inconsistent, but efficient. The sampling distribution of the Variance Ratio statistic assumes iid increments. If the returns are uncorrelated in nature, then we need a heteroscedasticity-consistent VR estimator.

$$\boldsymbol{\hat{\delta}_k = \frac{nq\sum_{j=k+1}^{nq}(p_j - p_{j-1} - \hat{\mu})^2\,(p_{j-k} - p_{k-1} - \hat{\mu})^2}{(\sum_{j=1}^{nq}(p_j - p_{j-1} - \hat{\mu})^2)^2}}$$

$$\boldsymbol{\hat{\theta}(q) = 4\sum_{k=1}^{q-1}(1 - \frac{k}{q})^2\hat{\delta}_k}$$

$$\boldsymbol{\psi^*(q) = \frac{\sqrt{nq}(VR(q) - 1)}{\sqrt{\hat{\theta}}} \sim N(0,1)}$$

$\psi^*(q)$ is the heteroscedasticity-consistent estimator which is also called Z2 statistic.

The 95% Variance-bounds are calculated as follows:

$$\boldsymbol{\mathbf{VR}(q) \in \left(1 + 1.96\sqrt{\frac{\hat{\theta}}{nq}}\right)}$$

DATA FOR THE ANALYSIS

The study employs Daily Market Capitalization and Average Daily Turnover of the stocks in NIFTY100 which have been extracted from Reuters, for a period of three months, 3rd December, 2019 to 3rd March, 2020.

For Market Capital, the average of all the 50 stocks have been computed and then on the basis of their values, they have been arranged into deciles. 10 deciles have been created, each consisting 5 stocks.

Decile 1 and Decile 10 as per Market Capitalization are shown as follows:

DECILE 1	AVG MARKET CAP (IN ₹ Mil)	DECILE 10	AVG MARKET CAP (IN ₹ Mil)
RELIANCE INDUSTRIES	9,505	PAGE INDUSTRIES	260
TATA CONSULTANCY	8,086	ORACLE	246
HDFC BANK	6,827	NHPC	240
HINDUSTAN UNILEVER	4,478	L&T FINANCE HOLDINGS	238
HDFC	4,121	ASHOK LEYLAND	237
ICICI BANK	3,453	NEW INDIA ASSURANCE	234
INFOSYS	3,204	VODAFONE IDEA	158
KOTAK MAHINDRA	3,195	INDIABULLS HOUSING	130
BHARTI AIRTEL	2,873	YES BANK	108
BAJAJ FINANCE	2,793	TATA MOTORS A DVR	37

For Average Daily Turnover, the average of all the 50 stocks have been computed and then on the basis of their market capital values, the stocks have been arranged into deciles. 10 deciles have been created, each consisting 5 stocks.

Decile 1 and Decile 10 as per Average Daily Turnover are shown as follows:

DECILE 1	AVG MARKET CAP (IN ₹ Mil)	DECILE 10	AVG MARKET CAP (IN ₹ Mil)
RELIANCE INDUSTRIES	12603	CADILA HEALTHCARE	437
STATE BANK OF INDIA	11800	TATA MOTORS A DVR	368
INDIABULLS HOUSING	11626	BOSCH	299
ICICI BANK	10666	NHPC	186
YES BANK	10197	BAJAJ HOLDINGS & INVS.	165
BHARTI AIRTEL	8208	HINDUSTAN ZINC	114
HDFC BANK	7400	GIC	112
HDFC	7375	NEW INDIA ASSURANCE	61
INDUSIND BANK	6791	P&G HYGIENE	58
TATA MOTORS	12603	ORACLE	53

The data is imported into RStudio, where the data is analyzed, converted into a time series. The test statistics and the auxiliary functions are written as R functions. The code and its corresponding comments have been mentioned in the appendix.

The summary statistics for the deciles according to Market Capitalization and Average Daily Turnover are:

As per Market Capitalization:

Decile	Min	Median	Mean	Max
Decile 1	-0.0385	0.0000	-0.0009	0.0209
Decile 2	-0.0033	0.0000	0.0000	0.0204
Decile 3	-0.02507	-0.0002	-0.0005	0.0274
Decile 4	-0.0336	-0.0008	-0.001	0.0160
Decile 5	-0.0358	-0.0011	-0.0008	0.0215
Decile 6	-0.0533	-0.0004	-0.0008	0.0325
Decile 7	-0.0439	-0.0008	-0.0024	0.0334
Decile 8	-0.0355	-0.0015	-0.0018	0.0259
Decile 9	-0.0406	0.0006	-0.0003	0.0362
Decile 10	-0.0511	-0.0025	-0.0031	0.0277

As per Average Daily Turnover:

Decile	Min	Median	Mean	Max
Decile 1	-0.0355	-0.0003	-0.0013	0.0286
Decile 2	-0.0448	0.0011	-0.0002	0.0173
Decile 3	0.0264	0.0000	0.0007	0.0266
Decile 4	-0.0387	0.0002	-0.0003	0.0269
Decile 5	-0.0399	0.0009	-0.0005	0.0259
Decile 6	0.0298	0.0000	0.0023	0.0258
Decile 7	-0.0307	0.0000	-0.0005	0.0268
Decile 8	-0.0352	0.0008	0.0005	0.0160
Decile 9	-0.0346	0.0000	-0.0007	0.0207
Decile 10	-0.0434	-0.0014	-0.0027	0.0228

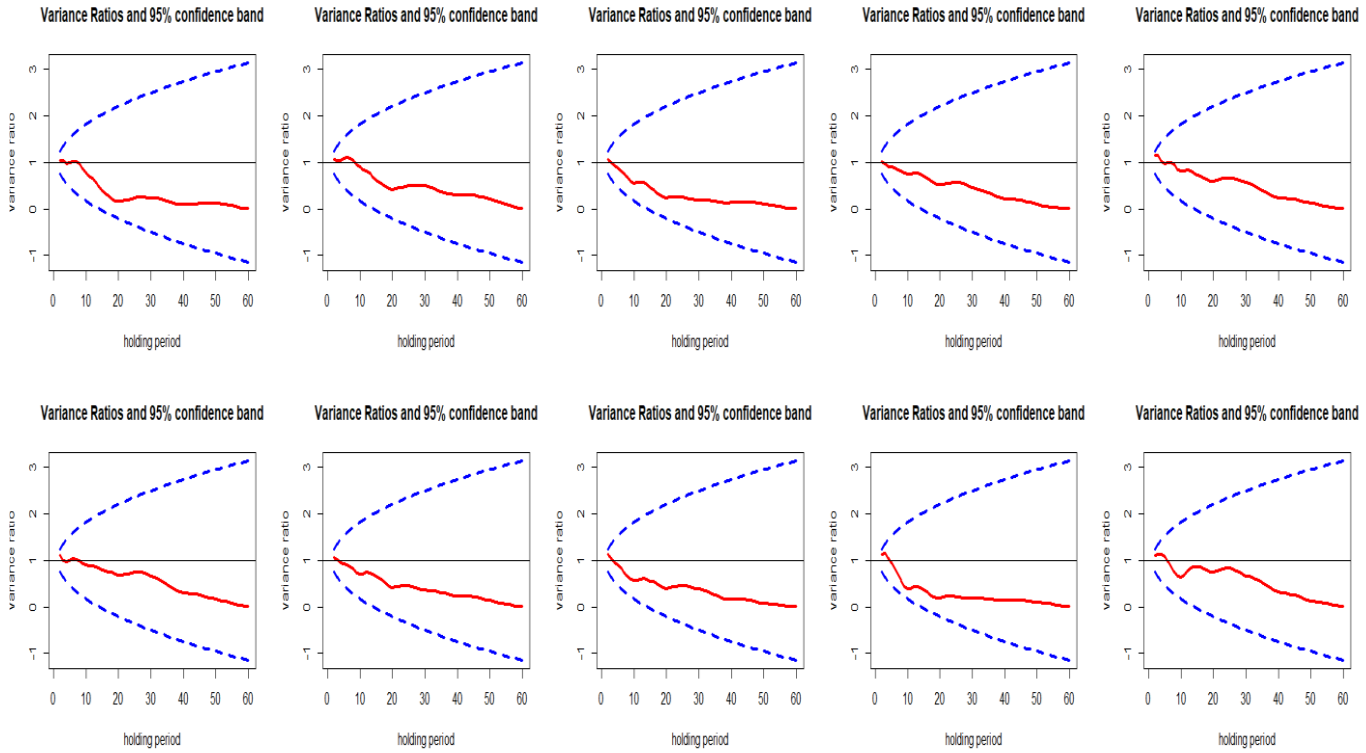
EMPIRICAL TEST RESULTS

This study has employed variance ratio tests for the null hypothesis, namely heteroskedastic increments random walk.

The Variance Ratio values for the Deciles as per Market Capitalization with ascending k values are as follows:

q/Deciles	1	2	3	4	5	6	7	8	9	10
5	0.9885	1.0829	0.8793	0.9001	0.9836	0.9930	0.9201	0.9122	0.9472	1.0907
10	0.7781	0.9074	0.5579	0.7598	0.8210	0.8988	0.7055	0.5728	0.3841	0.6495
15	0.3837	0.6259	0.4597	0.7027	0.7202	0.7932	0.6521	0.5609	0.3655	0.8674
20	0.1612	0.4207	0.2376	0.5192	0.5853	0.6825	0.4140	0.4039	0.1826	0.7505
25	0.2339	0.5030	0.2512	0.5808	0.6652	0.7532	0.4517	0.4595	0.2215	0.8320
30	0.2440	0.4967	0.1918	0.4584	0.5681	0.6713	0.3609	0.3834	0.1990	0.6679
35	0.1698	0.3731	0.1668	0.3458	0.3977	0.4825	0.3109	0.2576	0.1686	0.5337
40	0.0936	0.3090	0.1367	0.2244	0.2455	0.3115	0.2356	0.1657	0.1437	0.3295
45	0.1192	0.3041	0.1513	0.1891	0.2055	0.2602	0.2260	0.1541	0.1448	0.2520
50	0.1318	0.2138	0.1081	0.1000	0.1318	0.1684	0.1419	0.0799	0.1008	0.1336
55	0.0708	0.0967	0.0516	0.0375	0.0561	0.0844	0.0647	0.0423	0.0625	0.0789
60	0.0070	0.0119	0.0066	0.0058	0.0045	0.0093	0.0058	0.0061	0.0041	0.0137
65	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

From the table depicting variance ratios for the first decile within market capitalization across holding periods(k), we see conformity to Efficient Market Hypothesis (EMH) in the first few holding periods (k=5,10) due to the fact that variance ratio values (VR) lie close to unity. Subsequently, there is departure from market efficiency and mean reversion on account of VR's being below unity. Going to the second decile, we see a tendency for persistence in the first holding period i.e k=5, as given by VR>1. Subsequently, there is market efficiency in the second holding period i.e k=10, given by the fact the VR is close to unity. For all the subsequent holding periods, there is a tendency of mean version as VR stays below unity from hereon. The third decile starts with market efficiency (k=5), however subsequently there is evidence against EMH and mean reversion is seen as given by VR<1.Within the fourth through sixth decile, market efficiency stays on for longer, starting from holding period k=5 to k=15. Post that there is departure from EMH and mean reversion as given by VR<1. Within the seventh decile, market efficiency returns back to lasting two holding periods and posting inefficiency and mean reversion thereon. For deciles eight and nine, market remains efficient for just the starting holding period of k=5 and reporting a departure from EMH from there. Mean reversion exists post k=5. For the last decile, VR lies above unity in k=5 holding period, indicating persistence. Post k=5 there is mean reversion, with efficiency seen returning in holding period k=15 through k=25.

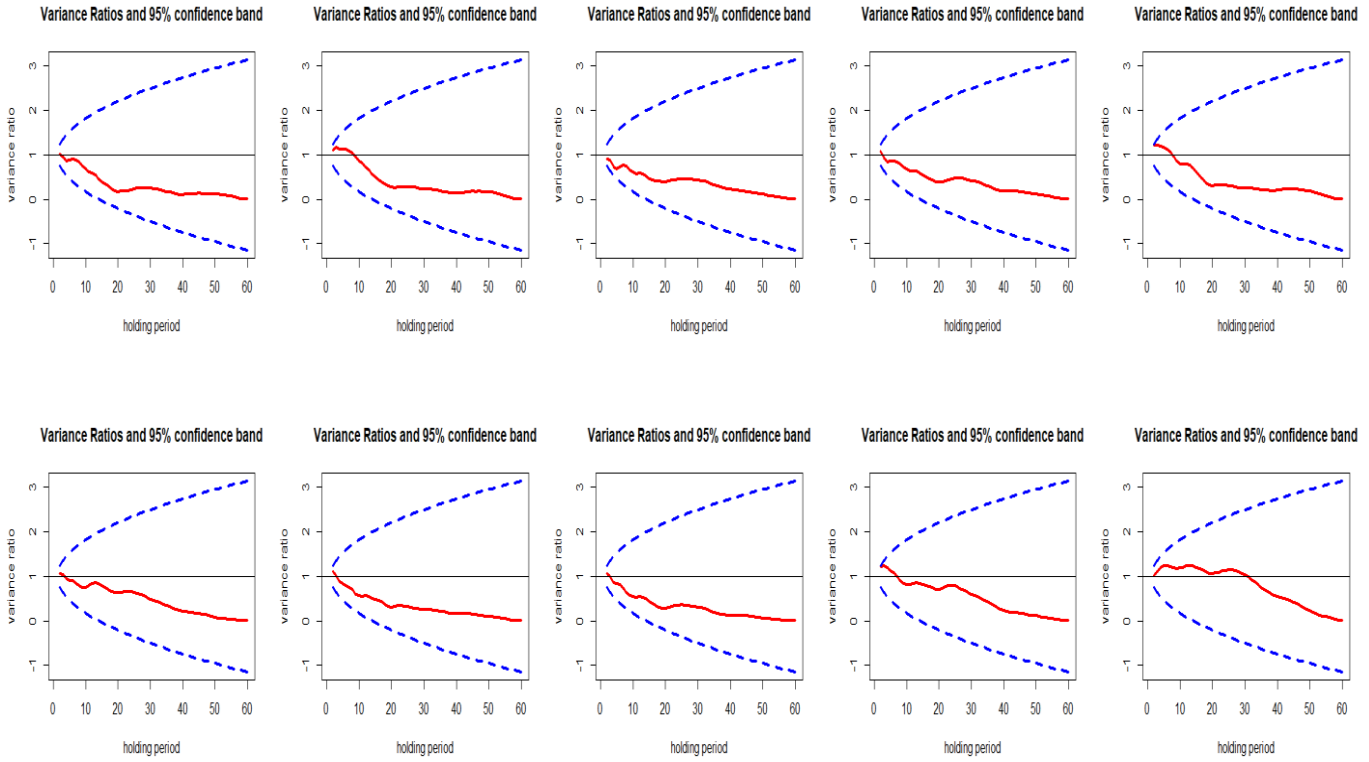


Now looking at the Variance Ratio plots for deciles within Market Capitalization, where VR are calculated against 95% confidence bands for each holding period, using standard errors under i.i.d returns. The plots corroborate the findings of the table. For the first decile, VR plot starts horizontal at unity and then turns downwards (mean reversion) remaining within the bands. The second decile starts with the persistence, goes on to efficiency and then mean reversion, while remaining within the bands. This is shown by a slight hump in the plot in the beginning and then a downward turn. The third decile, like the first decile starts with being horizontal at unity, although for just one holding period and then turns downward on account of mean reversion. For fourth through sixth decile, the plot stays horizontal at unity for longer and then turns downward depicting mean reversion. For the seventh decile, like the first one, the plot lies horizontal at unity for just two periods, before turning downward. For deciles eight and nine, there are sharp downward turns due to the fact that efficiency lasts only one period before showing mean reversion. Last decile starts with persistence with the plot showing a climb above unity in the beginning, and then turning downwards due to mean reversion. However, for holding periods k= 15 through 25, the plot returns to efficiency as seen by a jump to near unity until it again turns downwards to show mean reversion.

The Variance Ratio values for the Deciles as per Average Daily Turnover with ascending k values are as follows:

q/Deciles	1	2	3	4	5	6	7	8	9	10
5	0.8767	1.1368	0.6813	0.8622	1.1754	0.9075	0.8326	0.8506	1.1407	1.2366
10	0.677	0.8693	0.6139	0.6827	0.7948	0.7409	0.564	0.5402	0.8144	1.1918
15	0.389	0.498	0.4651	0.5439	0.6064	0.793	0.4821	0.4455	0.8227	1.2039
20	0.1744	0.2715	0.3939	0.3871	0.3017	0.6313	0.3044	0.287	0.7165	1.0658
25	0.2395	0.2922	0.4627	0.478	0.3156	0.6472	0.3191	0.3621	0.7888	1.1569
30	0.2496	0.2427	0.4373	0.4225	0.254	0.4945	0.2643	0.3134	0.6036	1.0322
35	0.1707	0.1934	0.3324	0.2993	0.2248	0.3494	0.2174	0.1953	0.4286	0.7593
40	0.1093	0.1475	0.2300	0.1857	0.2117	0.2149	0.1682	0.1299	0.2357	0.5474
45	0.1173	0.1826	0.1754	0.1774	0.2400	0.1661	0.1531	0.1197	0.1759	0.4239
50	0.1249	0.1737	0.1173	0.1212	0.1857	0.0835	0.1047	0.0662	0.1175	0.2314
55	0.0708	0.0842	0.0545	0.0592	0.0867	0.0346	0.0501	0.028	0.0567	0.0923
60	0.009	0.0081	0.0063	0.0071	0.0102	0.0028	0.0038	0.0038	0.0048	0.0102
65	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

From the table showing Variance Ratio values (VR) across deciles (q) for difference holding periods (k), we decipher the following. The first decile starts with conformity to EMH, as given by closeness of VR to unity and remains efficient in k=5 holding period before showing tendency for mean reversion for all holding period thereon. The second decile starts with persistence, as given by VR>1 in holding period k=5 and then turns to efficiency in k=10 holding period. Beyond this holding period, there is mean reversion as shown by VR<1. For decile three, efficiency is not seen across any holding period and in fact, there is pure mean reversion throughout with VR<1. Decile four, like decile one starts with market efficiency in holding period k=5, as seen by closeness of VR to unity and then turns to mean reversion as depicted by VR<1. Decile five starts with persistence as seen by VR>1, and then turns to efficiency with VR close to unity before showing mean reversion i.e VR<1. For decile six, efficiency lasts longer i.e holding periods k=5 through k=15, before showing a departure from EMH and then depicting mean reversion. For deciles seven and eight, departure from efficiency is seen post holding period k=5 and conforming to EMH is seen for only the starting holding period. There is evidence for mean reversion as seen by all VR<1 post k=5. For decile nine, the plot begins with persistence or VR>1 and then shows efficiency from holding period k=10 through k=25. Efficiency is seen for four holding periods. Post holding period k=25, there is mean reversion as seen by VR<1. For the last decile, there is persistence or VR>1 from the beginning i.e holding period k=5 through k=30, for five holding periods, and then showing efficiency as given by VR being close to 1. For periods hereon there is mean reversion.



Now looking at the Variance Ratio plots for deciles within Average Daily Turnover, where VR are calculated against 95% confidence bands for each holding period, using standard errors under i.i.d returns. The plots corroborate the findings of the table. The plot of first decile shows conformity to efficiency by being horizontal to unity for holding period k=5 and then turning downward showing mean reversion. The plot of second decile starts by being above unity, and then falling back in line with unity before turning downward to show mean reversion. As a result, the plot shows a little hump in the beginning, showing persistence and efficiency. The plot of decile three remains below unity throughout, indicating inefficiency or mean reversion. Plot of decile four looks like that of decile one, being horizontal at unity for just the first holding period k=5 before turning downward showing mean reversion. Plot of decile five, starts by being above unity in the first holding period k=5, then returning to efficiency and finally turning downward to indicate mean reversion. Plots for decile six, efficiency lasts longer and thus the plot is horizontal at unity for three holding periods, before turning downward to indicate mean reversion. For deciles seven and eight, like decile one, the plot starts by being horizontal at unity before turning downward indicating a departure from efficiency and showing mean reversion. For decile nine, the plot begins above unity and then returns to efficiency by being horizontal at unity for four holding periods, before turning downward to indicate mean reversion. For decile ten, the plot stays above unity from the beginning till five holding periods, then being horizontal at unity for one holding period before turning downward and depicting mean reversion.

CONCLUSION

The paper looks at market efficiency by examining a connection between Size of a Company with th e help of Market Capitalization and Liquidity of a Company with the help of Average Daily Turnover with the weak form efficiency respectively in the stocks of NIFTY100. The Variance Ratios were calculated for portfolios of stocks and observed that there is an association of size and liquidity of a company with market efficiency. The results of this study indicate that the stocks in NIFTY100 are efficient suggesting that past movements in stock prices cannot be used to predict their future movements. It is also observed that VR values tend to reduce with increase in k values.

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