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Serial correlation in high-frequency data and the link with liquidity

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2. Introduction

2.1. Market Efficiency

Market efficiency is defined as being of the

Weak-form Prices in the market incorporate all information in past prices.

Semi-Strong form Prices in the market incorporate all available public information.

Strong form Prices in the market incorporate all available public and private information.

In one version of an informationaly efficient market, changes in prices are expected to be random. Lo and MacKinlay (1989)

Prices are then expected to follow a *martingale difference sequence.* or (specifically) a *random walk.*

The RWH is controversial and extensively tested.

2.2. Random Walks

There are three versions of the random walk

IID increments: RW1

$$p_t = \mu + p_{t-1} + \epsilon_t, \quad \epsilon_t \overset{i.i.d}{\sim} (0, \sigma^2) \tag{1}$$

Independent increments: RW2

$$p_t = \mu + p_{t-1} + \epsilon_t, \quad \epsilon_t \overset{indep}{\sim} (0, \sigma_t^2) \tag{2}$$

Uncorrelated increments: RW3

$$p_t = \mu + p_{t-1} + \epsilon_t, \quad \epsilon_t \sim (0, \sigma_t^2), \quad Cov(\epsilon_t, \epsilon_{t-k}) = 0 \, \forall \, k \neq 0 \tag{3}$$

There are different tests for each version of the random walk.

RW1 Sequences and Reversals, Runs tests.

RW2 Filter Rules, technical analysis.

RW3 *Portmanteau* (Box-Pierce, Ljung-Box) tests, Variance Ratios.

3. Methodology

3.1. Variance Ratios

A Variance-Ratio test is a test of RW3. For a random walk,

$$p_t = \mu + p_{t-1} + r_t,$$

$$\begin{aligned} Var[r_t(2)] &= Var[r_t + r_{t-1}] \\ &= 2Var[r_t] + 2Cov[r_t, r_{t-1}] \end{aligned} \tag{4}$$

$$\begin{aligned} \text{Define} \quad VR(2) &= \frac{Var[r_t(2)]}{2Var[r_t]} \\ &= 1 + \rho_1 \end{aligned}$$

$\rho_1 = 0$ for a random walk. Hence $VR(2) = 1$.

$$\text{Similarly,} \quad VR(q) = \frac{Var[r_t(q)]}{qVar[r_t]} \tag{5}$$

For a random walk (RW3), $\rho_k = 0 \; \forall \; k$. Hence $VR(q) = 1$.

3.2. Variance-Ratios. contd.

The Lo and MacKinlay (1988) $VR(q)$ statistic

Say the sample has $nq+1$ observations. Define

$$\hat{\mu} = \frac{1}{nq}\sum_{k=1}^{nq}(p_k - p_{k-1}) \tag{6}$$

$$\overline{\sigma}_a^2 = \frac{1}{nq-1}\sum_{k=1}^{nq}(p_k - p_{k-1} - \hat{\mu})^2 \tag{7}$$

$$\overline{\sigma}_c^2 = \frac{1}{m}\sum_{k=q}^{nq}(p_k - p_{k-q} - q\hat{\mu})^2 \tag{8}$$

$$m = q(nq-q+1)(1-\frac{q}{nq}) \tag{9}$$

3.3. Variance Ratio Tests. Contd.

Now define the statistic

$$\overline{VR}(q) = \frac{\overline{\sigma}_c^2(q)}{\overline{\sigma}_a^2} \tag{10}$$

The distribution of the statistic is

$$\begin{aligned}
\Psi(q) &= \sqrt{nq}(\overline{VR}(q) - 1)\left(\frac{2(2q-1)(q-1)}{3q}\right)^{-1/2} && (11)\\
&= && (12)\\
&\sim N(0,1) && (13)
\end{aligned}$$

The $\overline{VR}$ is not unbiased and inconsistent, but efficient. The choice of alternatives to the null of the random walk affects the size and power of the Variance-Ratio test.

3.4. Heteroscedasticity-consistent estimators

The above sampling distribution of the Variance-Ratio stat assumes iid increments. If returns are merely uncorrelated, then we need a heteroscedasticity-consistent VR estimator. Define

$$\hat{\delta}_k = \frac{nq \sum_{j=k+1}^{nq} (p_j - p_{j-1} - \hat{\mu})^2 (p_{j-k} - p_{k-1} - \hat{\mu})^2}{\left[\sum_{j=1}^{nq} (p_j - p_{j-1} - \hat{\mu})^2\right]^2} \tag{14}$$

$$\hat{\theta}(q) = 4 \sum_{k=1}^{q-1} \left(1 - \frac{k}{q}\right)^2 \hat{\delta}_k \tag{15}$$

$$\Psi^*(q) = \frac{\sqrt{nq}(\overline{VR}(q) - 1)}{\sqrt{\hat{\theta}}} \tag{16}$$

$$\sim N(0, 1). \tag{17}$$

Here $\Psi^*(q)$ is the heteroscedasticity-consistent estimator. It's also called the $Z2$ statistic.

3.5. Calculations

Define

$$M_r(q) = VR(q) - 1 \tag{18}$$

It can be shown that

$$\begin{aligned} M_r(q) &= \hat{\rho}_1 - \frac{1}{4n\hat{\sigma}_a{}^2}\left[(r_1 - \hat{\mu})^2 + (r_n - \hat{\mu}^2)\right] \qquad (19)\\ &\simeq \hat{\rho}_1 \qquad (20) \end{aligned}$$

$$\text{Hence}\quad M_r(q) \simeq \frac{2(q-1)}{q}\hat{\rho}_1 + \frac{2(q-2)}{q}\hat{\rho}_2 + \ldots + \frac{2}{q}\hat{\rho}(q-1)$$

$$M_r(q) \overset{a}{=} \sum_{j=1}^{q-1} \frac{2(q-j)}{q}\hat{\rho}(j) \tag{21}$$

The 95% variance-bounds were calculated as

$$VR(q) \in \left(1 + 1.96\sqrt{\frac{\hat{\theta}}{nq}}\right) \tag{22}$$

3.6. Literature

- The VR test was popularized by Cochrane (1988). He also showed that the $VR(q)$ is a linear combination of autocorrelations.
- Lo and MacKinlay (1988) extended it to the more general null of an uncorrelated random walk. They also worked out the sampling distribution of the VR statistic under iid and uncorrelated increments.
- They also showed that the Variance-Ratio test had more power than the Dickey-Fuller, or the Box-Pierce tests.
- Standard practice in the literature *vis-a-vis* Variance-Ratio tests is to take weekly (sometimes daily) data, and run the Lo & MacKinlay (1988) test.
- The exercise is initially done assuming iid increments, and then uncorrelated increments (heteroscedasticity-consistent).

3.7. Long-horizon returns and Asymptotics

The two variable quantities in the $VR(q)$ test are q and T.

- Lo and MacKinlay (1988) assume that q is fixed, $T \rightarrow \infty$, but $q/T \rightarrow 0$. This has problems when q rises $w.r.t$ T. The asymptotics given above depend on this assumption. They show that the VR statistic is normal.
- Richardson and Stock (1989) assume that $q \rightarrow \infty$, $T \rightarrow \infty$, but $q/T \rightarrow \infty$. In their formulation, q is no longer fixed, but a random variable. They show that $VR(q)$ is a functional of Brownian Motion. This approximation has better asymptotic properties, than the Lo and MacKinlay (1988) approximation.
- Priestly (1982) shows that when $q \rightarrow \infty$, $T \rightarrow \infty$, and $q/T \rightarrow 0$, the $\hat{V}R(q)$ statistic is normal with mean $VR(q)$, and variance $(4q/3T)VR(q)$.

3.8. Is the Gaussian assumption of errors tenable?

- Return innovations need not be normal, in financial time series. Lo and MacKinlay assume normal innovations.
- Tse and Zhang (2002) have used Stable Paretian innovations to get the asymptotic distribution of the OVR statisic. [Monte Carlo only].
- Pan et al. (1997) have used a bootstrap technique to approximate the empirical distribution of the $VR(q)$ statistic. Others with similar approximations are Poterba and Summers (1988), Kim et al. (1991) (Use a randomization technique similar to the bootstrap), Cecchetti and sang Lam (1994), and many others.

4. Liquidity

- A market is said to be liquid if a “large volume of trades can be immediately executed with minimum effect on prices”.
- A liquid market has *depth*—A large volume has to be transacted to bring about a unit change in the price of the asset.
- A liquid market has *tightness*—There are minimum costs to trading positions in the market.
- A liquid market is *resilient*—The time to taken for prices to get back to their equilibrium levels (or a new equilibrium) after a shock, is minimal.
- Liquidity is not directly observable. There are various proxies to measuring it
  - bid-ask spreads
  - trading volume
  - impact-cost.

5. Empirical Investigation

- We test the Random Walk version of the EMH, on selected stocks from the NSE.
- How does liquidity affect market efficiency and how is this apparent across a cross-section of stocks, arranged by their liquidity?
- Liquidity reduces the uncertainty in prices (extraneous information, temporary changes in the price discovery function, etc).
- Prices would then assimilate new information quickly, and converge to their equilibrium values faster.

6. Data and Work

- Trades data for all NSE stocks in the period Mar1999-Feb 2001.
- 253,717,939 trades in 514 days. 24.56 trades per second.
- The trade data is discretized at a base frequency of 300 seconds.
- This gives us 67 data points per day.
- There were missing data to the tune of about 7%.
- Selection of the top 100 stocks using a liquidity measure: Impact Cost.
- Median non-trading probability: 3%.

6.1. Issues with HF data

- HF data have been relatively recently available.
- Irregular data. (Clock time *vs.* Trade time.)
- Choice of discretizing frequency.
- Choice of concatenation of data across days.

6.2. Selection of stocks by Impact Cost

- Impact Cost is a measure of the costs of trading. It is the premium that a buyer pays over and above the price (mid-quote) of a stock, and a seller loses when he sells a stock.
- A liquid stock has a very low impact cost.
- To calculate the IC for a stock, we calculate the cost incurred for a trade of a given size, on the buy and the sell side.
- Orders data is required for the calculation of this cost.
- Snapshots of the complete order-book (MBP) of the NSE are available four times a day.
- These snapshots are used to calculate the IC for a given stock at that time, on the buy side and the sell side.

6.3. Selection of stocks by IC. contd.

- The order size to calculate the IC was for Rs. 10,000.
- We calculate the ICs for 1382 stocks over 514 days, four times a day, during the period Mar1999-Feb2001.
- We took the median buy and sell ICs for a stock, and then took the average, to get the IC for that stock.
- We then took the hundred most liquid stocks in this period.
- These 100 stocks accounted for 83.32% of all the trades, and each stock traded about 4211 times/ day on an average.
- Each of the stocks were then cleaned of outliers.
- We divided the 100 stocks into ten deciles, based on their liquidity.
- As a comparative measure of liquidity, we also selected the most liquid stocks by trading intensity.

6.4. Impact Costs and trades for the first and the tenth deciles

Rank	Stock	#Trades	Stock	Buy	Sell	Avg
First Decile						
1	SATYAMCOMP	17971195	RELIANCE	0.07	0.07	0.07
2	ZEETELE	14746798	INFOSYSTCH	0.08	0.08	0.08
3	HIMACHLFUT	12609773	HINDLEVER	0.09	0.09	0.09
4	SILVERLINE	10987497	ITC	0.09	0.09	0.09
5	PENTSFWARE	10606971	SBIN	0.09	0.09	0.09
6	GLOBALTELE	10415715	ACC	0.11	0.11	0.11
7	SQRDSFWARE	9982348	L&T	0.11	0.11	0.11
8	RELIANCE	9204846	RANBAXY	0.11	0.11	0.11
9	ROLTA	7125411	SATYAMCOMP	0.11	0.11	0.11
10	INFOSYSTCH	7024350	MTNL	0.12	0.12	0.12
Tenth Decile						
90	HINDALC0	312177	JAINSTUDIO	0.25	0.25	0.25
91	BOMDYEING	311934	ORCHIDCHEM	0.25	0.25	0.25
92	GOLDTECHNO	311070	ORIENTINFO	0.25	0.25	0.25
93	BEL	307093	PHILIPS	0.25	0.25	0.25
94	STERLITIND	303246	POLARIS	0.25	0.25	0.25
95	ORCHIDCHEM	302334	PUNJABTRAC	0.25	0.25	0.25
96	POLARIS	299354	SMITHKLPHA	0.25	0.25	0.25
97	ORIENTBANK	295265	TATAPOWER	0.25	0.25	0.25
98	LML	294189	WOCKPHARMA	0.25	0.25	0.25
99	BANKBARODA	293264	AUROPHARMA	0.26	0.26	0.26
100	DABUR	290017	CORPBANK	0.26	0.26	0.26

6.5. Summary statistics for liquidity across portfolio-deciles.

Decile	Mean Trades	Mean IC	Minimum	Maximum
1	12323.29	0.098	0.07	0.12
2	11878.61	0.131	0.12	0.14
3	6615.22	0.144	0.14	0.15
4	4587.33	0.158	0.15	0.17
5	1783.87	0.174	0.17	0.18
6	2115.46	0.191	0.18	0.21
7	1146.91	0.219	0.21	0.23
8	594.96	0.231	0.23	0.24
9	904.73	0.245	0.24	0.25
10	666.27	0.252	0.25	0.26

7. Estimations and Results

- Variance Ratios were calculated for all stocks in each of the deciles.
- The aggregation q was from $q = 2$ (10 minutes), to $q = 3350$ (Two months).
- Cross-sectional averages of the variance ratios were calculated for each of the deciles (portfolios).
- Variance ratios were also calculated for the index NIFTY.
- Next, variance ratios were calculated for the ten *portfolio returns* formed by the deciles.
- For each of the VRs, heteroscedasticity-consistent variance bounds and the $Z2$ statistics were also calculated.

7.1. Results. contd

- We see that the null hypothesis of a random walk, of the RW3 kind can NOT be rejected for NSE stocks.
- Almost all the *liquid* stocks' prices behave like a random walk, individually.
- There is a some evidence of mean-reversion for the less actively traded stocks in the short-term.
- The hypothesis of weak-form market efficiency cannot be rejected for the most-liquid stocks.
- The index NIFTY is indistinguishable from a random walk even in the very short run.

7.2. Results: Liquidity

- We find that there's a direct correspondence between liquidity and random walk behaviour of stocks.
- The prices of the most liquid stocks should behave like random walks (RW3).
- This behaviour should decline with liquidity. This shows up in the time taken for the variance-ratio of a decile to be insignificantly different from unity.
- Some mean-reversion is expected at the least liquid cross-sectional average Variance-Ratio.
- The first decile average is a random walk with a few hours. The last decile takes more than a week.
- Positive cross-correlation among stocks taint the results if portfolio-returns are selected instead of average variance-ratios.

7.3. Cross-Sectional portfolio-wise Averages of Variance-Ratios of top ten portfolios of NSE stocks arranged by liquidity

Duration	q	Portfolios (Deciles) Cross-sectional averages of $VR(q)$'s.									
		1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}
10 min	2	0.955*	0.931*	0.899*	0.910*	0.895*	0.887*	0.866*	0.841*	0.827*	0.830*
15 min	3	0.916*	0.886*	0.850*	0.860*	0.840*	0.835*	0.807*	0.780*	0.750*	0.766*
20 min	4	0.891*	0.862*	0.820*	0.830*	0.810*	0.804*	0.776*	0.744*	0.710*	0.734*
25 min	5	0.878*	0.850*	0.799*	0.813*	0.792*	0.786*	0.759*	0.722*	0.685*	0.716*
30 min	6	0.871*	0.845*	0.788*	0.806*	0.779*	0.773*	0.748*	0.709*	0.670*	0.703*
35 min	7	0.866*	0.843*	0.781*	0.801*	0.769*	0.765*	0.740*	0.699*	0.659*	0.695*
40 min	8	0.862*	0.840*	0.774*	0.798*	0.762*	0.759*	0.734*	0.692*	0.652*	0.689*
45 min	9	0.862*	0.839*	0.770*	0.797*	0.757*	0.754*	0.730*	0.686*	0.646*	0.683*
50 min	10	0.861*	0.838*	0.766*	0.797*	0.752*	0.750*	0.728*	0.683*	0.641*	0.679*
55 min	11	0.859*	0.836*	0.762*	0.795*	0.747*	0.747*	0.726*	0.678*	0.636*	0.676*
1 hour	12	0.859*	0.834*	0.759*	0.794*	0.743*	0.744*	0.724*	0.674*	0.632*	0.672*
2 hours	24	0.854*	0.822*	0.742*	0.784*	0.718*	0.724*	0.707*	0.640*	0.604*	0.645*
3 hours	36	0.875*	0.833*	0.747*	0.792*	0.717*	0.725*	0.717*	0.629*	0.606*	0.641*
1 day	67	0.934	0.870	0.779*	0.827*	0.731*	0.732*	0.757*	0.647*	0.629*	0.663*
2 days	134	0.911	0.849	0.778	0.802	0.725*	0.711*	0.787	0.665*	0.628*	0.666*
1 week	335	0.915	0.817	0.748	0.761	0.713	0.665	0.793	0.643	0.625*	0.621*
2 weeks	670	0.931	0.797	0.736	0.699	0.713	0.619	0.810	0.629	0.645	0.566
1 month	1675	0.953	0.746	0.765	0.625	0.706	0.551	0.702	0.602	0.703	0.525
2 months	3350	0.910	0.683	0.787	0.558	0.702	0.453	0.652	0.594	0.739	0.489

7.4. Variance Ratios of top ten portfolios of NSE stocks arranged by liquidity.

Duration	q	$VR(q)$'s of Portfolios (Deciles) returns									
		1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}
10 minutes	2	1.014	1.021	1.027	1.041	1.071	1.024	1.026	0.937	0.980	0.998
15 minutes	3	0.965	0.991	1.008	1.032	1.076	1.024	1.022	0.914	0.951	0.995
20 minutes	4	0.927	0.971	0.985	1.019	1.068	1.021	1.020	0.904	0.939	0.999
25 minutes	5	0.905	0.965	0.970	1.012	1.065	1.025	1.021	0.899	0.931	1.007
30 minutes	6	0.895	0.967	0.963	1.014	1.065	1.031	1.028	0.900	0.930	1.019
35 minutes	7	0.890	0.973	0.963	1.018	1.066	1.036	1.037	0.901	0.932	1.030
40 minutes	8	0.883	0.973	0.959	1.021	1.064	1.039	1.039	0.904	0.931	1.039
45 minutes	9	0.882	0.979	0.958	1.024	1.065	1.042	1.046	0.906	0.932	1.047
50 minutes	10	0.881	0.981	0.956	1.027	1.065	1.045	1.053	0.908	0.934	1.054
55 minutes	11	0.878	0.982	0.951	1.025	1.062	1.045	1.059	0.908	0.934	1.056
1 hour	12	0.878	0.982	0.948	1.024	1.060	1.045	1.063	0.908	0.933	1.059
2 hours	24	0.874	0.985	0.923	1.017	1.035	1.055	1.067	0.897	0.932	1.061
3 hours	36	0.912	1.016	0.940	1.033	1.057	1.094	1.080	0.903	0.969	1.077
1 day	67	0.993	1.079	0.993	1.079	1.113	1.148	1.139	0.955	1.050	1.150
2 days	134	0.902	0.980	0.939	0.947	1.106	1.117	1.120	0.966	1.058	1.188
1 week	335	0.923	0.892	0.972	0.866	1.200	1.123	1.193	1.108	1.138	1.205
2 weeks	670	1.012	0.857	0.976	0.755	1.261	1.052	1.240	1.181	1.144	1.235
1 month	1675	1.050	0.742	1.022	0.662	1.501	0.754	1.285	1.521	1.389	1.466
2 months	3350	1.005	0.523	0.943	0.524	1.797	0.641	1.116	1.856	1.640	1.639

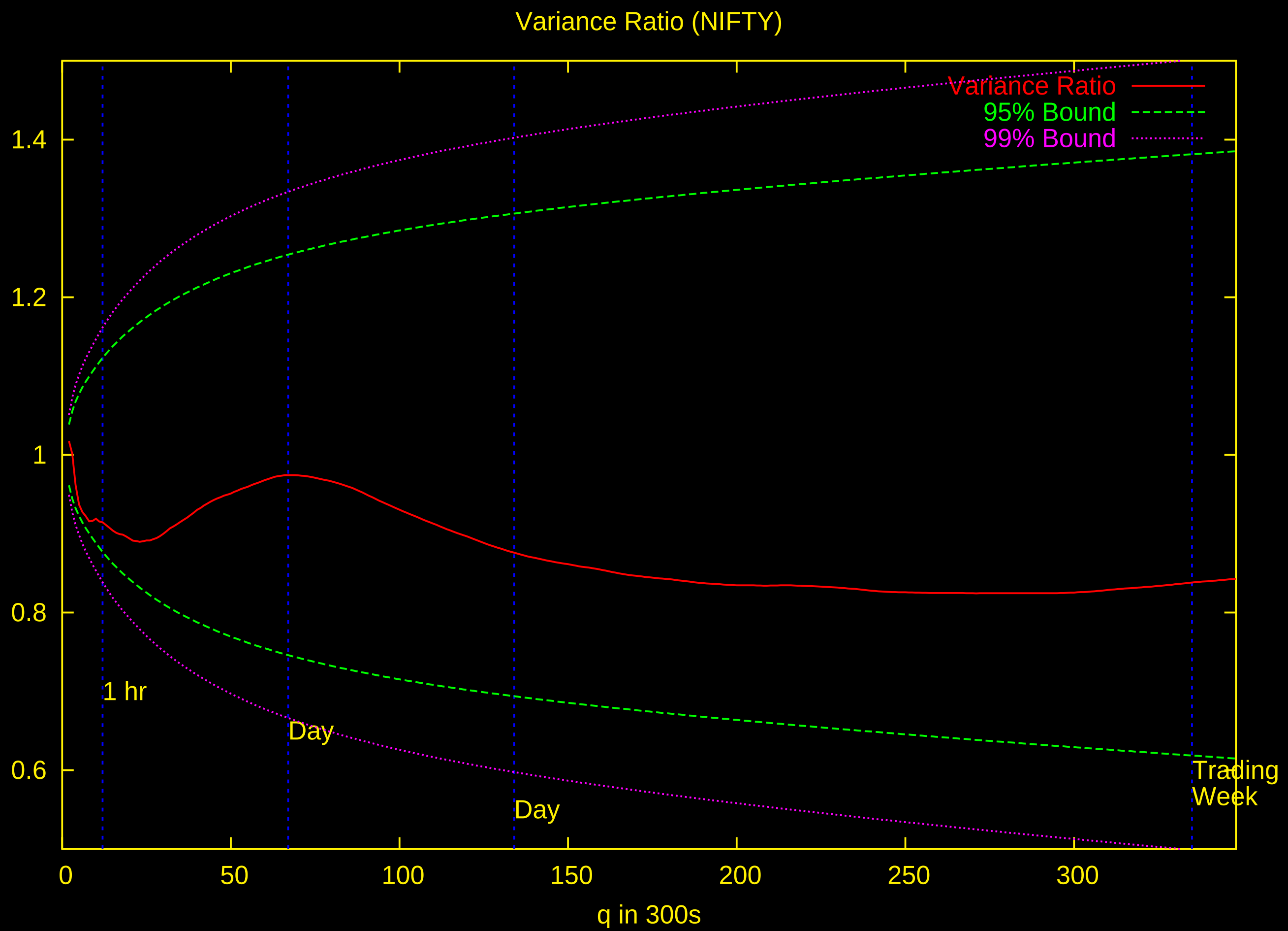
7.5. $Z1(q)$ statistics of top ten NSE liquidity portfolios

Duration	q	Homoscedastic $Z1$ statistics of Portfolios (Deciles) returns.									
		1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}
10 minutes	2	2.37	3.75	4.71	7.24	12.51	4.28	4.57	-11.05	-3.56	-0.29
15 minutes	3	-4.05	-1.11	0.92	3.76	8.97	2.86	2.62	-10.14	-5.79	-0.64
20 minutes	4	-6.80	-2.73	-1.44	1.81	6.41	1.93	1.86	-9.01	-5.77	-0.10
25 minutes	5	-7.62	-2.80	-2.42	0.92	5.23	2.01	1.72	-8.16	-5.54	0.54
30 minutes	6	-7.41	-2.33	-2.59	0.96	4.62	2.22	2.02	-7.12	-4.97	1.36
35 minutes	7	-7.03	-1.73	-2.36	1.13	4.22	2.34	2.37	-6.38	-4.43	1.95
40 minutes	8	-6.94	-1.58	-2.45	1.24	3.80	2.30	2.32	-5.73	-4.12	2.31
45 minutes	9	-6.48	-1.17	-2.31	1.31	3.59	2.35	2.53	-5.24	-3.77	2.63
50 minutes	10	-6.15	-0.96	-2.29	1.38	3.39	2.34	2.78	-4.81	-3.43	2.80
55 minutes	11	-5.97	-0.88	-2.41	1.22	3.03	2.24	2.89	-4.57	-3.29	2.79
1 hour	12	-5.71	-0.83	-2.40	1.13	2.81	2.13	2.94	-4.33	-3.15	2.77
2 hours	24	-4.01	-0.48	-2.46	0.54	1.10	1.78	2.16	-3.32	-2.18	1.96
3 hours	36	-2.26	0.41	-1.55	0.85	1.46	2.44	2.08	-2.51	-0.80	2.01
1 day	67	-0.12	1.48	-0.13	1.48	2.11	2.78	2.62	-0.85	0.94	2.83
2 days	134	-1.29	-0.27	-0.80	-0.69	1.40	1.55	1.59	-0.45	0.76	2.49
1 week	335	-0.63	-0.89	-0.23	-1.11	1.66	1.03	1.61	0.90	1.15	1.72
2 weeks	670	0.07	-0.83	-0.13	-1.44	1.53	0.30	1.41	1.06	0.85	1.39
1 month	1675	0.18	-0.95	0.08	-1.25	1.86 -	0.92	1.06	1.94	1.45	1.74
2 months	3350	0.01	-1.25	-0.14	-1.25	2.09 -	0.94	0.30	2.25	1.69	1.69

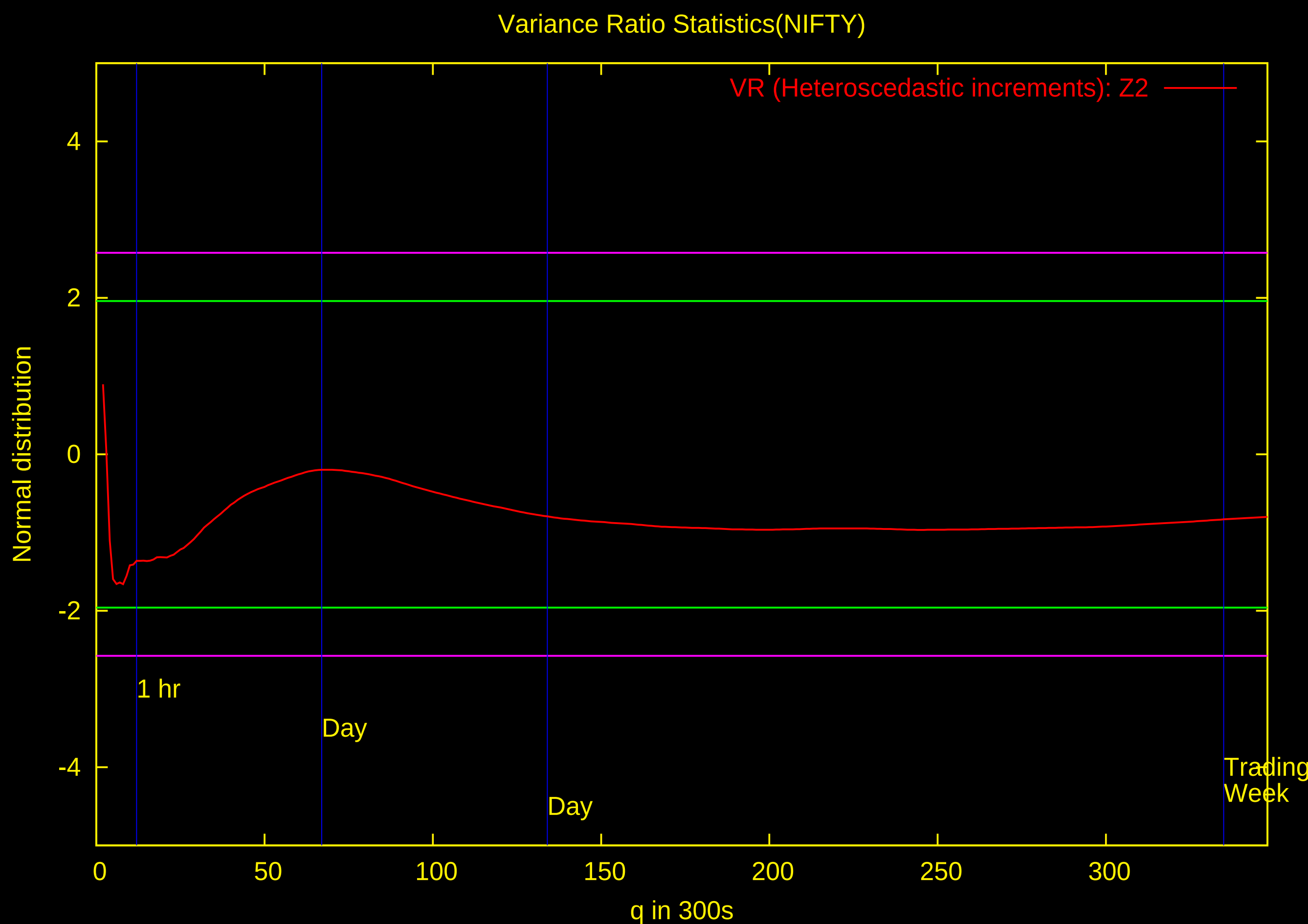
7.6. $Z2(q)$ statistics of top ten NSE liquidity portfolios

Duration	q	Heteroscedasticity-consistent $Z2$ statistics of Portfolios (Deciles) returns.									
		1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}
10 minutes	2	0.79	1.23	1.31	3.45	5.84	2.22	1.81	-3.32	-1.61	-0.15
15 minutes	3	-1.46	-0.39	0.28	1.83	4.28	1.56	1.10	-3.31	-2.73	-0.35
20 minutes	4	-2.63	-1.02	-0.47	0.90	3.15	1.10	0.83	-3.18	-2.81	-0.06
25 minutes	5	-3.12	-1.11	-0.85	0.47	2.65	1.19	0.80	-3.07	-2.78	0.31
30 minutes	6	-3.18	-0.97	-0.96	0.50	2.40	1.36	0.98	-2.83	-2.57	0.81
35 minutes	7	-3.13	-0.75	-0.92	0.60	2.25	1.47	1.20	-2.64	-2.34	1.18
40 minutes	8	-3.18	-0.70	-0.99	0.67	2.06	1.47	1.20	-2.46	-2.22	1.42
45 minutes	9	-3.02	-0.52	-0.95	0.71	1.97	1.53	1.32	-2.33	-2.08	1.65
50 minutes	10	-2.89	-0.43	-0.94	0.75	1.88	1.54	1.46	-2.19	-1.92	1.77
55 minutes	11	-2.81	-0.40	-0.99	0.66	1.70	1.49	1.52	-2.14	-1.87	1.78
1 hour	12	-2.69	-0.37	-0.98	0.62	1.58	1.43	1.55	-2.07	-1.81	1.78
2 hours	24	-1.95	-0.22	-1.02	0.31	0.64	1.27	1.19	-1.87	-1.40	1.34
3 hours	36	-1.15	0.20	-0.68	0.51	0.88	1.82	1.22	-1.54	-0.54	1.43
1 day	67	-0.07	0.84	-0.07	0.99	1.39	2.23	1.73	-0.59	0.69	2.15
2 days	134	-0.82	-0.17	-0.47	-0.50	1.01	1.29	1.16	-0.34	0.60	1.98
1 week	335	-0.46	-0.66	-0.16	-0.89	1.34	0.91	1.31	0.77	0.99	1.46
2 weeks	670	0.05	-0.67	-0.10	-1.23	1.32	0.27	1.21	0.96	0.77	1.23
1 month	1675	0.16	-0.84	0.07	-1.13	1.69	-0.85	0.96	1.84	1.37	1.62
2 months	3350	0.01	-1.16	-0.13	-1.17	1.96	-0.90	0.28	2.20	1.65	1.62

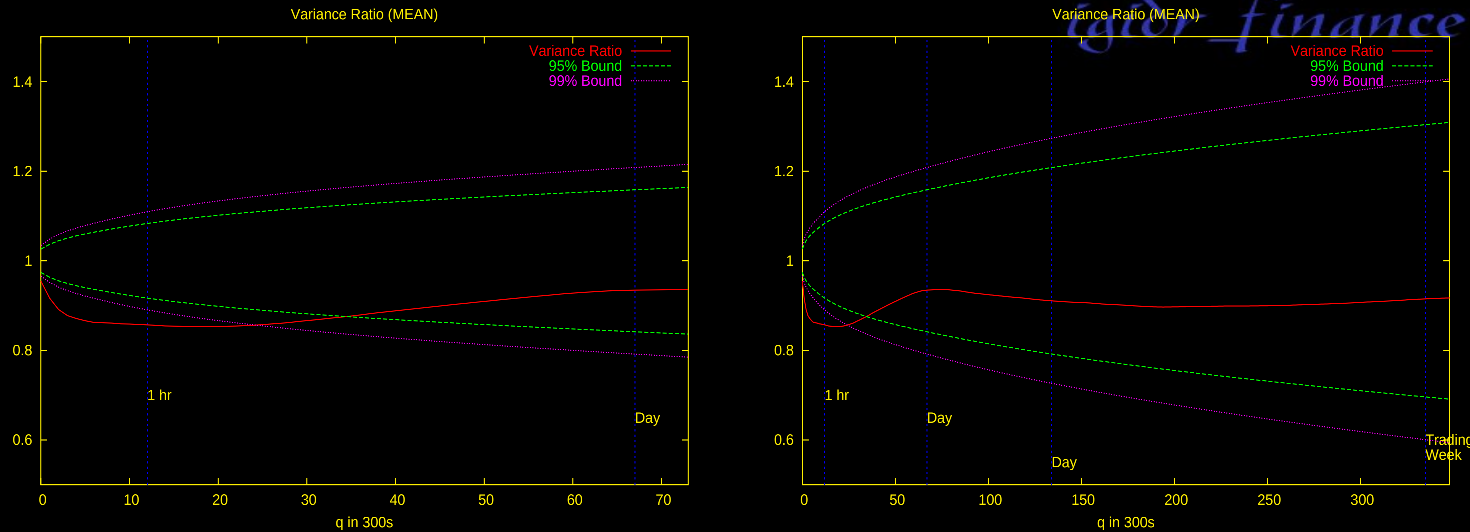
7.7. Variance Ratio of NIFTY upto a Week. $q \in [2, 350]$



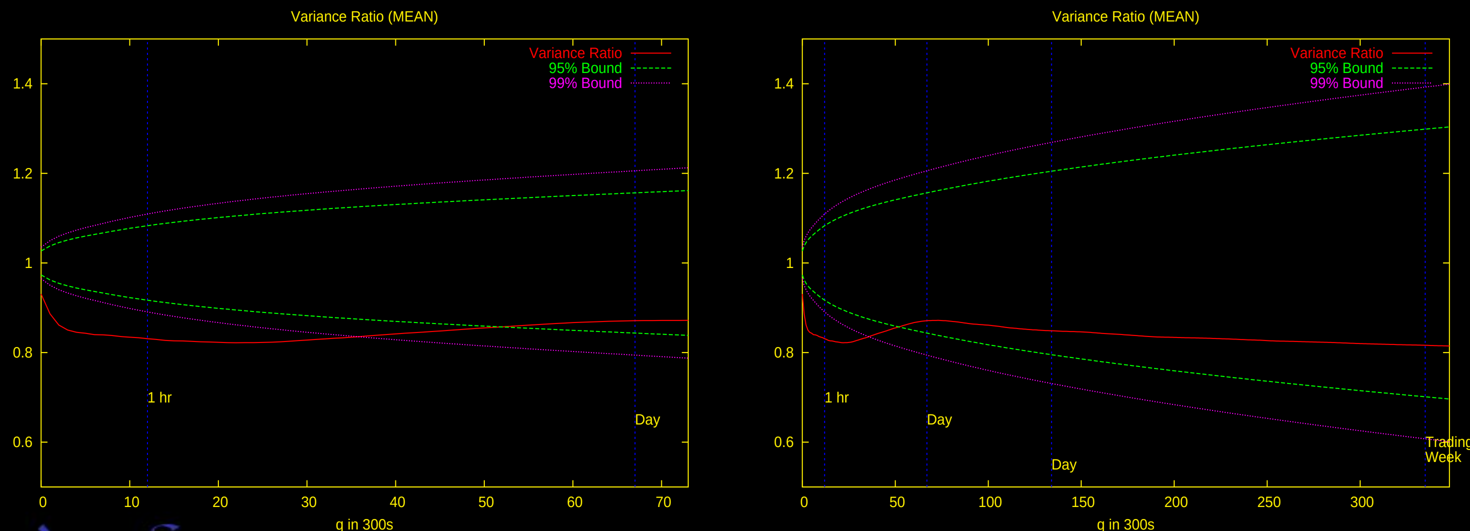
7.8. $Z1$ and $Z2$ statistics of NIFTY upto a Week. $q \in [2, 350]$



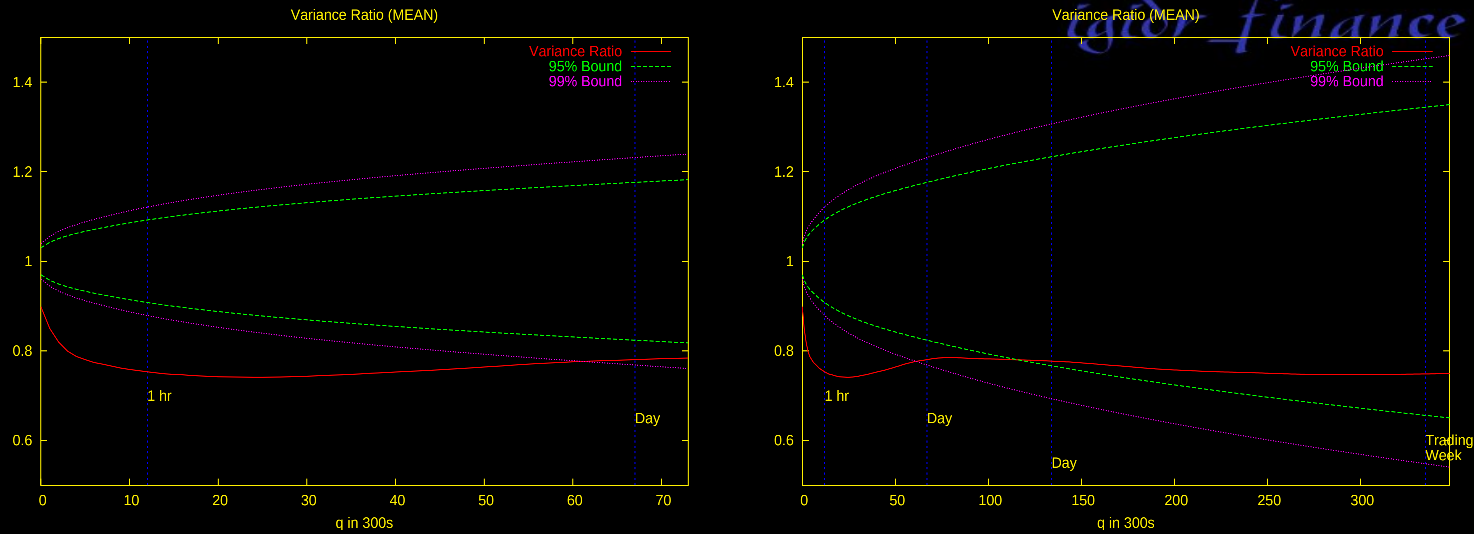
Heteroscedasticity-consistent $Z2$ statistic for NIFTY with aggregation up to 1 week.



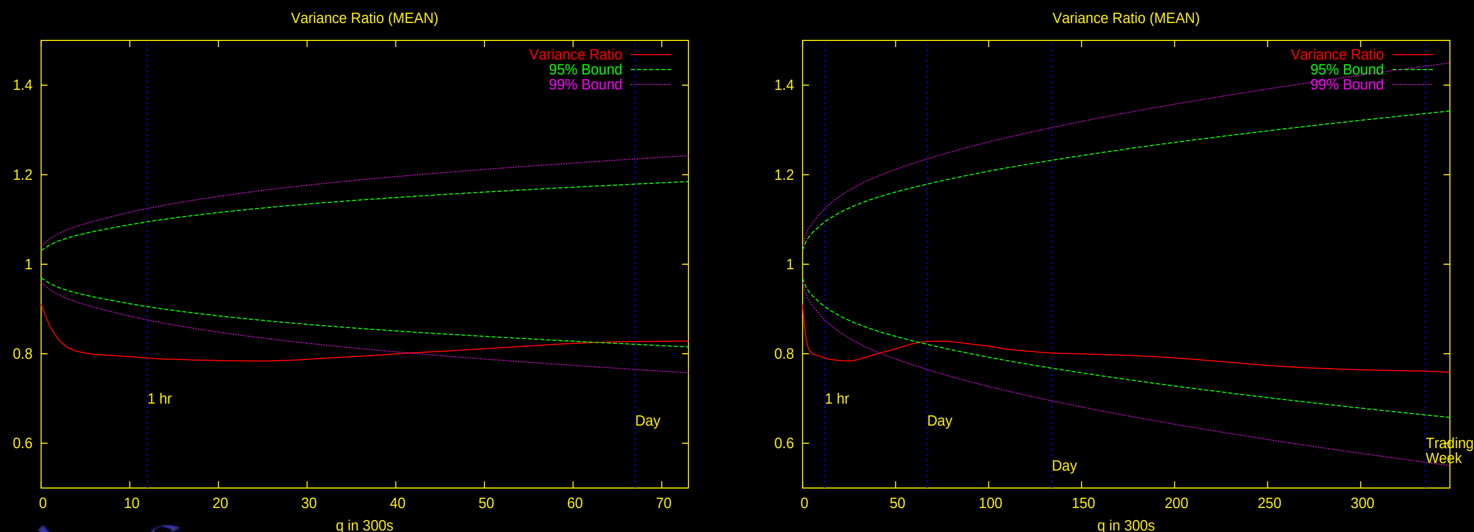
Cross sectional average of the stocks in the first decile. Day and Week.



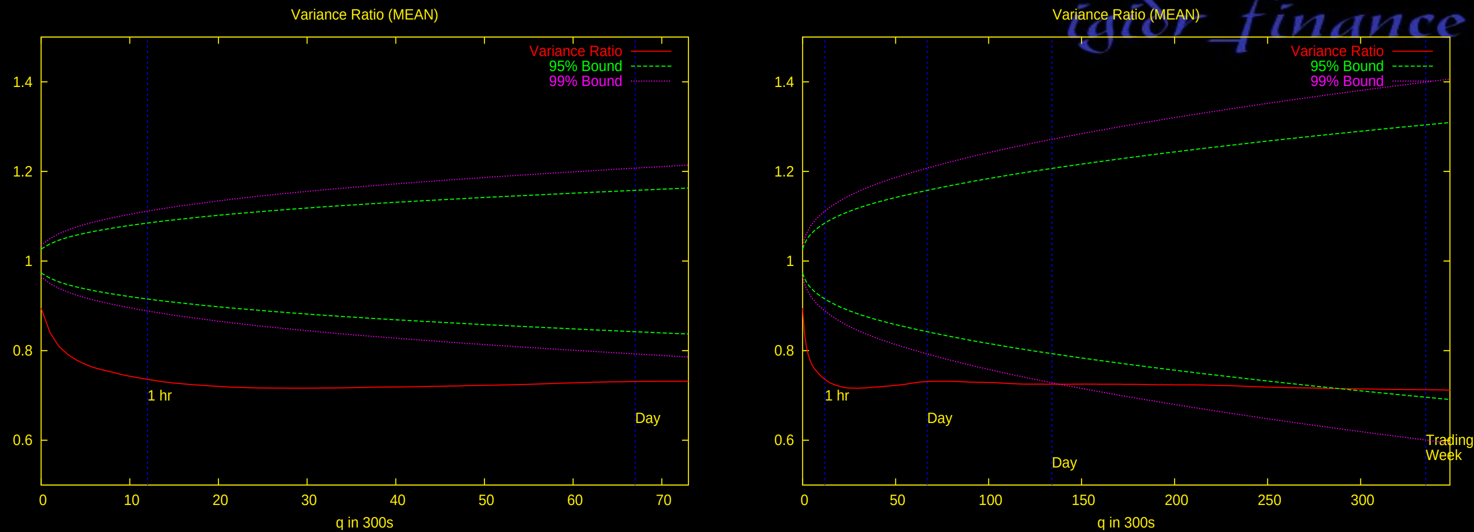
Cross sectional average of the stocks in the second decile. Day and Week.



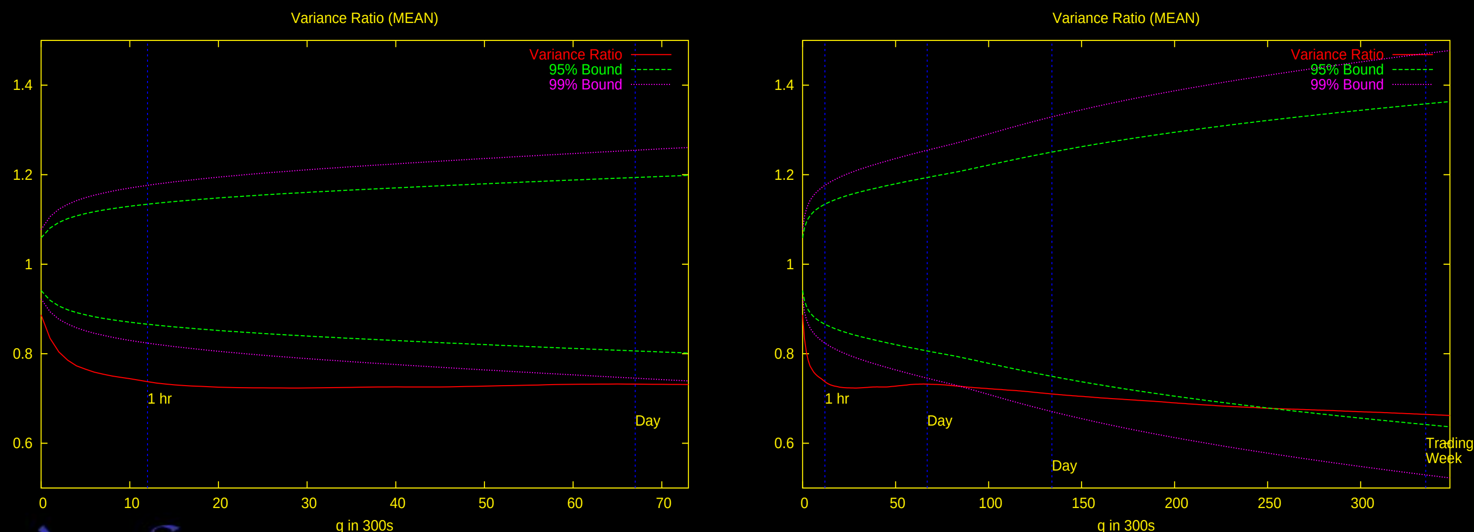
Cross sectional average of the stocks in the third decile. Day and Week.



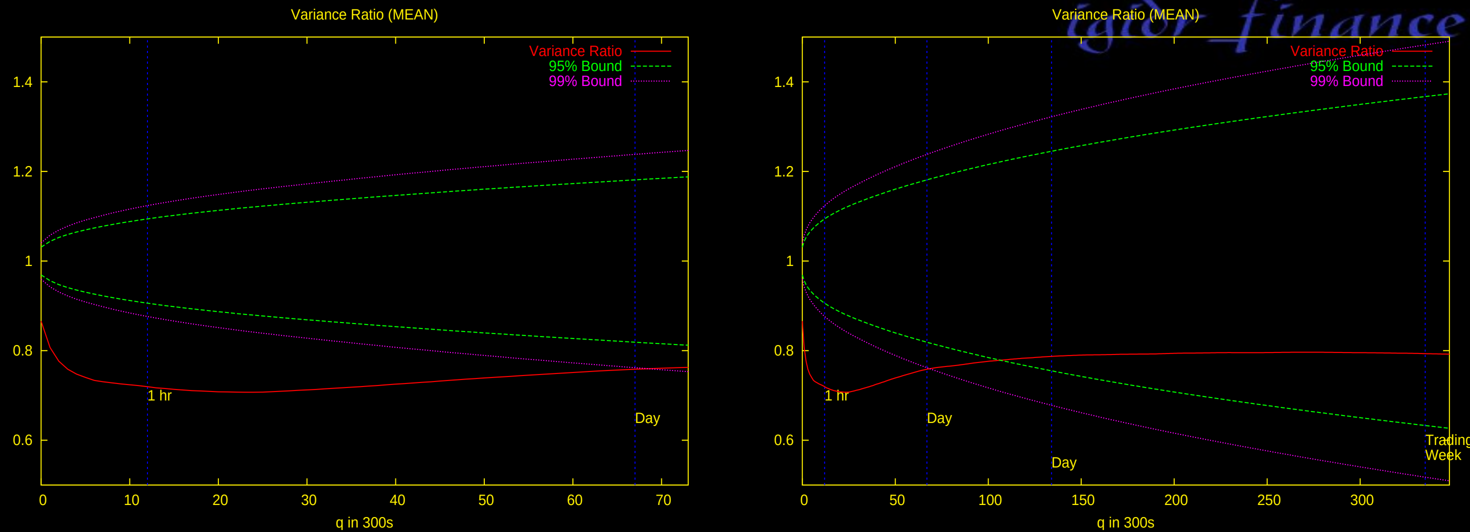
Cross sectional average of the stocks in the fourth decile. Day and Week.



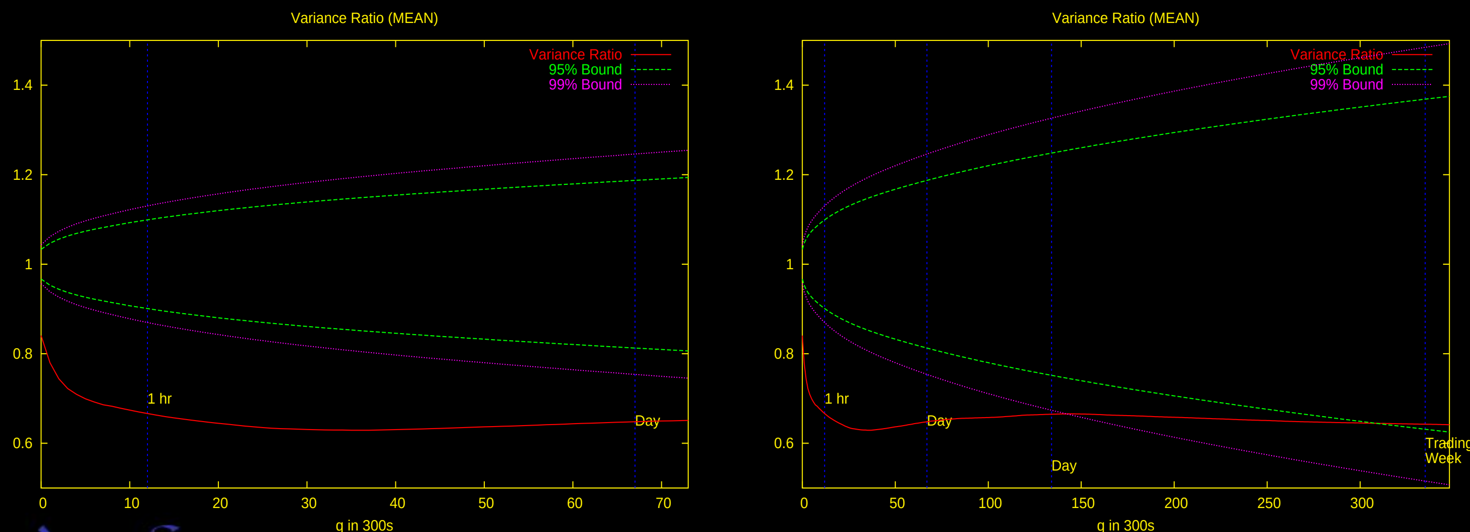
Cross sectional average of the stocks in the fifth decile. Day and Week.



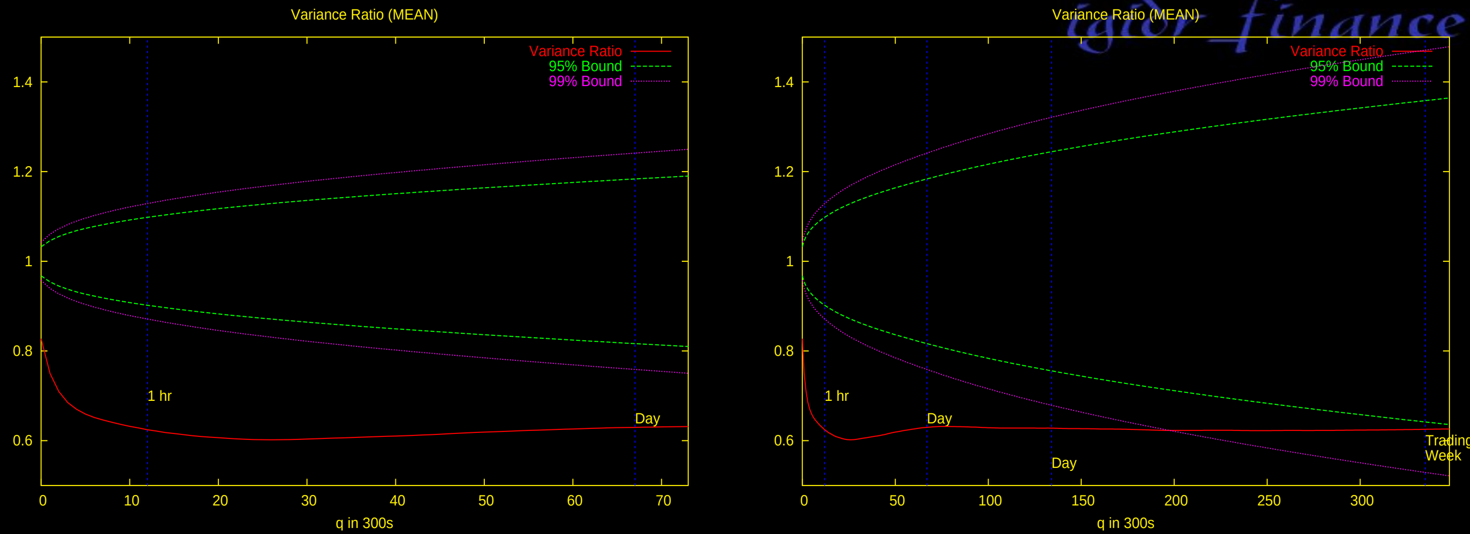
Cross sectional average of the stocks in the sixth decile. Day and Week.



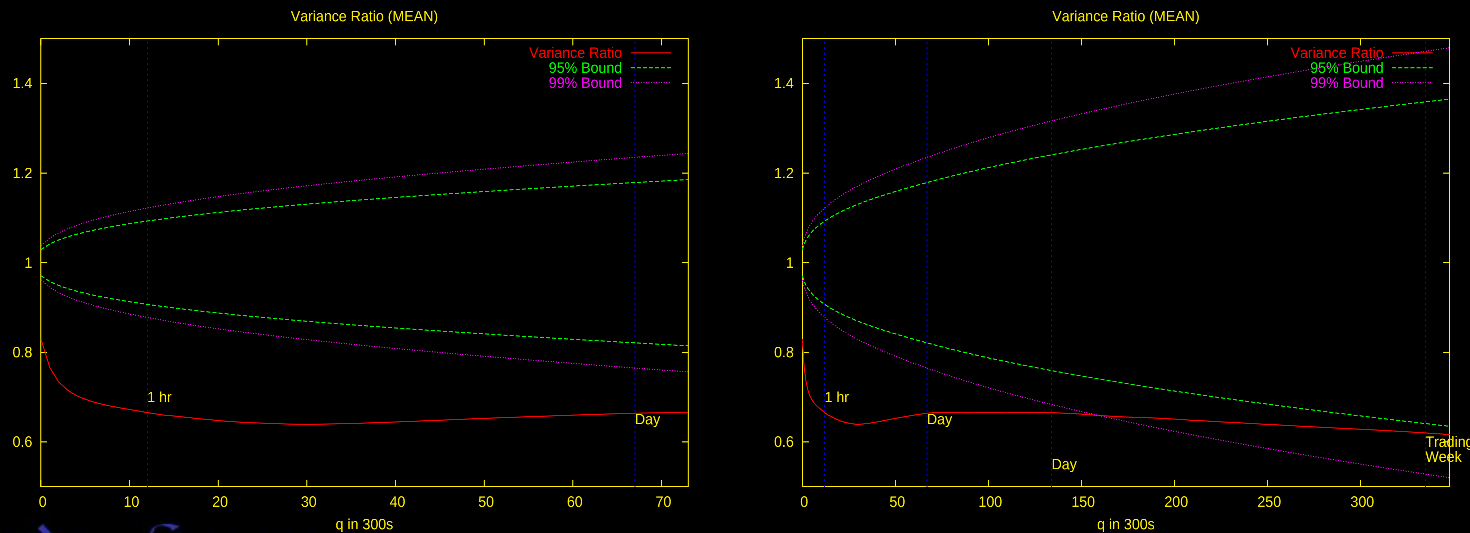
Cross sectional average of the stocks in the seventh decile. Day and Week.



Cross sectional average of the stocks in the eighth decile.Day and Week.



Cross sectional average of the stocks in the ninth decile.Day and Week.



Cross sectional average of the stocks in the tenth decile. Day and Week.

7.9. Time taken to for a Cross-sectional average to be insignificant from unity

Portfolio/ Decile	Time in 300s
1	36
2	54
3	118
4	64
5	290
6	253
7	108
8	309
9	370
10	451

8. Extensions

- Lo and MacKinlay (1988)'s VR test has better size and power properties if the lag length is potimally chosen.
- Cochrane (1988) has shown that the estimator of $VR(q)$ which using the usual consistent estimators of variances is asymptotically equivalent to 2π times the normalized spectral density estimator at the zero frequency which uses the Bartlett kernel.
- Andrews (1991) has shown that in the Quadratic Spectral kernel is optimal in estimating the spectral density at the zero frequency.
- Choi (1999) uses the Quadratic Spectral kernel to get the Variance-Ratio statistic. He uses Priestly (1982)'s asymptotic approximation of normality. This technique uses an optimal lag-selection (Selection of q, in the $VR(q)$), which is completely data-dependent.
- Wright (2000) has proposed alternative definitions of Variance-Ratio statistics using ranks and signs. He uses Monte Carlo techniques to test the size and power of his tests against conventional Lo and MacKinlay tests, and shows that the rank and sign tests have better size and power. Also, in contrast to standard VR procedures, these procedures are exact. The findings from these tests should be compared with the existing findings.

8.1. Extensions. *contd.*

- If the time to become indistinguishable from a random walk, is long, then one might design a trading strategy to take advantage of this, and see if opportunities exist, net of transaction costs.
- Mean-reversion or persistence of a stock price could thus be used to design an appropriate contrarian or momentum strategy.

Thank You.

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