Replies to comments on Manuscript ENGI–D–13–00133 ”Fourier Methods for Harmonic Scalar Waves in General Waveguides”

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We would like to thank the two reviewers for many valuable comments that have helped us improving the paper. Based on these comments, the paper is completely rewritten. Special attention has been paid to motivating the method and the presentation and the discussion of the results. We agree with the opinion that the motivations for the method were absent and that the presentation of the results was poor in the previous version.

Referee 1

Referee's comment
The presentation of the math problem is poor. Eqs (2) and (3) seem to be an eigenvalue problem. But the author actually solved a boundary value problem. In the numerical example, the definition of boundary conditions is misleading. A source at left is mentioned in page 15, while in section 5.1, boundary conditions are defined on elements of conformal mappings. Please unify the definitions. Moreover, can these conditions be rewritten in the form of Eq. (3). If not, the original math problem has to be re-defined.
Authors' comments and actions
The former implicit definition of the source as an incident wave is now made explicit in section 2 making it clear that a scattering problem is formulated and later solved. Then it should not cause confusion in keeping the use of the conformal mapping in section 5.1 to define the intervals where the non-zero admittance is applied.

Referee's comment

Many different methods are mentioned in Introduction section. Please provide key references and a brief description of these methods.
Authors' action
The description of different methods in the introduction is expanded and key references are added.

Referee 2

IMPORTANT POINTS: *in italics text from the manuscript;*
Referee's comment
(1): Introduction: *The problem appears in many applications, in acoustics, optics, electrodynamics and quantum physics.* The authors should provide one reference for each theme mentioned. I would add water waves and the reference:
Shi, A., Teng, M.H. & Wu, T.Y., Propagation of solitary waves through significantly curved shallow water channels. J. Fluid. Mech., 362 (1998) 157176.
Note this is nonlinear, fully time dependent problem.
Authors' action
References for the themes are mentioned and the new theme water waves is added together with the suggested reference.

Referee's comment
(2): Preliminaries: *The purpose of this article is exactly this, to investigate and to show how Fourier methods can be used to solve wave scattering problems in a waveguide with geometry and boundary conditions that exceed the ordinary school book examples.* I believe there has to be a stronger motivation for this article. Note at the end, by the time we reach the Conclusions we have: We can only conclude that the combination of semi-analytic techniques evolved here is a well working alternative to different FEM- based numerical methods. As an interested reader I felt frustrated of not having any idea of which method is better: this or the FEM? Is the present one faster? Is there any comparison in terms of performance regarding computer time? If not, then why is this an ALTERNATIVE?! What is the benefit if a computer has to be used anyway with the FEM or this one?
Authors' comment
We agree that the motivations for the method was absent in the previous version of the paper.
Authors' actions
The introduction has been rewritten in order to present motivations of non-mathematical nature for the method. It is based on industrial requirements on wave simulation tools originating partially from the second author's previous industrial experience.
In brief, more than one type of simulation tools is required.

From these motivations the revised purpose for the paper is . . . *to demonstrate that semi-analytical methods based on Fourier Analysis can solve time harmonic scattering problems in waveguides with complicated geometry and general normal impedance or admittance boundary conditions.*

An extended summary of the paper is presented including a balanced conclusion. The importance of improved physical understanding is stressed. The revised conclusion and summary is: *In summary, we conclude, based primarily on requirements from industry, that more than one type of time harmonic waveguide simulation tool is required. It is demonstrated that Fourier methods based on Fourier Analysis provides one such tool. Its accuracy is checked against FE Simulations for a general two-dimensional waveguide with normal admittance boundary conditions at low and medium frequencies. For the current investigation with non-zero normal boundary admittance, the Fourier method, with its present implementation, is considerably slower than the FE method that is more memory demanding. However, for inverse engineering involving tuning of straight waveguides, the Fourier method is an attractive alternative including time aspects. The prime motivation for the Fourier method is its added physical understanding primarily at low frequencies.*

Referee's comment
(3): Page 5: *When performing these calculations, interactions from the two ends of the block must be avoided, and hence, each block is assumed to be an infinitely long waveguide (...).* How was this infinitely long dealt with in the simulations. Since one the authors has experience with the Schwarz-Christoffel Toolbox by Driscoll [1] this could have been tested and displayed here. One does not need to go too far, correct? How far?
Authors' comment
This text was intended to motivate why the BB method requires that both ends of each block is an infinitely long waveguide corresponding to anechoic terminations. The text was not intended to connect to the Schwarz-Christoffel Mapping.
Authors' action
The following text is added for clarification: *It is assumed that the scattering matrix for each block is determined for anechoic terminations. Otherwise, the interactions due to the termination should have been included in the Building Block Method making it much more complicated. To this end each block is assumed to be an infinitely long waveguide with parallel straight walls and constant boundary conditions outside some bounded transition region, when determining its scattering properties; in applications the straight waveguide parts may be of shorther or even vanishing length.*
In the end of Section 5.2, it is now clearly stated which intervals are used for the two blocks in the model example.

Referee's comment
(4): Page 8: *Both methods are built on the Schwarz-Christoffel mapping, (...), mean- ing that no singularities are introduced by the mapping.* This is along the lines of the item above. Note that in
Nachbin, A. & Simões, V.S. 2012 Solitary Waves in Open Channels with Abrupt Turns and Branching Points. J. Nonlin. Math. Phys., Vol. 19, Suppl. 1, 1240011, the same strategy was used for the singularities of the conformal mapping. Also note that the reflection-transmission problem at sharp turns was done in a very straightforward and accurate fashion for a nonlinear evolution problem, namely more complex.
Authors' action
The suggested reference is added.

Referee's comment
(5): Page 11: *It is worth noticing that the Riccati equations (...) can for a countable set of k contain singularities (...), for details and examples see for example [6].* The authors should mention in words what is this singularity problem. And why they resist a numerical solution .
Authors' comment and action
The wording "resist a numerical solution" was unfortunate. We have clarified the situation also mentioning that the stiffness was tractable with standard methods for our example.

Referee's comment
(6): Page 16: Figure 6 is very bad. I can not read the numbers on either axis, nor know exactly what-is-what?! How many Φ_j are there? What do I conclude from this figure?
Authors' action
By using better type sizes on axes and in legends, many figures in Section 5.4 have been made more readable. The caption in Figure 6 now includes further relevant information and additional comments on the figure are included in the text.

Referee's comment
(7): Page 16-17: Why k = 15 was chosen, was a significant mode? As mentioned in the Conclusions why is this a low frequency mode? Why is figure 7 a good display of the results, or of the quality of the method? Why in the top of table 1 three Φ's are considered while at the bottom only 2? Why figure 8 has only Φ_0?
Authors' comment and action
The model problem has been solved for frequencies in the interval $0 < k \leq 20$. All statements, indicating the upper part of this interval as beeing "low frequen-

cies” have been removed. Instead, we use terms like “low and medium” frequencies.
The former Figure 7 is removed from the article.
Table 1 and its caption is slightly changed to increase its understandability.

Referee’s comment
(8): Page 18: *Finally, it is possible to extend the techniques to cover three-dimensional problems(...)* Nowhere 3D is done so it should not be mentioned at the start at page 3 and beyond. Also the doubly connected region in Figure 1 is misleading since holes are NOT considered. This should be removed.
Authors’ comment and action
Three-dimensional problems are not any longer included in the initial formulation of the problem. However, a note on the possibilty to extend the methods to cover 3D is still present in the discussion section. A reference to an article which discusses the matter is included.

Referee’s comment
(9): Conclusion: At this point as a reader I am not sure I want to use the method proposed, which might be useful. It seems time consuming. Is it? It has restrictions. So we go back to item 1 above and read *The purpose of this article is exactly this, to investigate and to show how Fourier methods can be used to solve wave scattering problems in a waveguide with geometry and boundary conditions that exceed the ordinary school book examples.*

I would like to be convinced that a reader would want to use this method and why. Or else the paper should not be published.
Authors’ action
See comments on point (2).

Minor points:
Referee’s comment
(a): typo/Abstract: equtions
Authors’ action
The abstract is rewritten.

Referee’s comment
(b) Intro: This sentence is quite long and confusing. *For more complex geometries or boundary conditions, purely numerical methods or more precisely, finite element methods, have often during recent years, due to evolution of both the methods and the computers, become the natural choice for solving Helmholtz equation.*

Authors' action
The sentence is reformulated.

Referee's comment
(c): *We can only conclude that the combination of semi-analytic techniques* **evolved** *here is a well working alternative to different FEM-based numerical methods.*
Authors' action
The sentence is no longer present in the article.