Fourier Methods for Harmonic Scalar Waves in General Waveguides

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Abstract.
Keywords:

1. Introduction

$$(\nabla^2 + k^2)p = 0 \tag{1}$$

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2. Preliminaries

$V \mathbb{R}^2 \mathbb{R}^3$,$p$

$$(\nabla^2 + k^2)p(\boldsymbol{r}) = 0, \boldsymbol{r} \in V, \tag{2}$$

V

$$\frac{\partial p}{\partial n} = \mathrm{i}k\beta p,\ \boldsymbol{r} \in \partial V, \tag{3}$$

$\partial V k \in \mathbb{R}^{+} \beta \in \mathbb{C} \mathrm{Re}\,\beta \geq \widehat{0 \boldsymbol{n} \partial} V \beta = 0 \beta = \mathrm{i}\infty \exp(-\mathrm{i}kc_0 t) c_0$,$\rho_0$.$c_0\rho_0$
$V \subset \mathbb{R}^n, n = 2, 3$,$V_{\mathrm{i}} V_{\mathrm{L}} V_{\mathrm{R}} u V_{\mathrm{R}} V_{\mathrm{L}}$.$\mathbb{R}^{\pm}_{u_0} = \{u \in \mathbb{R} : u \gtrless u_0\}$,$V_{\mathrm{L}} = \Omega_{\mathrm{L}} \times \mathbb{R}^{-}_{u_{\mathrm{L}}}$

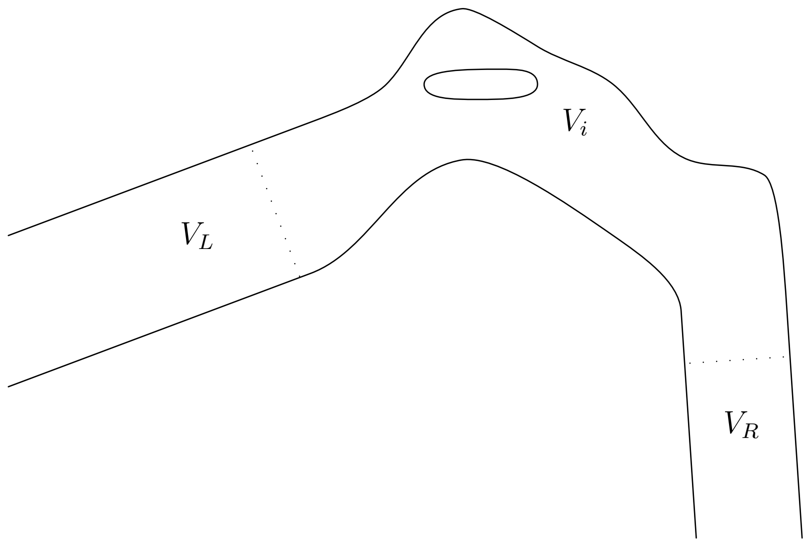

Figure 1. Waveguide $V = V_L \cup V_i \cup V_R$ consisting of straight parts V_L and V_R open to infinity and a bounded connecting part V_i.

$V_{\mathrm{R}} = \Omega_{\mathrm{R}} \times \mathbb{R}^{+}_{u_{\mathrm{R}}}, \Omega_{\mathrm{L}} \Omega_{\mathrm{R}} C\, \beta = 0\, \partial V_{\mathrm{L}} \cap \partial V . \partial V_{\mathrm{R}} \cap \partial V\, \partial V_{\mathrm{i}}$

$V_\mathrm{s} = \Omega \times \mathbb{R}$,$\Omega \Omega_\mathrm{L}$ Ω_R .$p V_\mathrm{s}$,p

$$p = p_- + p_+, \tag{4}$$

$$p_\pm = \sum_n p_n^\pm \mathrm{e}^{\pm \mathrm{i}\alpha_n u} \varphi_n(\boldsymbol{r}_\perp),\ \boldsymbol{r}_\perp \in \Omega. \tag{5}$$

$\varphi_n \lambda_n \geq 0$

$$\begin{cases} (\nabla_\perp^2 + \lambda_n^2)\varphi_n(\boldsymbol{r}_\perp) = 0,\ \boldsymbol{r}_\perp \in \Omega \\ \dfrac{\partial \varphi_n}{\partial n}(\boldsymbol{r}_\perp) = 0,\ \boldsymbol{r}_\perp \in \partial\Omega \\ \int_\Omega \varphi_n^2(\mathbf{r}_\perp)\mathrm{d}\Omega = 1 \end{cases}, \tag{6}$$

$\nabla_\perp^2 \Omega$,

$$\alpha_n = \begin{cases} \sqrt{k^2 - \lambda_n^2},\ k \geq \lambda_n \\ \mathrm{i}\sqrt{\lambda_n^2 - k^2},\ k < \lambda_n \end{cases}. \tag{7}$$

p

$$X_\pm = \left\{ p_\pm = \textstyle\sum_n p_n^\pm \mathrm{e}^{\pm \mathrm{i}\alpha_n u} \varphi_n(\boldsymbol{r}_\perp) : \sum_n |\alpha_n| \left| p_n^\pm \right|^2 < \infty \right\} \tag{8}$$

$X = X_- \oplus X_+$.$p_- \in X_-$,$p_+ \in X_+ p \in X$.

$p_+ \neq 0$,$u = -\infty$,$p_- \neq 0$,$u = +\infty$.$V_0 \subset V$,$(\nabla^2 + k^2)p \neq 0$,$u_- < u < u_+$.
$p_- p_+$,

AXIOM 1.

For the solution to $(\nabla^2 + k^2)p = -q$ in V_s, where $q = 0$ for $u < u_-$ and $u > u_+$,

$$\begin{matrix} p = p_-, \text{ when } u < u_- \text{ and there is no source in } u = -\infty \\ p = p_+, \text{ when } u > u_+ \text{ and there is no source in } u = +\infty \end{matrix}.$$

Comment 1$p_- p_+$

Comment 2$k k + \mathrm{i}\varepsilon$,$\varepsilon > 0$,$p \varepsilon t (\nabla^2 - c_0^{-2}\partial^2/\partial t^2)p = -q(\boldsymbol{r})(t)\sin(kc_0 t)(t)p_\pm$

3. Solving the one-block problems

$$\begin{cases} \left(\nabla^2 + k^2\right) p(x,y) = 0 \text{in the waveguide,} \\ \dfrac{\partial p}{\partial n} = \mathrm{i}k\beta(t)p \qquad \text{on the boundary,} \end{cases} \tag{9}$$

βt

$$F : w = u + \mathrm{i}v \to z = x + \mathrm{i}y$$

$\{u \in \mathbb{R}, 0 \leq v \leq 1\}(u,v)$

$$\begin{cases} \left(\nabla^2 + k^2 \mu(u,v)\right) \Phi(u,v) = 0 \\ \left.\dfrac{\partial \Phi(u,v)}{\partial v}\right|_{v=1} = \mathrm{i}kY(u)\Phi(u,1) \\ \left.\dfrac{\partial \Phi(u,v)}{\partial v}\right|_{v=0} = 0 \end{cases} \tag{10}$$

$\mu(u,v) = |F'(w)|^2 Y(u) = \beta(u)\,|F'(u+i)|$

$\Phi(u,v) v$

$$\Phi(u,v) = \sum_n \Phi_n(u)\varphi_n(u,v), \tag{11}$$

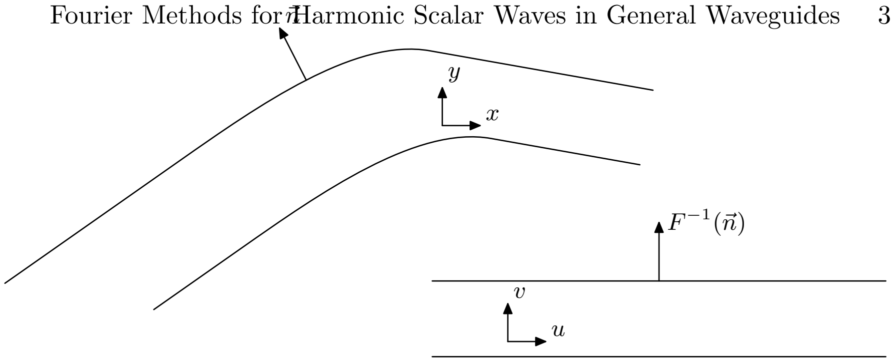

Figure 2. A single block in the $z = x + \mathrm{i}y$ plane and the $w = u + \mathrm{i}v$ plane.

$\varphi_n(u,v) = \cos(v\lambda_n(u))\lambda_n(u), n \in \mathbb{N}$
$\lambda_n \lambda_n(u) n = 0, 1, \dots$

$$\lambda_n(u)\tan\big(\lambda_n(u)\big) = -\,\mathrm{i}kY(u). \tag{12}$$

$$\lambda_n'(u) = \frac{-\,\mathrm{i}kY'(u)}{Q(u)} \tag{13}$$

$$\lambda_n''(u) = -\,\mathrm{i}k\left(\frac{Y''(u)Q(u) - Y'(u)Q'(u)}{\big(Q(u)\big)^2}\right) \tag{14}$$

$(')u$

$$Q(u) = \tan\big(\lambda(u)\big) + \lambda(u)\Big(1 + \tan^2\big(\lambda(u)\big)\Big).$$

$$v\sin(v\lambda_n(u)) = \sum_m \alpha_{mn}(u)\cos(v\lambda_m(u)), \tag{15}$$

$$v^2\cos(v\lambda_n(u)) = \sum_m \beta_{mn}(u)\cos(v\lambda_m(u)), \tag{16}$$

$$\mu(u,v)\cos(v\lambda_n(u)) = \sum_m \mu_{mn}(u)\cos(v\lambda_m(u)), \tag{17}$$

$$\boldsymbol{\Phi}''(u) - A(u)\boldsymbol{\Phi}'(u) - B^2(u)\boldsymbol{\Phi}(u) = 0, \tag{18}$$

$\boldsymbol{\Phi} = (\Phi_1\Phi_2\Phi_3\dots)^T AB^2$

$$A_{mn}(u) = 2\alpha_{mn}(u)\lambda_n'(u) \tag{19}$$

$$B_{mn}^2(u) = \alpha_{mn}(u)\lambda_n''(u) + \beta_{mn}(u)\left(\lambda_n'(u)\right)^2 + \delta_{mn}\left(\lambda_n(u)\right)^2 - k^2\mu_{mn}(u). \tag{20}$$

$(\varphi_m)\varphi_m(v) = \cos v\lambda_m(u) Y(\varphi_m)(\overline{\varphi_m})\bar{\ }$

$$\langle\varphi_m, \varphi_n\rangle = (\varphi_m, \overline{\varphi_n}) = \int_0^1 \varphi_m(v)\varphi_n(v)\mathrm{d}v, \tag{21}$$

$n \neq m f g (f,g) \langle f,g\rangle \alpha_{mn}(u), \beta_{mn}(u) \mu_{mn}(u)$

$$a_m = \frac{\langle f, \varphi_m \rangle}{\langle \varphi_m, \varphi_m \rangle} \tag{22}$$

$$f = \sum_m a_m \varphi_m . \tag{23}$$

$(\varphi_m)^2 (0,1) (\varphi_m) Y Y$

3.1. Conformal mapping techniques

AB^2

3.2. Accomplishing stable equations

$k k \notin \{k_1, k_2, k_3, \dots\} \boldsymbol{\Phi}$

$\Omega_L \Omega_R \Omega_L \Omega_R A B B = B_- \Omega_L B = B_+ \Omega_R \Omega_L \Omega_R B^2 B^2 B_- B_+$

3.2.1. *Determining Reflection and Transmission operators (The RT method)*

$p = p_- + p_+ u \in \mathbb{R}$

$$\boldsymbol{\Phi}(u) = (\Phi_1(u) \Phi_2(u) \dots)^T = \boldsymbol{\Phi}^+(u) + \boldsymbol{\Phi}^-(u), \tag{24}$$

$\boldsymbol{\Phi}^+(u) \boldsymbol{\Phi}^-(u)$

$C D u$

$$\frac{\partial \boldsymbol{\Phi}}{\partial u}(u) = -C(u) \boldsymbol{\Phi}^+(u) + D(u) \boldsymbol{\Phi}^-(u), \tag{25}$$

$u \in \mathbb{R} C D u \Omega_L \Omega_R A(u) = 0 C = D = B_- \Omega_L C = D = B_+ \Omega_R$

$$C(u) = D(u) = B_- + f(u)(B_+ - B_-), \tag{26}$$

$f 0 \Omega_L 1 \Omega_R$

$R^+ R^- T^+ T^- u_1 < u_2$

$$\begin{pmatrix} \boldsymbol{\Phi}^+(u_2) \\ \boldsymbol{\Phi}^-(u_1) \end{pmatrix} = \begin{pmatrix} T^+(u_2,u_1) & R^-(u_1,u_2) \\ R^+(u_2,u_1) & T^-(u_1,u_2) \end{pmatrix} \begin{pmatrix} \boldsymbol{\Phi}^+(u_1) \\ \boldsymbol{\Phi}^-(u_2) \end{pmatrix} . \tag{27}$$

$T^- R^- \boldsymbol{\Phi}^- T^+ R^+ \boldsymbol{\Phi}^+$

$$\frac{\partial}{\partial u} \begin{pmatrix} \boldsymbol{\Phi}^+ \\ \boldsymbol{\Phi}^- \end{pmatrix} = \begin{pmatrix} J & K \\ L & M \end{pmatrix} \begin{pmatrix} \boldsymbol{\Phi}^+ \\ \boldsymbol{\Phi}^- \end{pmatrix} , \tag{28}$$

$$\begin{aligned} J &= (C+D)^{-1} \left(-C' - B^2 + (A-D)C \right), \\ K &= (C+D)^{-1} \left(D' - B^2 - (A-D)D \right), \\ L &= (C+D)^{-1} \left(C' + B^2 - (A+C)C \right), \\ M &= (C+D)^{-1} \left(-D' + B^2 + (A+C)D \right). \end{aligned} \tag{29}$$

$T^+ R^+ \Omega_R u_2 \in \Omega_R u = u_1 \boldsymbol{\Phi}^-(u_2) = 0$

$$\begin{cases} T^+(u_2,u) \boldsymbol{\Phi}^+(u) = \boldsymbol{\Phi}^+(u_2), \\ R^+(u_2,u) \boldsymbol{\Phi}^+(u) = \boldsymbol{\Phi}^-(u). \end{cases} \tag{30}$$

u

$$\begin{cases} \dfrac{\partial T^+}{\partial u}(u_2,u)\boldsymbol{\Phi}^+(u) + T^+(u_2,u)\dfrac{\partial \boldsymbol{\Phi}^+}{\partial u}(u) = 0, \\ \dfrac{\partial R^+}{\partial u}(u_2,u)\boldsymbol{\Phi}^+(u) + R^+(u_2,u)\dfrac{\partial \boldsymbol{\Phi}^+}{\partial u}(u) = \dfrac{\partial \boldsymbol{\Phi}^-}{\partial u}(u), \end{cases} \tag{31}$$

$$\frac{\partial R^+}{\partial u}(u_2,u) = -R^+(u_2,u)\big(J(u) + K(u)R^+(u_2,u)\big) + L(u) + M(u)R^+(u_2,u) \tag{32}$$

$$\frac{\partial T^+}{\partial u}(u_2,u) = -T^+(u_2,u)\big(J(u) + K(u)R^+(u_2,u)\big) \tag{33}$$

$R^-T^-\Omega_L$

$$\frac{\partial R^-}{\partial u}(u,u_1) = -R^-(u,u_1)\big(M(u) + L(u)R^-(u,u_1)\big) + K(u) + J(u)R^-(u,u_1), \tag{34}$$

$$\frac{\partial T^-}{\partial u}(u,u_1) = -T^-(u,u_1)\big(M(u) + L(u)R^-(u,u_1)\big). \tag{35}$$

$JKLMR^+(u_2,u_2) = 0T^+(u_2,u_2) = IR^-(u_1,u_1) = 0T^-(u_1,u_1) = I$

3.2.2. *Determining the field (The DtN method)*

$\boldsymbol{\Phi}$

$$\boldsymbol{\Phi} = \boldsymbol{\Phi}_R + \boldsymbol{\Phi}_L, \tag{36}$$

$\boldsymbol{\Phi}_R + \infty \boldsymbol{\Phi}_L - \infty \Lambda_R \Lambda_L$

$$\boldsymbol{\Phi}'_R(u) = -\Lambda_R(u)\boldsymbol{\Phi}_R(u), \tag{37}$$

$$\boldsymbol{\Phi}'_L(u) = \Lambda_L(u)\boldsymbol{\Phi}_L(u). \tag{38}$$

$\boldsymbol{\Phi}_R\boldsymbol{\Phi}_L$

$$\Lambda'_R(u) = \big(A(u) + \Lambda_R(u)\big)\Lambda_R(u) - B^2(u) \tag{39}$$

$$\Lambda'_L(u) = \big(A(u) - \Lambda_L(u)\big)\Lambda_L(u) + B^2(u) \tag{40}$$

$A = 0\Omega_L\Omega_R$

$$\boldsymbol{\Phi}'_R(u) + B_-\boldsymbol{\Phi}_R(u) = 0, \boldsymbol{\Phi}'_L(u) - B_-\boldsymbol{\Phi}_L(u) = 0, \qquad u \in \Omega_L, \tag{41}$$

$$\boldsymbol{\Phi}'_R(u) - B_+\boldsymbol{\Phi}_R(u) = 0, \boldsymbol{\Phi}'_L(u) + B_-\boldsymbol{\Phi}_L(u) = 0, \qquad u \in \Omega_R, \tag{42}$$

$AB^2\Lambda_R(u_2) = B_+\Lambda_L(u_1) = B_-u_1 \in \Omega_L u_2 \in \Omega_R$
kuk

4. Combining the Blocks - the Building Block Method

$\Omega_1\Omega_3\Omega_2\Omega_1\Omega_3$

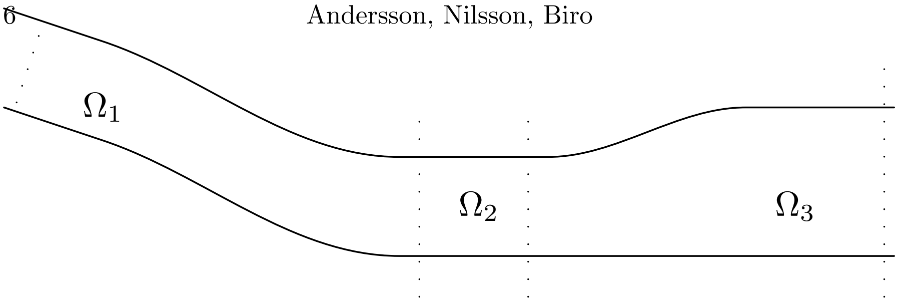

Figure 3. Waveguide divided in blocks. Ω_2 is straight and with constant cross-section.

$\boldsymbol{\Phi} = \boldsymbol{\Phi}^{+} + \boldsymbol{\Phi}^{-} abbT^{+}(= T^{+}(b,a))\boldsymbol{\Phi}^{+} a\boldsymbol{\Phi}^{+}(b) = T^{+}\boldsymbol{\Phi}^{+}(a)btt = t_0\Omega_1 t = t_1 = 0\Omega_1\Omega_2 t = t_2 = \ell\Omega_2\Omega_3\Omega_2\ell t = t_3\Omega_3$

$$\begin{aligned}
R_1^{+} &= R^{+}(t_1,t_0), & T_1^{+} &= T^{+}(t_1,t_0),\\
R_1^{-} &= R^{-}(t_0,t_1), & T_1^{-} &= T^{-}(t_0,t_1),\\
R_3^{+} &= R^{+}(t_3,t_2), & T_3^{+} &= T^{+}(t_3,t_2),\\
R_{\text{tot}}^{+} &= R^{+}(t_3,t_0), & T_{\text{tot}}^{+} &= T^{+}(t_3,t_0).
\end{aligned}$$

$\Omega_2\ell a$

$$S(t) = \begin{pmatrix} e^{i\alpha_0 t} & 0 & 0 & \cdots \\ 0 & e^{i\alpha_1 t} & 0 & \cdots \\ 0 & 0 & e^{i\alpha_2 t} & \\ & & & \end{pmatrix}, 0 \le t \le \ell, \tag{43}$$

$\alpha_n = \sqrt{k^2 - \dfrac{n^2\pi^2}{a^2}}$

$\boldsymbol{\Phi}^{\text{in}} = \boldsymbol{\Phi}^{+}(t_0)\Omega_1\Omega_3 C^{\pm}\Omega_1\Omega_2\boldsymbol{\Phi}^{+}(0) = C^{+}\boldsymbol{\Phi}^{\text{in}}\boldsymbol{\Phi}^{-}(0) = C^{-}\boldsymbol{\Phi}^{\text{in}}\Omega_2 t$

$$\boldsymbol{\Phi}(t) = (S(t)C^{+} + S^{-1}(t)C^{-})\boldsymbol{\Phi}^{\text{in}}. \tag{44}$$

$\Omega_2\Omega_3 S(\ell)C^{+}\boldsymbol{\Phi}^{\text{in}} S^{-1}(\ell)C^{-}\boldsymbol{\Phi}^{\text{in}}$
$\Omega_3 S^{-1}(\ell)C^{-} = R_3^{+}S(\ell)C^{+}C^{-} = S(\ell)R_3^{+}S(\ell)C^{+}C^{+} = T_1^{+} + R_1^{-}C^{-}$

$$\begin{aligned}
C^{+} &= \left(I - R_1^{-}S(\ell)R_3^{+}S(\ell)\right)^{-1} T_1^{+},\\
C^{-} &= S(\ell)R_3^{+}S(\ell)C^{+},\\
T_{\text{tot}}^{+} &= T_3^{+}S(\ell)C^{+},\\
R_{\text{tot}}^{+} &= R_1^{+} + T_1^{-}C^{-}.
\end{aligned} \tag{45}$$

5. A numerical example

5.1. Boundary conditions

$\Omega_1\Omega_3 F_j([-2,2] + \text{i})j \in \{1,2\}\beta = 0.5 + 0.5\,\text{i}F_j([-1,1] + \text{i})F_j$

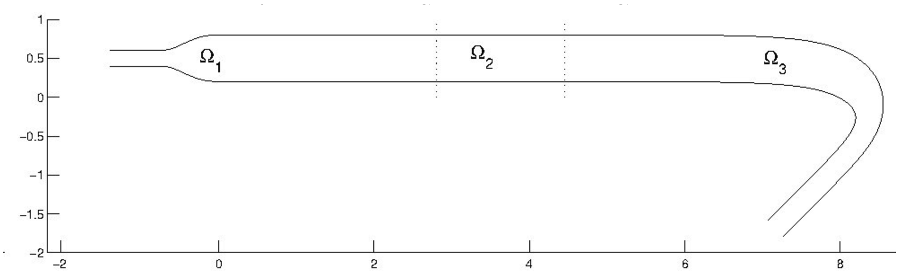

Figure 4. The waveguide in the example.

5.2. Conformal mappings

$\Omega_1\Omega_2\Omega_3$
$\Omega_1 F_1 = f_1 \circ g_1$

$$f_1(w) = A\int_{w_0}^{w}\prod_{j=1}^{4}\left(\sqrt{(\omega + b_k\,\mathrm{i} - w_k)^2 - c_k^2} - b_k\,\mathrm{i}\right)^{\alpha_k - 1}\omega^{-1}d\omega + z_0, \quad (46)$$

$$g_1(w) = \exp(\pi w). \quad (47)$$

$A = 0.6/\pi 0.6\boldsymbol{\alpha} = (0.85, 1.15, 1.15, 0.85)^t 1.15\pi 0.85\pi\boldsymbol{b} = \boldsymbol{c} = (1, 0.05, 0.05, 1)^t$
$\boldsymbol{w} = (-1, -a, a, 1)^t a = 0.0087400.2 w_0 2 z_0 1 + 0.2i$

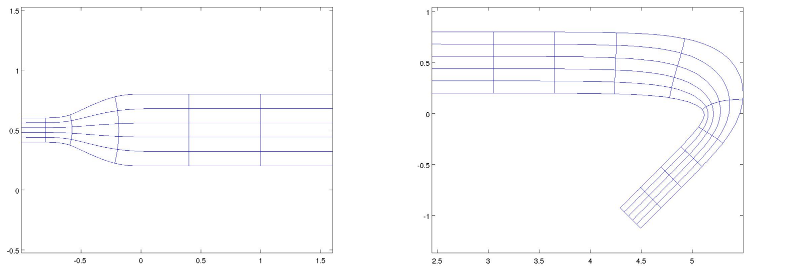

Figure 5. The two building blocks. The grid lines are images under the conformal mappings of $u = -5, -4, ..., 4, 5$ and $v = 0, 0.2, \ldots, 1$.

$\Omega_3 F_2 = f_2 \circ g_2$

$$f_2(w) = \int_{g_2(w_0)}^{w}\frac{(\omega - 1)^{\alpha-1}}{(\omega + 1)^{\alpha-1}(\omega - a)}\,d\omega + z_0 \quad (48)$$

$$g_2(w) = w^{(\varphi_2 - \varphi_1)/\pi}\,\mathrm{e}^{\mathrm{i}\varphi_1} + a, \quad (49)$$

$A = 0.1501\exp(3\pi\,\mathrm{i}/4)\alpha = 7/4\varphi_1 = 3\pi/10\varphi_2 = 7\pi/10 a = -0.4632 w_0 = -7$
$z_0 = 4.4485 + 0.2\,\mathrm{i}$

5.3. Determination of the field, reflection and transmission operators

$\Omega_1\Omega_3\Omega_1\Omega_3 10 \times 10$`ode45`

$A(u) B^2(u) JKLMu = -5, -4.99, ..., 5 \Omega_1 u = -7, -6.99, ..., 7 \Omega_3 JKLMA$
$B^2 u$
Ω_2

5.4. Results

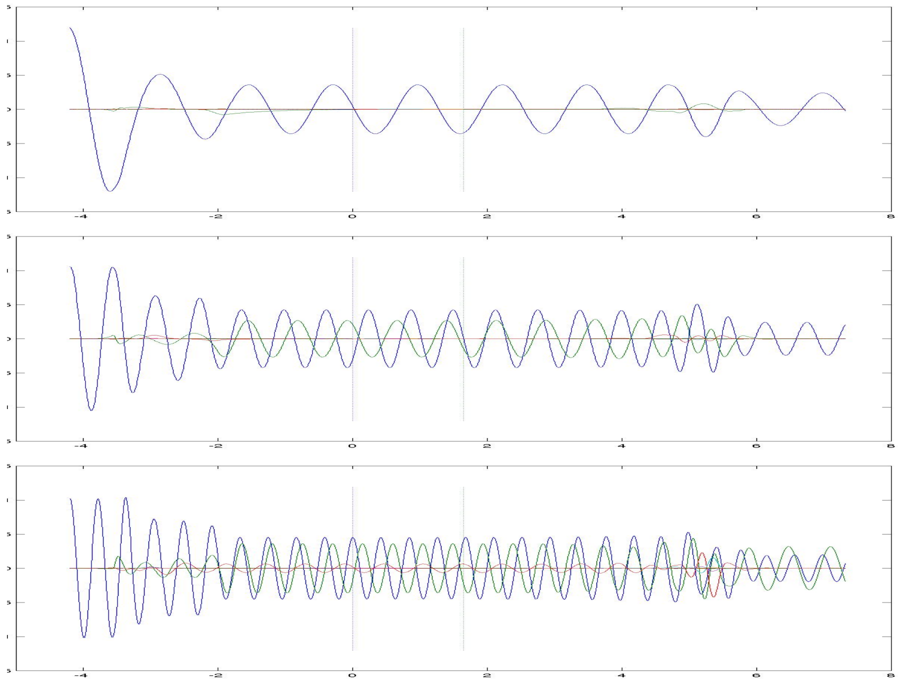

Figure 6. $\mathrm{Re}\,\Phi_1, \mathrm{Re}\,\Phi_2, \mathrm{Re}\,\Phi_3, \dots$ for $k = 5$, $k = 10$ and $k = 15$. Dotted vertical lines indicate borders between Ω_1, Ω_2 and Ω_3.

$\mathrm{Re}\,\Phi_1, \mathrm{Re}\,\Phi_2, \dots k = 5 k = 10 k = 15 \mathrm{Re}\, p(x,y) k = 15$

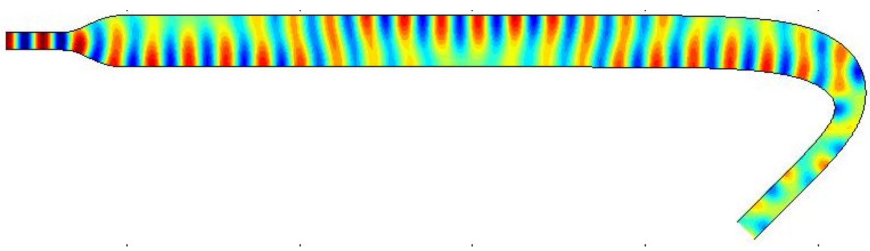

Figure 7. $\mathrm{Re}\, p(x,y)$ plotted for $k = 15$.

$\Omega_1 \boldsymbol{\Phi} \Omega_2 \boldsymbol{\Phi} \boldsymbol{\Phi} \Omega_1 \boldsymbol{\Phi} \Omega_2 (C^+ + C^-) \boldsymbol{\Phi}^{\text{in}} \boldsymbol{\Phi} \Omega_3 T^+_{\text{tot}} \boldsymbol{\Phi}^{\text{in}} T^+_{\text{tot}}$

5.5. A FEM solution to the problem

$k = 0 \dots 20 k 4 \cdot 10^{-3}$

6. Discussion and conclusion

Table I. Comparing the RF and DtN methods. Above: Φ_1, Φ_2 and Φ_3 at the border between Ω_1 and Ω_2 calculated with the two different methods. Below: Φ_1 and Φ_2 at the end of Ω_3 calculated with the two different methods. All calculations are made for $k = 15$.

	$\boldsymbol{\Phi}_{\Omega_1}(\text{end})$	$\boldsymbol{\Phi}_{\Omega_2}(0)$	$\lvert\text{difference}\rvert$
Φ_1	$0.4521 - 0.0448\,\mathrm{i}$	$0.4521 - 0.0449\,\mathrm{i}$	$3.744 \cdot 10^{-5}$
Φ_2	$-0.1873 - 0.2693\,\mathrm{i}$	$-0.1873 - 0.2693\,\mathrm{i}$	$7.971 \cdot 10^{-6}$
Φ_3	$0.0190 + 0.0203\,\mathrm{i}$	$0.0190 + 0.0203\,\mathrm{i}$	$7.730 \cdot 10^{-6}$
	$\boldsymbol{\Phi}_{\Omega_3}(\text{end})$	$T^{+}_{\text{tot}}\boldsymbol{\Phi}^{\text{in}}$	
Φ_1	$0.0671 - 0.1847\,\mathrm{i}$	$0.0671 - 0.1848\,\mathrm{i}$	$2.792 \cdot 10^{-5}$
Φ_2	$-0.1926 + 0.2528\,\mathrm{i}$	$-0.1926 + 0.2528\,\mathrm{i}$	$2.812 \cdot 10^{-5}$

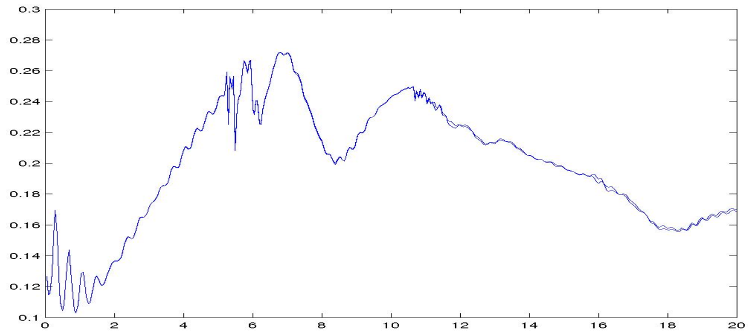

Figure 8. $|\Phi_0|$ at the end of Ω_3 calculated for $k = 0.05, 0.1, \ldots, 20$ with the Fourier methods described in the article and with the Finite Element Method.

$AB^2 uu\lambda_n(u) n = 0, \ldots, N-1 \alpha\beta u N^2 \mu_{mn}(u) u \lambda_n = n\pi\mu u$
$k|T^{+}_{\text{tot}}(0,0)||T^{+}_{\text{tot}}(1,0)| 2 \times 250 \times 507 \times 7$

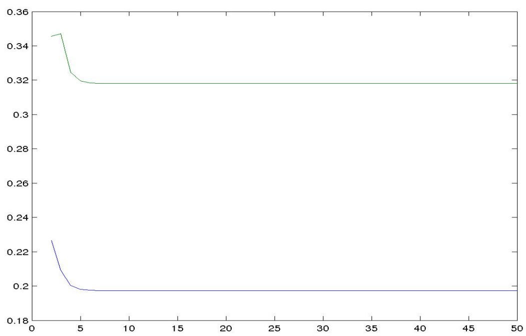

Figure 9. $|T^{\text{tot}}(0,0)|$ and $|T^{\text{tot}}(1,0)|$ for $k = 15$ calculated using matrices of size $N \times N$ with $N = 2 \ldots 50$.

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