

Newton's Reciprocal Approximation Algorithm with Errors

Visualization Tool to Accompany A Balanced Introduction to Computer Science, 3rd ed. David Reed, Prentice Hall, © 2010.

In order to approximate the reciprocal of some number N , Newton's algorithm starts with an initial approximation of 1, then repeatedly refines the approximation as a function of the old approximation with N .
More succinctly,

$$\text{newApproximation} = \text{oldApproximation} * (2 - N * \text{oldApproximation})$$

To see Newton's algorithm in action, enter the number for which you want to compute the reciprocal, then click the button to see each successive approximation.

Find the reciprocal of in the range (0, 1).

```
approximation 1 = 1
    absolute error: 4
    relative error: 0.8
approximation 2 = 1.8
    absolute error: 3.2
    relative error: 0.64
approximation 3 = 2.952
    absolute error: 2.048
    relative error: 0.4096
approximation 4 = 4.1611392
    absolute error: 0.8388608
    relative error: 0.16777216
approximation 5 = 4.859262511644672
    absolute error: 0.1407374883553283
    relative error: 0.02814749767106566
approximation 6 = 4.996038591874287
    absolute error: 0.003961408125713106
    relative error: 0.0007922816251426212
approximation 7 = 4.99996861449131
    absolute error: 0.0000031385508689751873
    relative error: 6.277101737950375e-7
approximation 8 = 4.999999999998029
    absolute error: 1.970867913314578e-12
    relative error: 3.941735826629156e-13
approximation 9 = 5
    absolute error: 0
    relative error: 0
```