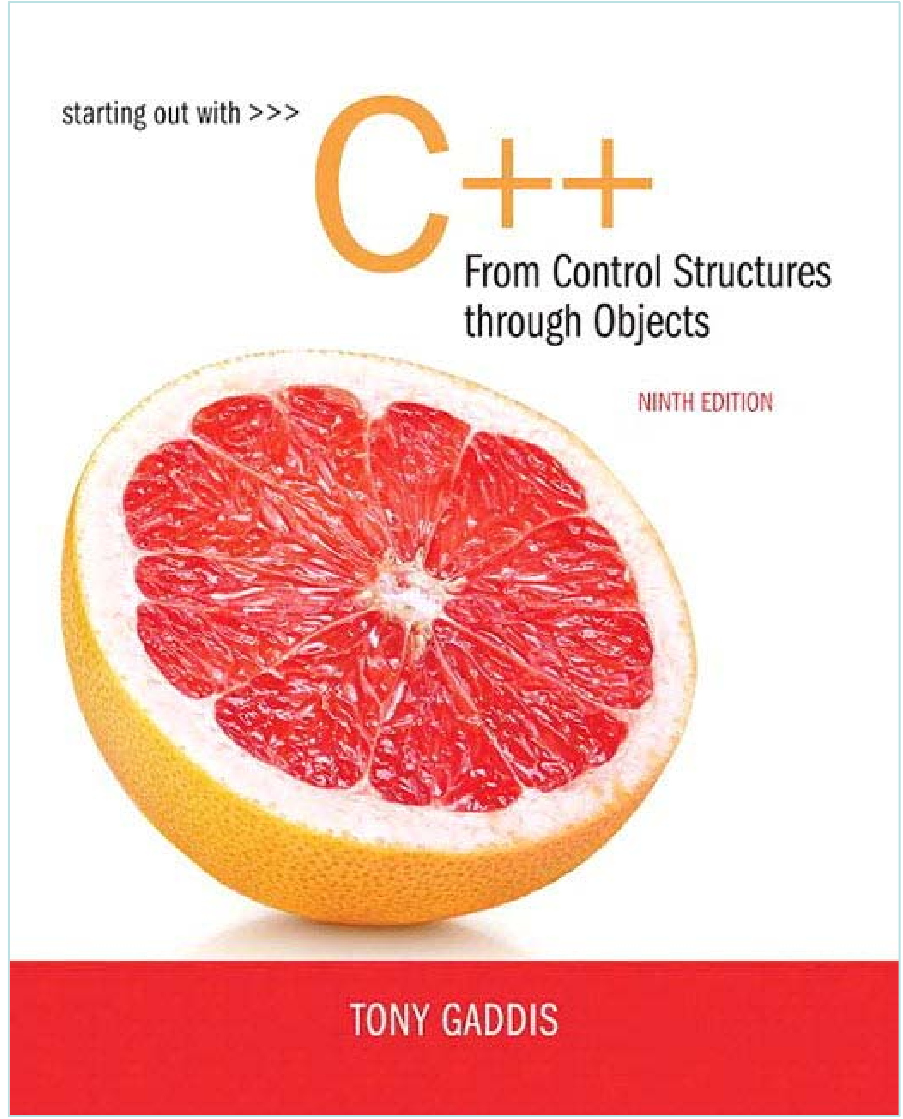
Chapter 20:

Recursion



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Introduction to Recursion

- A <u>recursive function</u> contains a call to itself:

```
void countDown(int num)
{
   if (num == 0)
      cout << "Blastoff!";
   else
   {
      cout << num << "...\n";
      countDown(num-1); // recursive
   }                    // call
}
```

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What Happens When Called?

If a program contains a line like `countDown(2);`

1. `countDown(2)` generates the output `2...`, then it calls `countDown(1)`
2. `countDown(1)` generates the output `1...`, then it calls `countDown(0)`
3. `countDown(0)` generates the output `Blastoff!`, then returns to `countDown(1)`
4. `countDown(1)` returns to `countDown(2)`
5. `countDown(2)`returns to the calling function

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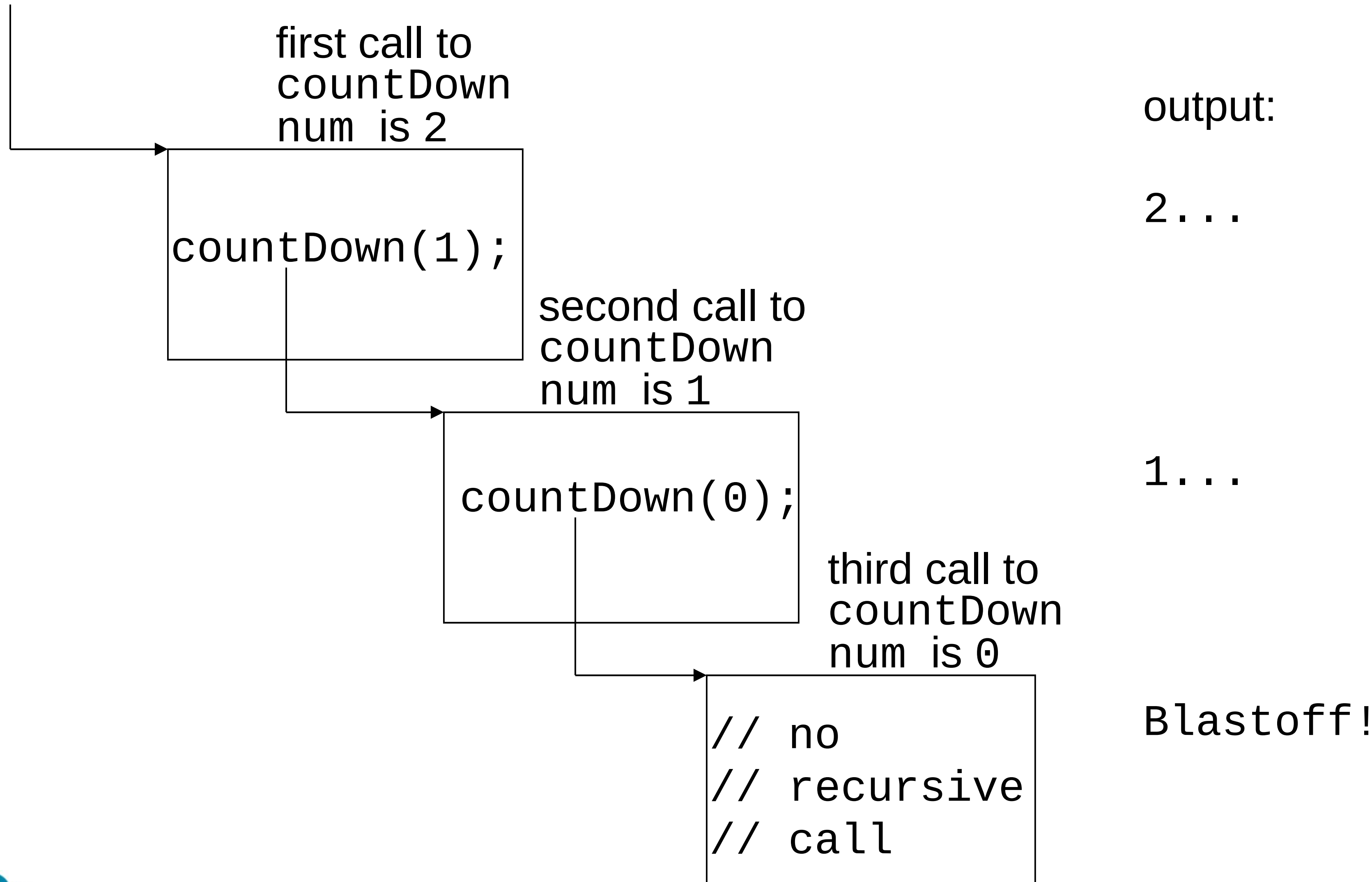
What Happens When Called?



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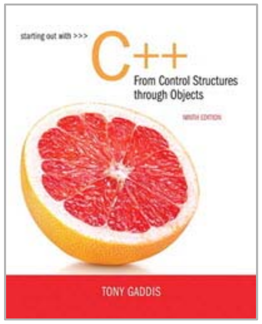

20.2

Solving Problems with Recursion

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Recursive Functions - Purpose

- Recursive functions are used to reduce a complex problem to a simpler-to-solve problem.
- The simpler-to-solve problem is known as the <u>base case</u>
- Recursive calls stop when the base case is reached

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Stopping the Recursion

- A recursive function must always include a test to determine if another recursive call should be made, or if the recursion should stop with this call
- In the sample program, the test is:

```
if (num == 0)
```

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Stopping the Recursion

```
void countDown(int num)
{
   if (num == 0) // test
      cout << "Blastoff!";
   else
   {
      cout << num << "...\n";
      countDown(num-1); // recursive
   }                    // call
}
```

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Stopping the Recursion

- Recursion uses a process of breaking a problem down into smaller problems until the problem can be solved
- In the `countDown` function, a different value is passed to the function each time it is called
- Eventually, the parameter reaches the value in the test, and the recursion stops

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Stopping the Recursion

```
void countDown(int num)
{
   if (num == 0)
      cout << "Blastoff!";
   else
   {
      cout << num << "...\n";
      countDown(num-1);// note that the value
   }                    // passed to recursive
}                       // calls decreases by
                        // one for each call
```

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What Happens When Called?

- Each time a recursive function is called, a new copy of the function runs, with new instances of parameters and local variables created
- As each copy finishes executing, it returns to the copy of the function that called it
- When the initial copy finishes executing, it returns to the part of the program that made the initial call to the function

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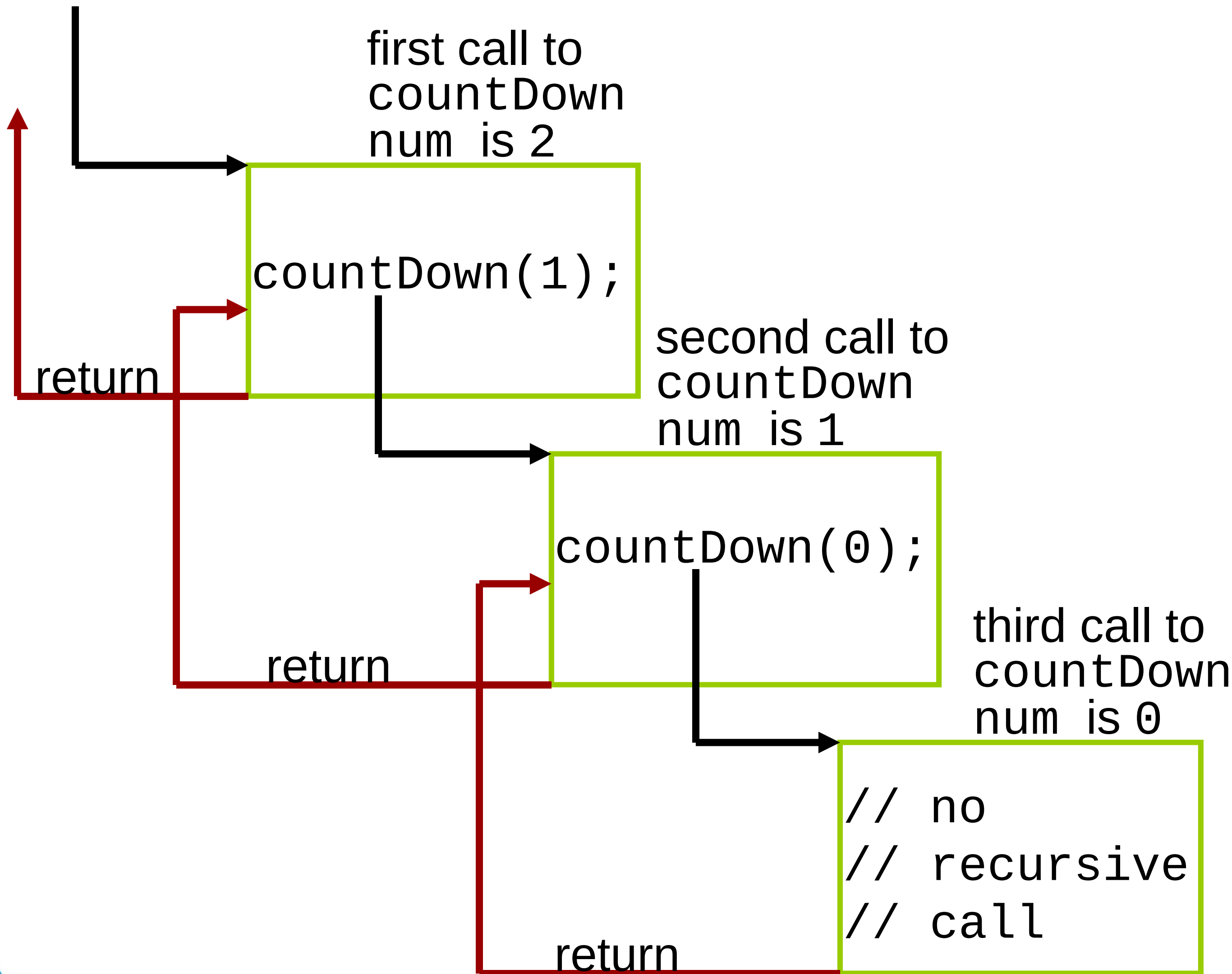
What Happens When Called?



output:

```
2...

1...

Blastoff!
```

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Types of Recursion

- Direct
  - a function calls itself
- Indirect
  - function A calls function B, and function B calls function A
  - function A calls function B, which calls …, which calls function A

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The Recursive Factorial Function

- The factorial function:

  `n! = n*(n-1)*(n-2)*...*3*2*1` if `n > 0`

  `n! = 1` if `n = 0`

- Can compute factorial of n if the factorial of `(n-1)` is known:

  `n! = n * (n-1)!`

- `n = 0` is the base case

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The Recursive Factorial Function

```
int factorial (int num)
{
  if (num > 0)
    return num * factorial(num - 1);
  else
    return 1;
}
```

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Program 20-3

```
// This program demonstrates a recursive function to
// calculate the factorial of a number.
#include <iostream>
using namespace std;

// Function prototype
int factorial(int);

int main()
{
    int number;

```

(program continues)

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Program 20-3 (Continued)

```
    // Get a number from the user.
    cout << "Enter an integer value and I will display\n";
    cout << "its factorial: ";
    cin >> number;

    // Display the factorial of the number.
    cout << "The factorial of " << number << " is ";
    cout << factorial(number) << endl;
    return 0;
}

//**************************************************************
// Definition of factorial. A recursive function to calculate *
// the factorial of the parameter n.                          *
//**************************************************************

int factorial(int n)
{
    if (n == 0)
        return 1;                          // Base case
    else
        return n * factorial(n - 1);  // Recursive case
}
```

Program Output with Example Input Shown in Bold

```
Enter an integer value and I will display
its factorial: 4 [Enter]
The factorial of 4 is 24
```

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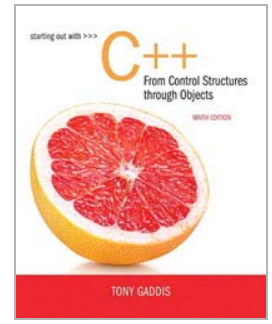

20.3

The Recursive gcd Function

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The Recursive gcd Function

- Greatest common divisor (gcd) is the largest factor that two integers have in common
- Computed using Euclid's algorithm:

  `gcd(x, y) = y` if y divides x evenly

  `gcd(x, y) = gcd(y, x % y)` otherwise
- `gcd(x, y) = y` is the base case

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The Recursive gcd Function

```
int gcd(int x, int y)
{
    if (x % y == 0)
        return y;
    else
        return gcd(y, x % y);
}
```

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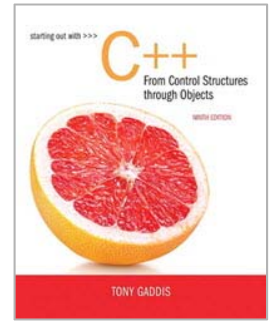

20.4

Solving Recursively Defined Problems

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Solving Recursively Defined Problems

- The natural definition of some problems leads to a recursive solution
- Example: Fibonacci numbers:

  `0, 1, 1, 2, 3, 5, 8, 13, 21, ...`
- After the starting `0, 1`, each number is the sum of the two preceding numbers
- Recursive solution:

  `fib(n) = fib(n – 1) + fib(n – 2);`
- Base cases: `n <= 0, n == 1`

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Solving Recursively Defined Problems

```
int fib(int n)
{
   if (n <= 0)
       return 0;
   else if (n == 1)
       return 1;
   else
         return fib(n – 1) + fib(n – 2);
}
```

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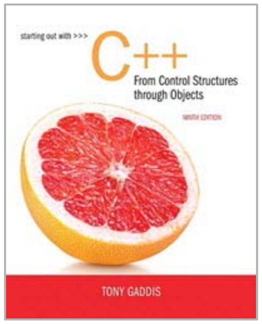

20.5

Recursive Linked List Operations

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Recursive Linked List Operations

- Recursive functions can be members of a linked list class
- Some applications:
  - Compute the size of (number of nodes in) a list
  - Traverse the list in reverse order

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Counting the Nodes in a Linked List

- Uses a pointer to visit each node
- Algorithm:
  - pointer starts at head of list
  - If pointer is null pointer, return 0 (base case)

    else, return 1 + number of nodes in the list pointed to by current node

- See the `NumberList` class in Chapter 19

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The `countNodes` function, a private member function

```
int NumberList::countNodes(ListNode *nodePtr) const
{
    if (nodePtr != nullptr)
        return 1 + countNodes(nodePtr->next);
    else
        return 0;
}
```

- The `countNodes` function is executed by the public `numNodes` function:

```
int numNodes() const
   { return countNodes(head); }
```

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Contents of a List in Reverse Order

- Algorithm:
  - pointer starts at head of list
  - If the pointer is null pointer, return (base case)
  - If the pointer is not null pointer, advance to next node
  - Upon returning from recursive call, display contents of current node

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The `showReverse` function, a private member function

```
void NumberList::showReverse(ListNode *nodePtr) const
{
    if (nodePtr != nullptr)
    {
        showReverse(nodePtr->next);
        cout << nodePtr->value << " ";
    }
}
```

- The `showReverse` function is executed by the public `displayBackwards` function:

```
void displayBackwards() const
   { showReverse(head); }
```

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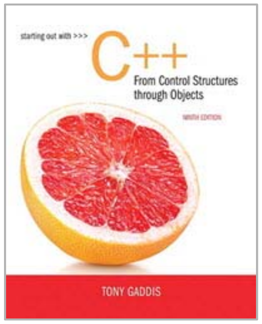

20.6

A Recursive Binary Search Function

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A Recursive Binary Search Function

- Binary search algorithm can easily be written to use recursion
- Base cases: desired value is found, or no more array elements to search
- Algorithm (array in ascending order):
  - If middle element of array segment is desired value, then done
  - Else, if the middle element is too large, repeat binary search in first half of array segment
  - Else, if the middle element is too small, repeat binary search on the second half of array segment

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A Recursive Binary Search Function (Continued)

```
int binarySearch(int array[], int first, int last, int value)
{
   int middle;    // Mid point of search

   if (first > last)
      return -1;
   middle = (first + last) / 2;
   if (array[middle] == value)
      return middle;
   if (array[middle] < value)
      return binarySearch(array, middle+1,last,value);
   else
      return binarySearch(array, first,middle-1,value);
}
```

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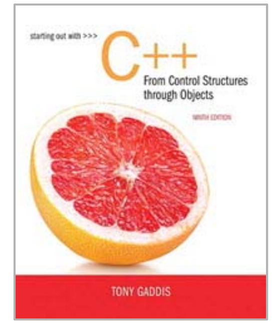

20.7

The Towers of Hanoi

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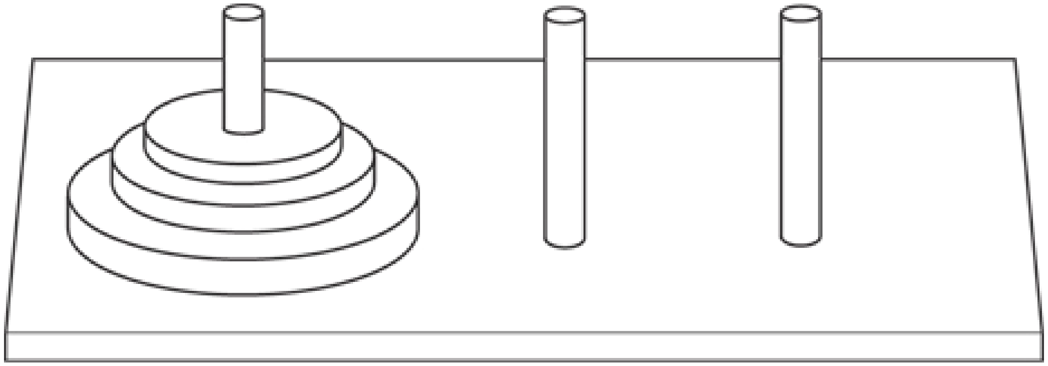
The Towers of Hanoi

- The Towers of Hanoi is a mathematical game that is often used to demonstrate the power of recursion.
- The game uses three pegs and a set of discs, stacked on one of the pegs.



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The Towers of Hanoi

- The object of the game is to move the discs from the first peg to the third peg. Here are the rules:
  - Only one disc may be moved at a time.
  - A disc cannot be placed on top of a smaller disc.
  - All discs must be stored on a peg except while being moved.

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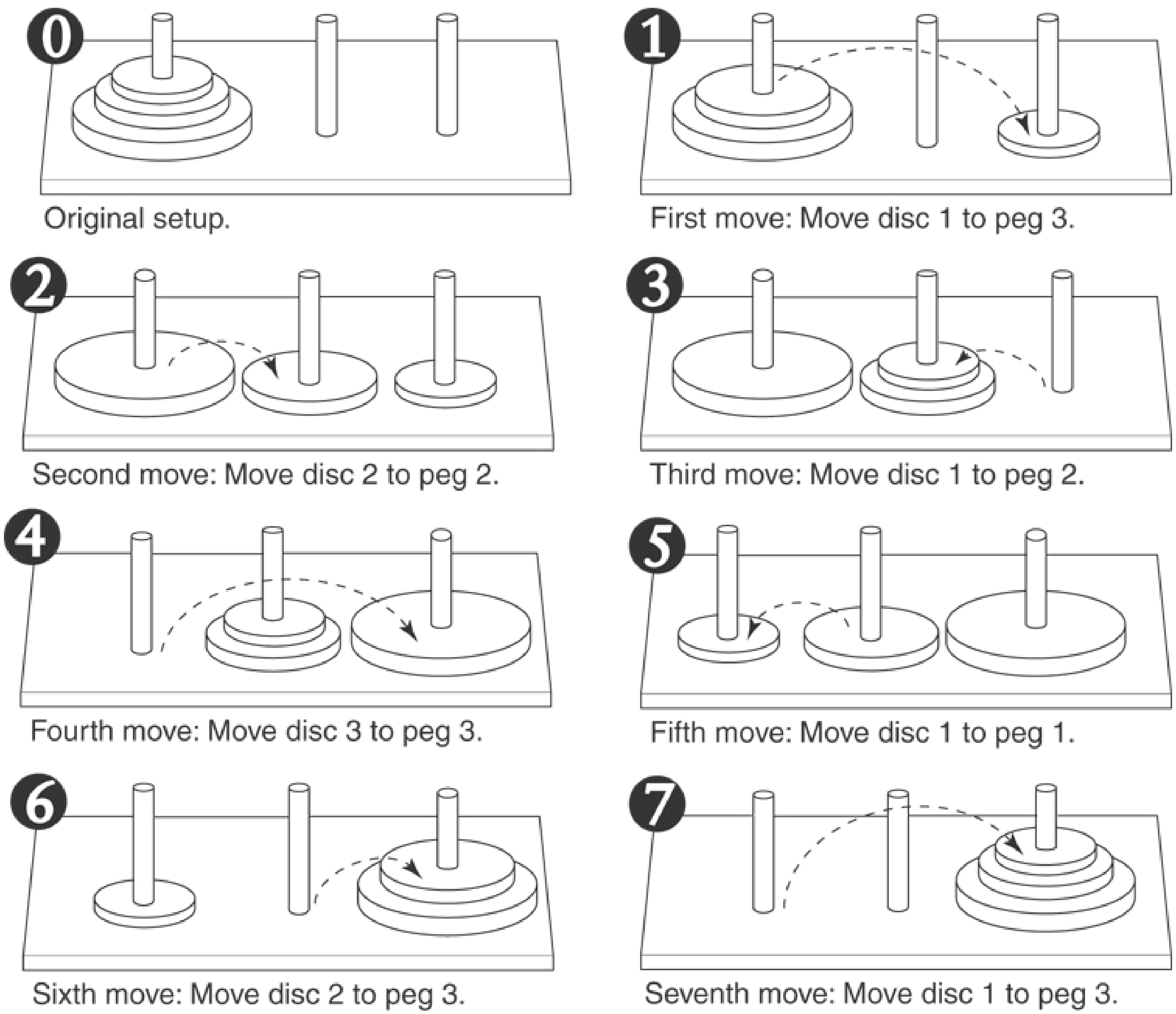
Moving Three Discs



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The Towers of Hanoi

- The following statement describes the overall solution to the problem:
  - *Move n discs from peg 1 to peg 3 using peg 2 as a temporary peg.*

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The Towers of Hanoi

- Algorithm
  - *To move n discs from peg A to peg C, using peg B as a temporary peg:*
    If n > 0 Then
    Move n – 1 discs from peg A to peg B, using peg C as a temporary peg.

    Move the remaining disc from the peg A to peg C.

    Move n – 1 discs from peg B to peg C, using peg A as a temporary peg.

    End If

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Program 20-10

```
// This program displays a solution to the Towers of
// Hanoi game.
#include <iostream>
using namespace std;

// Function prototype
void moveDiscs(int, int, int, int);

int main()
{
    const int NUM_DISCS = 3;  // Number of discs to move
    const int FROM_PEG = 1;   // Initial "from" peg
    const int TO_PEG = 3;     // Initial "to" peg
    const int TEMP_PEG = 2;   // Initial "temp" peg

    // Play the game.
    moveDiscs(NUM_DISCS, FROM_PEG, TO_PEG, TEMP_PEG);
    cout << "All the pegs are moved!\n";
```

(program continues)

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Program 20-10 *(continued)*

```
    return 0;
}

//*****************************************************
// The moveDiscs function displays a disc move in   *
// the Towers of Hanoi game.                         *
// The parameters are:                               *
// num: The number of discs to move.                 *
// fromPeg: The peg to move from.                    *
// toPeg: The peg to move to.                        *
// tempPeg: The temporary peg.                       *
//*****************************************************

void moveDiscs(int num, int fromPeg, int toPeg, int tempPeg)
{
    if (num > 0)
    {
        moveDiscs(num - 1, fromPeg, tempPeg, toPeg);
        cout << "Move a disc from peg " << fromPeg
             << " to peg " << toPeg << endl;
        moveDiscs(num - 1, tempPeg, toPeg, fromPeg);
    }
}
```

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Program 20-10 (Continued)

Program Output

```
Move a disc from peg 1 to peg 3
Move a disc from peg 1 to peg 2
Move a disc from peg 3 to peg 2
Move a disc from peg 1 to peg 3
Move a disc from peg 2 to peg 1
Move a disc from peg 2 to peg 3
Move a disc from peg 1 to peg 3
All the pegs are moved!
```

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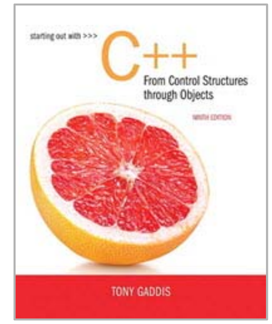

20.8

The QuickSort Algorithm

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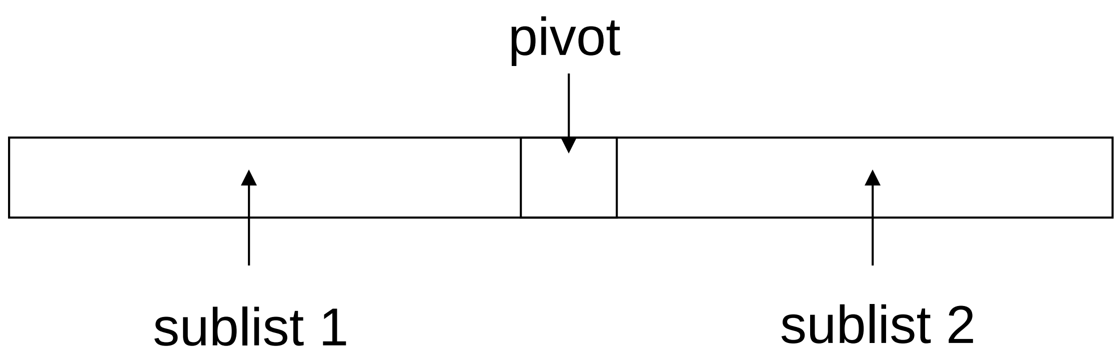
The QuickSort Algorithm

- Recursive algorithm that can sort an array or a linear linked list
- Determines an element/node to use as pivot value:



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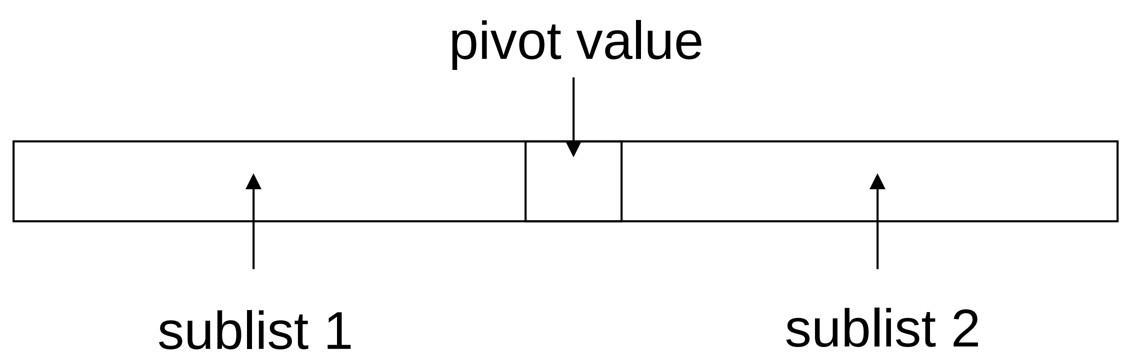
The QuickSort Algorithm



- Once pivot value is determined, values are shifted so that elements in sublist1 are < pivot and elements in sublist2 are > pivot
- Algorithm then sorts sublist1 and sublist2
- Base case: sublist has size 1

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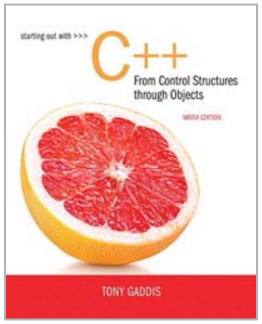

20.9

Exhaustive and Enumeration Algorithms

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Exhaustive and Enumeration Algorithms

- Exhaustive algorithm: search a set of combinations to find an optimal one
  - Example: change for a certain amount of money that uses the fewest coins
- Uses the generation of all possible combinations when determining the optimal one.

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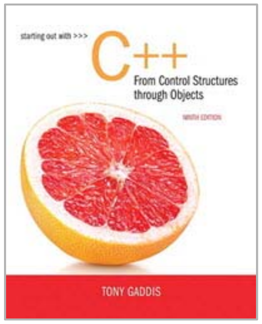

20.10

Recursion vs. Iteration

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Recursion vs. Iteration

- Benefits (+), disadvantages(-) for recursion:
  - \+ Models certain algorithms most accurately
  - \+ Results in shorter, simpler functions
  - May not execute very efficiently
- Benefits (+), disadvantages(-) for iteration:
  - \+ Executes more efficiently than recursion
  - Often is harder to code or understand

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