A Continuous Version of the Newsvendor Problem

1. *Problem Statement*: A vendor orders a fixed quantity x of a certain perishable liquid to be sold each day. The cost to the vendor, and the selling price respectively, of each unit volume of the liquid is c and s price units. The unsold liquid is salvaged each day at the unit price w. The daily demand D has a Burr Type XII distribution with parameters α and β. Recall that the Burr Type XII distribution is supported on $[0, \infty)$ and has the cumulative distribution function (cdf) $F(x) = 1 - (1 + x^{\alpha})^{-\beta}$, where $x, \alpha, \beta > 0$. A simulation that generates random variates from the specified demand distribution is available. What is the quantity x that maximizes the expected profit for the vendor? Assume that the parameter values and the fact that the demand is Burr distributed are unknown to the solution procedure.

2. *Recommended Parameter Settings*: $c = 5, s = 9, w = 1, \alpha = 2, \beta = 20, z_{\text{bad}} = 0$.

3. *Measurement of time*: Number of demand random variates generated. Take desired budgets to be $100, 3000, 9000, 15000$.

4. *Starting Solution(s)*: 0.

5. *Optimal Solution(s)*: Global minimum at $\inf\{x : F(x) \geq (s-c)/(s-w)\}$ where F is the Burr Type XII cdf with parameters α and β. Since F is invertible, the global minimum is at

$$x^* = \left(\frac{1}{(1-r)^{1/\beta}} - 1 \right)^{1/\alpha},$$

   where $r = (s-c)/(s-w)$. For the recommended parameter settings $c = 5, s = 9, w = 1$, the optimal solution $x^* = 0.1878$, and the maximum expected profit is 0.4635.

6. *Known Structure*: The objective function is concave, equals 0 for $x = 0$, and tends to $-\infty$ as $x \to \infty$.

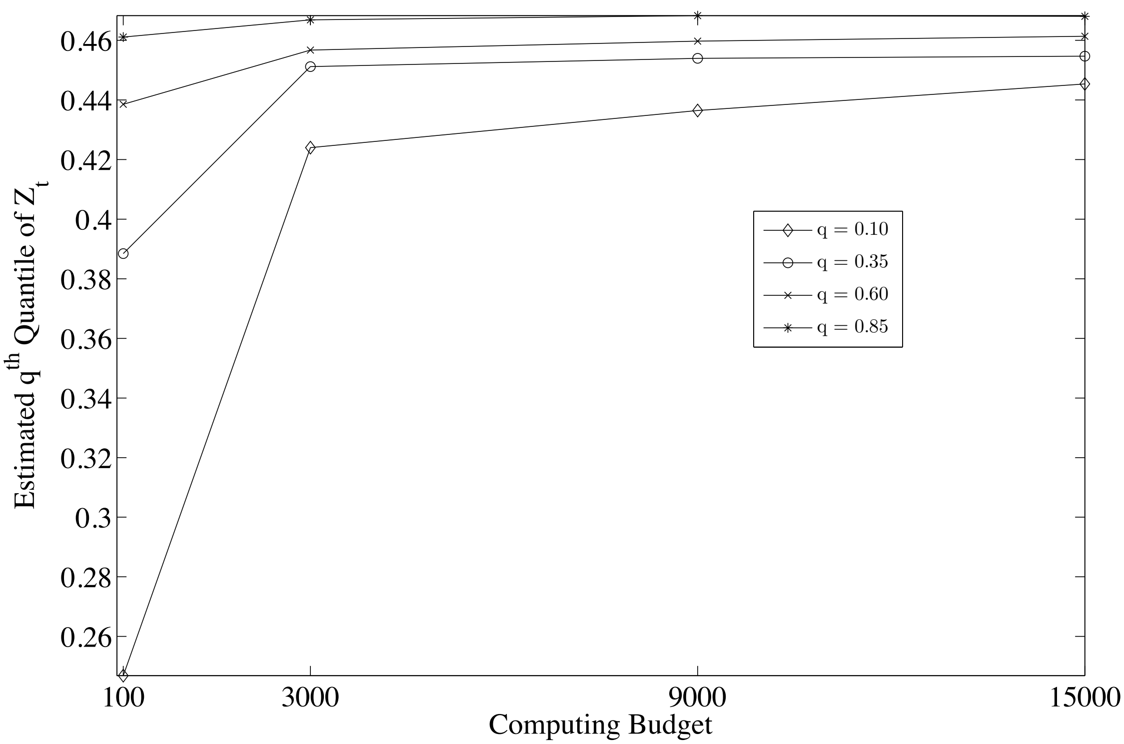
7. *Recommended Quantiles*: $0.10, 0.35, 0.6, 0.85$.

Solution Procedure

The above SO problem instance was solved using a Retrospective Approximation (RA) implementation. The sample-path problems that were generated were solved using a version of the derivative-free Nelder-Mead numerical procedure. Details about the algorithm, including an actual implementation, can be obtained through https://filebox.vt.edu/users/pasupath/pasupath.htm.

Finite-Time Performance Based on Computing Budget

For this problem, the computing budget t is measured as the number of demand random variates generated. The random variable Z_t denotes the true objective function value at the solution returned upon expending t units of computing effort.



Finite-Time Performance Based on a General Stopping Criterion

For generating the curves displayed in the figure below, the algorithm was terminated when the estimated standard error of the reported solution was less than or equal to 0.005.