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MATRÍCULA: 422029

$\Delta x^{(k)}$	f(x)	f''(x)	e(x)
0.5	20.47996	47.219433	--
0.25	--	45.602379	0.035460
0.125	--	45.205280	0.008784
0.0625	--	45.106449	0.002191
0.03125	--	45.081769	0.000547
0.015625	--	45.075601	0.000137
0.007812	--	45.074059	0.000034

Pela derivada segunda encontrada analiticamente, temos:

$f''(2) = 45.073545$

OBS: Foram consideradas 6 casas decimais de precisão.

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a)

1) $f(x+\Delta x) = f(x) + f'(x)\Delta x + \frac{1}{2} f''(x)(\Delta x)^2 + \frac{1}{3!} f'''(x)(\Delta x)^3 + \frac{1}{4!} f^{(iv)}(x)(\Delta x)^4 + \frac{1}{5!} f^{(v)}(x)(\Delta x)^5 + \frac{1}{6!} f^{(vi)}(x)(\Delta x)^6$

2) $f(x-\Delta x) = f(x) - f'(x)\Delta x + \frac{1}{2} f''(x)(\Delta x)^2 - \frac{1}{3!} f'''(x)(\Delta x)^3 + \frac{1}{4!} f^{(iv)}(x)(\Delta x)^4 - \frac{1}{5!} f^{(v)}(x)(\Delta x)^5 + \frac{1}{6!} f^{(vi)}(x)(\Delta x)^6$

3) $f(x+2\Delta x) = f(x) + f'(x)2\Delta x + \frac{1}{2} f''(x)(2\Delta x)^2 + \frac{1}{3!} f'''(x)(2\Delta x)^2 + \frac{1}{4!} f^{(iv)}(x)(2\Delta x)^4 + \frac{1}{5!} f^{(v)}(x)(2\Delta x)^5 + \frac{1}{6!} f^{(vi)}(x)(2\Delta x)^6$

4) $f(x-2\Delta x) = f(x) - f'(x)2\Delta x + \frac{1}{2} f''(x)(2\Delta x)^2 - \frac{1}{3!} f'''(x)(2\Delta x)^2 + \frac{1}{4!} f^{(iv)}(x)(2\Delta x)^4 - \frac{1}{5!} f^{(v)}(x)(2\Delta x)^2 + \frac{1}{6!} f^{(vi)}(x)(2\Delta x)^6$

$(1) + \alpha(2) + \beta(3) + \gamma(4)$.

Como queremos eliminar $f'(x)$, $f'''(x)$, $f^{(iv)}(x)$ e $f^{(v)}(x)$ seguimos os mesmos passos já apresentados em aula, logo:

$\alpha = 1$, $\beta = -\frac{1}{16}$ e $\gamma = -\frac{1}{16}$

Logo,

$$(1)+\alpha(2)+\beta(3)+\gamma(4)=$$

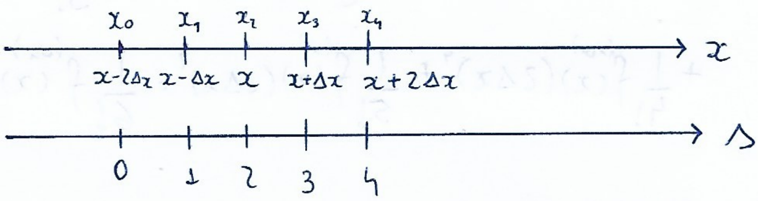
$$f(x+\Delta x)+f(x-\Delta x)+f(x+2\Delta x)+f(x-2\Delta x)=$$

$$f(x)(1+\alpha+\beta+\gamma)+\frac{1}{2}f''(x)(\Delta x)^2(1+\alpha+4\beta+4\gamma)+\frac{1}{6!}f^{(vi)}(x)(\Delta x)^6(1+\alpha+32\beta+32\gamma)$$

Isolando $f''(x)$ e substituindo os valores de α, β e γ temos:

$$f''(x)=\frac{1}{(\Delta x)^2}\left(-\frac{1}{12}f(x+2\Delta x)+\frac{4}{3}f(x+\Delta x)+\frac{4}{3}f(x-\Delta x)-\frac{1}{12}f(x-2\Delta x)-\frac{5}{2}f(x)\right)+\frac{8}{6!}f^{(vi)}(x)(\Delta x)^4$$

b)



(1) $x(s)=x_0+(s-2)\Delta x$

(2) $s(x)=\frac{(x-x_0)}{\Delta x}+2$

Derivando (2) dos dois lados, temos: $ds=\frac{dx}{\Delta x}\rightarrow\frac{ds}{dx}=\frac{1}{\Delta x}$

$$f(x) \approx g(s) = \sum_{j=0}^{N} \binom{s}{j} \Delta^j f_0$$

No nosso probleme temos 5 pontos ($f(x), f(x+\Delta x), f(x-\Delta x), f(x+2\Delta x), f(x-2\Delta x)$) logo nosso polinômio será de Grau 4. Então $N=4$

$$g(s) = \sum_{j=0}^{4} \binom{s}{j} \Delta^j f_0$$

$$= \binom{s}{0}\Delta^0 f_0 + \binom{s}{1}\Delta^1 f_0 + \binom{s}{2}\Delta^2 f_0 + \binom{s}{3}\Delta^3 f_0 + \binom{s}{4}\Delta^4 f_0$$

$$= \Delta^0 f_0 + s\Delta^1 f_0 + \frac{(s^2-s)}{2}\Delta^2 f_0 + \frac{(s^3-3s^2+2s)}{3!}\Delta^3 f_0 + \frac{(s^4-6s^3+11s^2-6s)}{4!}\Delta^4 f_0$$

Derivando $f(x)$ 2 vezes e usando a Regra da Cadeia; além do Fato de que $\frac{ds}{dx} = \frac{1}{\Delta x}$.

$$f''(x) = \frac{1}{(\Delta x)^2} g''(s)$$

$$g''(s) = \Delta^0 f_0 + (s-1)\Delta^3 f_0 + \frac{(6s^2-18s+11)}{12}\Delta^4 f_0$$

Como queremos o ponto central, logo $s=2$.

$$g''(2) = \Delta^2 f_0 + \Delta^3 f_0 - \frac{1}{12}\Delta^4 f_0$$

Como:

$$\Delta^2 f_0 = f_2 - 2f_1 + f_0$$

$$\Delta^3 f_0 = f_3 - 3f_2 + 3f_1 - f_0$$

$$\Delta^4 f_0 = f_4 - 4f_3 + 6f_2 - 4f_1 + f_0$$

Logo:

$$g''(2) = -\frac{5}{2}f_2 + \frac{4}{3}f_1 - \frac{1}{12}f_0 + \frac{4}{3}f_3 - \frac{1}{12}f_4$$

Então

$$f''(x) = \frac{1}{(\Delta x)^2}\left(-\frac{5}{2}f_2 + \frac{4}{3}f_1 - \frac{1}{12}f_0 + \frac{4}{3}f_3 - \frac{1}{12}f_4\right)$$

Substituindo f_0, f_1, f_2, f_3 e f_4:

$$\boxed{f''(x) = \frac{1}{(\Delta x)^2}\left(-\frac{5}{2}f(x) + \frac{4}{3}f(x-\Delta x) - \frac{1}{12}f(x-2\Delta x) + \frac{4}{3}f(x+\Delta x) - \frac{1}{12}f(x+2\Delta x)\right)}$$