Tarefa 01 de Métodos II - Anderson Costa

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a)

Pelas contas do professor temos que:

$$f''(x) = \frac{1}{(\Delta x)^2}(\frac{-1}{12}f(x+2\Delta x)+\frac{4}{3}f(x+\Delta x)+\frac{4}{3}f(x-\Delta x)+\frac{-1}{12}f(x-2\Delta x)+\frac{-5}{2}f(x))+\frac{8}{6!}f^{(6)}(\Delta x)^4$$

A função $f(x)$ é definida como:

$$f(x) = \sqrt{e^{3x} + 4x^2}$$

A derivada segunda de $f(x)$ é dada por:

$$f''(x) = \frac{9e^{3x} + 8}{2\sqrt{e^{3x} + 4x^2}} - \frac{(3e^{3x} + 8x)^2}{4(e^{3x} + 4x^2)^{3/2}}$$

Programa para ver os resultados...

b) Faremos:

$$x - 2\Delta x = x_0 \rightarrow s = 0$$

$$x - \Delta x = x_1 \rightarrow s = 1$$

$$x = x_2 \rightarrow s = 2$$

$$x + \Delta x = x_3 \rightarrow s = 3$$

$$x + 2\Delta x = x_4 \rightarrow s = 4$$

Logo:

$$x(s) = x_0 + (s - 2)\Delta x$$

$$s(x) = \frac{(x - x_0)}{\Delta x} + 2$$

Derivando a última equação, temos:

$$ds = \frac{dx}{\Delta x}$$

Logo:

$$\frac{ds}{dx} = \frac{1}{\Delta x}$$

A aproximação de $f(x)$ pelo polinômio $g(s)$:

$$f(x) \approx g(s) = \sum_{j=0}^{N} \binom{s}{j} \Delta^j f_0$$

No nosso caso temos 5 pontos $f(x), f(x+\Delta x), f(x+2\Delta x), f(x-\Delta x), f(x-2\Delta x)$, logo nosso polinômio terá grau 4. Logo, N = 4.

$$f(x) \approx g(s) = \sum_{j=0}^{4} \binom{s}{j} \Delta^j f_0$$

$$g(s) = \binom{s}{0}\Delta^0 f_0 + \binom{s}{1}\Delta^1 f_0 + \binom{s}{2}\Delta^2 f_0 + \binom{s}{3}\Delta^3 f_0 + \binom{s}{4}\Delta^4 f_0$$

$$= \Delta^0 f_0 + s\Delta^1 f_0 + \frac{(s^2-s)}{2!}\Delta^2 f_0 + \frac{(s^3-3s^2+2s)}{3!}\Delta^3 f_0 + \frac{(s^4-6s^3+11s^2-6s)}{4!}\Delta^4 f_0$$

Como queremos a derivada segunda de $f(x)$ temos que derivar $g(s)$ em relação x, logo temos que usar a Regra da Cadeia:

$$f'(x) = \frac{df(x)}{dx} = \frac{dg(s)}{dx} = \frac{dg(s)}{ds}\frac{ds}{dx}$$

Porém:

$$\frac{ds}{dx} = \frac{1}{\Delta x}$$

Logo:

$$f'(x) = \frac{df(x)}{dx} = \frac{dg(s)}{dx} = \frac{dg(s)}{ds}\frac{ds}{dx} = \frac{1}{\Delta x}\frac{dg(s)}{ds}$$

Derivando novamente:

$$f''(x) = \frac{d^2 f(x)}{dx^2} = \frac{d^2 g(s)}{dx^2} = \frac{1}{(\Delta x)^2}\frac{d^2 g(s)}{ds^2}$$

Logo:

$$f''(x) = \frac{1}{(\Delta x)^2} g''(s)$$

Derivando $g(s)$:

$$g'(s) = \Delta^1 f_0 + \frac{(2s-1)}{2!}\Delta^2 f_0 + \frac{(3s^2-6s+2)}{3!}\Delta^3 f_0 + \frac{(4s^3-18s^2+22s-6)}{4!}\Delta^4 f_0$$

Derivando novamente:

$$g''(s) = \frac{2}{2!}\Delta^2 f_0 + \frac{(6s-6)}{3!}\Delta^3 f_0 + \frac{(12s^2-36s+22)}{4!}\Delta^4 f_0$$

$$= \Delta^2 f_0 + (s-1)\Delta^3 f_0 + \frac{(6s^2 - 18s + 11)}{12}\Delta^4 f_0$$

Como queremos a derivada de x_2, logo $s = 2$, então:

$$g''(2) = \Delta^2 f_0 + (2-1)\Delta^3 f_0 + \frac{(6(2)^2 - 18(2) + 11)}{12}\Delta^4 f_0$$
$$= \Delta^2 f_0 + \Delta^3 f_0 - \frac{1}{12}\Delta^4 f_0$$

Encontrando os Δ's:

$$\Delta^2 f_0 = [\Delta^1 f_1] - [\Delta^1 f_0] = (\Delta^0 f_2 - \Delta^0 f_1) - (\Delta^0 f_1 - \Delta^0 f_0)$$
$$\Delta^2 f_0 = f_2 - 2f_1 + f_0$$

Analogamente (e pelas contas do professor):

$$\Delta^3 f_0 = f_3 - 3f_2 + 3f_1 - f_0$$
$$\Delta^4 f_0 = f_4 - 4f_3 + 6f_2 - 4f_1 + f_0$$

Logo:

$$g''(2) = (f_2 - 2f_1) + (f_3 - 3f_2 + 3f_1 - f_0) - \frac{1}{12}(f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$$

Fazendo as contas:

$$g''(2) = \frac{-5}{2}f_2 + \frac{4}{3}f_1 - \frac{1}{12}f_0 + \frac{4}{3}f_3 - \frac{1}{12}f_4$$

Substituindo $g''(2)$ na fórmula de $f''(2)$:

$$f''(x) = \frac{1}{(\Delta x)^2}(\frac{-5}{2}f_2 + \frac{4}{3}f_1 - \frac{1}{12}f_0 + \frac{4}{3}f_3 - \frac{1}{12}f_4)$$

Substituindo os f_i's:

$$f''(x) = \frac{1}{(\Delta x)^2}(\frac{-5}{2}f(x) + \frac{4}{3}f(x-\Delta x) - \frac{1}{12}f(x-2\Delta x) + \frac{4}{3}f(x+\Delta x) - \frac{1}{12}f(x+2\Delta x))$$