Tarefa 05 de Métodos II - Antônio Anderson Costa Pereira - 422029

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Fórmula da Quadratura de Gauss-Legendre de 4 pontos

Elementos da fórmula de Gauss-Legendre com 4 pontos de interpolação:

$$I = \int_{x_i}^{x_f} f(x)\ dx \approx \frac{x_f - x_i}{2} \sum_{k=1}^{4} f(x(\alpha_k)) w_k$$

Valores de $\alpha_1, \alpha_2, \alpha_3$ **e** α_4:

$$P_4(\alpha) = \frac{1}{2^4 4!} \frac{d^4}{d\alpha^4}[(\alpha^2 - 1)^4] = \frac{35\alpha^4 - 30\alpha^2 + 3}{8} = 0$$

Logo resolvendo $35\alpha^4 - 30\alpha^2 + 3 = 0$, temos:

$$\alpha_1 = \sqrt{\frac{15 + 2\sqrt{30}}{35}}$$

$$\alpha_2 = -\sqrt{\frac{15 + 2\sqrt{30}}{35}}$$

$$\alpha_3 = \sqrt{\frac{15 - 2\sqrt{30}}{35}}$$

$$\alpha_4 = -\sqrt{\frac{15 - 2\sqrt{30}}{35}}$$

Cálculo de $x(\alpha_1), x(\alpha_2), x(\alpha_3)$ **e** $x(\alpha_4)$:

$$x(\alpha_1) = \frac{x_f + x_i}{2} + \frac{x_f - x_i}{2}(\sqrt{\frac{15 + 2\sqrt{30}}{35}})$$

$$x(\alpha_2) = \frac{x_f + x_i}{2} + \frac{x_f - x_i}{2}(-\sqrt{\frac{15 + 2\sqrt{30}}{35}})$$

$$x(\alpha_3) = \frac{x_f + x_i}{2} + \frac{x_f - x_i}{2}(\sqrt{\frac{15 - 2\sqrt{30}}{35}})$$

$$x(\alpha_4) = \frac{x_f + x_i}{2} + \frac{x_f - x_i}{2}(-\sqrt{\frac{15 - 2\sqrt{30}}{35}})$$

Cálculo de w_1, w_2, w_3 e w_4: Como α_1 e α_2 são simétricos em relação a origem então w_1 e w_2 são iguais, o mesmo se repete para α_3 e α_4 com w_3 e w_4. Logo basta calcular w_1 e w_3.

$$L_1(\alpha) = \frac{(\alpha-\alpha_2)(\alpha-\alpha_3)(\alpha-\alpha_4)}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)(\alpha_1-\alpha_4)} =$$

$$= \frac{(\alpha+\sqrt{\frac{15+2\sqrt{30}}{35}})(\alpha-\sqrt{\frac{15-2\sqrt{30}}{35}})(\alpha+\sqrt{\frac{15-2\sqrt{30}}{35}})}{(\sqrt{\frac{15+2\sqrt{30}}{35}}+\sqrt{\frac{15+2\sqrt{30}}{35}})(\sqrt{\frac{15+2\sqrt{30}}{35}}-\sqrt{\frac{15-2\sqrt{30}}{35}})(\sqrt{\frac{15+2\sqrt{30}}{35}}+\sqrt{\frac{15-2\sqrt{30}}{35}})}$$

$$= \frac{\alpha^3+\alpha^2\sqrt{\frac{15+2\sqrt{30}}{35}}-\alpha(\frac{15-2\sqrt{30}}{35})-(\frac{15-2\sqrt{30}}{35})\sqrt{\frac{15+2\sqrt{30}}{35}}}{\frac{8\sqrt{30}}{35}\sqrt{\frac{15+2\sqrt{30}}{35}}}$$

$$L_3(\alpha) = \frac{(\alpha-\alpha_1)(\alpha-\alpha_2)(\alpha-\alpha_4)}{(\alpha_3-\alpha_1)(\alpha_3-\alpha_2)(\alpha_3-\alpha_4)} =$$

$$= \frac{(\alpha-\sqrt{\frac{15+2\sqrt{30}}{35}})(\alpha+\sqrt{\frac{15+2\sqrt{30}}{35}})(\alpha+\sqrt{\frac{15-2\sqrt{30}}{35}})}{(\sqrt{\frac{15-2\sqrt{30}}{35}}-\sqrt{\frac{15+2\sqrt{30}}{35}})(\sqrt{\frac{15-2\sqrt{30}}{35}}+\sqrt{\frac{15+2\sqrt{30}}{35}})(\sqrt{\frac{15-2\sqrt{30}}{35}}+\sqrt{\frac{15-2\sqrt{30}}{35}})} =$$

$$= \frac{\alpha^3+\alpha^2\sqrt{\frac{15-2\sqrt{30}}{35}}-\alpha(\frac{15+2\sqrt{30}}{35})-(\frac{15+2\sqrt{30}}{35})\sqrt{\frac{15-2\sqrt{30}}{35}}}{\frac{-8\sqrt{30}}{35}\sqrt{\frac{15-2\sqrt{30}}{35}}}$$

$$w_1 = w_2 = \int_{-1}^{1} L_1(\alpha)\, d\alpha = \frac{\frac{2}{3}\sqrt{\frac{15+2\sqrt{30}}{35}}-2\sqrt{\frac{15+2\sqrt{30}}{35}}(\frac{15-2\sqrt{30}}{35})}{\frac{8\sqrt{30}}{35}\sqrt{\frac{15+2\sqrt{30}}{35}}} =$$

$$= \frac{2(\frac{-10+6\sqrt{30}}{105})\sqrt{\frac{15+2\sqrt{30}}{35}}}{\frac{8\sqrt{30}}{35}\sqrt{\frac{15+2\sqrt{30}}{35}}} = \frac{-5+3\sqrt{30}}{6\sqrt{30}}$$

$$w_3 = w_4 = \int_{-1}^{1} L_3(\alpha)\, d\alpha = \frac{\frac{2}{3}\sqrt{\frac{15-2\sqrt{30}}{35}}-2(\frac{15+2\sqrt{30}}{35})\sqrt{\frac{15-2\sqrt{30}}{35}}}{\frac{-8\sqrt{30}}{35}\sqrt{\frac{15-2\sqrt{30}}{35}}} =$$

$$= \frac{-2\sqrt{\frac{15-2\sqrt{30}}{35}}(\frac{10+6\sqrt{30}}{105})}{\frac{-8\sqrt{30}}{35}\sqrt{\frac{15-2\sqrt{30}}{35}}} = \frac{5+3\sqrt{30}}{6\sqrt{30}}$$

Assim temos todos os elementos para formarmos a Quadratura de Gauss-Legendre com 4 pontos.