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Tarefa 04

Estimativa do erro da Fórmula de Newton-Cotes de Grau 2 abordagem aberta.

Fórmula de Newton-Cotes de Grau 2 abordagem aberta:

$$I_f = \frac{4h}{3}\left(2f(x+h) - f(x+2h) + 2f(x+3h)\right)$$

Fazendo o ponto central $\bar{x} = x+2h$, logo

$$I_f = \frac{4h}{3}\left(2f(\bar{x}-h) - f(\bar{x}) + 2f(\bar{x}+h)\right)$$

Recorrendo à série de Taylor para encontrarmos $f(\bar{x}-h)$ e $f(\bar{x}+h)$, temos:

$$f(\bar{x}-h) = f(\bar{x}) - f'(\bar{x})h + \frac{f''(\bar{x})h^2}{2!} - \frac{f'''(\bar{x})h^3}{3!} + \frac{f^{(4)}(\bar{x})h^4}{4!}$$

$$f(\bar{x}+h) = f(\bar{x}) + f'(\bar{x})h + \frac{f''(\bar{x})h^2}{2!} + \frac{f'''(\bar{x})h^3}{3!} + \frac{f^{(4)}(\bar{x})h^4}{4!}$$

Somando $f(\bar{x}-h)$ e $f(\bar{x}+h)$, temos:

$$f(\bar{x}-h) + f(\bar{x}+h) = 2f(\bar{x}) + \frac{2f''(\bar{x})h^2}{2!} + \frac{2f^{(4)}(\bar{x})h^4}{4!} + \dots$$

Dobrando o valor e diminuindo $f(\bar{x})$:

$$2f(\bar{x}-h)+2f(\bar{x}+h)-f(\bar{x}) = 3f(\bar{x}) + 4\frac{f''(\bar{x})h^2}{2!} + 4\frac{f^{(4)}(\bar{x})h^4}{4!} + \dots$$

logo:

$$I_f = \frac{4h}{3}\left(3f(\bar{x}) + 4\frac{f''(\bar{x})h^2}{2!} + 4\frac{f^{(4)}(\bar{x})h^4}{4!} + \dots\right)$$

$$I_f = 4f(\bar{x})h + 8\frac{f''(\bar{x})h^3}{3} + 16\frac{f^{(4)}(\bar{x})h^5}{4!\ 3} + \dots \qquad (\mathrm{I})$$

Da equação (9) da aula 09, temos:

$$I_e = \int_a^b f(x)\,dx = p\int_{-1}^{1} f(\bar{x}+\xi h)\,d\xi$$

$$= p\int_{-1}^{1}\left(f(\bar{x}) + f'(\bar{x})(\xi p) + \frac{f''(\bar{x})(\xi p)^2}{2!} + \frac{f'''(\bar{x})(\xi p)^3}{3!} + \frac{f^{(4)}(\bar{x})(\xi p)^4}{4!} + \dots\right)d\xi \qquad (\mathrm{II})$$

Integrando (II):

$$I_e = p\left(2f(\bar{x}) + \frac{p^2}{2!}f''(\bar{x})\frac{2}{3} + \frac{p^4}{4!}f^{(4)}(\bar{x})\frac{2}{5} + \dots\right)$$

Onde $p = \frac{\Delta x}{2}$. Já $h = \frac{\Delta x}{4}$, logo $p = 2h$, então:

$$I_e = 2h\left(2f(\bar{x}) + \frac{(2h)^2}{2!} f''(\bar{x}) \frac{2}{3} + \frac{(2h)^4}{4!} f^{(4)}(\bar{x}) \frac{2}{5} + \ldots\right)$$

$$I_e = 4h f(\bar{x}) + \frac{(2h)^3}{2!} f''(\bar{x}) \frac{2}{3} + \frac{(2h)^5}{4!} f^{(4)}(\bar{x}) \frac{2}{5} + \ldots \qquad (\mathrm{III})$$

Fazendo (III) − (I) e nos concentrando no termo dominante:

$$I_e - I_f = \frac{2^6 f^{(4)}(\bar{x}) h^5}{4!\,5} - \frac{16 f^{(4)}(\bar{x}) h^5}{4!\,3} = \frac{2^4 f^{(4)}(\bar{x}) h^5}{4!}\left(\frac{4}{5} - \frac{1}{3}\right)$$

$$= +\frac{14 h^5 f^{(4)}(\bar{x})}{45}$$