Tarefa 02 de Métodos II - Antônio Anderson Costa Pereira - 422029

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FÓRMULA DE NEWTON-COTES PARA O GRAU 4 ABORDAGEM FECHADA

Polinômio de grau 4:

$$g(s) = \binom{s}{0}\Delta^0 r_0 + \binom{s}{1}\Delta^1 r_0 + \binom{s}{2}\Delta^2 r_0 + \binom{s}{3}\Delta^3 r_0 + \binom{s}{4}\Delta^4 r_0$$

Realizando as contas para cada termo temos:

$$g(s) = (\frac{s^4}{24} - \frac{10s^3}{24} + \frac{35s^2}{24} - \frac{25s}{12} + 1)r_0 + (\frac{-s^4}{6} + \frac{3s^3}{2} - \frac{26s^2}{6} + 4s)r_1$$
$$+(\frac{s^4}{4} - 2s^3 + \frac{19s^2}{4} - 3s)r_2 + (\frac{-s^4}{6} + \frac{7s^3}{6} - \frac{14s^2}{6} + \frac{8s}{6})r_3 + (\frac{s^4}{24} - \frac{6s^3}{24} + \frac{11s^2}{24} - \frac{6s}{24})r_4$$

Onde:

$$f(x_i) = r_0$$
$$f(x_i + h) = r_1$$
$$f(x_i + 2h) = r_2$$
$$f(x_i + 3h) = r_3$$
$$f(x_i + 4h) = r_4$$

Com $h = \frac{\Delta x}{4}$ e integrando $g(s)$ temos:

$$\int_0^4 g(s)\ ds = \int_0^4 h((\frac{s^4}{24} - \frac{10s^3}{24} + \frac{35s^2}{24} - \frac{25s}{12} + 1)r_0 + (\frac{-s^4}{6} + \frac{3s^3}{2} - \frac{26s^2}{6} + 4s)r_1$$
$$+(\frac{s^4}{4} - 2s^3 + \frac{19s^2}{4} - 3s)r_2 + (\frac{-s^4}{6} + \frac{7s^3}{6} - \frac{14s^2}{6} + \frac{8s}{6})r_3 + (\frac{s^4}{24} - \frac{6s^3}{24} + \frac{11s^2}{24} - \frac{6s}{24})r_4)ds$$

$$= h((\frac{s^5}{120} - \frac{10s^4}{96} + \frac{35s^3}{72} - \frac{25s^2}{24} + s)r_0 + (-\frac{s^5}{30} + \frac{3s^4}{8} - \frac{13s^3}{9} + 2s^2)r_1 + (\frac{s^5}{20} - \frac{2s^4}{4} + \frac{19s^3}{12} - \frac{3s^2}{2})r_2$$
$$+(\frac{-s^5}{30} + \frac{7s^4}{24} - \frac{14s^3}{18} + \frac{4s^2}{6})r_3 + (\frac{s^5}{120} - \frac{6s^4}{96} + \frac{11s^3}{72} - \frac{6s^2}{48})r_4)|_0^4$$

$$p(x(s)) = \int_0^4 g(s)\ ds = \frac{2h}{45}(7r_0 + 32r_1 + 12r_2 + 32r_3 + 7r_4)$$

Substituindo os $r_i's$ pelos $f(x)'s$ temos:

$$\boxed{\int_{x_i}^{x_f} f(x)\ dx \approx \frac{2h}{45}(7f(x_i) + 32f(x_i+h) + 12f(x_i+2h) + 32f(x_i+3h) + 7f(x_i+4h))}$$

FÓRMULA DE NEWTON-COTES PARA O GRAU 4 ABORDAGEM ABERTA

Polinômio de grau 4:

$$g(s) = \binom{s}{0}\Delta^0 r_0 + \binom{s}{1}\Delta^1 r_0 + \binom{s}{2}\Delta^2 r_0 + \binom{s}{3}\Delta^3 r_0 + \binom{s}{4}\Delta^4 r_0$$

$$g(s) = (\frac{s^4}{24} - \frac{10s^3}{24} + \frac{35s^2}{24} - \frac{25s}{12} + 1)r_0 + (\frac{-s^4}{6} + \frac{3s^3}{2} - \frac{26s^2}{6} + 4s)r_1$$
$$+(\frac{s^4}{4} - 2s^3 + \frac{19s^2}{4} - 3s)r_2 + (\frac{-s^4}{6} + \frac{7s^3}{6} - \frac{14s^2}{6} + \frac{8s}{6})r_3 + (\frac{s^4}{24} - \frac{6s^3}{24} + \frac{11s^2}{24} - \frac{6s}{24})r_4$$

Onde:

$$f(x_i + h) = r_0$$
$$f(x_i + 2h) = r_1$$
$$f(x_i + 3h) = r_2$$
$$f(x_i + 4h) = r_3$$
$$f(x_i + 5h) = r_4$$

Com $h = \frac{\Delta x}{6}$ e integrando $g(s)$ temos:

$$\int_{-1}^{5} g(s)\ ds = \int_{-1}^{5} h((\frac{s^4}{24} - \frac{10s^3}{24} + \frac{35s^2}{24} - \frac{25s}{12} + 1)r_0 + (\frac{-s^4}{6} + \frac{3s^3}{2} - \frac{26s^2}{6} + 4s)r_1$$
$$+(\frac{s^4}{4} - 2s^3 + \frac{19s^2}{4} - 3s)r_2 + (\frac{-s^4}{6} + \frac{7s^3}{6} - \frac{14s^2}{6} + \frac{8s}{6})r_3 + (\frac{s^4}{24} - \frac{6s^3}{24} + \frac{11s^2}{24} - \frac{6s}{24})r_4)ds$$

$$= h((\frac{s^5}{120} - \frac{10s^4}{96} + \frac{35s^3}{72} - \frac{25s^2}{24} + s)r_0 + (-\frac{s^5}{30} + \frac{3s^4}{8} - \frac{13s^3}{9} + 2s^2)r_1 + (\frac{s^5}{20} - \frac{2s^4}{4} + \frac{19s^3}{12} - \frac{3s^2}{2})r_2$$
$$+(\frac{-s^5}{30} + \frac{7s^4}{24} - \frac{14s^3}{18} + \frac{4s^2}{6})r_3 + (\frac{s^5}{120} - \frac{6s^4}{96} + \frac{11s^3}{72} - \frac{6s^2}{48})r_4)|_{-1}^{5}$$

$$p(x(s)) = \int_{-1}^{5} g(s)\ ds = \frac{3h}{10}(11r_0 - 14r_1 + 26r_2 - 14r_3 + 11r_4)$$

Substituindo os $r_i's$ pelos $f(x)'s$ temos:

$$\boxed{\int_{x_i}^{x_f} f(x)\ dx \approx \frac{3h}{10}(11f(x_i+h) - 14f(x_i+2h) + 26f(x_i+3h) - 14f(x_i+4h) + 11f(x_i+5h))}$$