Nome: Antônio Anderson Costa Pereira.
Matrícula: 422029.

FÓRMULA DE NEWTON-COTES PARA O GRAU 4 ABORDAGEM FECHADA.

Polinômio de grau 4:

$$g(s) = \binom{s}{0}\Delta^0 r_0 + \binom{s}{1}\Delta^1 r_0 + \binom{s}{2}\Delta^2 r_0 + \binom{s}{3}\Delta^3 r_0 + \binom{s}{4}\Delta^4 r_0$$

Realizando as contas para cada termo temos:

$$g(s) = \left(\frac{s^4}{24} - \frac{10s^3}{24} + \frac{35s^2}{24} - \frac{25s}{12} + 1\right) r_0 + \left(-\frac{s^4}{6} + \frac{3s^3}{2} - \frac{26s^2}{6} + 4s\right) r_1$$
$$+ \left(\frac{s^4}{4} - 2s^3 + \frac{19s^2}{4} - 3s\right) r_2 + \left(-\frac{s^4}{6} + \frac{7s^3}{6} - \frac{14s^2}{6} + \frac{8s}{6}\right) r_3$$
$$+ \left(\frac{s^4}{24} - \frac{6s^3}{24} + \frac{11s^3}{24} - \frac{6s}{24}\right) r_4 \qquad (*)$$

Onde $f(x_i) = r_0$

$f(x_i + h) = r_1$

$f(x_i + 2h) = r_2$

$f(x_i + 3h) = r_3$

$f(x_i + 4h) = r_4$

com $h = \frac{\Delta x}{4}$ e integrando $g(s)$ temos:

$$\int_0^4 g(s)\,ds = \int_0^4 h\Bigg(\left(\frac{s^4}{24} - \frac{10s^3}{24} + \frac{35s^2}{24} - \frac{25s}{12} + 1\right) r_0$$

$$+ \left(-\frac{s^4}{6} + \frac{3s^3}{2} - \frac{26s^2}{6} + 4s\right) r_1 + \left(\frac{s^4}{4} - 2s^3 + \frac{19s^2}{4} - 3s\right) r_2$$

$$+ \left(-\frac{s^4}{6} + \frac{7s^3}{6} - \frac{14s^2}{6} + \frac{8s}{6}\right) + \left(\frac{s^4}{24} - \frac{6s^3}{24} + \frac{11s^2}{24} - \frac{6s}{24}\right) r_4\Bigg) ds$$

$$= h\Bigg(\left(\frac{s^5}{120} - \frac{10s^4}{96} + \frac{35s^3}{72} - \frac{25s^2}{24} + s\right) r_0 + \left(-\frac{s^5}{30} + \frac{3s^4}{8} - \frac{13s^3}{9} + 2s^2\right) r_1$$

$$+ \left(\frac{s^5}{20} - \frac{2s^4}{4} + \frac{19s^3}{12} - \frac{3s^2}{12}\right) r_2 + \left(-\frac{s^5}{30} + \frac{7s^4}{24} - \frac{14s^3}{18} + \frac{4s^2}{6}\right) r_3$$

$$+ \left(\frac{s^5}{120} - \frac{6s^4}{96} + \frac{11s^3}{72} - \frac{6s^2}{48}\right) r_4\Bigg)\Bigg|_0^4 \qquad (**)$$

$$p(x(s)) = \int_0^4 g(s)\,ds = \frac{2h}{45}\left(7r_0 + 32r_1 + 12r_2 + 32r_3 + 7r_4\right)$$

substituindo os r_i's pelos $f(x_i)$'s temos

$$\int_{x_i}^{x_f} f(x)\,dx \approx \frac{2h}{45}\Big(7f(x_i) + 32f(x_i+h) + 12f(x_i+2h) + 32f(x_i+3h) + 7f(x_i+4h)\Big)$$

FÓRMULA DE NEWTON-COTES PARA O GRAU 4
ABORDAGEM ABERTA

Polinômio de grau 4 é o mesmo de (*).

Agora temos:

$$f(x_i+h)=r_0$$
$$f(x_i+2h)=r_1$$
$$f(x_i+3h)=r_2$$
$$f(x_i+4h)=r_3$$
$$f(x_i+5h)=r_4$$

Com $h=\frac{\Delta x}{6}$ e integrando $g(s)$ temos:

$$\int_{-1}^{5} g(s) = h\left((**)\right)\Big|_{-1}^{5}$$

$$= \frac{3h}{10}\left(11r_0 - 14r_1 + 26r_2 - 14r_3 + 11r_4\right)$$

substituindo os r_i's pelos $f(x_i)$'s temos.

$$\int_{x_i}^{x_f} f(x)\,dx \approx \frac{3h}{10}\Big(11f(x_i+h) - 14f(x_i+2h) + 26f(x_i+3h) - 14f(x_i+4h) + 11f(x_i+5h)\Big)$$