

I. Introduction

Greetings Congressional committee members. I bring this report to you today in order to show you the knowledge I possess in our progress towards the moon. I will demonstrate to you that I have the knowledge necessary to calculate and visualize the factors affecting a vessel that I plan to land on the moon using python.

II. The gravitational potential of the Earth-Moon system

Firstly I needed a way to understand the potential energy that would be acting on the shuttle from the Earth and the moon. The potential energy of the Earth and moon acting on the shuttle would be important in figuring out how much kinetic energy we must generate to get to the moon. In order to understand this I coded a function that takes the sum of the gravitational potential energy from the moon and the Earth. The function first takes a point as an x and y coordinate and uses the pythagorean theorem in order to find the distance from the point to the Earth and the moon. This is done using np.sqrt which takes the root of what is inside of it. Then, because these distances are used to solve for the gravitational potential energy, I use np.where in order to avoid using points that would cause division by 0. This function checks if the distance equals 0 and returns a "NaN" or "Not a Number" if it is. This is done to avoid multiplying by 0 which would not work in python causing the code to fail. After making sure I don't pass any 0s through the distances are then used as the respective radii in the functions for gravitational potential energy which are then summed. I then took this function and graphed it on a grid lying between negative one and a half times the distance to the moon and positive one and a half times the distance to the moon. This was done by creating two variables x and y which are both np.linspace which effectively act as number lines. These two number lines together create an effective grid for our function to lay on and be graphed over. Then I created Figure A that plots the absolute value of the function on this grid. I chose to graph the absolute value because it provides an easier understanding of the energy we're acting against. This graph also used a logarithmic scale which goes between "vmin" and "vmax" two values that are defined as the minimum and maximum of the equation avoiding negative values and non real numbers in order to avoid taking the logarithm of something that doesn't exist which would break the code. A colorbar is then added following the same log scale which shows what color represents the level of gravitational potential energy. The axes and title are then labeled accordingly with what they represent. After this I created Figure B which is a contour plot. This is exceptionally useful because it effectively creates a map of gravitational potential energy created by the Earth-Moon system within certain areas of space.

III. The Gravitational Force of the Earth-Moon System

Also necessary to landing a shuttle on the moon would be understanding the forces acting on the shuttle at any point in space. In order to do this we first create a function that takes a mass for the shuttle and an x and y coordinate for the shuttle at any point in space. Working backwards the function returns the force in the x direction and the force in the y direction which

is found by summing the force produced by the moon and Earth in the x and y directions. The force in the x and y directions from the earth and moon are both found using the same process. First the distance between the point of the shuttle and the respective body is found by using `np.sqrt` which takes the square root necessary to solve for distance using the pythagorean theorem. The distances are then filtered using `np.where` which checks to make sure the distance is not 0 in which case it returns a "NaN" or "Not a Number" in order to avoid dividing by zero later in the function which would break the code. The distance is then used to find directional unit vectors for the x and y directions of the force. This is done by dividing the distance in the x or y direction by the magnitude of the direction. The magnitude of the gravitational force is then solved for and multiplied by the directional unit vector in order to get the gravitational force of the Earth or moon in the x and y directions. It is then as previously mentioned summed and returned by the function. After defining the function I then plotted it as a stream plot by creating Figure C. I first defined X and Y as two line spaces that act as number lines ranging between negative one and half times the distance to the moon and positive one and a half times the distance to the moon. X and Y are then used to create a meshgrid which the function will compute over and lay on when graphed. I then created a streamplot which will be best showing the direction and magnitude of the force of gravity acting on the shuttle at all points. The streamplot uses a color scale with the magnitude of the force as calculated using `np.hypot` and the force in the x and y direction. This means that the arrows on the lines of the stream plot show the direction of the forces and the color shows the magnitude of the force as labeled by the color bar on the left side of Figure C.

IV. Projected performance of the Saturn V Stage 1

We can also demonstrate the expected performance of the shuttle in order to demonstrate that we accurately predict the motion of the shuttle in flight. Here I use python to solve for how long time it takes for the shuttle to burn through all of its fuel as well as the altitude when the rocket does run out of fuel. This is done first by inputting the mass of the rocket when filled and empty as well as the rate at which fuel burns out of the rocket. Using these values we can use python to compute the burn time that will be required for the rocket to burn through all its fuel. After that we can use inputted values for the exhaust velocity of the rocket as well as the gravity in order to create a function that computes Tsiolkovsky's rocket equation in order to solve for the velocity of the rocket at any given time. You may notice in this equation that `np.log` is used which is simply a function that takes the natural log of whatever terms are inside of it. After returning the result we then calculate the position of the rocket at a certain time by taking the integral of the velocity using the "quad" function which simply takes the integral of the function inside of its argument.

V. Discussion and Future Work

Although all of the data and calculations shown are very useful and accurate, there are a few approximations that could be done better in the future to ensure better accuracy and results. Firstly, all equations accounting for the Earth and the moon treat them as point masses. This does not account for the size of the Earth and the moon and how the rocket would move when in the atmosphere but not yet off of the planet fully. Secondly, The calculations made to estimate the total burn time of the rocket and the altitude at burnout of the rocket neglect the impact of

drag force on the rocket. This leads to incorrect predictions of the values. The reported total burn time and altitude when tested were 160 seconds and 70 kilometers respectively. The numbers that we found when calculating were 157 seconds and 74 kilometers. This difference shows that our calculations for distance were overestimated and time was underestimated. This makes sense because with the presence of drag force the rocket would not move as far upward because it has another force acting downwards on it. Although the calculations were very close it could be done better to ensure higher accuracy in the future.

Figure A

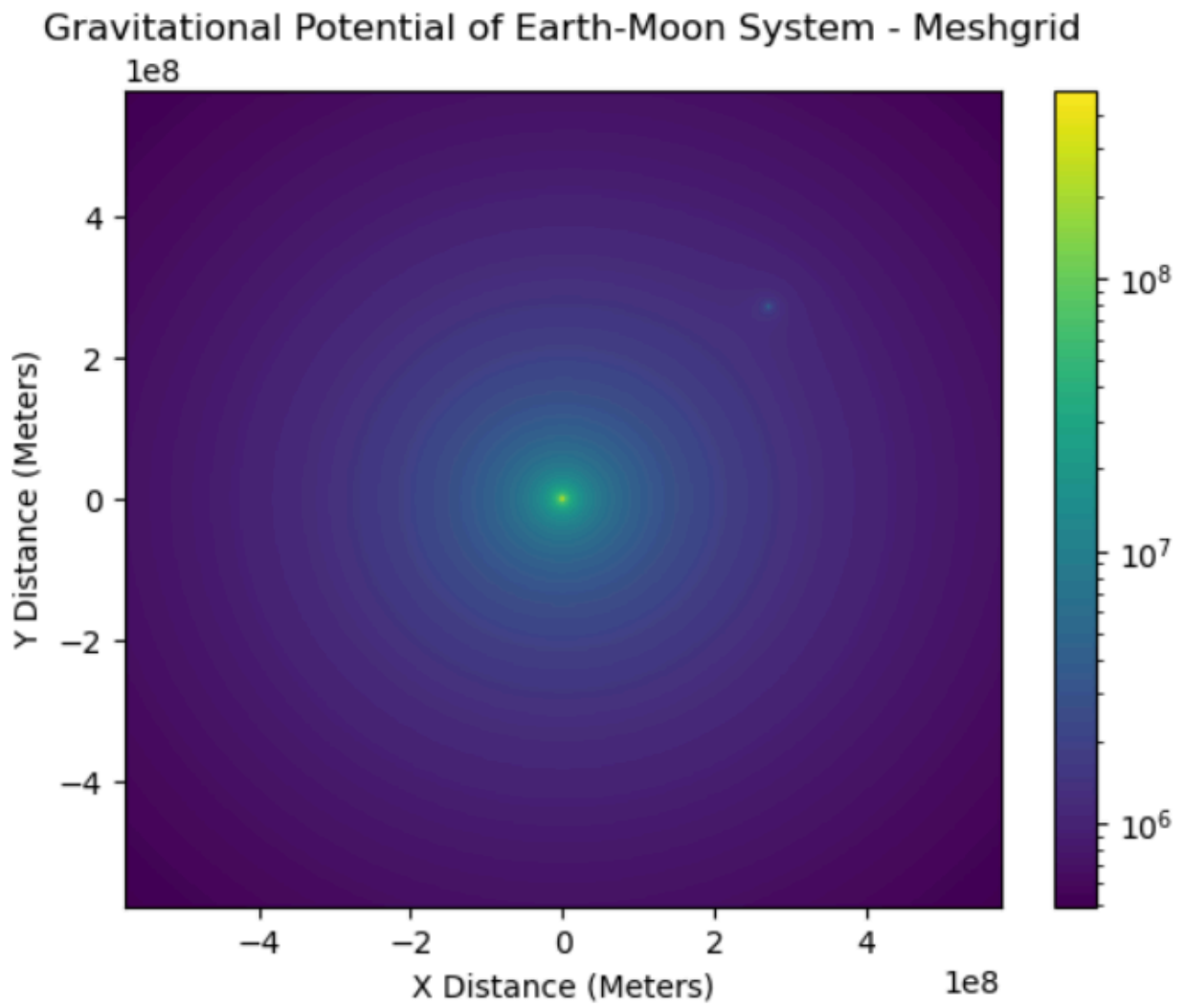


Figure B

Gravitational Potential of Earth-Moon System - Contour Plot

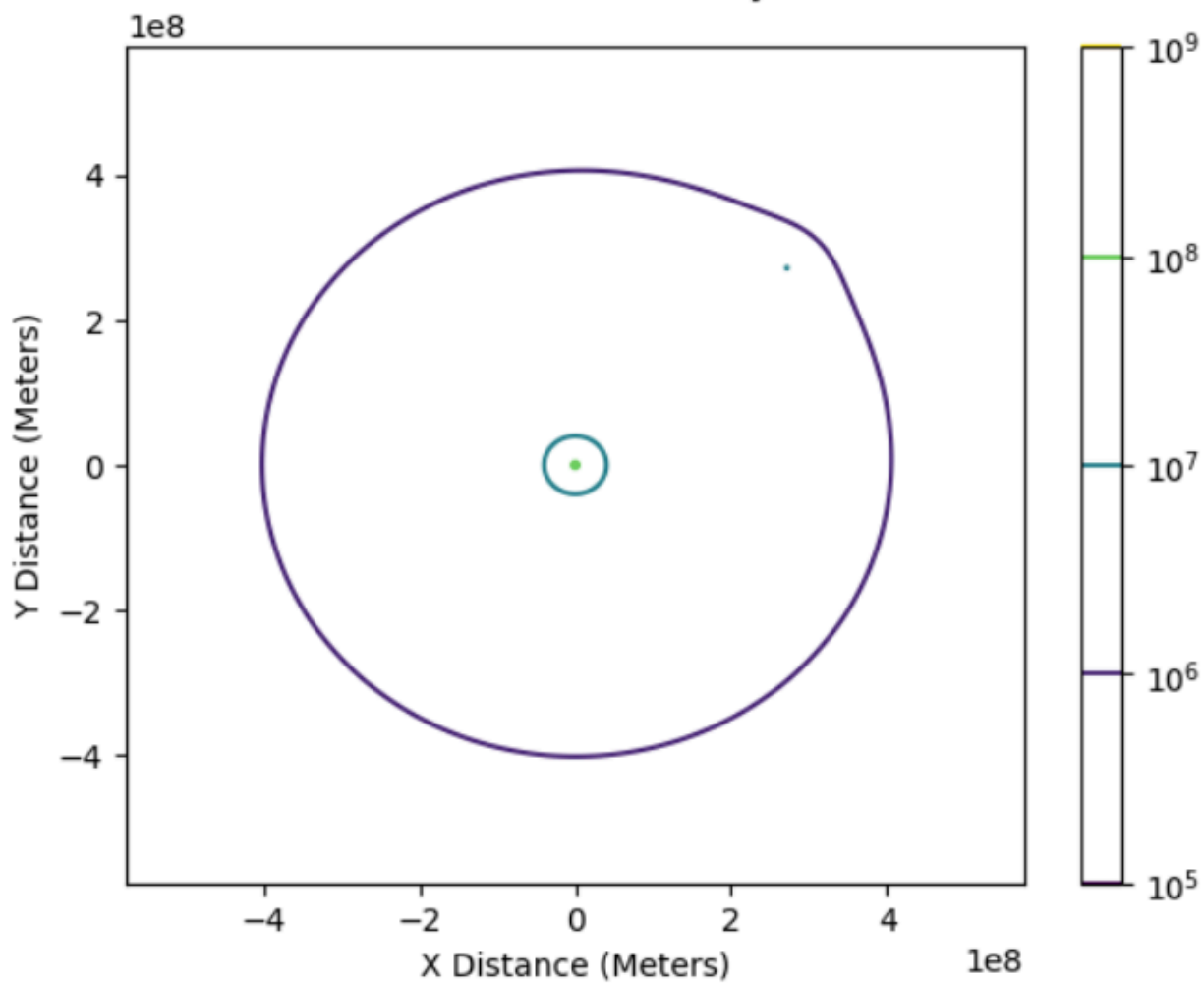


Figure C

Gravitational Force of Earth-Moon system on Apollo 11

