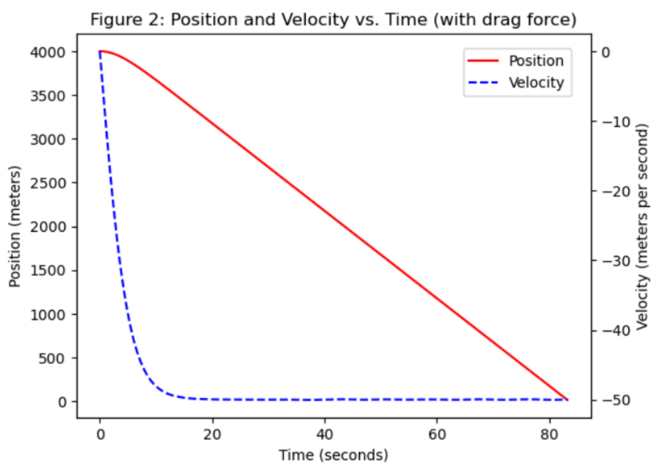
Report on Gravitational Fall Times in Terrestrial and Lunar Mines

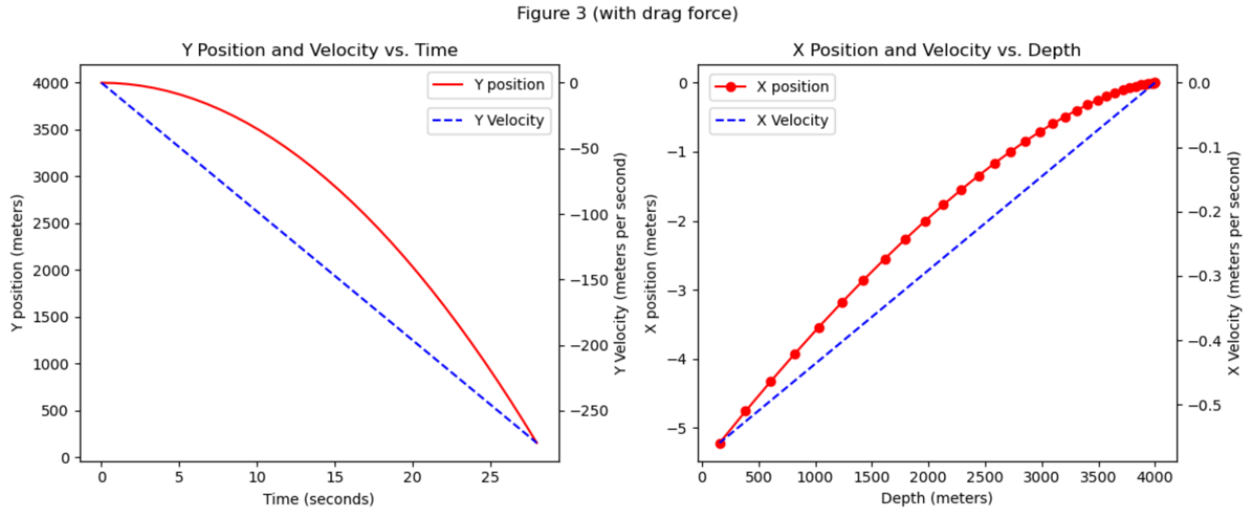
Author: Anthony Angarita
Date: 8 April 2025

In this project I am assigned the task of assessing the feasibility of measuring the vertical depth of a mine shaft by dropping a 1 kg mass and measuring the time to hit the bottom. I have created a notebook that estimates the fall times of the mass under various physical situations. First, the situation with no drag or varying gravity, second, the situation where drag and varying gravity are included, and finally a situation including the varying density of the Earth's mass. Most of the estimations are done using solve_ivp, which solves a set of differential equations for position, and then plotting the results to visualize the motion.

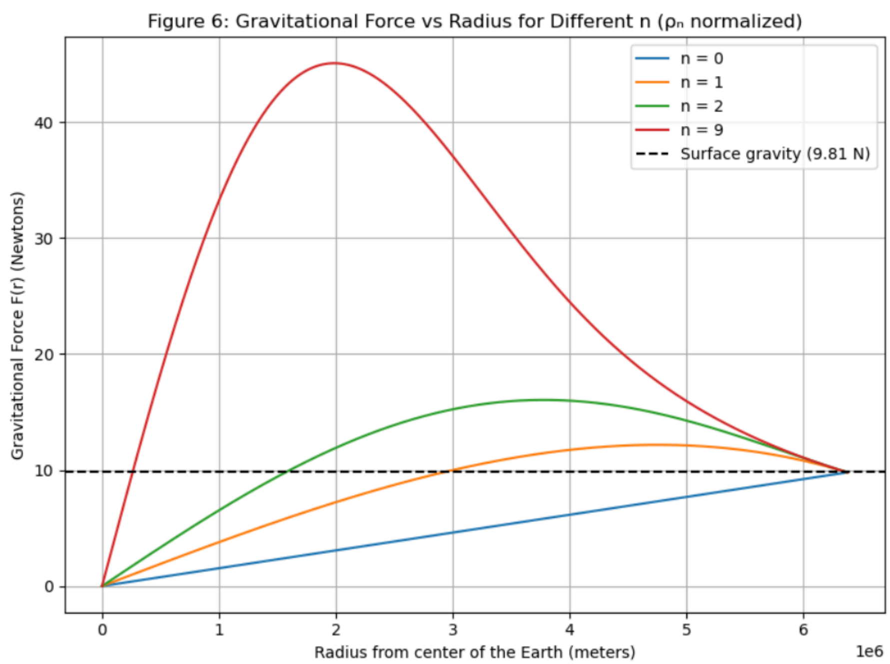
The first fall time where there was no drag and constant gravity was solved for two ways. Once where the kinematic equation for position was reorganized for time and and once where a set of differential equations were solved for position and then checked for the time when the object hit the bottom. Both of these processes found that the time for impact was 28.6 seconds. The next fall time was the situation where we assumed the Earth to be of homogeneously distributed mass. Because of this we could use an equation where the gravitational constant was proportional to how close the object was to the Earth's radius. This resulted in a time of 28.6 seconds which is the same as the first value. The next fall time that was calculated included the effects of drag force on the object. In order to solve for alpha in the given equation velocity was set to 50 meters per second as this was the given terminal velocity. Then the forces were balanced for when this velocity was achieved. This gave an alpha value of 0.003924 assuming that gravity is 9.81 meters per second squared. The time found for this situation was 83.5 meters per second. This adds a significant amount of time because it caps the speed of the falling object and forces it to fall at terminal velocity. It can be seen from the three times that the more forces taken into consideration seem to have an increased effect on the time it takes the object to reach the bottom of the shaft. The drag force had the greatest effect on the time of 54.9 seconds longer than the ideal situation.



Because of the Earth's rotation, the object being dropped would also experience a Coriolis force. This means that now we have an additional force acting in the y-direction as well as in the x-direction of which we must not track the motion. The transverse velocity of the object increases as the y-velocity increases. For a shaft with a width of 5 meters the object would not make it all the way to the bottom before it hit the wall. This is because if dropped from the center, the object would only have 2.5 meters of transversal movement before it hit a wall. The object moves 2.5 meters horizontally after 21.9 seconds and doesn't hit the ground until after 28.6 seconds. After factoring in drag it can be seen that the x displacement would be 23 meters if it was able to move that far. Because of this it would not be very feasible to attempt to measure the vertical depth by dropping a test object. For future measurements I would recommend finding a different measurement method



Next we consider that the density of the earth is not uniform and changes as we move inward. This means that a different amount of mass is acting on the object at different distances. In the given equation for density of the Earth as a function of radius there is a variable "n". This variable is representative of how core-heavy versus uniformly distributed the mass of the Earth is. In order to keep the model for density valid the other variable ρ_n is solved for in a way that makes sure that no matter what value of n is used that the constant value for acceleration of surface gravity is always 9.81 meters per second squared. In the case where n = 0, the object crosses the center of the Earth after 1267.3 seconds, at a speed of 7905.3 meters per second. In the case where n = 9, the object crosses the center of the Earth after 943.8 seconds, as a speed of 18370.7 meters per second This shows that when the mass of the Earth is assumed to be concentrated at the core of the Earth the object passes the center of the Earth 323.5 seconds sooner than the uniform density Earth at a speed 10465.4 meters per second faster than the uniform density Earth. Treating the moon as having constant density we can compute the time for an object to fall from the surface to its core in 1624.5 seconds. It was found that the ratio of the Moon's density to the Earth's density was 1 to 0.6. This is interesting because the ratio of the time it takes for the object to reach the center of the moon and the time it takes the object to reach the center of the Earth is 1 to 1.3 which is almost exactly doubled. The orbital period of the object around the Earth is computed to be 5069.4 seconds which is about 4 times the time that was calculated to reach the center of the Earth which was 1266.6 seconds.



In these findings a few notable things can be said. Firstly the assumptions that were made throughout the notebook. For the drag force it was assumed that gravity could be approximated to 9.81 meters per second squared. It was also assumed from the instructions that the terminal velocity was 50 meters per second. One notable assumption that was made was also that the Earth is perfectly spherical. These three things would have varying effects on our computations though I believe the assumption of a perfectly spherical Earth is the most notable. Secondly our results show that because of the Coriolis effects it will be more difficult to measure the depth of the shaft as it may, depending on the width and depth of the mineshaft, crash into the walls before it reaches the bottom. The density profile also heavily impacts motion of the object. For future work I think it would be most important that a correct value for “n” is found for the density profile because it had such a significant impact on the motion of the test mass. Also for future work it may be important to take into consideration the change in air pressure and temperature and how that may affect the object as it falls.