Department of Computer Science
Digital Image Processing

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Point-wise Intensity Transformations

The task of this assignment is to transform cardiac magnetic resonance images (MRI) in a way to emphasize areas of interest. To this end, you will make use of intensity transformations based on histogram matching. You will consider two models whose their probability density is obtained by mixing an exponential distribution p_e with Gaussian p_G or Rice distributions p_R, where the mixing factor π_0 will be computed from image data.

- Gaussian : $p_1(l) = \pi_0 p_e(l) + (1 - \pi_0) p_G(l), \quad 0 \leq l \leq 255$
- Rice : $p_2(l) = \pi_0 p_e(l) + (1 - \pi_0) p_R(l), \quad 0 \leq l \leq 255$

The following algorithmic steps are requested to be implemented for the point-wise intensity transformations and should applied to all images in the given data set.

1. Computation of:
   (a) Image histogram $\quad \hat{h}(l), \quad 0 \leq l \leq 255$
   (b) Probability density function $\quad \hat{p}(l), \quad 0 \leq l \leq 255$
   (c) Cumulative distribution function $\quad \hat{q}(l) = \sum_{l'=0}^{l'} \hat{p}(l), \quad 0 \leq l \leq 255$

   Provide your own implementation for each of the requested statistics.

   Compare your own implementation of (b) with an implementation that is based on the built-in Matlab function, *imhist()*. Utilize Root Mean Square Error (RMSE) to perform a comparison between the two resulting probability density functions.

2. Estimate the parameter $\pi_0 = \frac{\hat{q}(0)}{p_e(0)}$

3. Computation of the cumulative distribution functions for the two given models:

$$q_1(l) = \sum_{l'=0}^{l} p_1(l'), \quad q_2(l) = \sum_{l'=0}^{l} p_2(l'), \quad 0 \leq l \leq 255$$

4. Find the intensity transformations for the two models:

$$T_1(l) = \underset{q_1}{\operatorname{argmin}} |q_1 - \hat{q}(l)|, \quad T_2(l) = \underset{q_2}{\operatorname{argmin}} |q_2 - \hat{q}(l)|$$

5. Apply the transformations to the image:

$$Y_1 = T_1(X), \quad Y_2 = T_2(X),$$

6. Apply the image histogram equalization transform T.Provide your own implementation of T and qualitatively compare with the output of the built-in Matlab function *histeq*().

7. Create a figure with subplots which visualize the output of transformations T_1, T_2, T.

Image dataset and pre-calculated probability densities of p_G, p_R, p_e are provided in: http://www.csd.uoc.gr/~hy371/assignments/ask_02.rar.

Comment yours result in a brief report.