CS371 Digital Image Processing
Exercise 4 part 2
Discrete Fourier Transform & Image Compression

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December 23, 2022

1 The role of the phase and magnitude

1.1 Part 1 - Altering the magnitude

In this section we are requested to apply the inverse DFT of the image, by maintaining the phase as is and altering the magnitude using the following formula:

$$A(u,v) = \frac{1}{1-\alpha(cos(2\pi u)+cos(2\pi v))}$$

where (u,v) variables take values in the range $[0,1)$ as follows:

$$(u,v) = (\frac{m}{M},\frac{n}{N}), \text{m} = 0,\ldots,\text{M-1 and n} = 0,\ldots,\text{N-1}$$

where (M,N) denote the image dimensions.

We consider three different cases for the magnitude A, corresponding to the values of $\alpha = 0.45, 0.49, 0.495$. We will plot every transformed image and compare them.

First of all, we can see that the new images with their magnitude altered are different when we change the variable α. When we increase α, the Mean Absolute Error between the original image and the altered image is decreasing. For small α values, we can see that the new image has somewhat revealed the edges of the original images. Thus the outline of the image can be shown. Contrary to that, for larger α values, the image feels like it's a bleached out version of the original image, but has the least error. As we can see from the errors, the Barbara image is more affected by altering its magnitude.

All the altered images feels like they are a sketched version of the original one, but a lot of details and information has been lost. It also feels like the images are "burnt", as we increase the alpha.

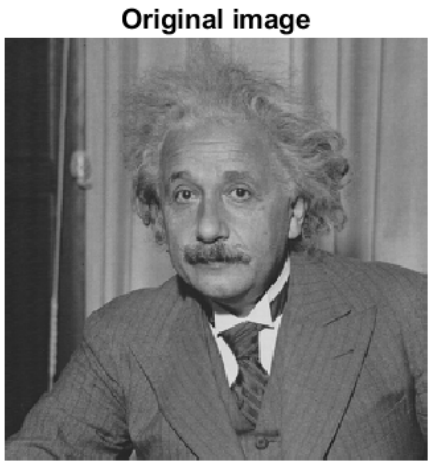

Figure 1: Original Image

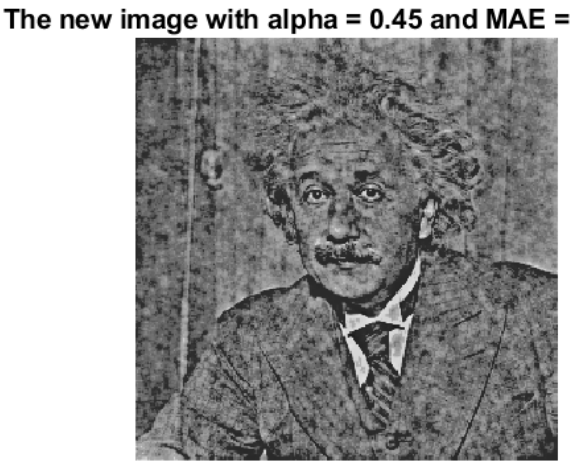

Figure 2: Altered image with alpha = 0.45

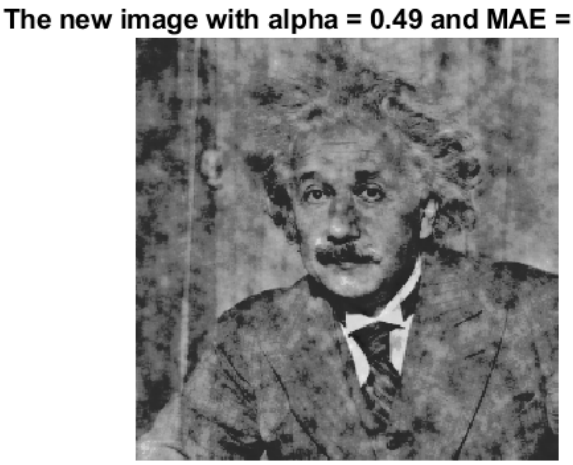

Figure 3: Altered image with alpha = 0.49

Figure 4: Altered image with alpha = 0.495

Figure 5: Original Image

Figure 6: Altered image with alpha = 0.45

Figure 7: Altered image with alpha = 0.49

Figure 8: Altered image with alpha = 0.495

Figure 9: Original Image

Figure 10: Altered image with alpha = 0.45

Figure 11: Altered image with alpha = 0.49

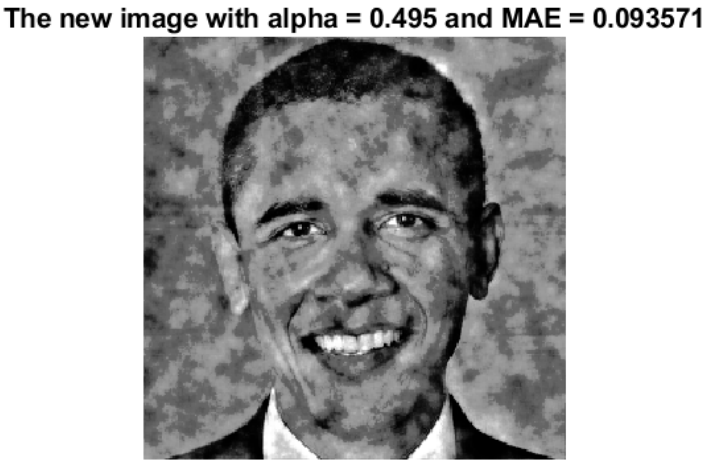

Figure 12: Altered image with alpha = 0.495

2 Part 2 - Phase Quantization

In this section, we are going to compare images that have their phase uniformly quantized with a set of intervals and have their normal magnitude, with images that again have their phase uniformly quantized with the same set, but have their magnitude altered, with the formula of part 1.

First of all, we can see that for each image that is quantized with 5 intervals, the mean absolute error is pretty high, approximately 35%, with and without altering their magnitude. We lose a lot of information and details from the original images, it feels like we have sketched the original images with a very dark pencil. Now, increasing our interval from 5 to 9, the difference is huge. We can now easily understand the image and the person that it depicts, the details are there (not all of them tho), and the mean absolute error have been decreased from 35% to approx. 15%. We can now see that the image with the altered magnitude has less details and worse resolution, it again feels like we have sketched the original image but with a less darker pencil. This "sketchy" feeling applies for all the images with their magnitude being altered, regardless of the intervals and it becomes less and less darker each time we increase those intervals. The most interesting thing here is how the images with their normal magnitude change, as we increase the number of intervals we quantize their phase with. We can see that their quality from 9 to 33 and so on, is very good, with mean absolute error approximately 1%. This means that there is no big differences between this image and the original one. The minimum mean absolute error between the original image and the image with their magnitude altered is 7.5%.

To sum up, by increasing the number of intervals we quantize the phase of the images with, we make the images more and more clear, decreasing their mean absolute error between them and the original image by a significant number. Of course, the most clear images are quantized with 33 and 65 intervals. The images with altered magnitude, have a lot of differences between those without being altered, for every interval, they have that "sketchy" feeling, and it becomes less and less visible as we increase the interval, but it never gets clear. The best we can do is 7.4% error.

We saw that the image with Barbara has the most error, for all of our experiments. This happens because the magnitude of its Fourier Transform is basically a very dark vertical line in the middle of the image, unlike the two other images that have their magnitude spreaded all over the image. To be more specific, if we take a look at A, the formula we use to alter the magnitude of the images, we can see that it mostly affect the magnitudes that are in an rectangular area from the center. This is why, Einstein's error has the least error, and the next is Obama's. The magnitude of their Fourier Transform are at the end of this report.

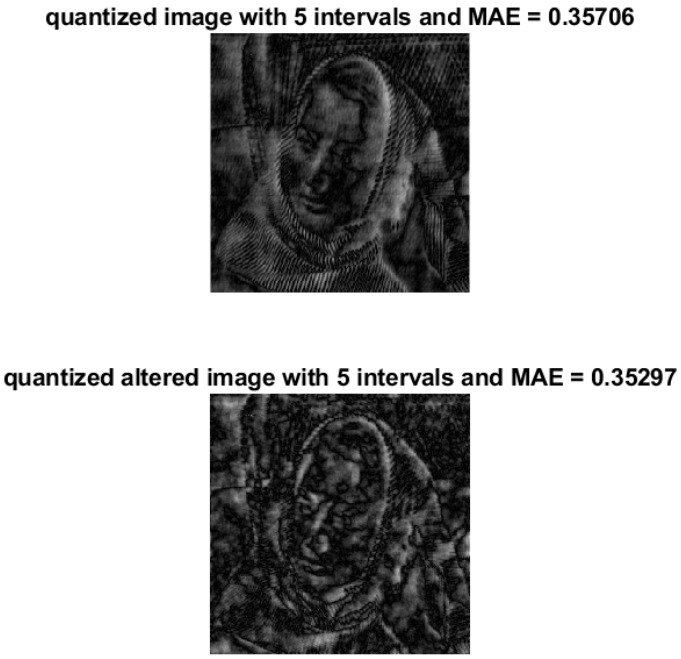

Figure 13: Upper image: Image with uniformly quantized phase with 5 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 5 intervals and altered magnitude.

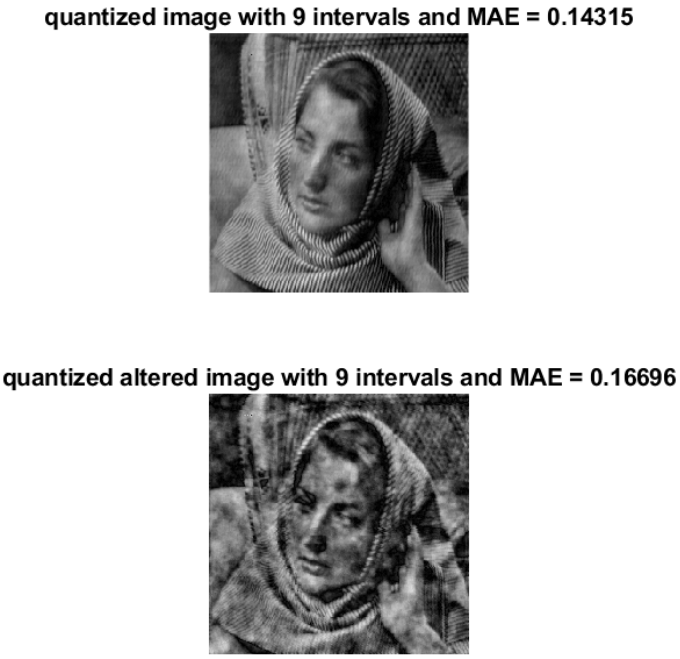

Figure 14: Upper image: Image with uniformly quantized phase with 9 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 9 intervals and altered magnitude.

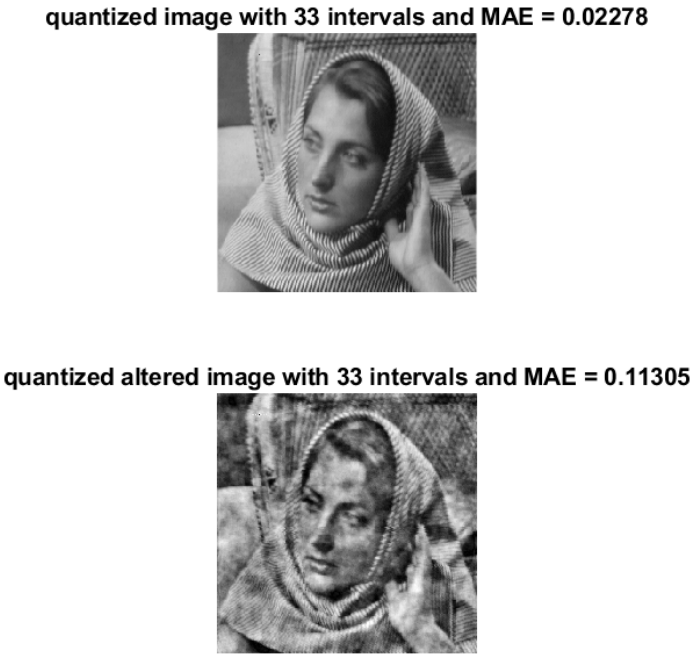

Figure 15: Upper image: Image with uniformly quantized phase with 33 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 33 intervals and altered magnitude.

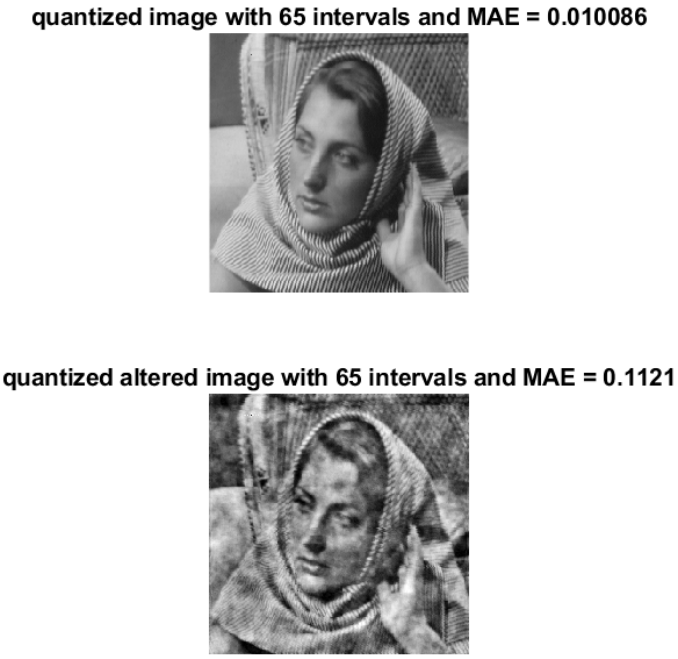

Figure 16: Upper image: Image with uniformly quantized phase with 65 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 65 intervals and altered magnitude.

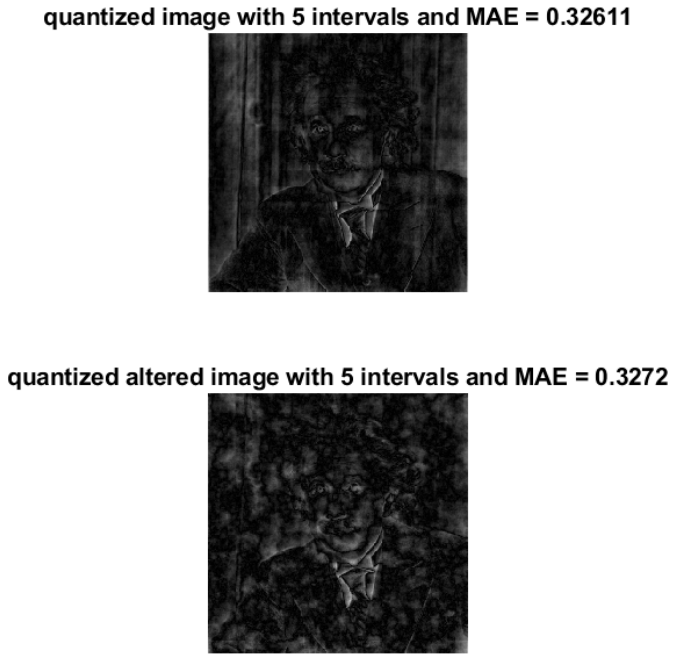

Figure 17: Upper image: Image with uniformly quantized phase with 5 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 5 intervals and altered magnitude.

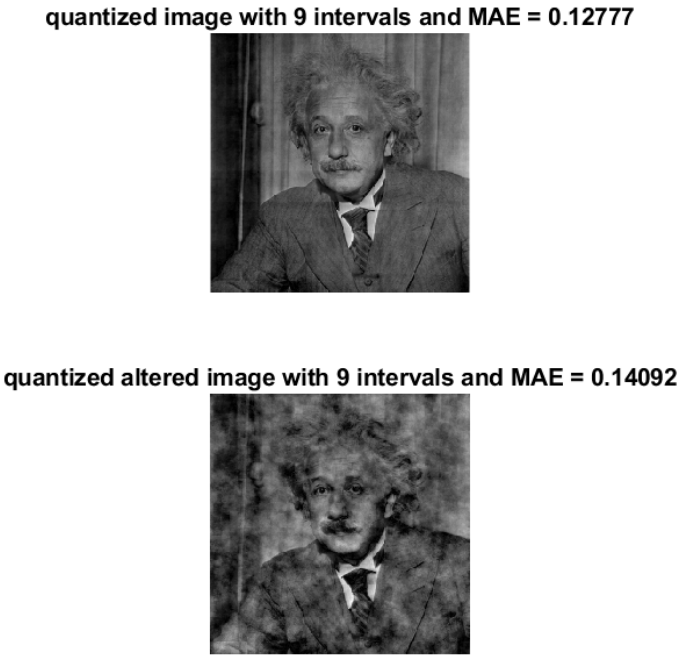

Figure 18: Upper image: Image with uniformly quantized phase with 9 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 9 intervals and altered magnitude.

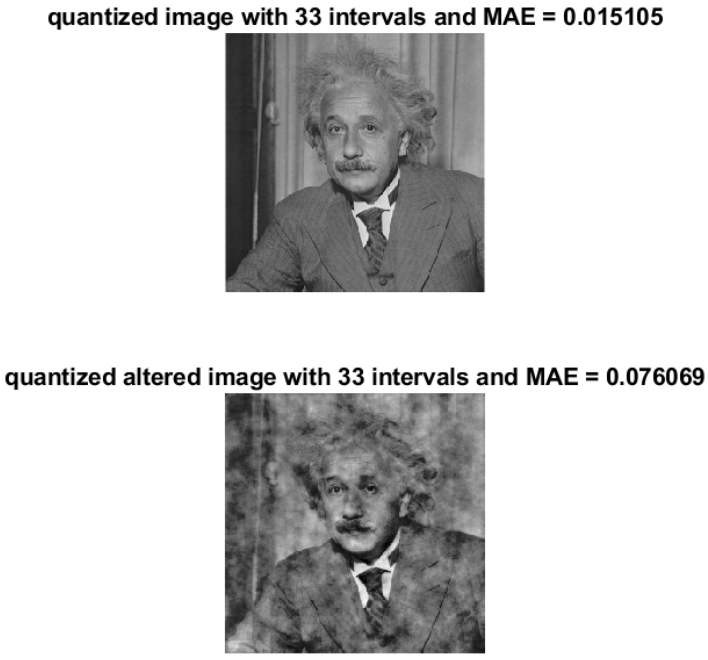

Figure 19: Upper image: Image with uniformly quantized phase with 33 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 33 intervals and altered magnitude.

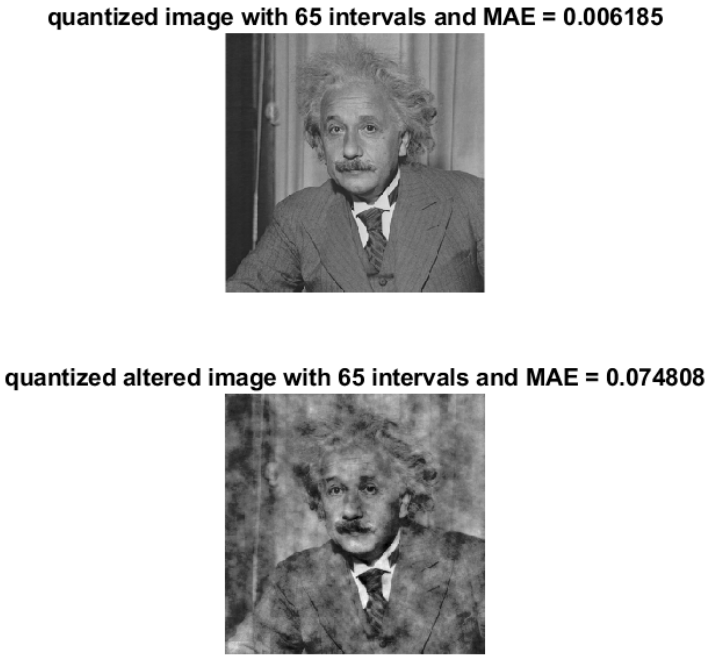

Figure 20: Upper image: Image with uniformly quantized phase with 65 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 65 intervals and altered magnitude.

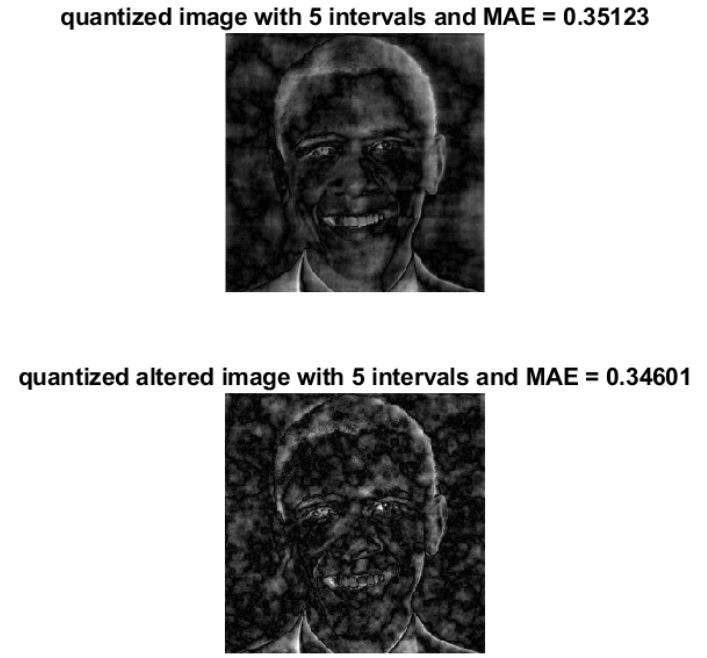

Figure 21: Upper image: Image with uniformly quantized phase with 5 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 5 intervals and altered magnitude.

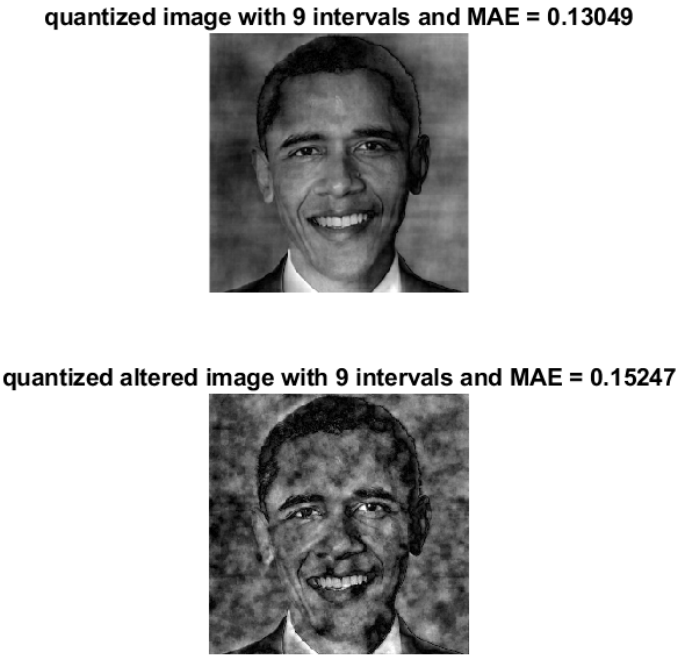

Figure 22: Upper image: Image with uniformly quantized phase with 9 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 9 intervals and altered magnitude.

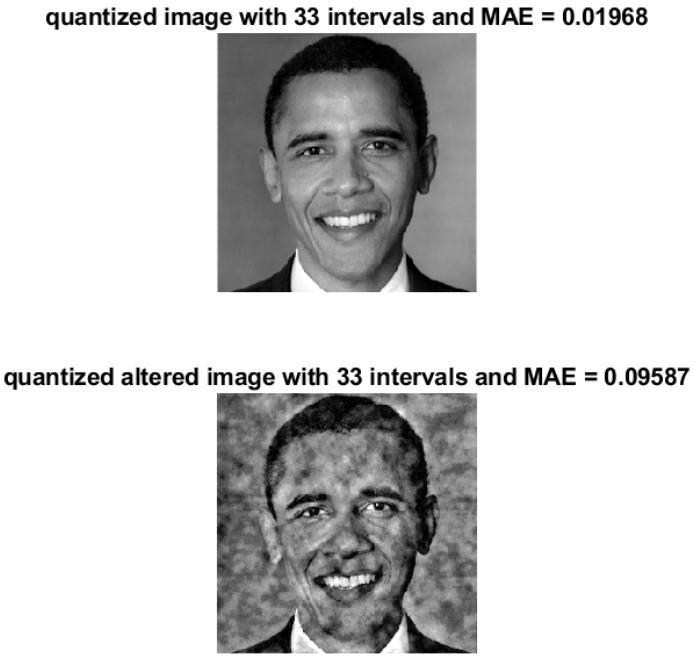

Figure 23: Upper image: Image with uniformly quantized phase with 33 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 33 intervals and altered magnitude.

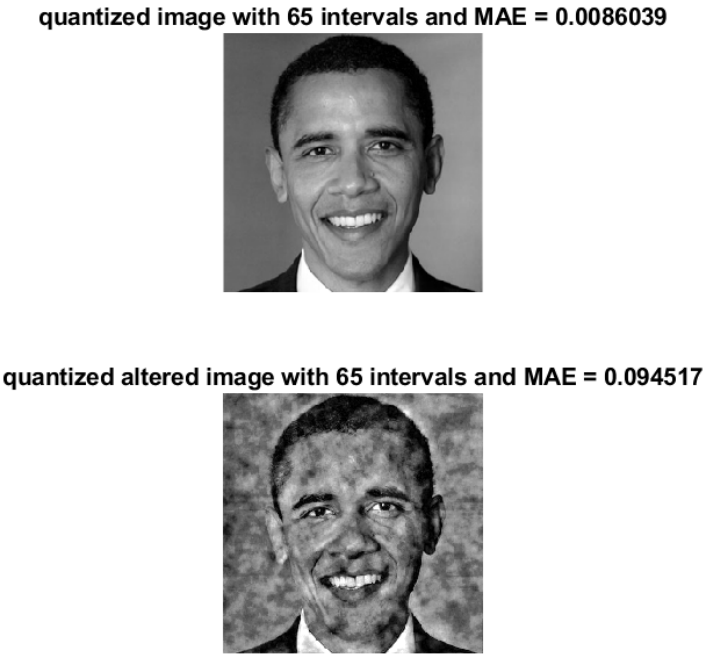

Figure 24: Upper image: Image with uniformly quantized phase with 65 intervals and normal magnitude. Lower image: Image with uniformly quantized phase with 65 intervals and altered magnitude.

3 Image Compression

In this section we are going to compare and comment on how the image changes, when we only consider a number of its spatial frequencies.

First of all, we can see that by considering only the 2.5% of the spatial frequencies, the image has lost a lot of details and information, if we are to say that the original image is FHD(1080p), the new image is 120p. A lot of edges have been also lost from the original one, it's a blurry version of the original image. It feels like we have sub sampled the image, like we did in assignment 1. As for the mean absolute error between the new image and the original one, it's only approximately 3%, with the image of the Barbara having the most error, 4.4%. Now by increasing the percentage of spatial frequencies we consider from 2.5% to 5%, the quality of the image is increasing, and now the true edges can be seen and a lot of details too (like wrinkles, eyes etc.). The details of the background are also lost, not only the details of the person that is depicted. Finally, when we consider only the 7.5% of the spatial frequencies, the images are a lot clearer, both the details of the person and of the background are pretty visible. This is of course the images with the least mean absolute error, with the image of Barbara again having the most error.

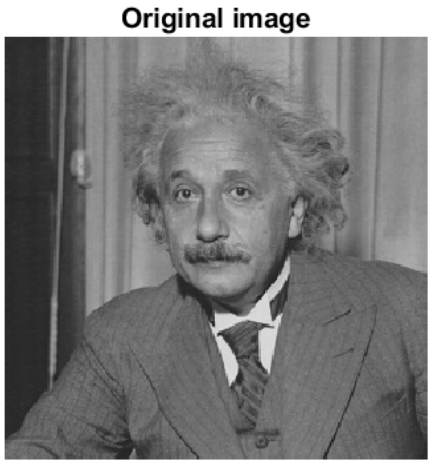

Figure 25: Original Image

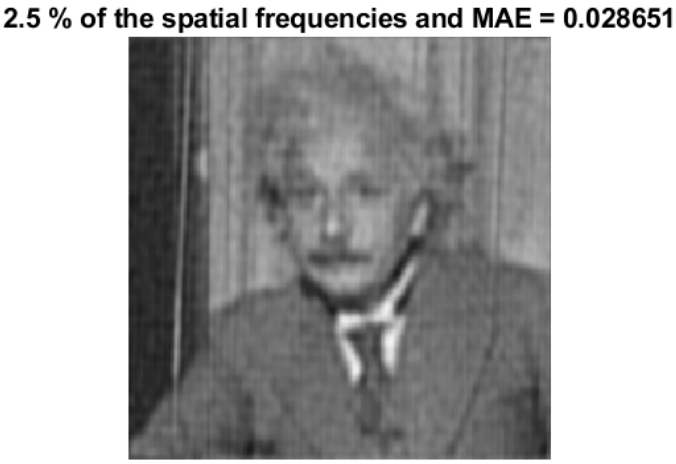

Figure 26: Image with 2.5% of spatial frequencies

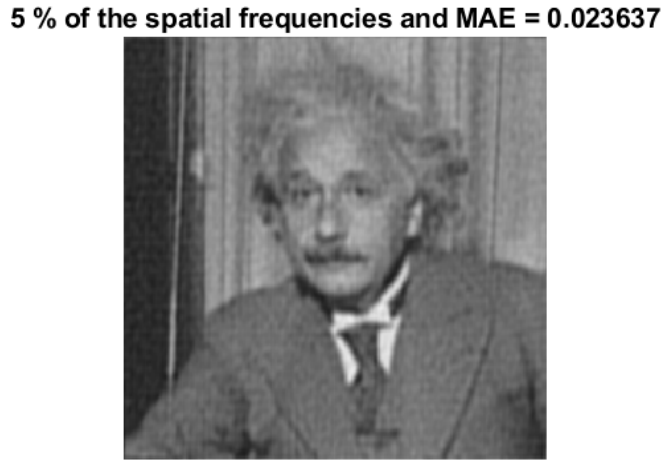

Figure 27: Image with 5% of spatial frequencies

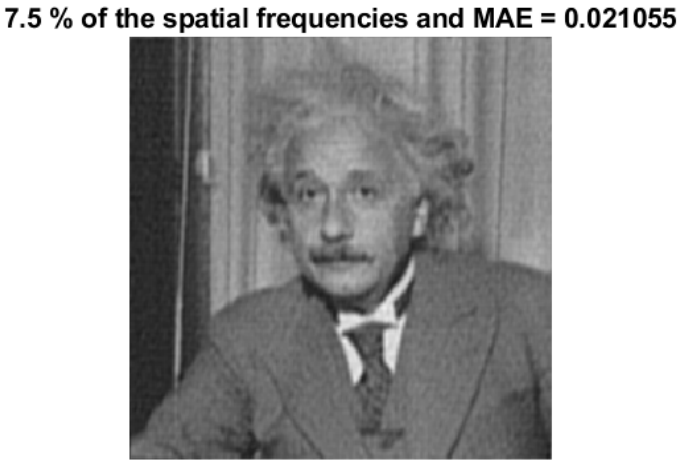

Figure 28: Image with 7.5% of spatial frequencies

Figure 29: Original Image

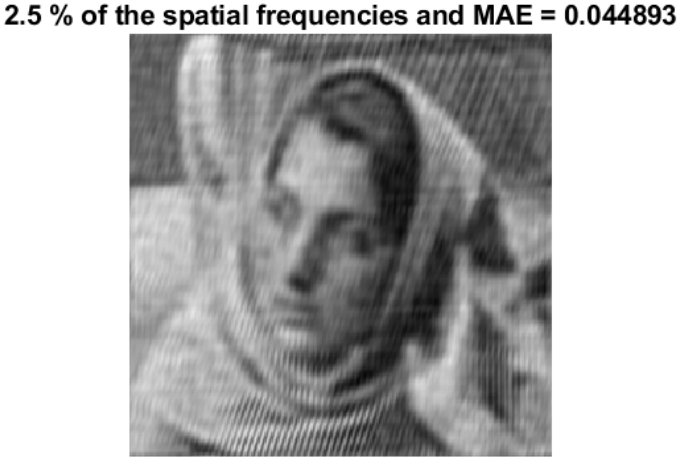

Figure 30: Image with 2.5% of spatial frequencies

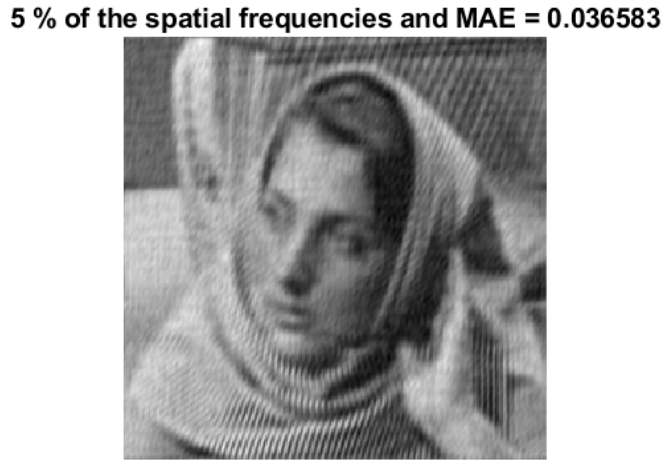

Figure 31: Image with 5% of spatial frequencies

Figure 32: Image with 7.5% of spatial frequencies

Figure 33: Original Image

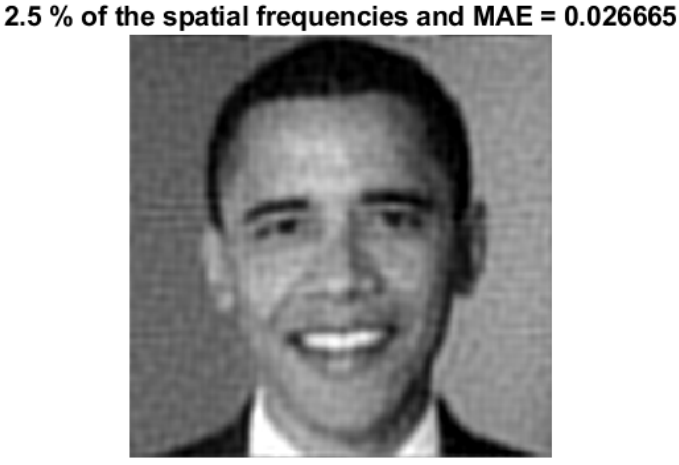

Figure 34: Image with 2.5% of spatial frequencies

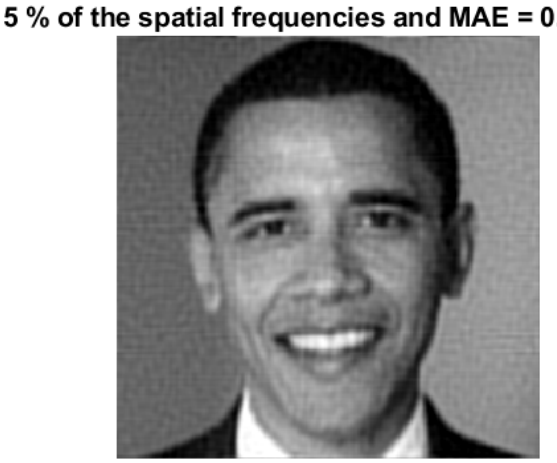

Figure 35: Image with 5% of spatial frequencies

Figure 36: Image with 7.5% of spatial frequencies

4 Plots of Magnitude and the A

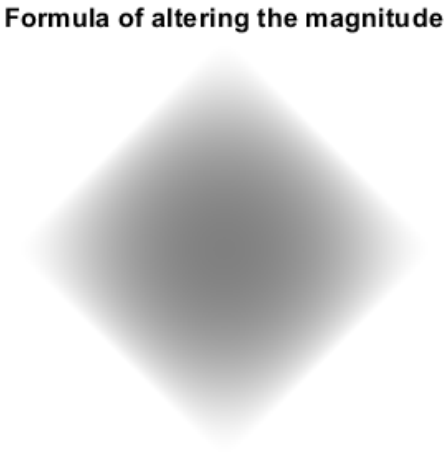

Figure 37: A

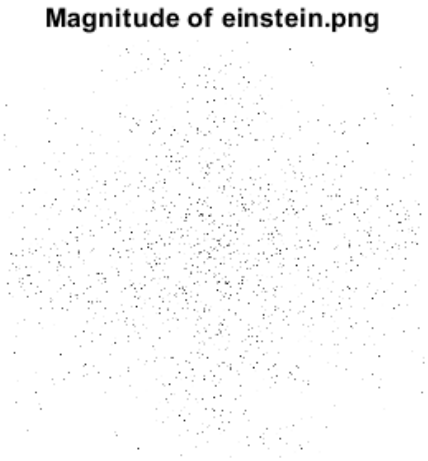

Figure 38: Magnitude of Einstein

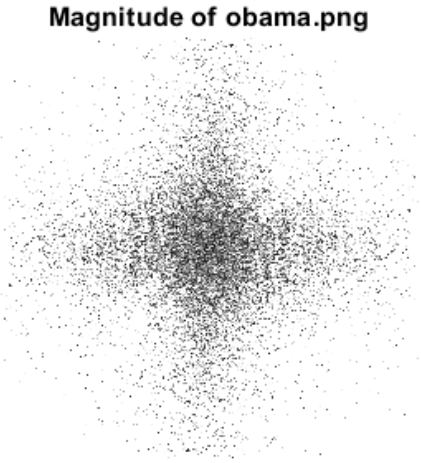

Figure 39: Magnitude of Obama

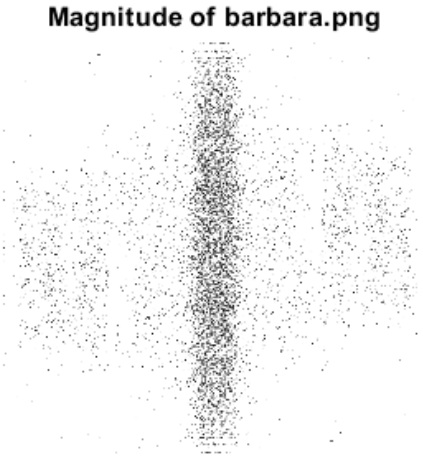

Figure 40: Magnitude of Barbara