ПЗ к вебинару 9.

№1. найти условный экстремум

$U = 3 - 8x + 6y \quad , \quad x^2 + y^2 = 36.$

$\varphi_1 = x^2 + y^2 - 36 = 0$

$L(\lambda_1, x, y) = 3 - 8x + 6y + \lambda_1(x^2 + y^2 - 36)$

$$\begin{cases} L'_x = -8 + 2\lambda_1 \cancel{y} x = 0 \\ L'_y = 6 + 2\lambda_1 y = 0 \\ L'_{\lambda_1} = x^2 + y^2 - 36 = 0 \end{cases} \Rightarrow \begin{cases} x = \frac{8}{2\lambda_1} = 4/\lambda_1 \\ y = \frac{-6}{2\lambda_1} = -3/\lambda_1 \Rightarrow \\ \frac{\cancel{64}^{16}}{\cancel{4}\lambda_1^2} + \frac{\cancel{36}^{9}}{\cancel{4}\lambda_1^2} - 36 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = 8/2\lambda_1 = 4/\lambda_1 \\ y = -6/2\lambda_1 = -3/\lambda_1 \\ \frac{25}{\lambda_1^2} = 36. \end{cases}$$

$$\begin{cases} \lambda_1 = \pm \frac{5}{6} \\ x = \pm \frac{4 \cdot 6}{5} = \pm \frac{24}{5} \\ y = \mp \frac{3 \cdot 6}{5} = \mp \frac{18}{5} \end{cases} \Rightarrow \left[\begin{array}{l} \left(\frac{5}{6}; \frac{24}{5}, \frac{-18}{5}\right) = A \\ \left(\frac{-5}{6}, \frac{-24}{5}, \frac{18}{5}\right) = B \end{array} \right.$$

стационарные точки

$$\begin{vmatrix} L''_{\lambda_1\lambda_1} & L''_{\lambda_1 x} & L''_{\lambda_1 y} \\ L''_{x\lambda_1} & L''_{xx} & L''_{xy} \\ L''_{y\lambda_1} & L''_{yx} & L''_{yy} \end{vmatrix} = \begin{vmatrix} 0 & 2x & 2y \\ 2x & 2\lambda_1 & 0 \\ 2y & 0 & 2\lambda_1 \end{vmatrix} =$$

$$= -8x^2\lambda_1 - 8y^2\lambda_1$$

$$A:\ -\frac{8 \cdot 24^2 \cdot 5}{25 \cdot 6} - \frac{8 \cdot (-18)^2 \cdot 5}{25 \cdot 6} =$$

$$= -\frac{\overset{4}{\cancel{8}} \cdot \cancel{5}}{\underset{5}{\cancel{25}} \cdot \underset{3}{\cancel{6}}}\left(24^2 + 18^2\right) < 0$$

$$B:\ -\frac{8 \cdot 24^2 \cdot (-5)}{25 \cdot 6} - \frac{8 \cdot 18^2 \cdot (-5)}{25 \cdot 6} =$$

$$= \frac{\overset{4}{\cancel{8}} \cdot \cancel{5}}{\underset{5}{\cancel{25}} \cdot \underset{3}{\cancel{6}}}\left(24^2 + 18^2\right) > 0$$

A – минимум $\left(\frac{5}{6}, \frac{24}{5}, \frac{-18}{5}\right)$

B – максимум $\left(\frac{-5}{6}, \frac{-24}{5}, \frac{18}{5}\right)$

№2

$$u = 2x^2 + 12xy + 32y^2 + 15$$

$$x^2 + 16y^2 = 64$$

$$\varphi_1 = x^2 + 16y^2 - 64 = 0$$

$$L(\lambda_1, x, y) = 2x^2 + 12xy + 32y^2 + 15 + \lambda_1(x_2 + 16y^2 - 64)$$

$$\begin{cases} L'_{\lambda_1} = x^2 + 16y^2 - 64 = 0 \\ L'_x = 4x + 12y + 2\lambda_1 x = 0 \;\Big|:2 \to 2x + 6y + \lambda_1 x = 0 \\ L'_y = 12x + 64y + 32y\lambda_1 = 0 \;\Big|:4 \Rightarrow 3x + 16y + 8\lambda_1 y = 0 \end{cases}$$

$$\begin{cases} L'_{\lambda_1} = x^2 + 16y^2 - 64 = 0 \\ -\dfrac{(2x+6y)}{x} = \lambda_1 \\ -\dfrac{(3x+16y)}{8y} = \lambda_1 \\ x \neq 0,\ y \neq 0 - \text{т.к. иначе система несовместна} \end{cases} \Leftrightarrow \begin{cases} \cancel{L'_{\lambda_1}} = x^2 + 16y^2 - 64 = 0 \\ -\dfrac{2x+6y}{x} = \lambda_1 \\ -(2x+6y)\cdot 8y = -(3x+16y)x \ (*) \end{cases}$$

(*)

$$\cancel{16xy} + 48y^2 = 3x^2 + \cancel{16xy} \quad \Big|:3$$

$$16y^2 = x^2$$

$$\begin{cases} x^2 + 16y^2 - 64 = 0 \\ 16y^2 = x^2 \\ \dfrac{-(2x+6y)}{x} = \lambda_1 \end{cases} \Rightarrow \begin{cases} \cancel{2}\cdot\cancel{16}y^2 = \cancel{64} \\ 16y^2 = x^2 \\ \cancel{2\lambda}\ \dfrac{-(2x+6y)}{x} = \lambda_1 \end{cases} \Rightarrow y^2 = 2 \quad y = \left[\begin{array}{l} -\sqrt{2} \\ \sqrt{2} \end{array}\right.$$

$$\begin{cases} \left[\begin{array}{l} \begin{cases} y=\sqrt{2} \\ x=\pm 4\sqrt{2}\end{cases} \\ \begin{cases} y=-\sqrt{2} \\ x=\pm 4\sqrt{2}\end{cases}\end{array}\right. \\ \dfrac{-(2x+6y)}{x}=\lambda_1\end{cases}$$

A:

$$\begin{cases} \lambda_1=\dfrac{-(8\sqrt{2}+6\sqrt{2})}{4\sqrt{2}}=-\dfrac{7}{2} \\ x=4\sqrt{2} \\ y=\sqrt{2}\end{cases}$$

B:

$$\begin{cases} \lambda_1=\dfrac{-(-8\sqrt{2}+6\sqrt{2})}{-4\sqrt{2}}=-\dfrac{1}{2} \\ x=-4\sqrt{2} \\ y=\sqrt{2}\end{cases}$$

C:

$$\begin{cases} \lambda_1=\dfrac{-(8\sqrt{2}-6\sqrt{2})}{4\sqrt{2}}=-\dfrac{1}{2} \\ x=4\sqrt{2} \\ y=-\sqrt{2}\end{cases}$$

D:

$$\lambda_1=\dfrac{-(-8\sqrt{2}-6\sqrt{2})}{-4\sqrt{2}}=-\dfrac{7}{2}$$

$$x=-4\sqrt{2}$$

$$y=-\sqrt{2}$$

$$\begin{vmatrix} L''_{\lambda_1\lambda_1} & L''_{\lambda_1 x} & L''_{\lambda_1 y} \\ L''_{x\lambda_1} & L''_{xx} & L''_{xy} \\ L''_{y\lambda_1} & L''_{yx} & L''_{yy}\end{vmatrix} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 0 & 2x & 32y \\ 2x & 4+2\lambda_1 & 12 \\ 32y & 12 & 64+32\lambda_1\end{vmatrix}\end{array} =$$

= [illegible]

= [illegible]

[illegible]

$A: \left(-\frac{7}{2}, 4\sqrt{2}, \sqrt{2}\right)$

$$\begin{vmatrix} 0 & 8\sqrt{2} & 32\sqrt{2} \\ 8\cdot\sqrt{2} & -3 & 12 \\ 32\sqrt{2} & 12 & -48 \end{vmatrix} = -8\sqrt{2}\left(-48\cdot 8\sqrt{2} + 12\cdot 32\sqrt{2}\right) \oplus$$

$$\oplus\ 32\sqrt{2}\left(12\cdot 8\sqrt{2} + 3\cdot 32\sqrt{2}\right) > 0$$

$\Rightarrow$ A - максимум

$B: \left(-\frac{7}{2}, -4\sqrt{2}, \sqrt{2}\right)$

$\Delta < 0$

B - минимум

$C: \left(-\frac{1}{2}, 4\sqrt{2}, \sqrt{2}\right)$

$\Delta < 0$

C - минимум

$D: \left(-\frac{7}{2}, -4\sqrt{2}, -\sqrt{2}\right)$

$\Delta > 0$

D - максимум

N3

$u = x^2 + y^2 + z^2$ по $\vec{c}(-9, 8, -12)$

в т. $M(8, -12, 9)$.

$|\bar{c}| = \sqrt{(-9)^2 + 8^2 + (-12)^2} = \sqrt{289} = 17$

$\bar{c}_0 = \frac{\bar{c}}{|\bar{c}|} = \left(\frac{-9}{17}, \frac{8}{17}, \frac{-12}{17}\right)$.

$u'_x = 2x$ $\qquad \frac{\partial u}{\partial x}\Big|_M = 16$

$u'_y = 2y$ $\qquad \frac{\partial u}{\partial y}\Big|_M = -24$ $\qquad \operatorname{grad} u\Big|_M = (16, -24, 18)$

$u'_z = 2z$ $\qquad \frac{\partial u}{\partial z}\Big|_M = 18$

$$\frac{\partial z}{\partial c_0}\Big|_M = \frac{-9 \cdot 16}{17} + \frac{-8 \cdot 24}{17} + \frac{-18 \cdot 12}{17} =$$

$$= -\frac{144 + 192 + 216}{17} = -\frac{552}{17} = -32\frac{8}{17}$$

№4

$u = e^{x^2+y^2+z^2}$ по $\bar{d} = (4, -13, -16)$

в т. $M(-16, 4, -13)$

$x^2+y^2+z^2 = 256+16+169 = 441$

$\frac{\partial u}{\partial x} = e^{x^2+y^2+z^2} \cdot 2x$ $\quad \frac{\partial u}{\partial x}\Big|_M = e^{441} \cdot (-32)$

$\frac{\partial u}{\partial y} = e^{x^2+y^2+z^2} \cdot 2y$ $\quad \frac{\partial u}{\partial y}\Big|_M = e^{441} \cdot 8$

$\frac{\partial u}{\partial z} = e^{x^2+y^2+z^2} \cdot 2z$ $\quad \frac{\partial u}{\partial z}\Big|_M = e^{441} \cdot (-26)$

$|\bar{d}| = \sqrt{16+169+256} = 21$

$\bar{d}_0 \quad \frac{\bar{d}}{|\bar{d}|} = \left(\frac{4}{21}, \frac{-13}{21}, \frac{-16}{21}\right)$

$grad(u)\Big|_M = (-32, 8, -26)$

$\frac{\partial u}{\partial d_0}\Big|_M = \frac{4}{21} \cdot (-16) + \frac{-13 \cdot 4}{21} + \frac{-16 \cdot (-13)}{21} =$

$= \frac{-64 - 52 + 208}{21} = \frac{92}{21}$