№1.

$$z = \sqrt{1-x^3} + \ln(y^2-1)$$

$$\begin{cases} 1-x^3 \geq 0 & (1) \\ y^2-1 > 0 & (2) \end{cases}$$

(1)
$x^3 \leq 1$
$x \leq 1$

(2) $y^2 > 1$
$|y| > 1$
$y \in (-\infty; -1) \cup (1; +\infty)$.

$$\begin{cases} x \in (-\infty; 1] \\ y \in (-\infty; -1) \cup (1; +\infty) \end{cases}$$

№2.

$$z = \left(1 + \frac{\ln x}{\ln y}\right)^3$$

$$\frac{\partial z}{\partial x} = 3\left(1 + \frac{\ln x}{\ln y}\right)^2 \cdot \frac{1}{x \cdot \ln y}$$

$$\frac{\partial z}{\partial y} = 3\left(1 + \frac{\ln x}{\ln y}\right)^2 \cdot \ln x \cdot \left(\frac{-1}{\ln^2 y}\right) \cdot \frac{1}{y}.$$

№3

$$z = \sqrt{2xy + \cos\frac{x}{y}} \quad \text{в т. } (1;1)$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{2xy + \cos\frac{x}{y}}} \cdot 2y + \left(-\sin\frac{x}{y}\right) \cdot \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{2xy + \cos\frac{x}{y}}} \cdot 2x + \left(-\sin\frac{x}{y}\right) \cdot \frac{-x}{y^2}$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = \left(\frac{y}{\sqrt{2xy + \cos\frac{x}{y}}} - \frac{\sin\frac{x}{y}}{y}\right) dx \oplus$$

$$\oplus \left(\frac{x}{\sqrt{2xy + \cos\frac{x}{y}}} + \frac{\sin\frac{x}{y} \cdot x}{y^2}\right) dy$$

$\cos(1) = 0{,}54$
$\sin(1) = 0{,}84.$

$$dz(1,1) = \left(\frac{1}{2 + 0{,}54} - \frac{0{,}84}{1}\right) dx +$$

$$+ \left(\frac{1}{2 + 0{,}54} + \frac{0{,}84}{1}\right) dy$$

№4

$$z = x^2 + xy + y^2 - 6x - 9y$$

$$\begin{cases} \frac{\partial z}{\partial x} = 2x + y - 6 = 0 \\ \frac{\partial z}{\partial y} = x + 2y - 9 = 0 \end{cases}$$

$$\begin{cases} y = 6-2x \\ x + 12 - 4x - 9 = 0 \end{cases} \Leftrightarrow \begin{cases} y = 6-2x \\ 3x = 3 \end{cases} \Leftrightarrow \begin{cases} y = 4 \\ x = 1 \end{cases}$$

критическая точка: $M(1;4)$.

$$\frac{\partial^2 z}{\partial x^2} = 2 \quad \frac{\partial^2 z}{\partial y^2} = 2 \quad \frac{\partial^2 z}{\partial x \partial y} = 1$$

$$\left.\begin{aligned} &\Delta = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = 4 - 2 = 2 > 0 \\ &\frac{\partial^2 z}{\partial^2 x} = 2 > 0 \end{aligned}\right\} \Rightarrow M(1;4)$$

↑
минимум