Huffman Coding

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What is Huffman Coding?

Huffman Coding

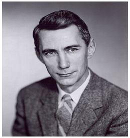
Huffman coding is an efficient method of compressing data without losing information. It provides an efficient, unambiguous code by analyzing the frequencies that certain symbols appear in a message. The algorithm is named after David A. Huffman, who developed it while he was a Sc.D student at MIT.

A little bit of History

In 1951, Professor Robert M. Fano assigned a term paper on the problem of finding the most efficient binary code. In his paper, David A. Huffman developed an algorithm where he assigned shorter codes to the most frequently occurring characters and longer codes to the less frequently occurring characters, thus employing a variable-length encoding system.

David A. Huffman

Robert M. Fano



Claude Shannon

In doing so, Huffman outdid Fano, who alongside Claude Shannon developed a similar method. Huffman's bottom-up approach turned out to be more optimal than the top-down approach of Shannon and Fano.

Where is Huffman Coding used?

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- **Audio Compression:** MP3, AAC
- **Network Communication:** Reduction of the size of data packets transmitted over networks

What's our objective?

The Problem

Efficient digital communication requires converting any type of information into binary strings. Since resource is limited, we must find out a way of conversion such that the message requires least possible bandwidth to transmit.

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Our Goal

Now we look forward to finding an encoding scheme that requires minimum binary characters on average to encode a message.

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Doing so, we obtain the following table.

Letter	**Binary Code**
A	000
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A Naive Solution

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- Each of the letters needs 3 bits to encode.
- Therefore, if the length of a message $= n$, the expected length of the binary code $= 3n$

We Can Do Better. . .

Here, as a saviour, arrives the ingenious idea of Huffman

Huffman's Idea

The Observation

- Assigning shorter codes for more-frequent characters decreases the average length of the encoded message

Huffman's Idea

A Greedy Algorithm

1. Maintain a min-heap of nodes for characters with their relative frequencies as weights. For instance, a character with relative frequency 17%, its weight is 0.17
2. Pop two nodes with minimum weights
3. Merge them into a single node whose weight will be the summation of the previous nodes
4. Push the newly created node back into the min-heap
5. Repeat until the min-heap contains only the root of the tree
6. Use the Huffman tree to encode any message

Why Greedy Technique Works in Huffman Coding

The Intuition Behind

- The greedy approach always places more-frequent characters closer to the root of the tree
- This ensures that shorter codes are assigned to more-frequent characters and vice versa

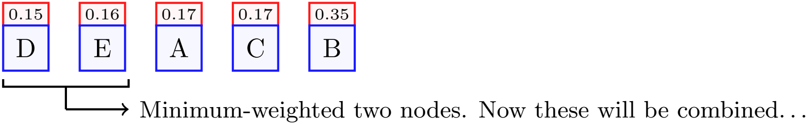
The Huffman Tree

Huffman Tree

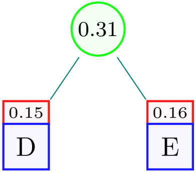
In Huffman encoding, symbols are represented by nodes in a binary tree structure. It is built using bottom-up greedy approach.

Construction of Huffman Tree

Construction of Huffman Tree



Construction of Huffman Tree



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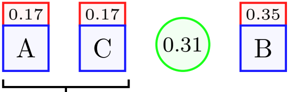
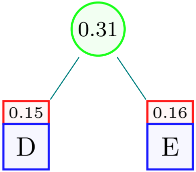


Minimum-weighted two nodes. Now these will be combined...



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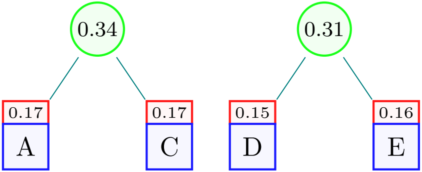

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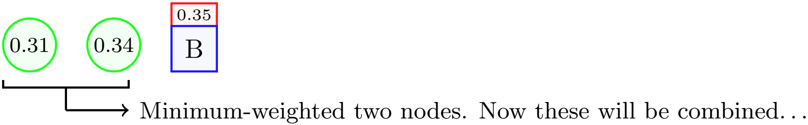

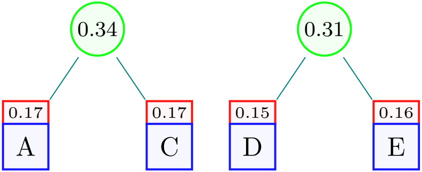

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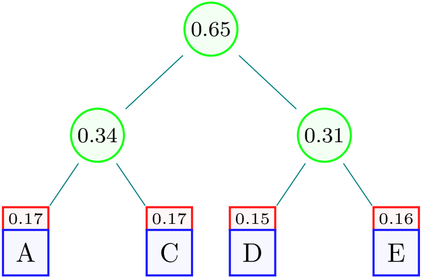

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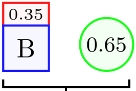

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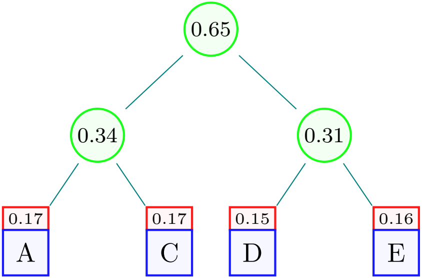

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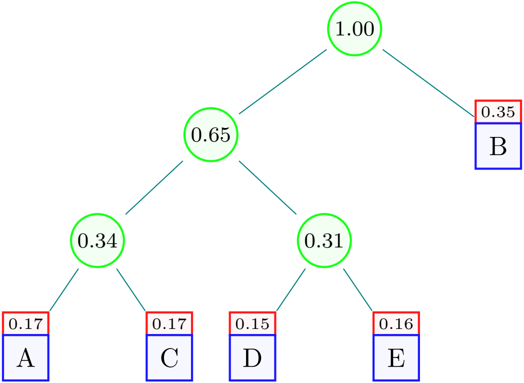


Interpretation of Huffman Tree

Example

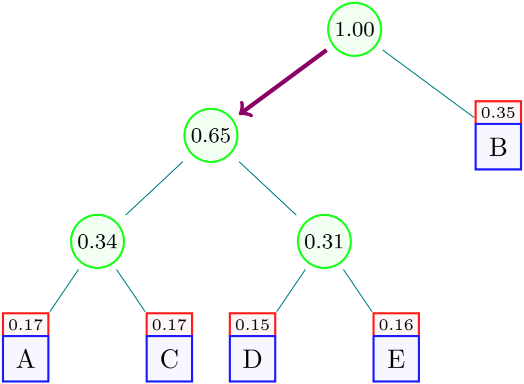
Let's say we want to find the binary encoding of the letter 'D'

Solution

We find the path from root to the leaf containing 'D'. For every left branch, we append a **0** and for every right branch, we append a **1** to the binary encoding.

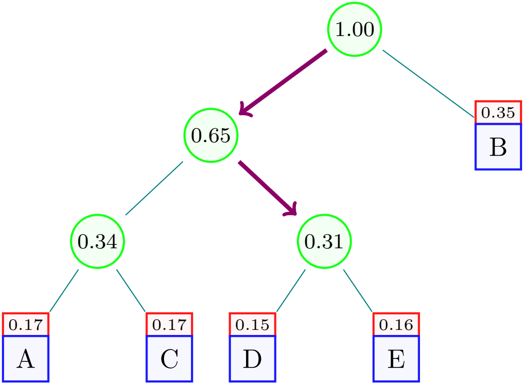
Interpretation of Huffman Tree

0



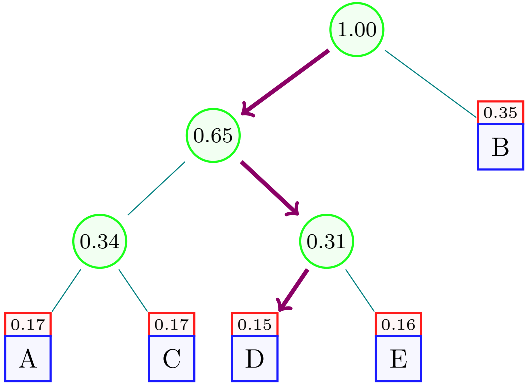
Interpretation of Huffman Tree

0 1



Interpretation of Huffman Tree

0 1 0



Interpretation of Huffman Tree

So, the binary encoding of 'D' = **010**

Interpretation of Huffman Tree

Similarly, we determine all the codes. . .

Letter	Binary Code
A	000
B	1
C	001
D	010
E	011

We have indeed done better ...

- Now let's calculate the average length of the binary encoding for a message.
- Average Length $= 0.17 \times 3 + 0.35 \times 1 + 0.17 \times 3 + 0.15 \times 3 + 0.16 \times 3$
  $= 2.3$
- Compression ratio $= \frac{3-2.3}{3} \times 100\% =$ **23.33 %**

How to Encode?

Encoding is easy, isn't it?

Idea: We may just look up the binary code for each of the letters and append them one by one to the result

Example

Example

Let's find the encoding for the word 'DECADE'.

Example

Letter	Binary Code
A	000
B	1
C	001
D	010
E	011

D E C A D E

010 011 001 000 010 011

Example

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D E C A D E

010 011 001 000 010 011

Efficiency of Encoding

$$\text{Entropy}, H = -\sum_{i=1}^{n} p_i \cdot \log_2(p_i)$$

Here p_i denotes the probability of ith symbol. For our word "DECADE", we calculate entropy as follows-

$$\begin{aligned} H = -\,(&0.17 \cdot \log_2(0.17) \\ &+ 0.35 \cdot \log_2(0.35) \\ &+ 0.17 \cdot \log_2(0.17) \\ &+ 0.15 \cdot \log_2(0.15) \\ &+ 0.16 \cdot \log_2(0.16)) = 2.23 \end{aligned}$$

Efficiency of Encoding

$$\text{Efficiency} = \frac{\text{Entropy}}{\text{Number of bits per symbol}} \times 100\%$$

Encoding of our word "DECADE" is - 010 011 001 000 010 011
Number of bits per symbol $= \frac{18}{6} = 3$

$$\text{Efficiency} = \frac{2.23}{3} \times 100\% = 74.33\%$$

How to Decode?

Decoding is a bit more interesting. . .

Idea: Keep scanning the binary string from *left to right* and upon decoding a letter, we append it to the result.

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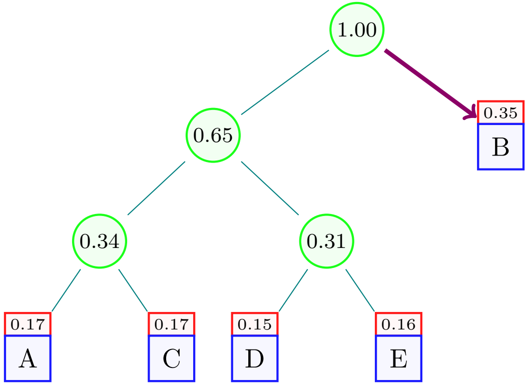
We may think of scanning the input as traversing the Huffman-Tree. When we read a **0**, we turn **left** and when **1**, we turn **right**. Upon reaching a leaf, we add the corresponding letter to our result.

Example

Let's now decode '**1011011**'.

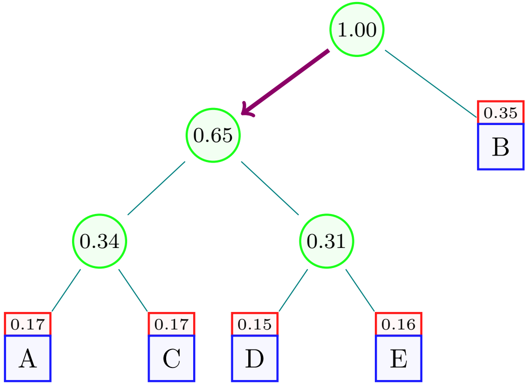
Example

1 011011 B



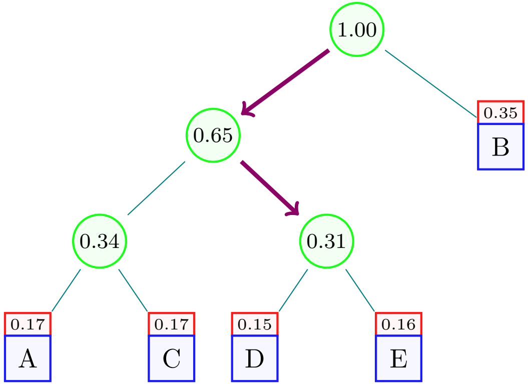
Example

1 0 11011 **B**



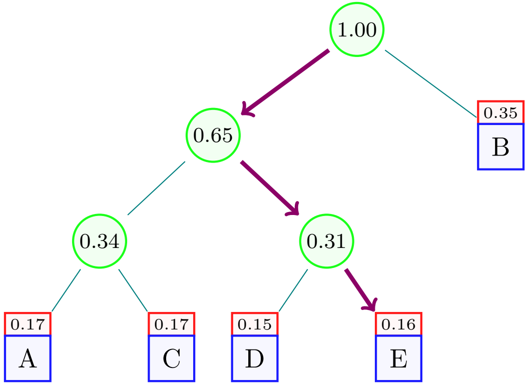
Example

1 01 1011 B



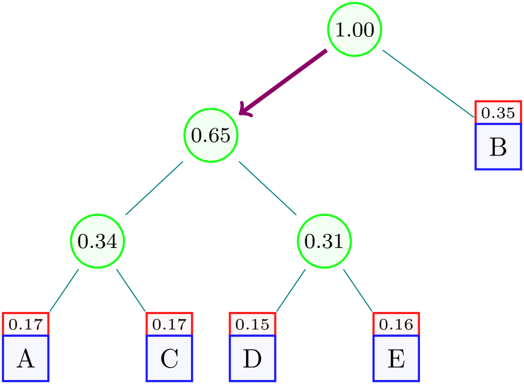
Example

1 011 011 **B E**



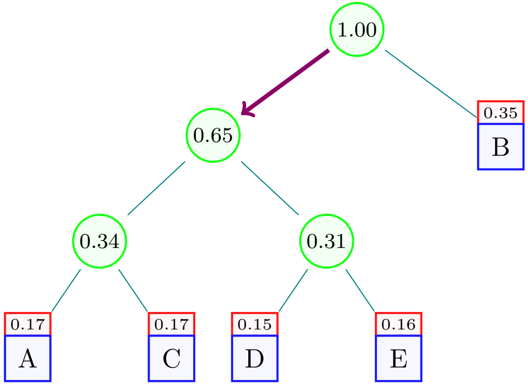
Example

1011 0 11 **BE**



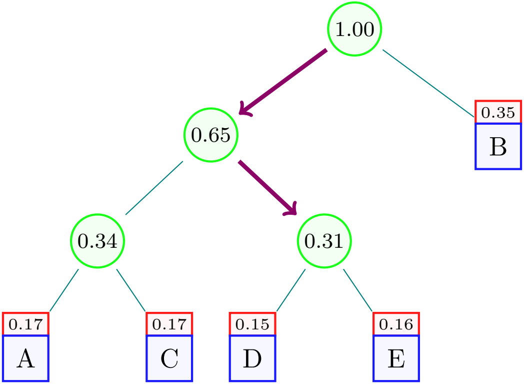
Example

1011 0 11 **BE**



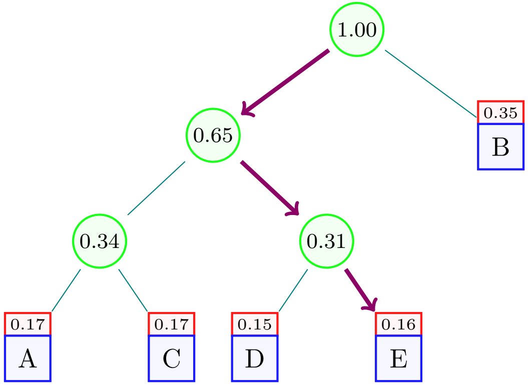
Example

1011 01 1 **BE**



Example

1011 011 **BE E**



Example

So, decoded message = '**BEE**'

Time Complexity Analysis of Huffman Coding

- **Building Frequency Table:** $O(n)$, where n is the number of symbols in the input.
- **Building the Huffman Tree:**
  - Constructing initial heap: $O(n)$
  - Merging nodes in the heap: $O(\log n)$ per merge operation
  - Total time: $O(n \log n)$
- **Generating Huffman Codes:** $O(n)$, where n is the number of symbols

Conclusion

- In conclusion, Huffman coding has been a fundamental stepping stone in the journey of data compression.
- It revolutionized the field by providing efficient and effective compression techniques based on symbol frequencies.
- Despite the emergence of more sophisticated compression algorithms, Huffman coding remains widely used in various applications.

Acknowledgments

- Special thanks to Brilliant for their informative article on Huffman Encoding.
  https://brilliant.org/wiki/huffman-encoding/
- Wikipedia for providing valuable insights into Huffman Coding.
  https://en.wikipedia.org/wiki/Huffman-Coding
- YouTube tutorials that greatly contributed to our understanding:
  - "Huffman Coding Explained" by Michael Sambol
    https://www.youtube.com/watch?v=umTbivyJoiI
  - "Huffman Coding - Greedy Algorithm" by mycodeschool
    https://www.youtube.com/watch?v=B3y0RsVCyrw