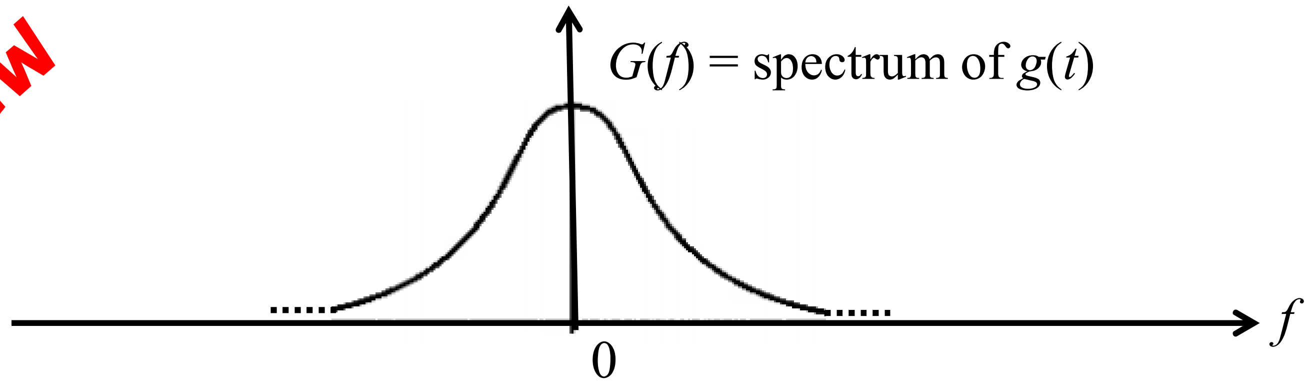
CSE 311:
Data Communication

Instructor:
Dr. Md. Monirul Islam

Sampling
and
Analog-to-Digital Conversion

Sampling Effect: Non-band-limited signal to band-limited signal

Review



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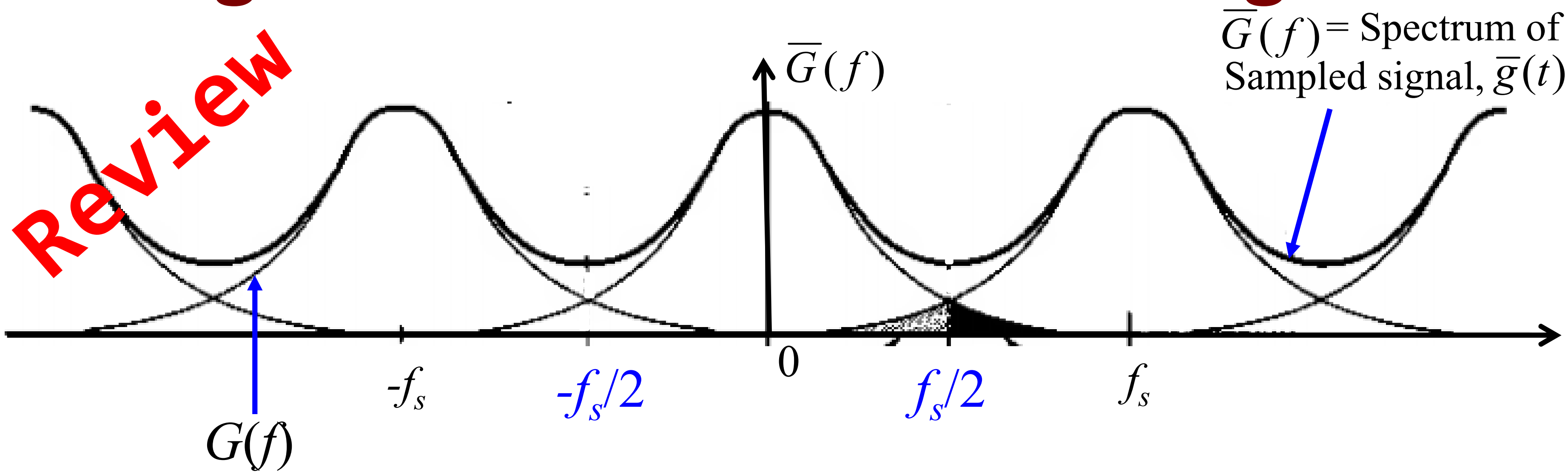

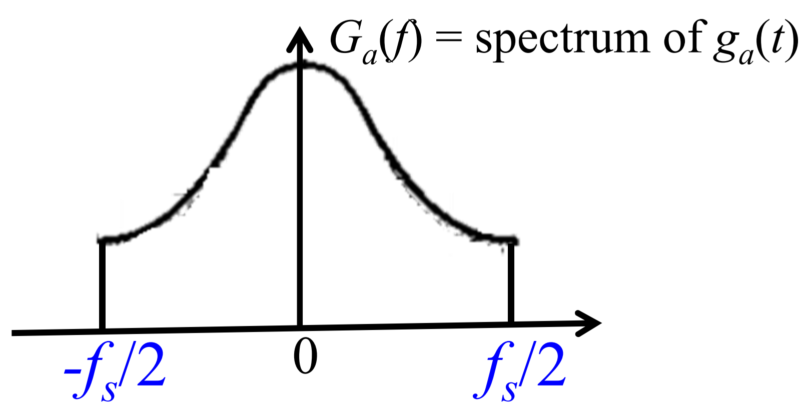

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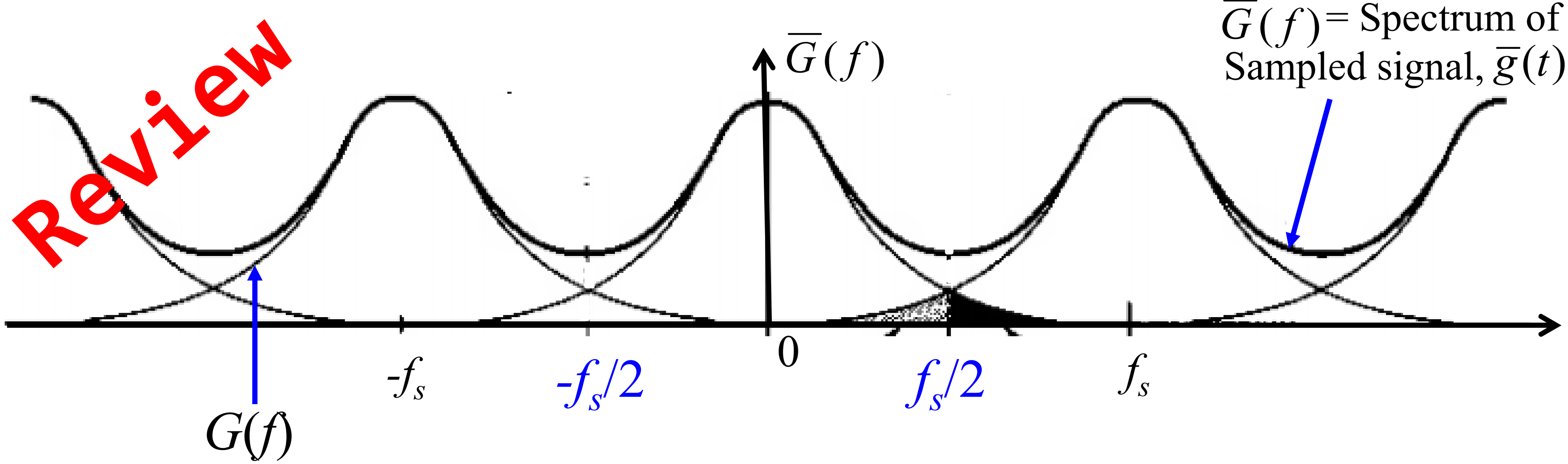

Sampling a non-band-limited signal $g(t)$ at f_s is equivalent to Nyquist sampling of some signal $g_a(t)$ band-limited to $f_s/2$

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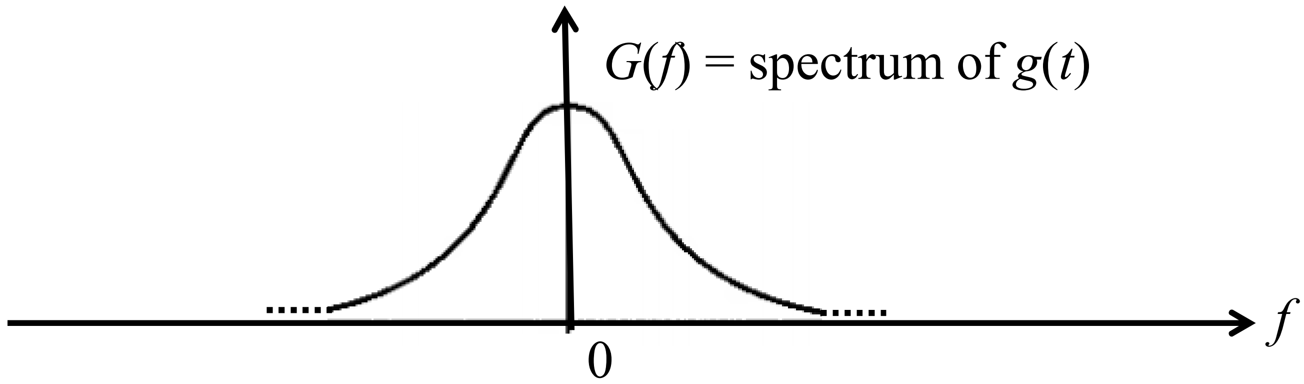

Let

sub-Nyquist Sampling of $g(t)$ at f_s generates samples $g(0)$, $g(T_s)$, $g(2T_s)$, $g(3T_s)$,, . . .

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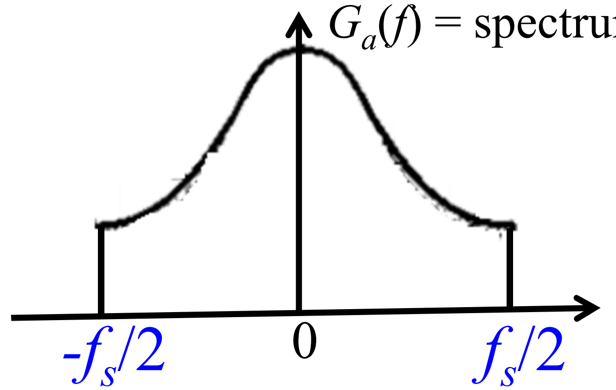

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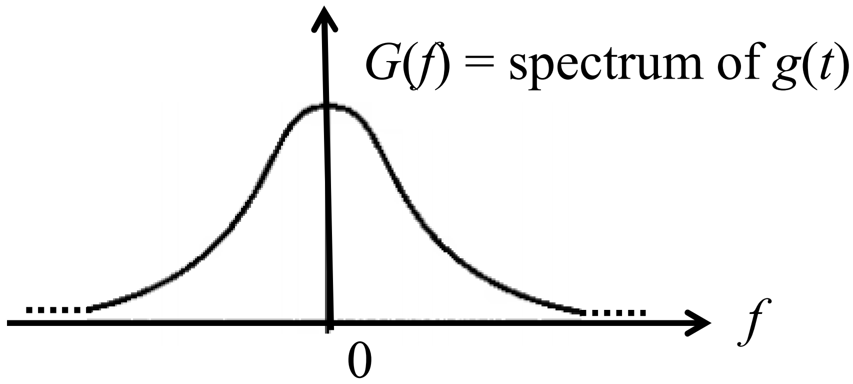

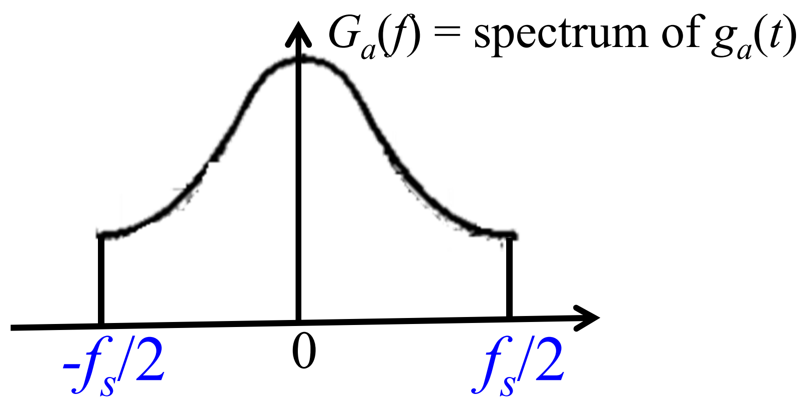

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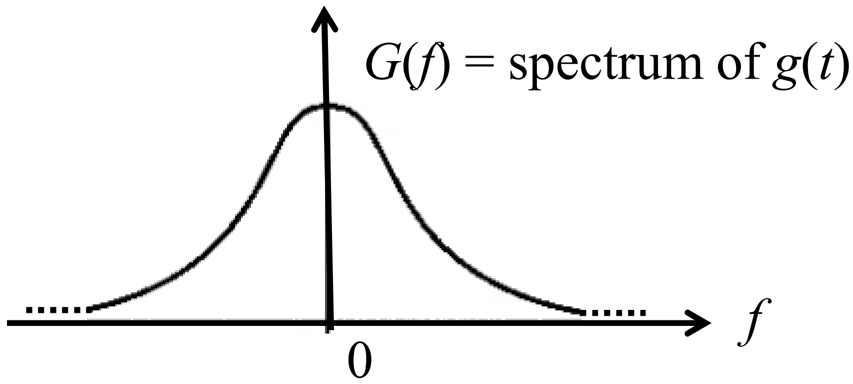

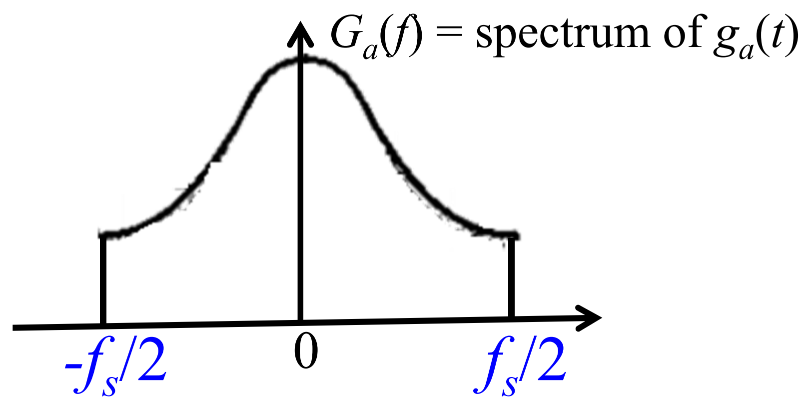

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According to sampling effect that we saw,

$g(0) = g_a(0)$, $g(T_s) = g_a(T_s)$, $g(2T_s) = g_a(2T_s)$, $g(3T_s) = g_a(3T_s)$, and so on

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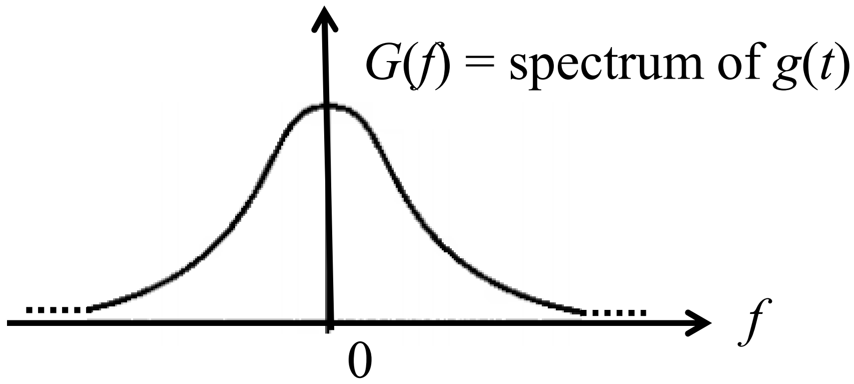

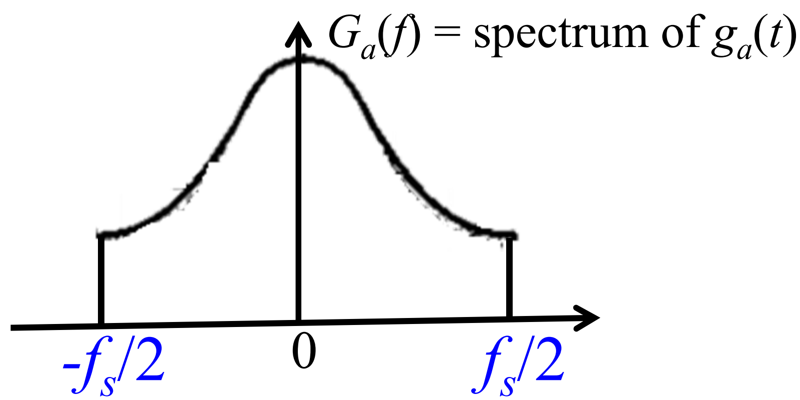

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Therefore,

$$g(nT_s) = g_a(nT_s) = g_n$$

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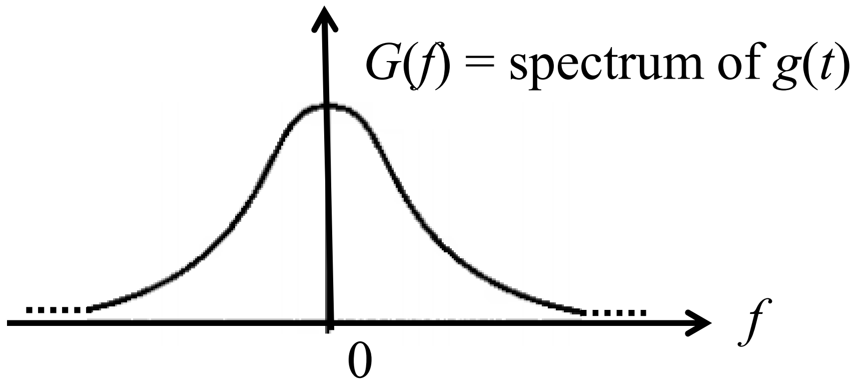

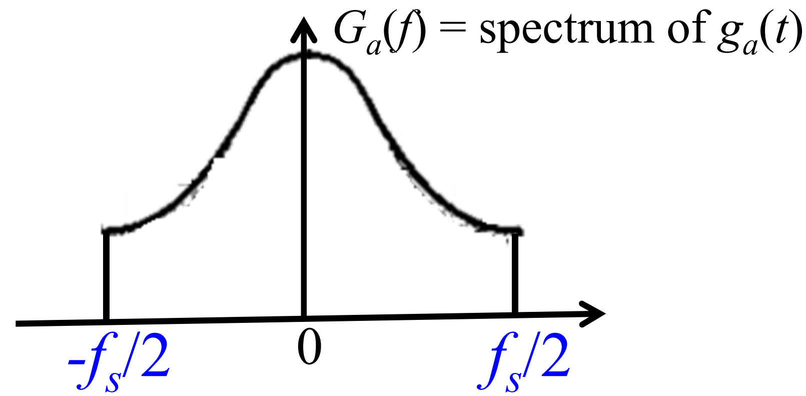

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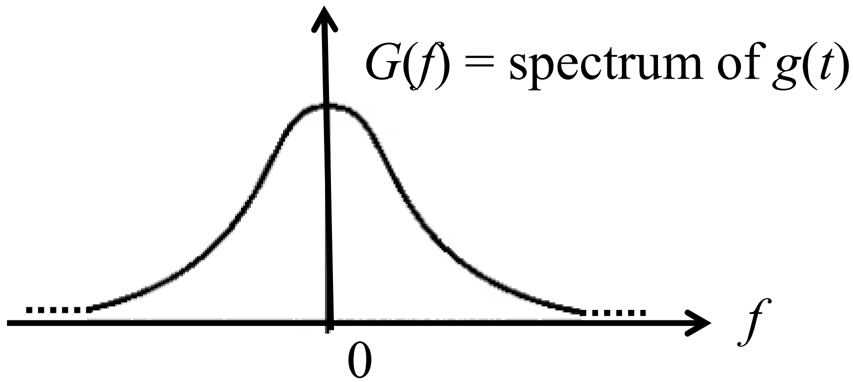

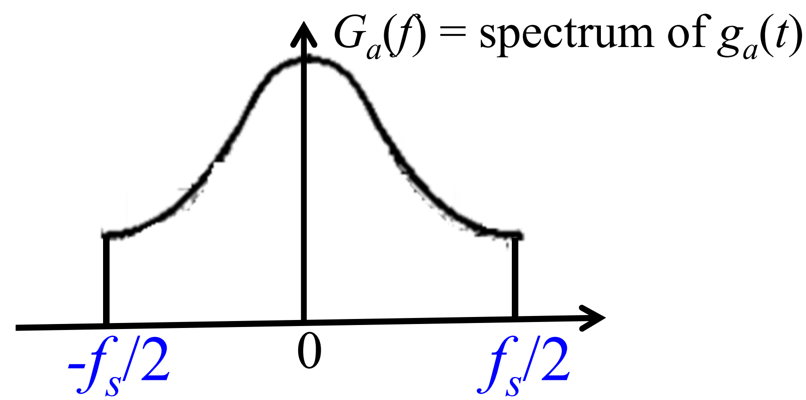

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Maximum Information Rate of a Channel with BW = *B* Hz

Assume

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We will prove

- Maximum 2*B* pieces of information can be sent per second

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Previous Knowledge

- Channel can send a low pass signal of B Hz
- This signal can be recovered from samples uniformly taken at $2B$ samples per second

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We have to prove that

- A sequence of data at the rate of $2B$ Hz can come from uniform sampling of a signal of bandwidth B Hz
- The signal can be recovered from this data sequence

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Assume a sequence of samples $g_0, g_1, g_2, g_3, \ldots$ denoted as $\{g_n\}$ at the rate of $2B$ s/s

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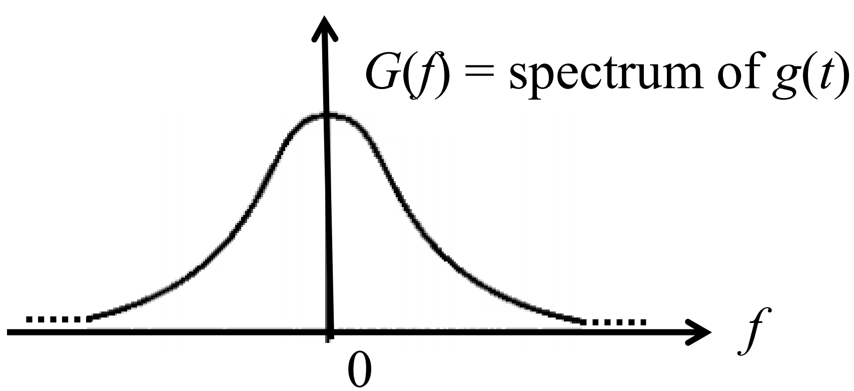
We will always find a signal $g(t)$ whose samples $g(0), g(T_s), g(2T_s), g(3T_s), \ldots$ matches with $\{g_n\}$.



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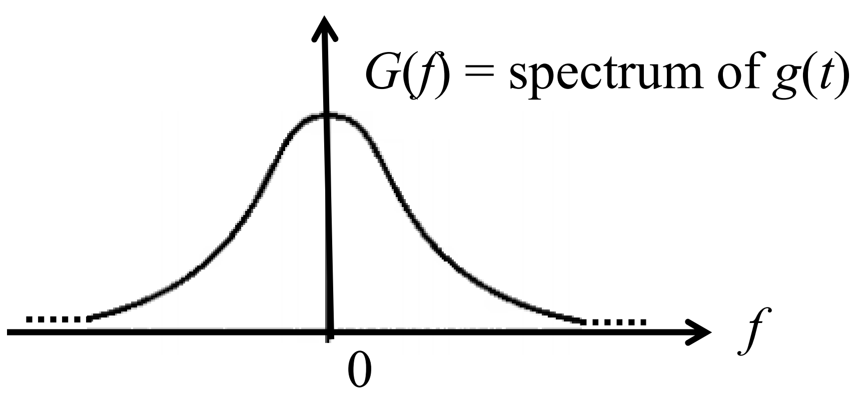

This means,

$$g_n = g(nT_s)$$

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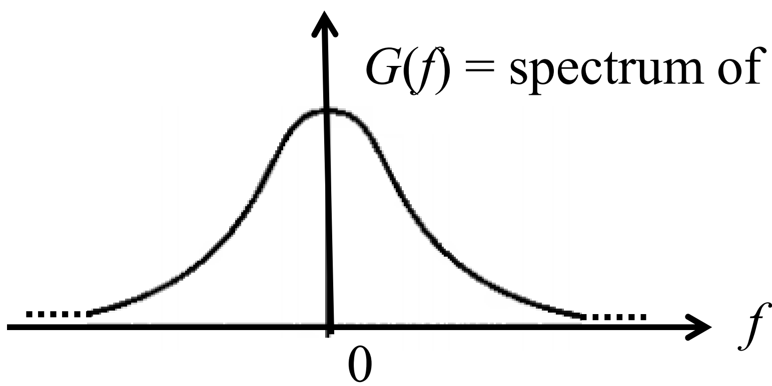

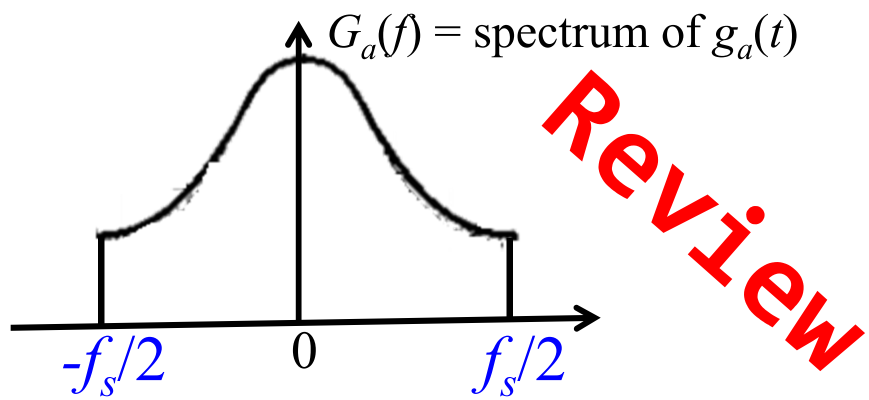

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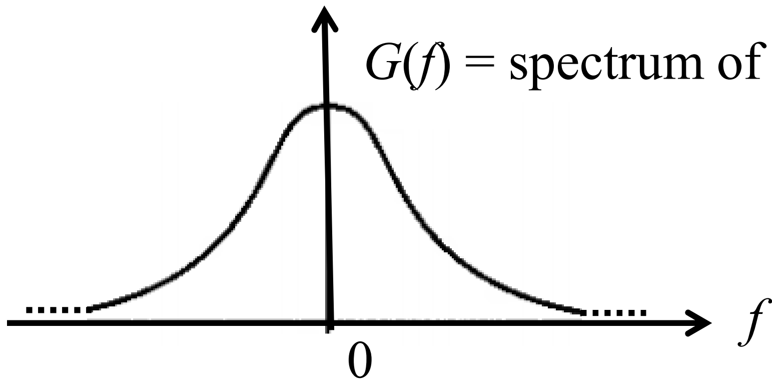

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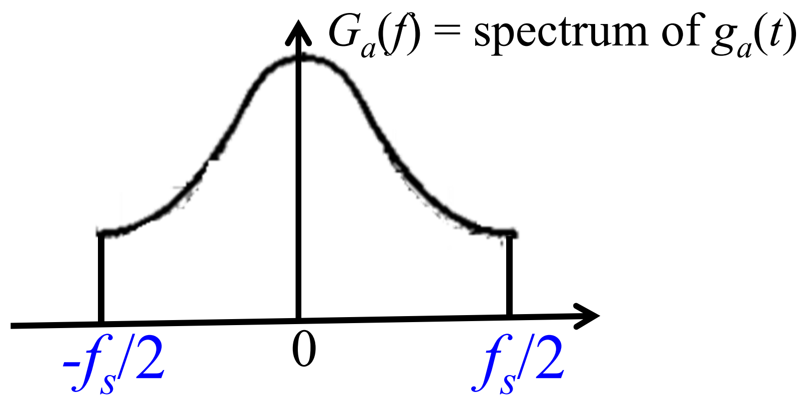

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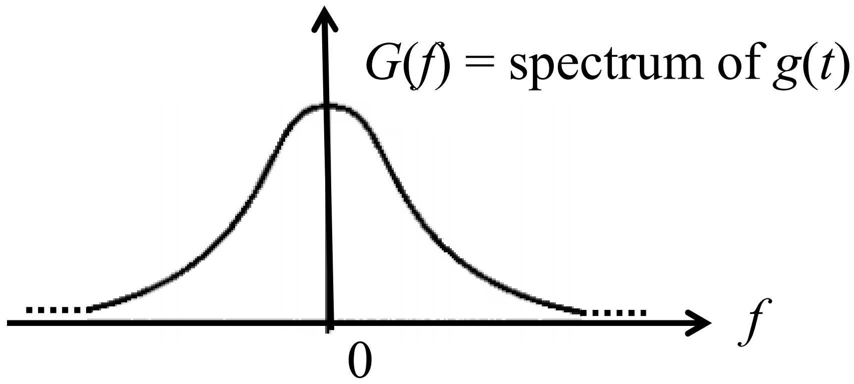

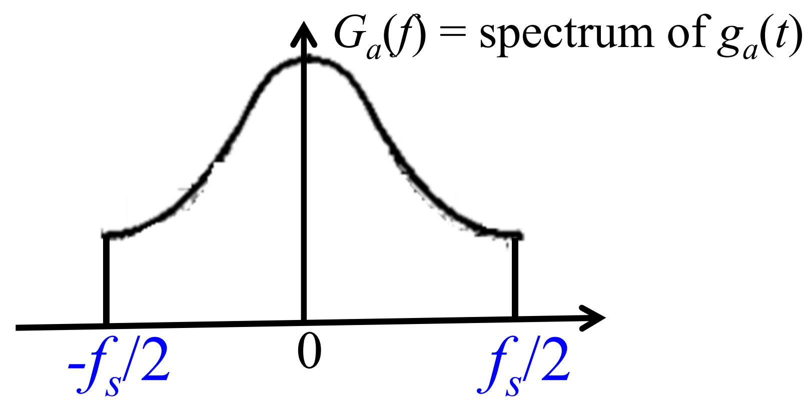

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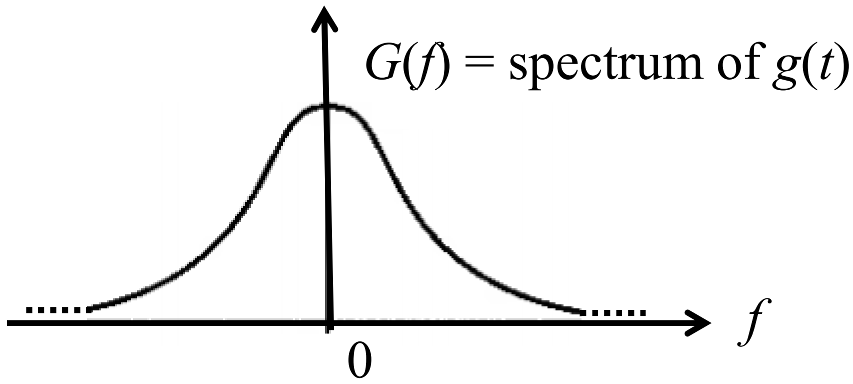

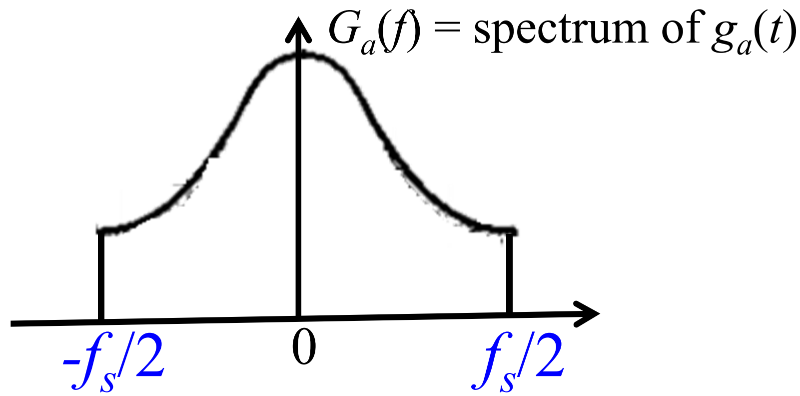

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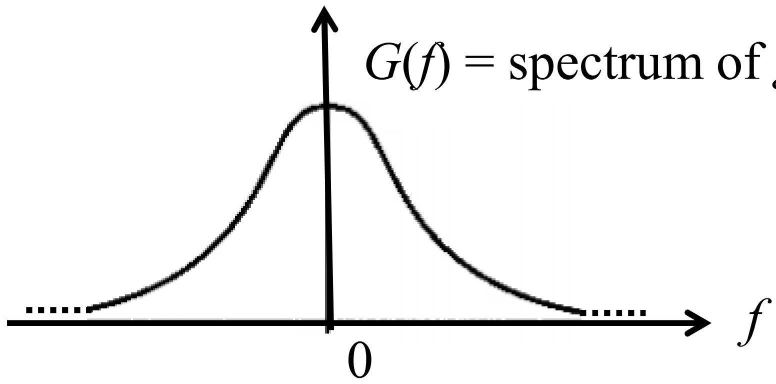

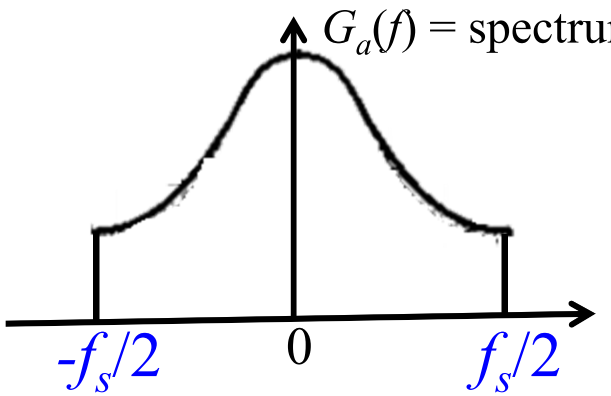

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$$= \sum_n g_a(nT_s)\delta(t - nT_s)$$
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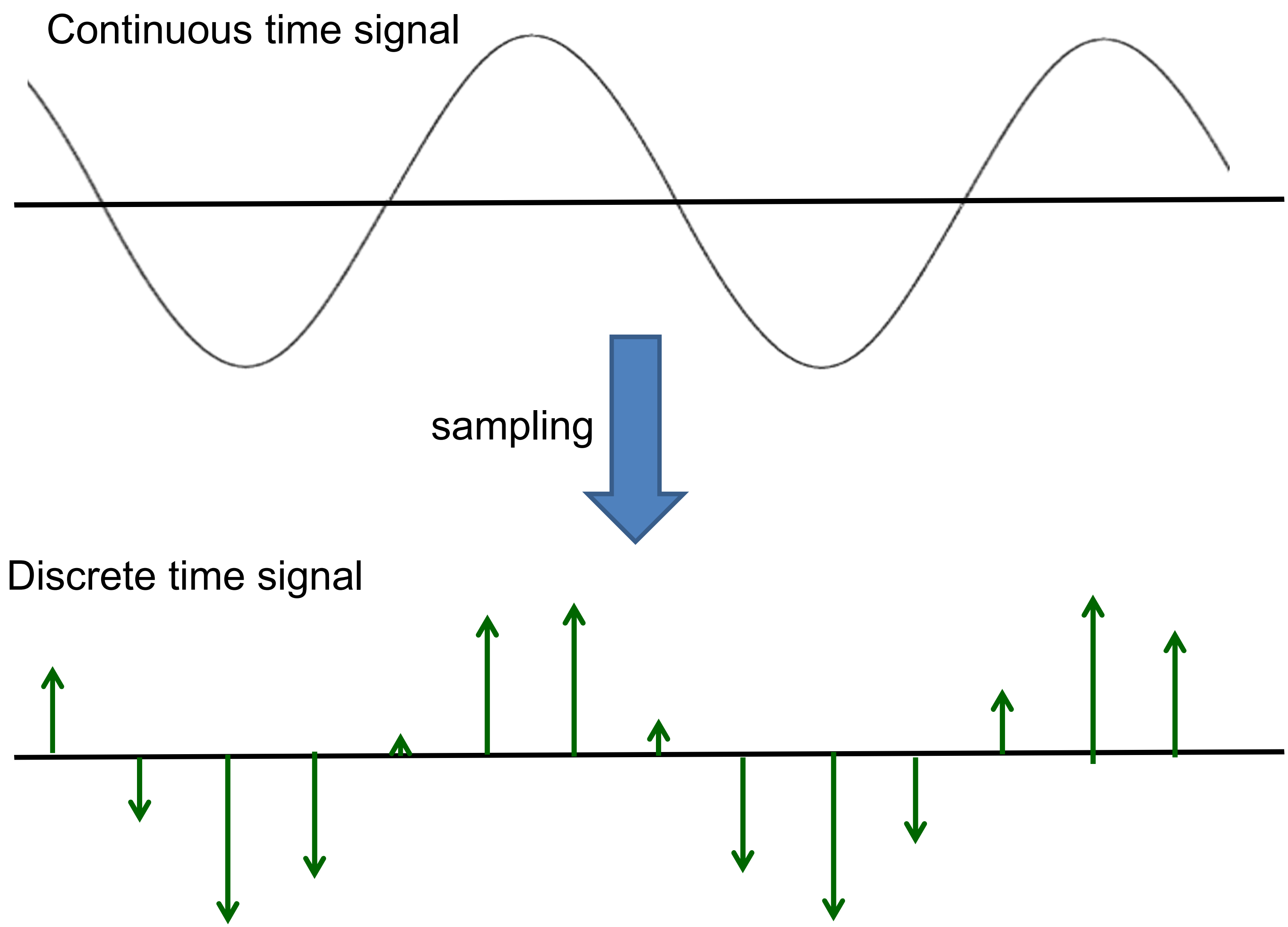
Nyquist Samples $g_a(0)$, $g_a(T_s)$, $g_a(2T_s)$, $g_a(3T_s)$,, . . .

To recover $g_a(t)$, we can use $\{g_n\}$ using,

$$g_a(t) = \sum_n g_n \mathrm{sinc}(2\pi Bt - n\pi)$$

Application of Sampling Theorem



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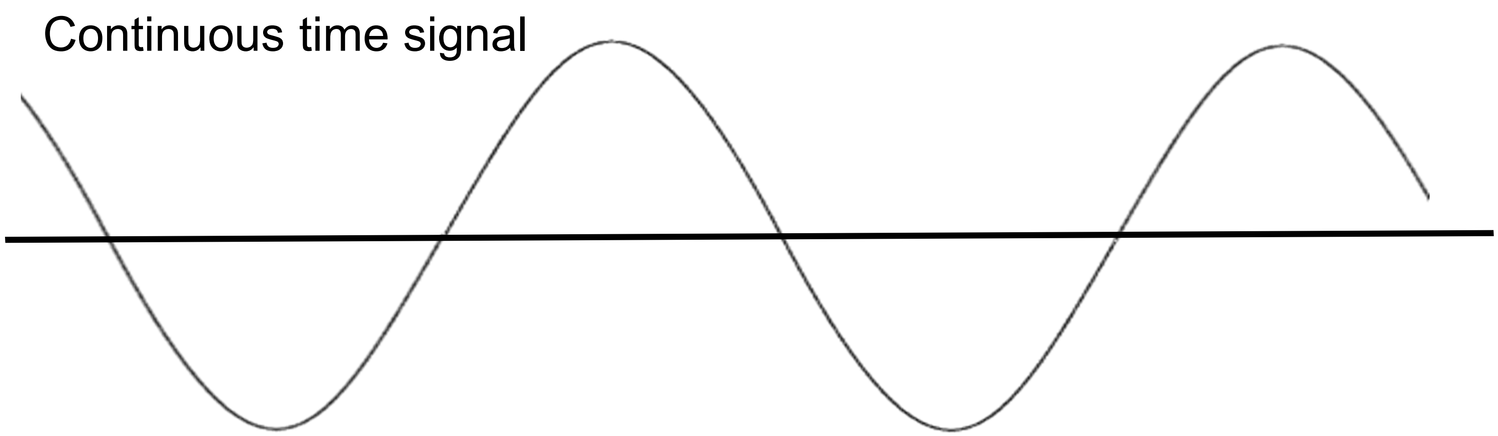
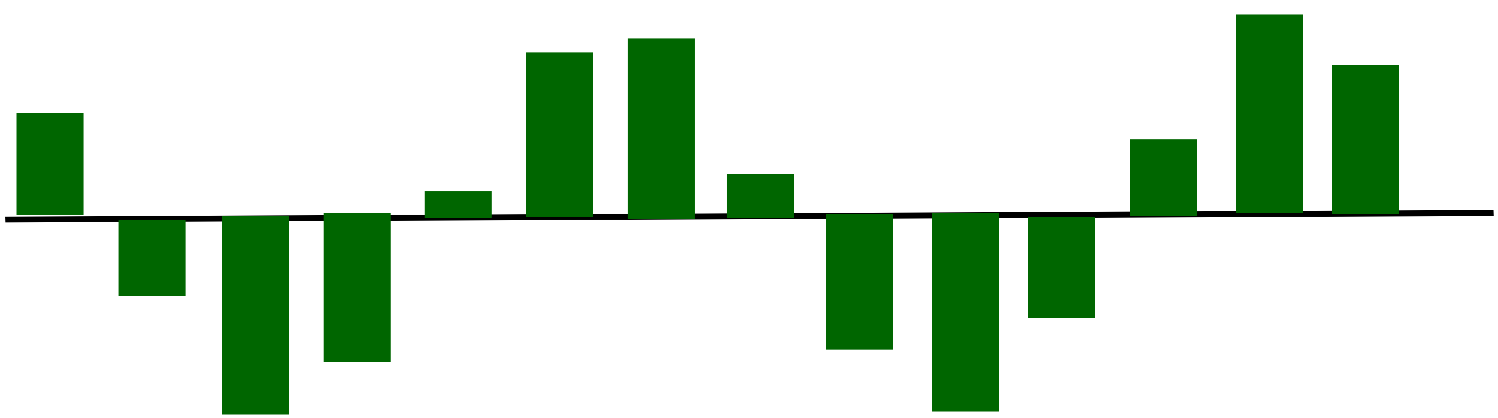


can be represented by pulse train and transmitted thereafter



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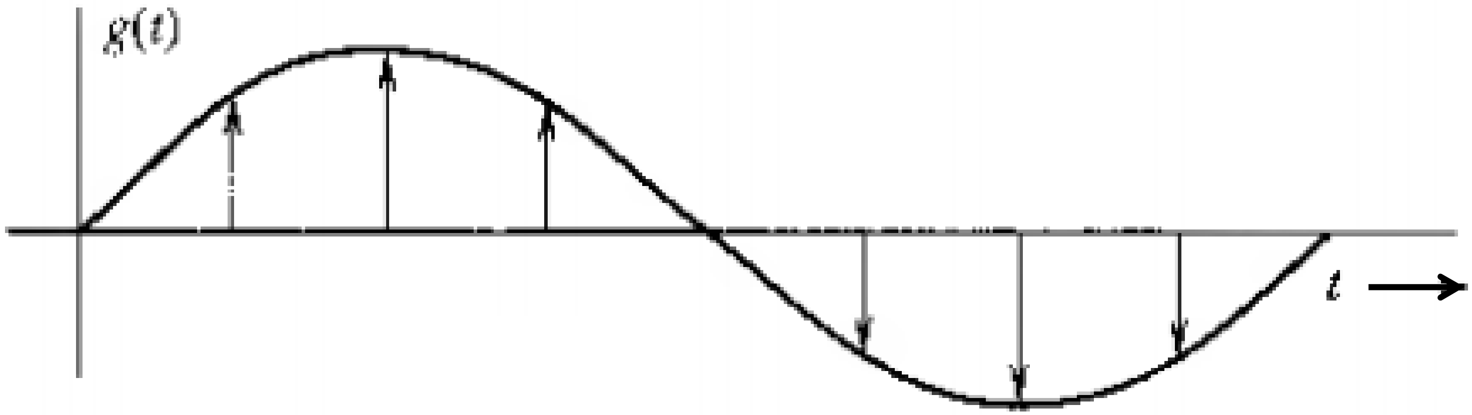
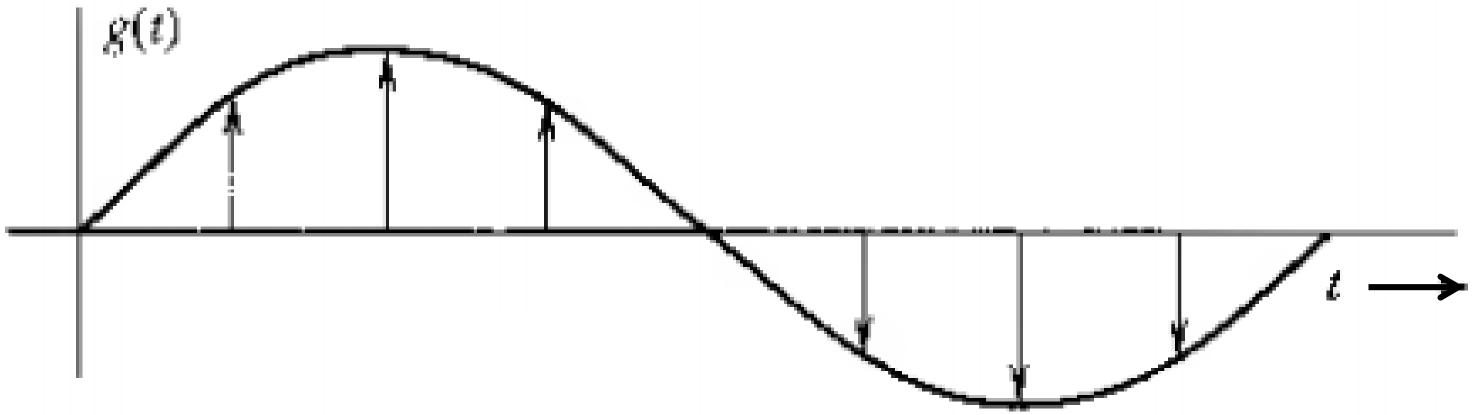
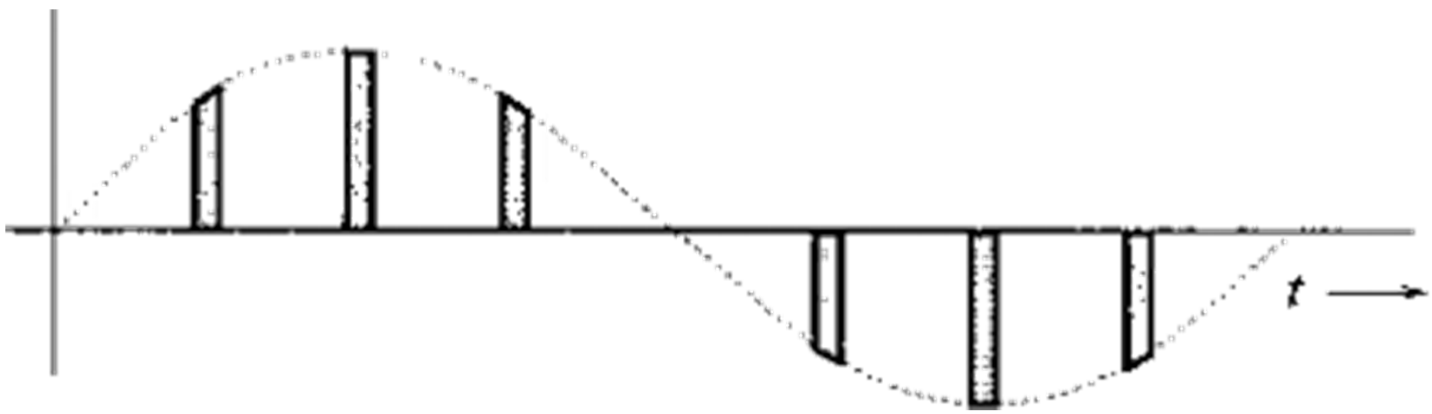


Pulse modulations: different ways to transmit sampled signal modifying pulse trains

Application of Sampling Theorem



PAM: Pulse amplitude is modulated



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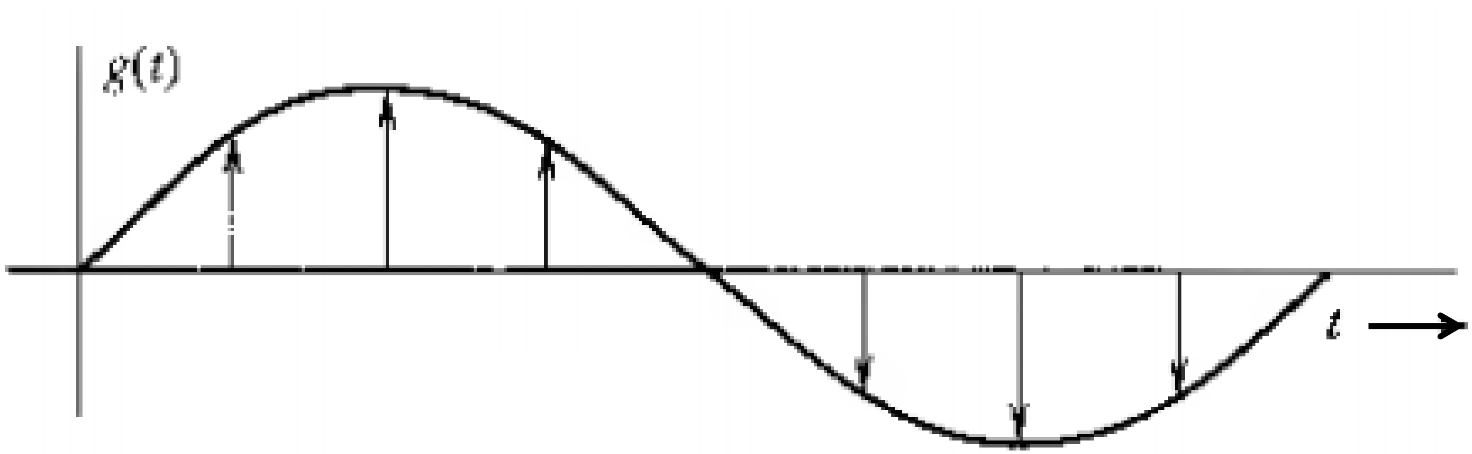
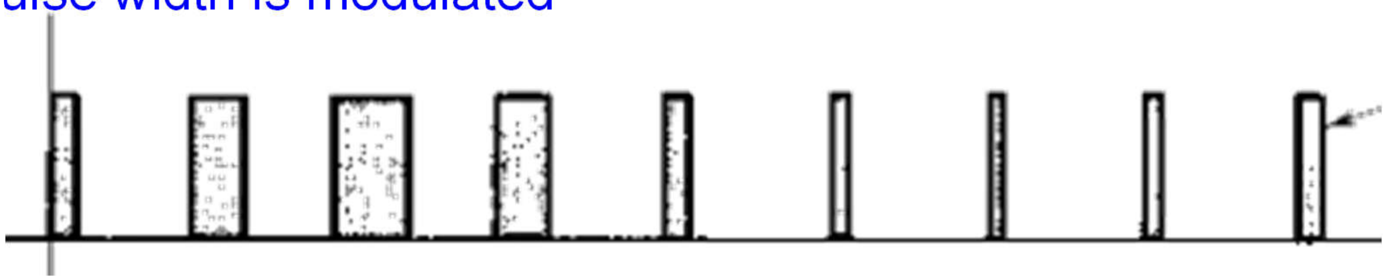
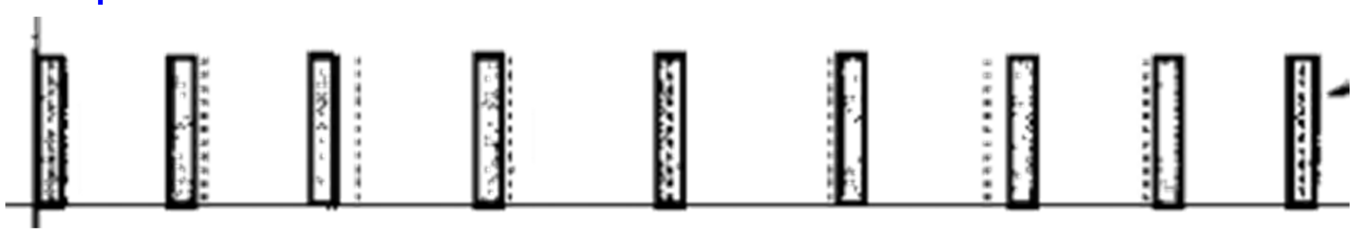


PWM: Pulse width is modulated



PPM: Pulse position is modulated



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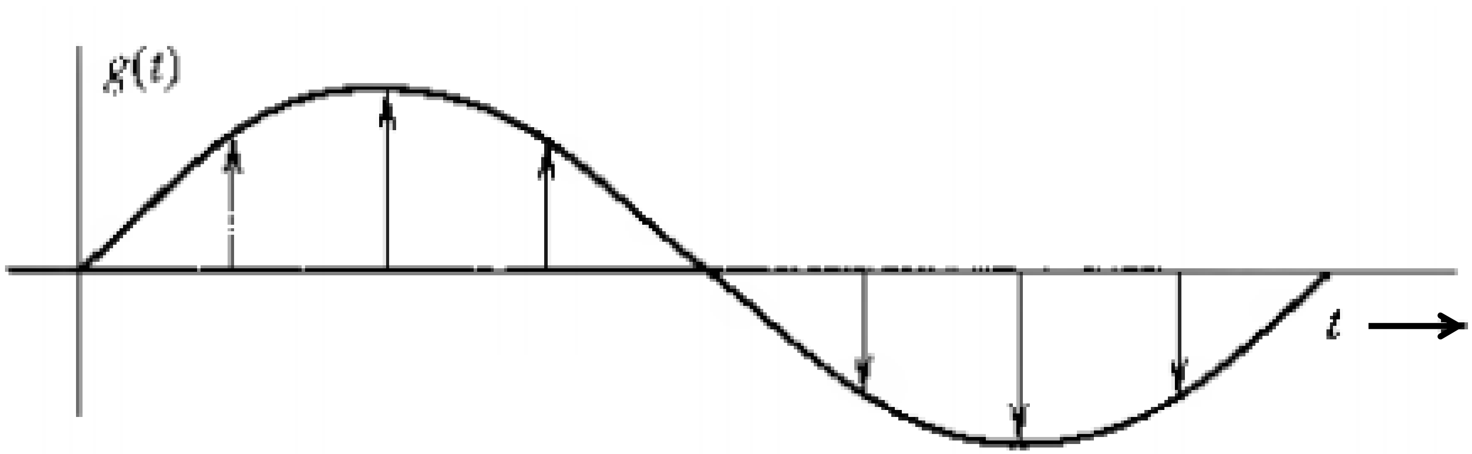
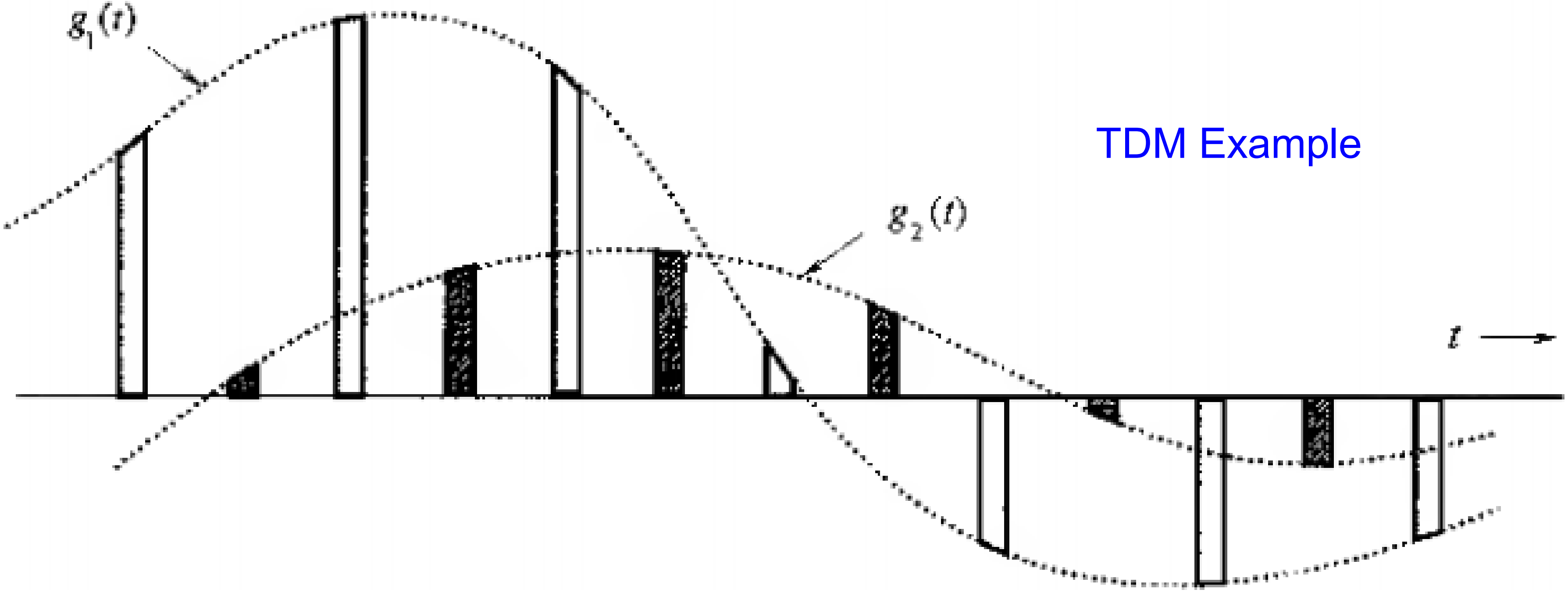


Pulse code modulation:

- most widely used pulse modulation
- each sample value is converted to a set of pulses.

Application of Sampling Theorem

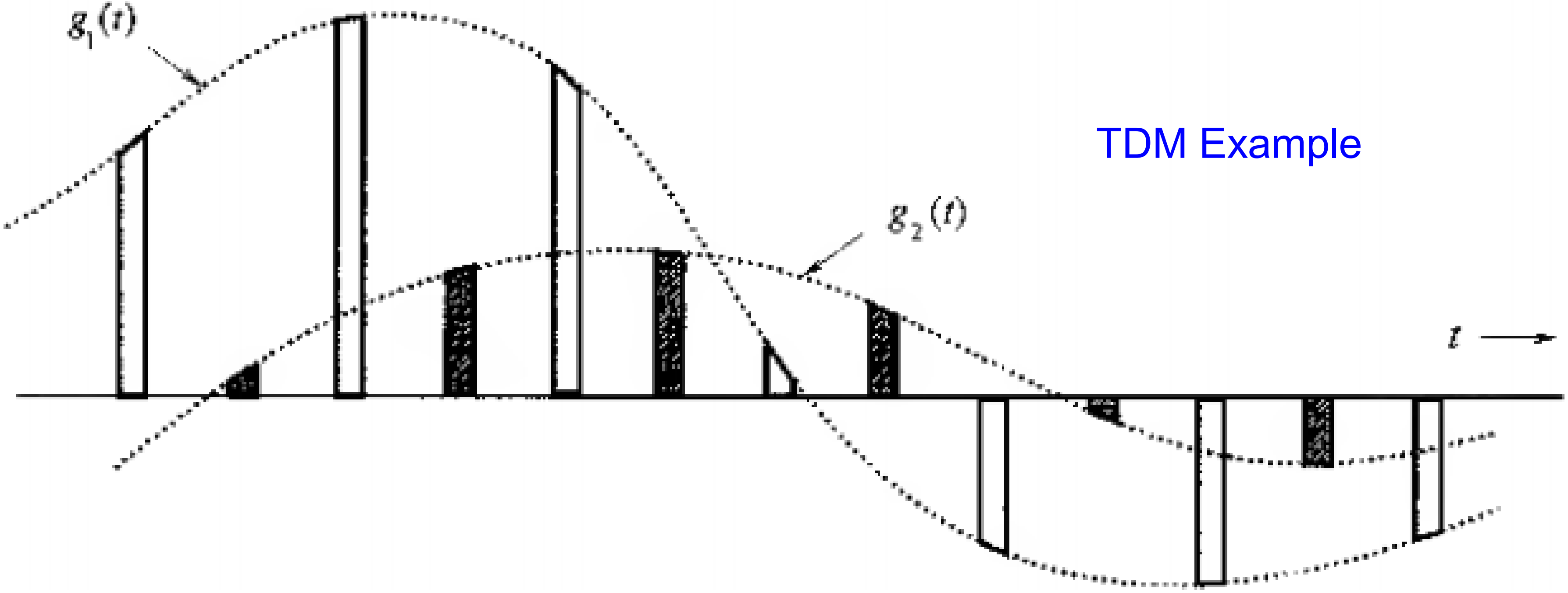
TDM: Pulses from multiple signals are interweaved on the same channel



TDM Example

Application of Sampling Theorem

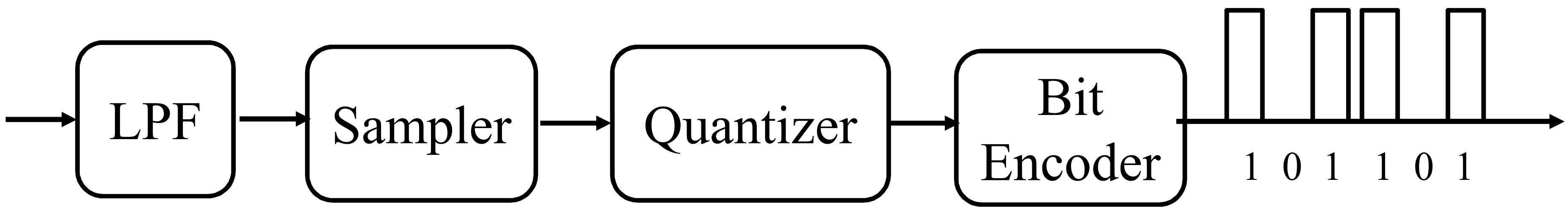
TDM: dual of FDM where different signals share channel bandwidth



TDM Example

FDM Example

Pulse Code Modulation (PCM)



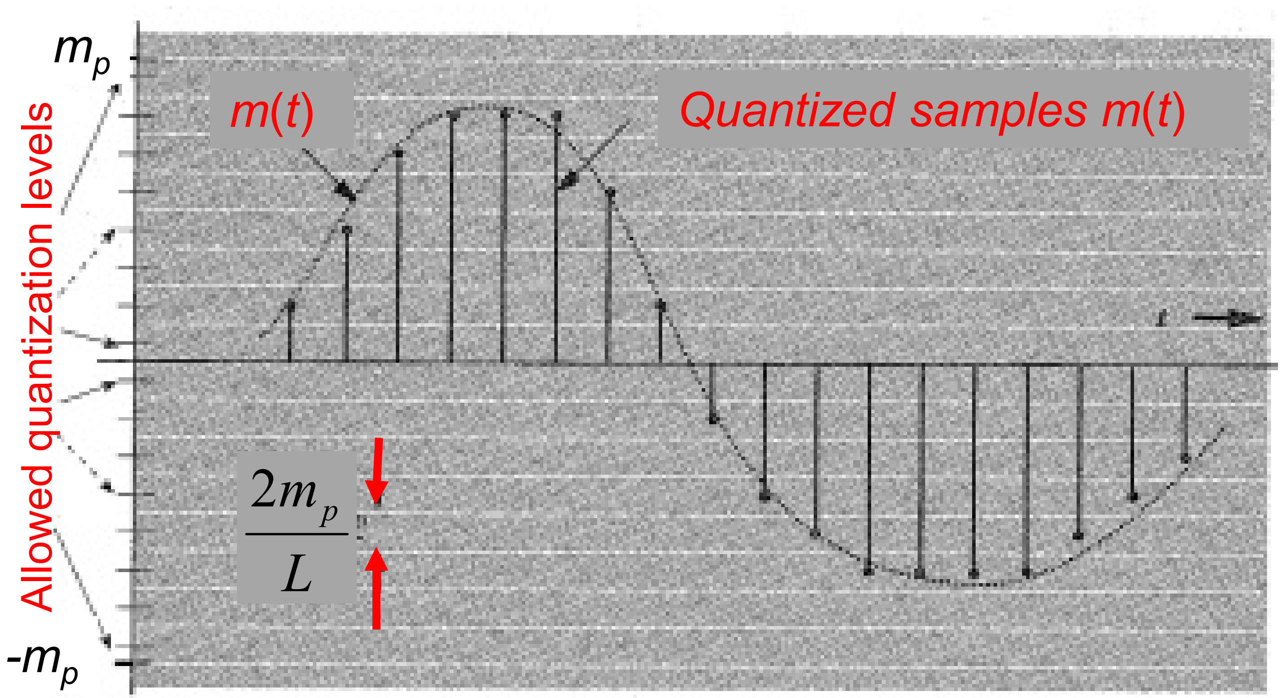
PCM system: basically an ADC

Two major Steps:

- Sampling and
- quantizing

Analog to Digital Conversion of Message Signal

- 2 major steps
  - Sampling
  - Quantizing



The range $(-m_p, m_p)$ is divided into L sub-intervals, each of magnitude Δv

$$\Delta v = \frac{2m_p}{L}$$

Analog to Digital Conversion of Message Signal

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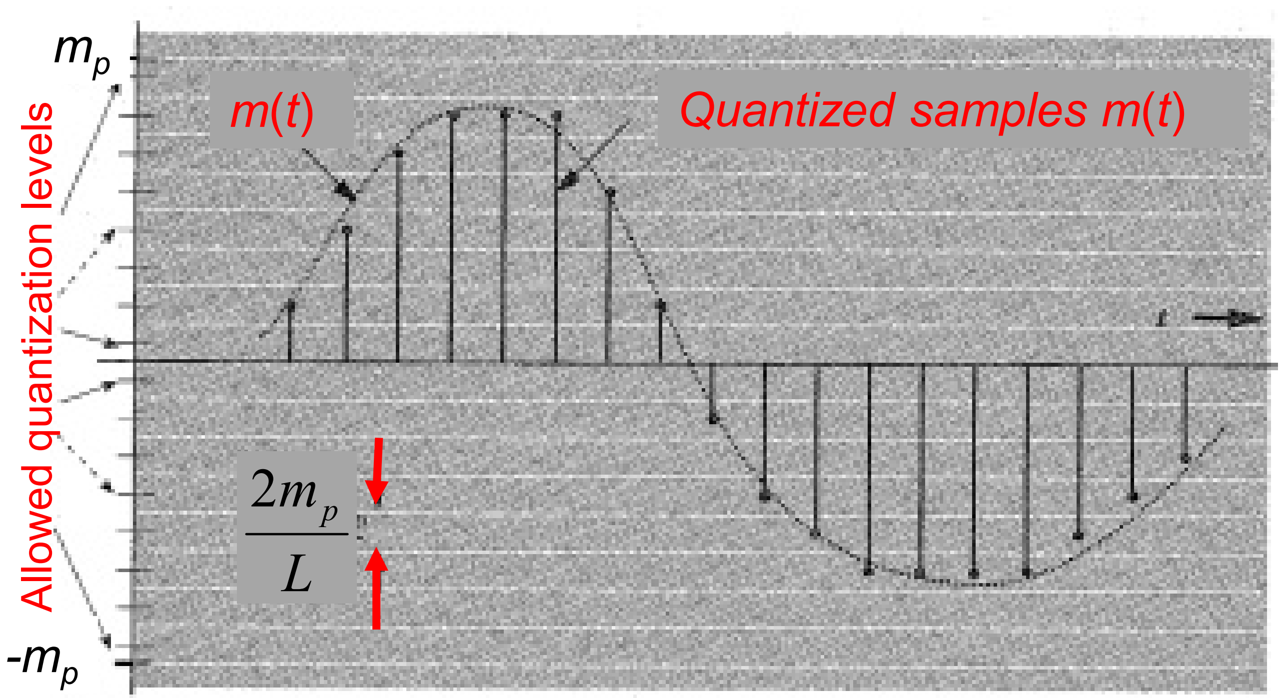

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L is known as quantization level

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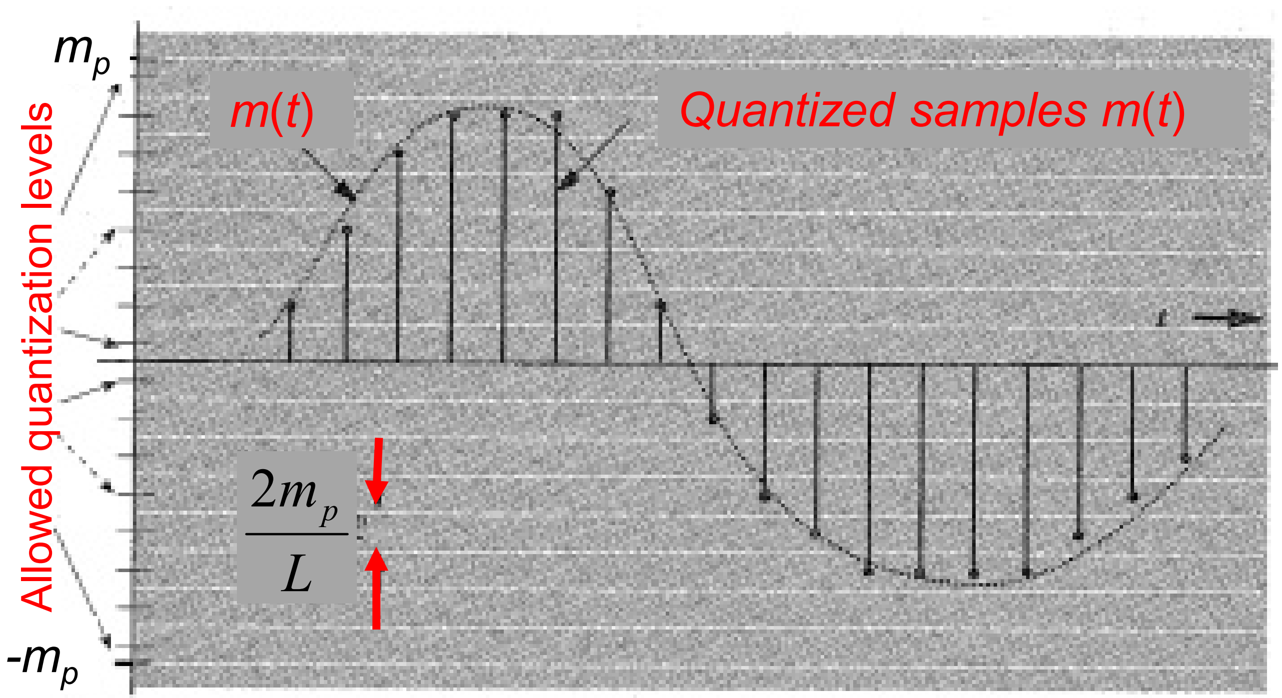

A sampled value is placed into one of these L sub-intervals, thus gets ONE of the L values

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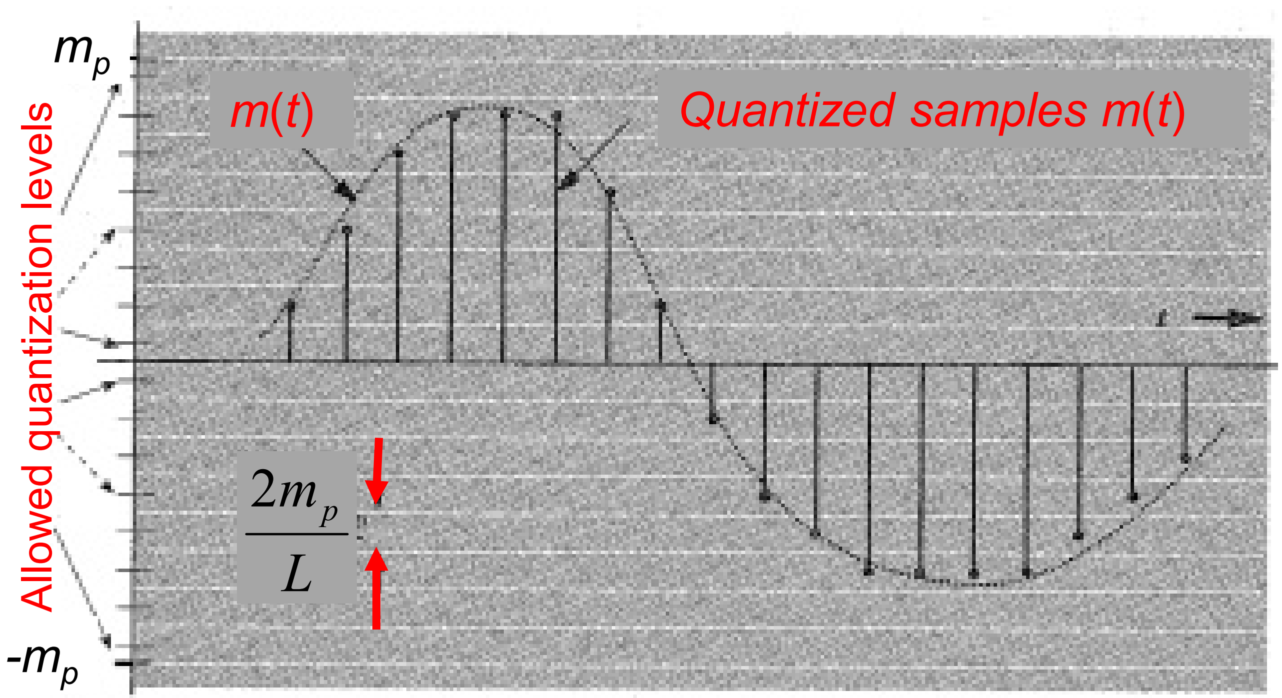

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Signal is known as L-ary digital signal

Analog to Digital Conversion of Message Signal

L-ary digital signal is converted to binary digital signal using pulse coding

Each of *L* values is encoded as a group of binary digits

Digit	Binary equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

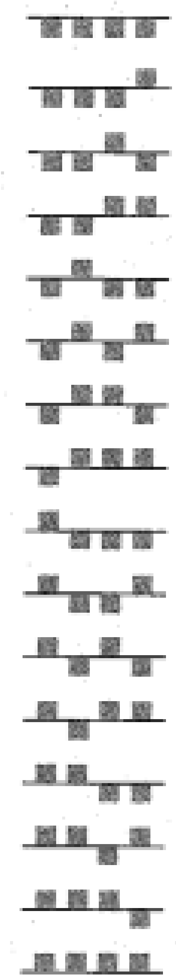
Analog to Digital Conversion of Message Signal

L-ary digital signal is converted to binary digital signal using pulse coding

Each of *L* values is encoded as a group of binary digits

Each bit is transmitted using a distinct pulse shape

Digit	Binary equivalent	Pulse code waveform
0	0000	
1	0001	
2	0010	
3	0011	
4	0100	
5	0101	
6	0110	
7	0111	
8	1000	
9	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	



Analog to Digital Conversion of Message Signal

Analog signal bandwidth to digital data rate

Audio signal b/w = 15 KHz

However, up to 3400 Hz is sufficient for articulation (intelligibility).

Fidelity is compromised!

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However, up to 3400 Hz is sufficient for articulation (intelligibility)

$B = 3400$ Hz
$f_s = 8000 > 2B$

Quantization level, $L = 256$ (8 bits)

Data rate = 8000*8 =64000 pulse/second = 64 Kbps

Analog to Digital Conversion of Message Signal

Example 2: data rate for compact disc

Fidelity is required!

Audio signal b/w = 20 KHz

$B = 20000$ Hz
$f_s = 44100\cancel{0}$ Hz $> 2B$

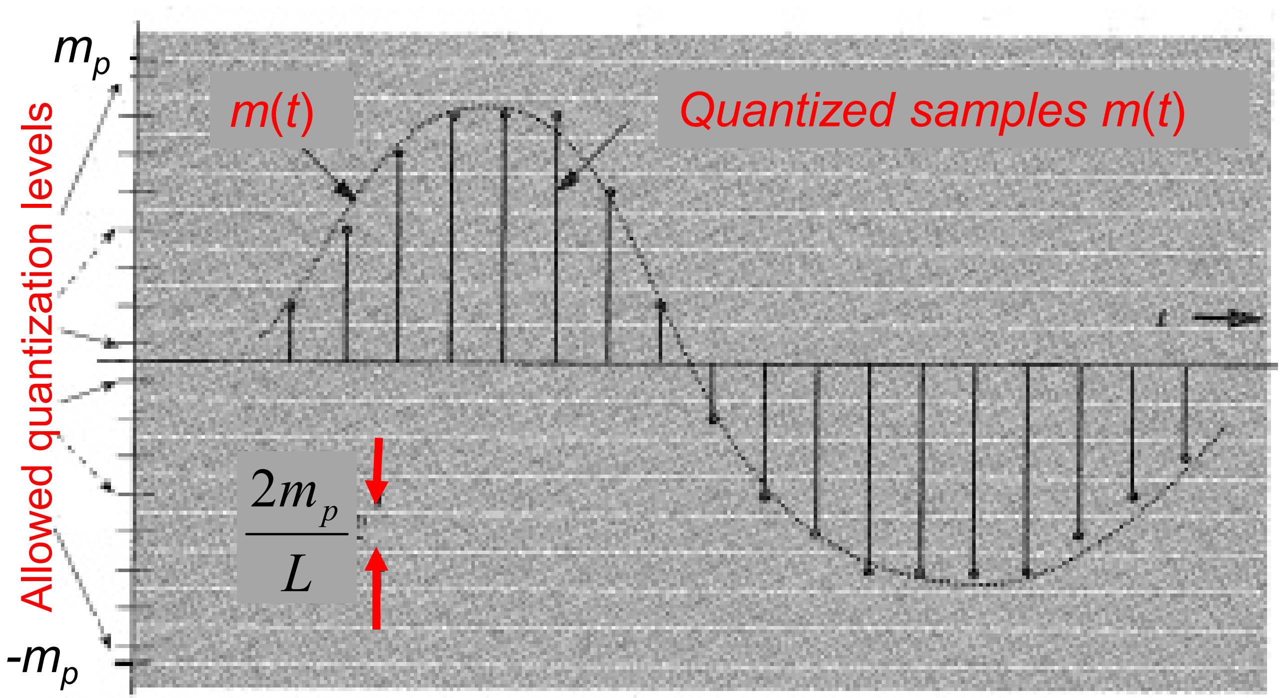
Quantization level, $L = 65{,}536$ (16 bits)

Data rate = 44100*16 = 1.4 **Mbps**

Advantages of Digital Communication

Self Study

Quantization



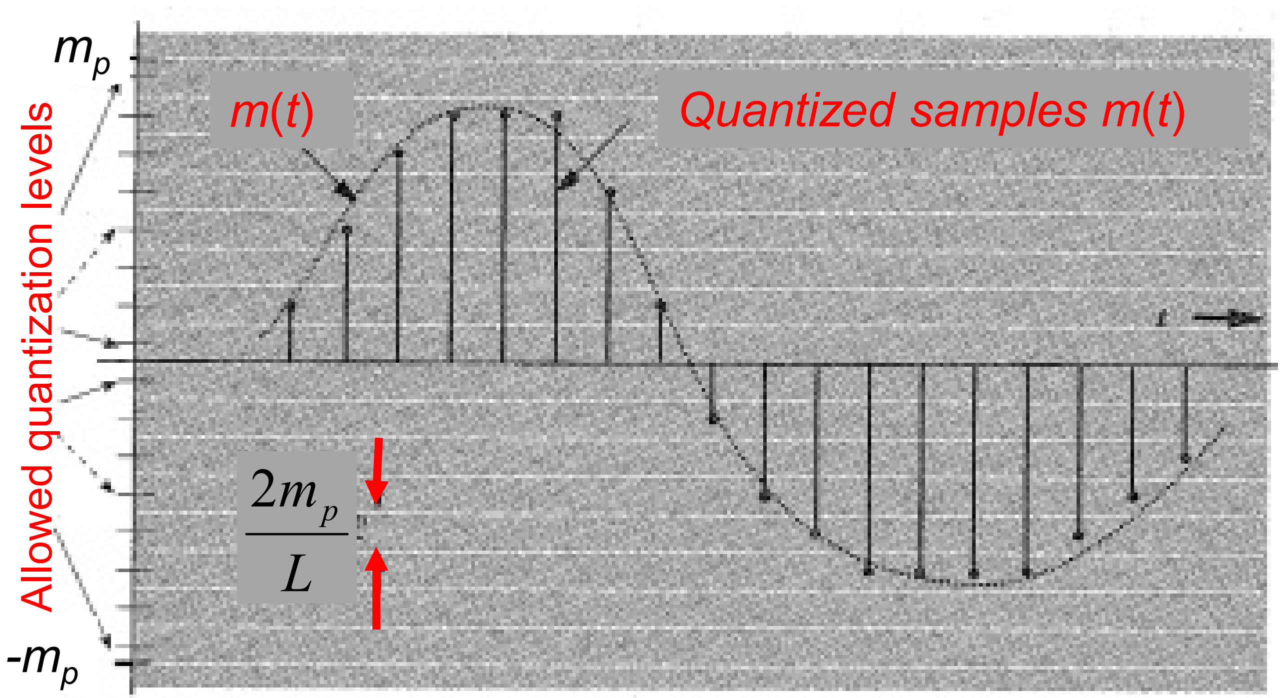
The range $(-m_p, m_p)$ is divided into L sub-intervals, each of magnitude Δv

$$\Delta v = \frac{2m_p}{L}$$

L is known as quantization level

Quantization

m_p is NOT the signal PEAK, rather is it's the LIMIT of the quantizer



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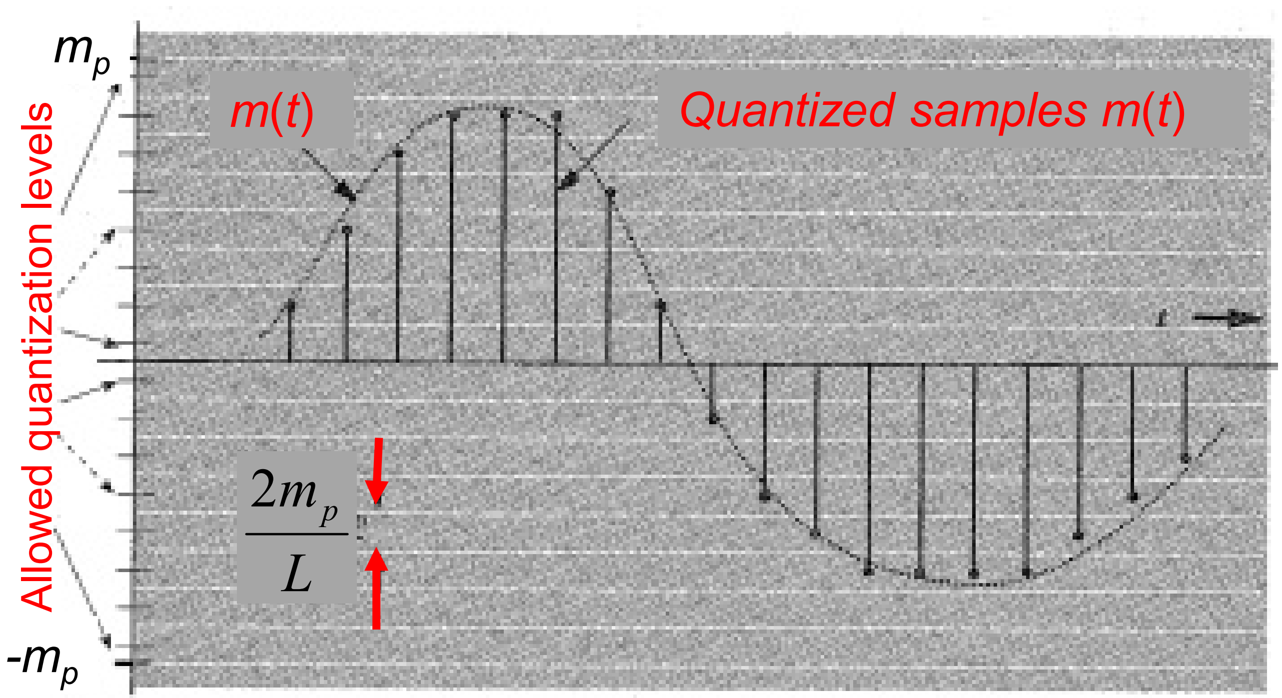
k-th sample value $m(kT_s)$ is replaced by the midpoint of an interval where it lies



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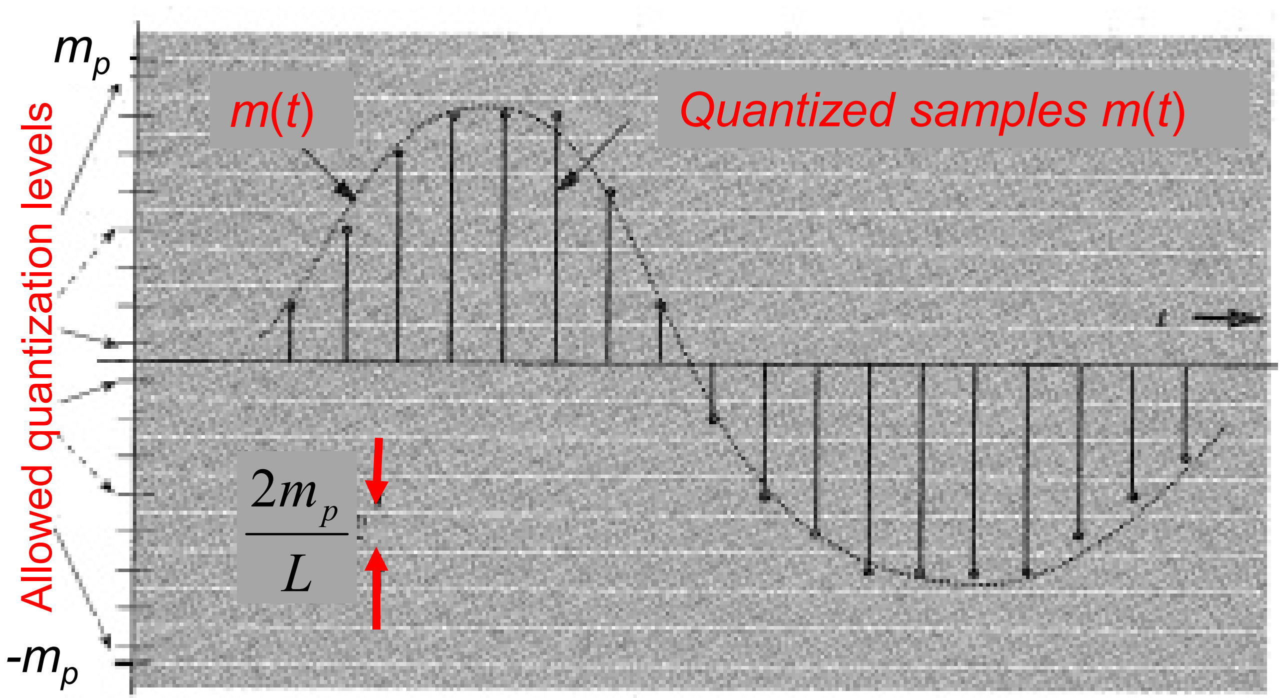

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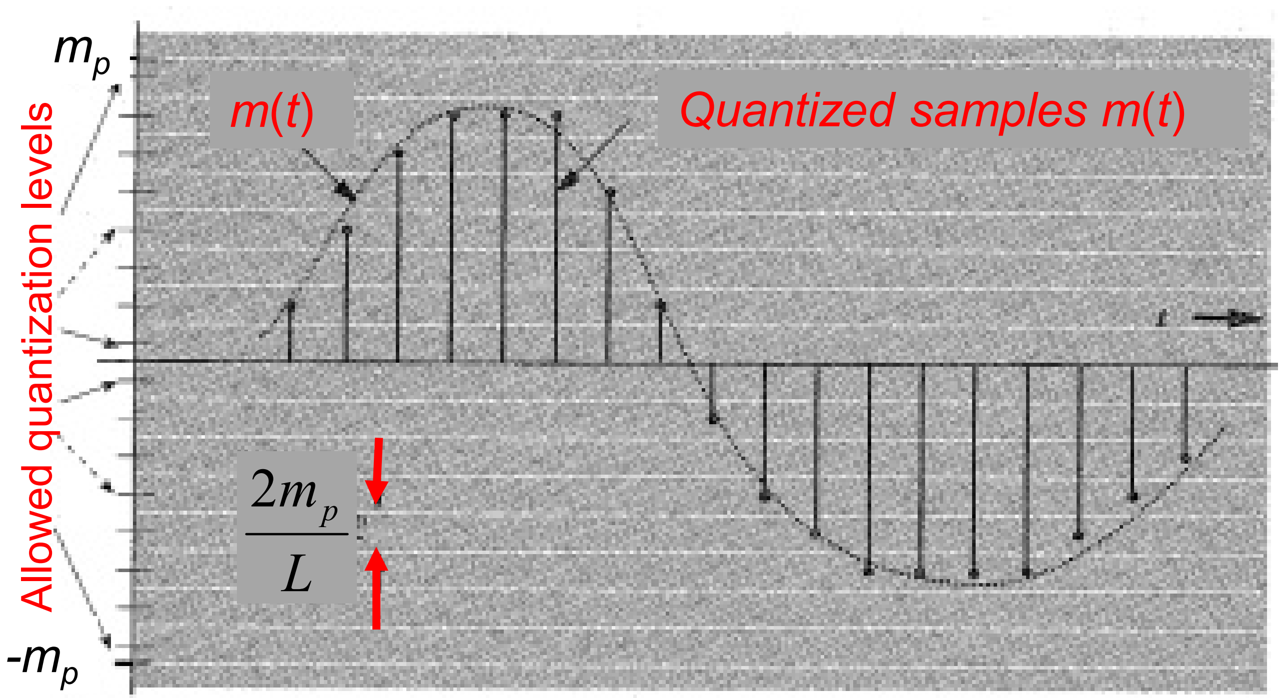
quantization error is unavoidable which is lies in (-Δv/2, Δv/2)



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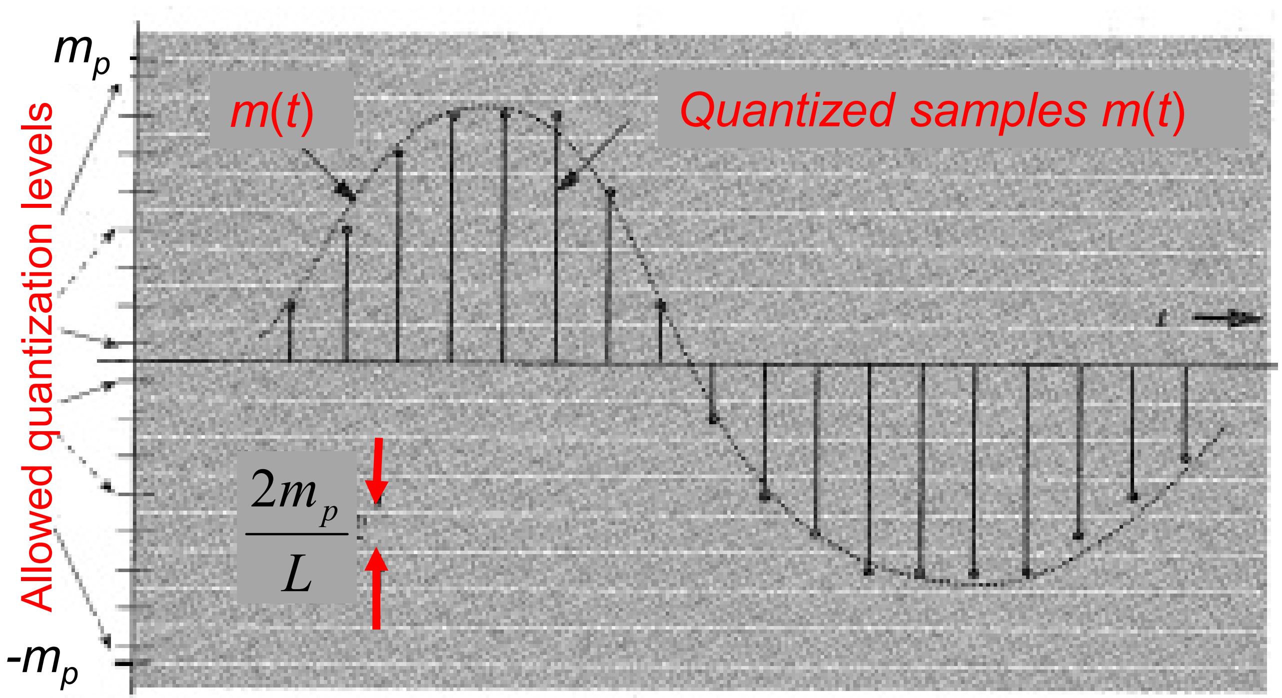

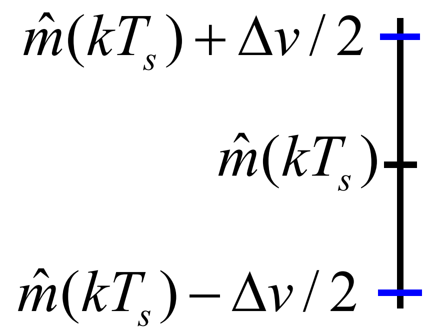

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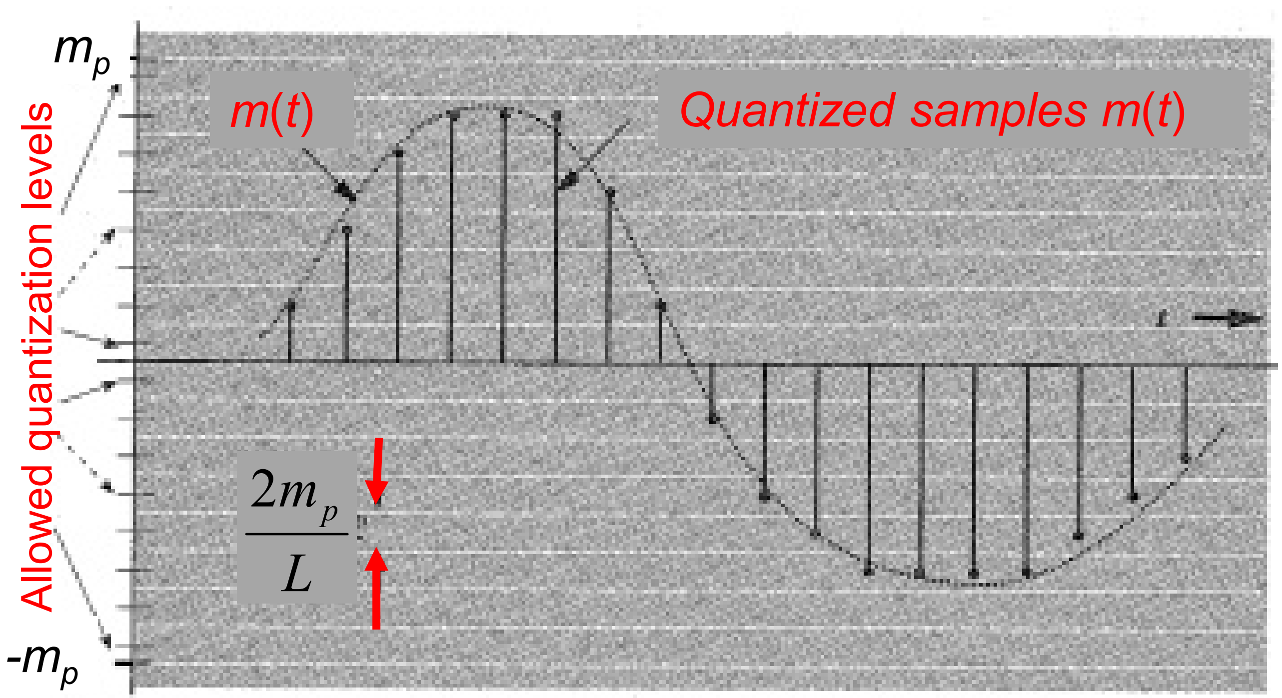

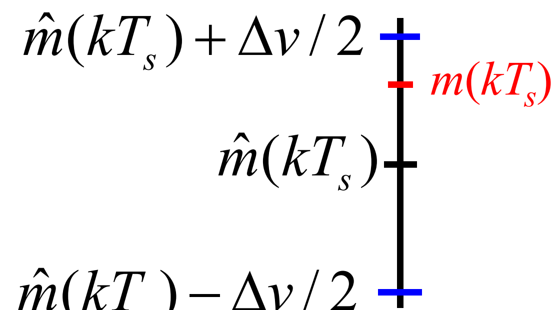

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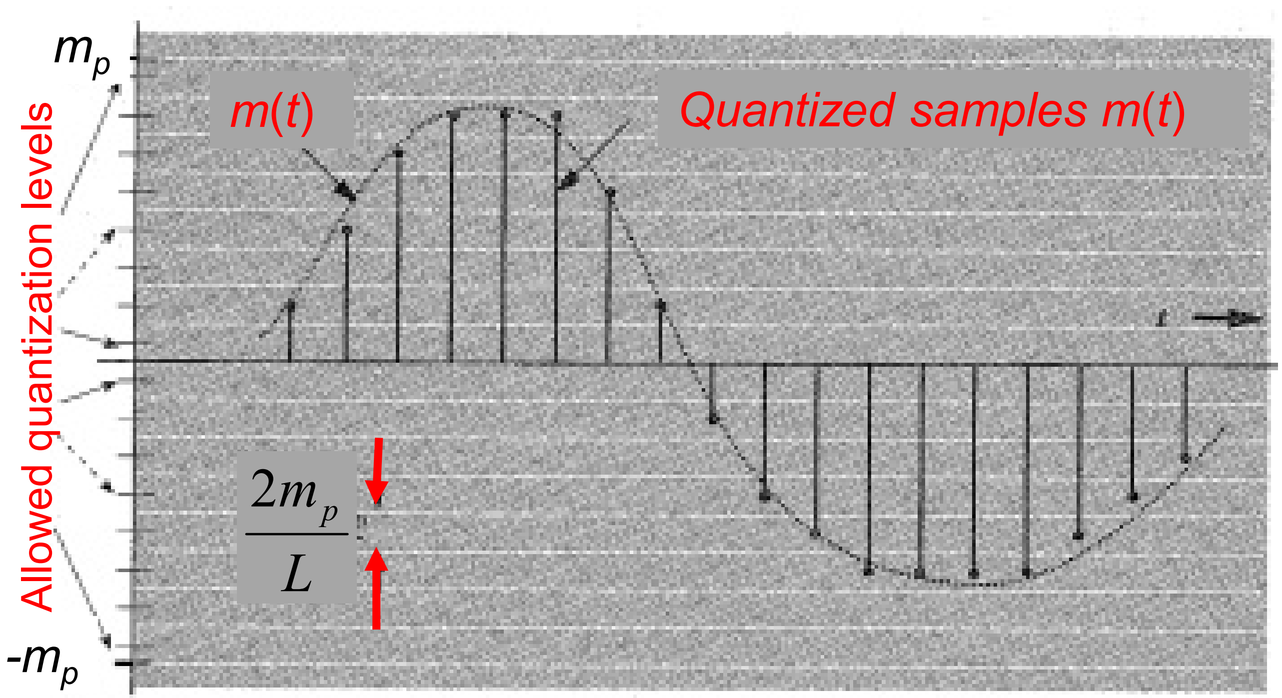

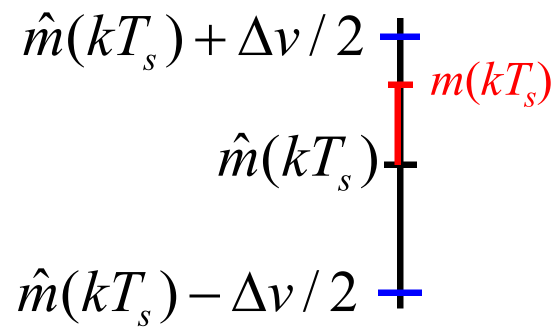

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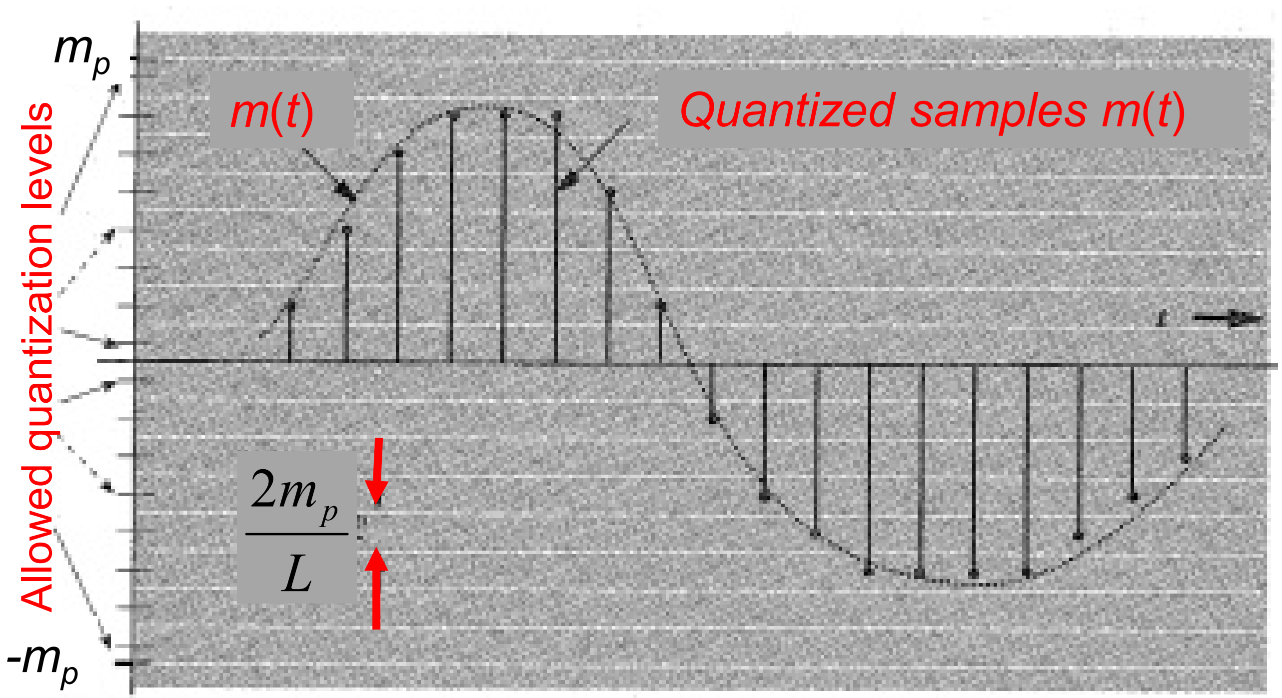

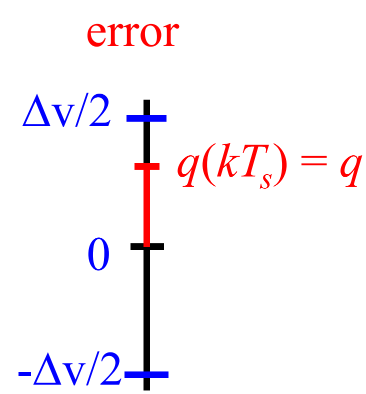

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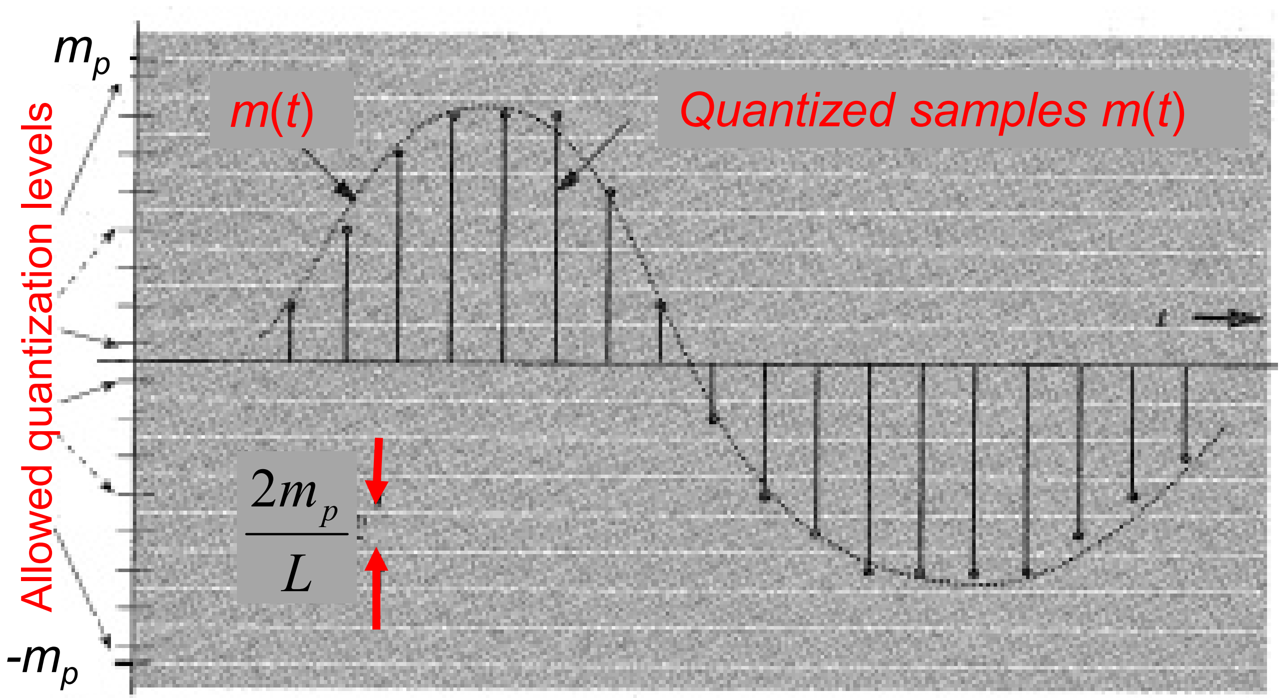

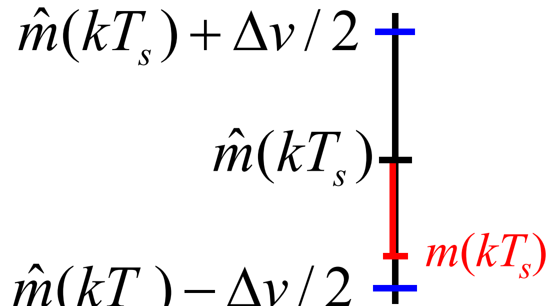

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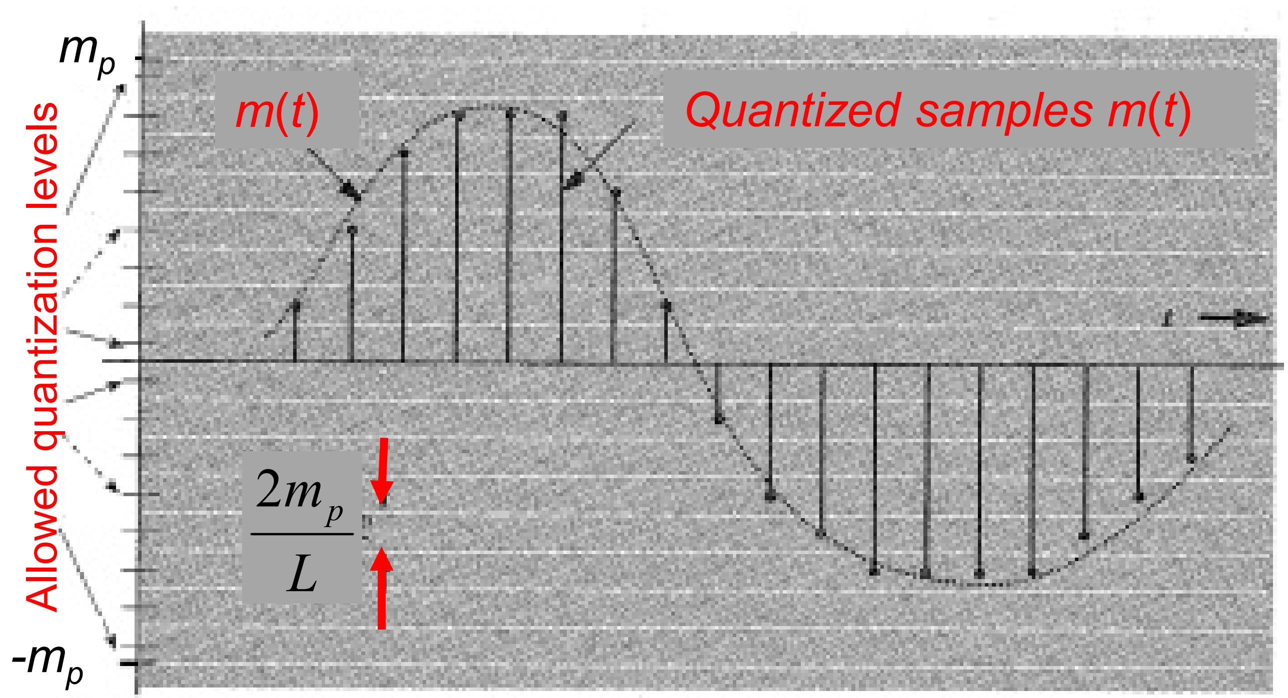

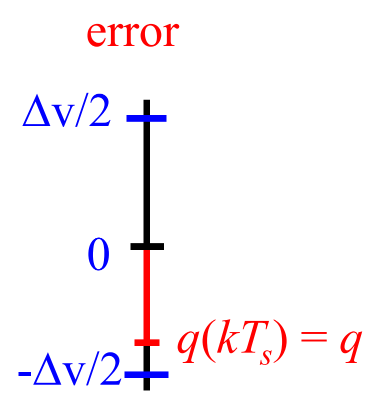

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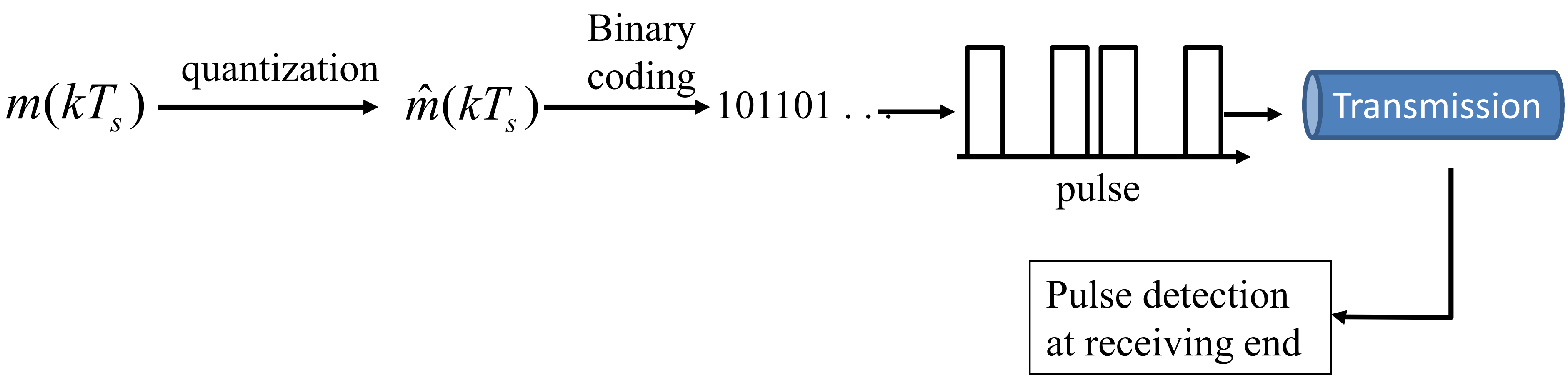

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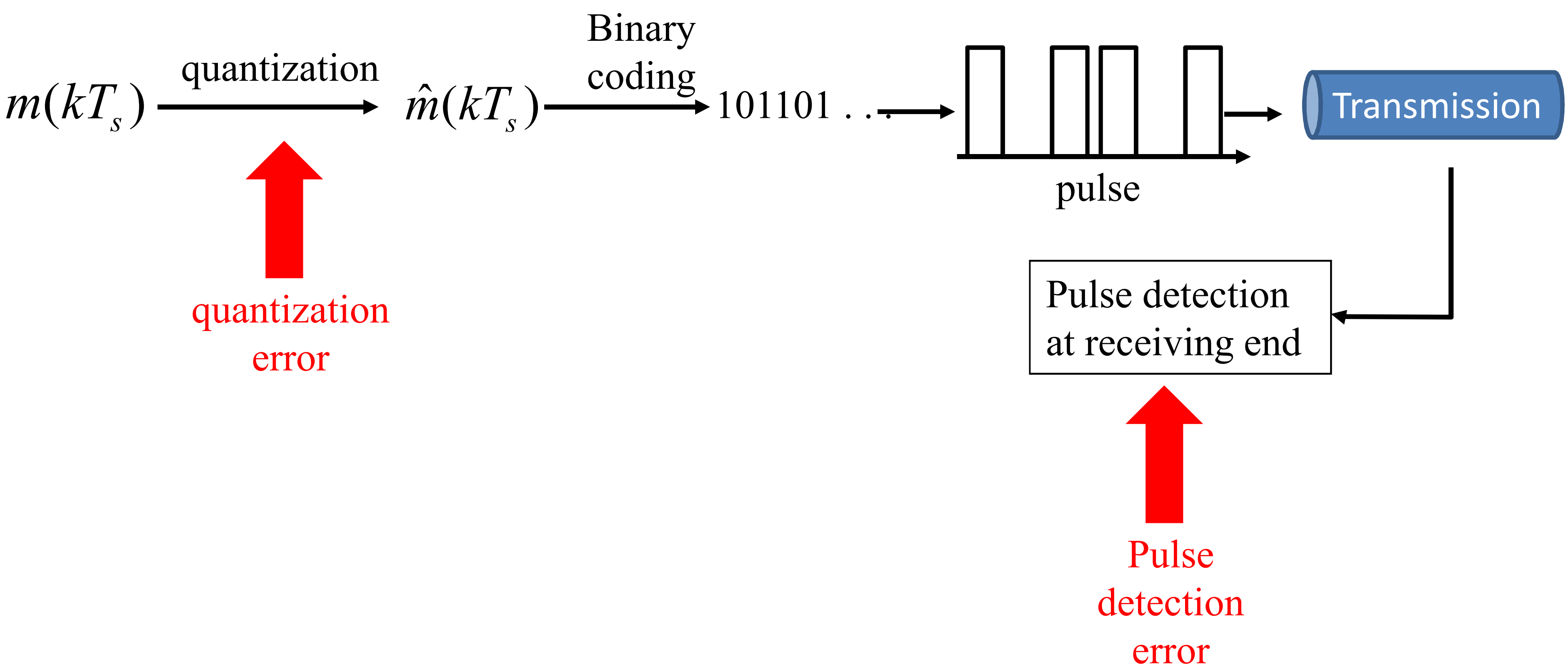

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If there were no quantization error,

$$m(t) = \sum_k m(kT_s)\,\mathrm{sinc}(2\pi Bt - k\pi)$$

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Quantization error $q(t)$,

$$q(t) = \hat{m}(t) - m(t)$$

Quantization

Quantization error or quantization noise or undesired signal,

$$q(t) = \sum_k \left[\hat{m}(kT_s) - m(kT_s)\right] \text{sinc}(2\pi Bt - k\pi)$$

$$= \sum_k q(kT_s) \text{sinc}(2\pi Bt - k\pi)$$

Quantization

Quantization noise,

$$q(t) = \sum_k \left[\hat{m}(kT_s) - m(kT_s)\right] \mathrm{sinc}(2\pi Bt - k\pi)$$

$$= \sum_k q(kT_s) \mathrm{sinc}(2\pi Bt - k\pi)$$

$q(kT_s)$ = Quantization error for kth sample

Quantization

Power or Mean square of Quantization noise,

$$\widetilde{q^2(t)} = \lim_{T \to \infty} \frac{1}{\mathrm{T}} \int_{-T/2}^{T/2} q(t)^2\, dt$$

$$= \lim_{T \to \infty} \frac{1}{\mathrm{T}} \int_{-T/2}^{T/2} \left[\sum_k q(kT_s)\,\mathrm{sinc}(2\pi Bt - k\pi) \right]^2 dt$$

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$$= \lim_{T \to \infty} \frac{1}{\mathrm{T}} \int_{-T/2}^{T/2} \left[a_1 + a_2 + a_3 + \cdots \right]^2 dt$$

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$$\overline{q^2(t)} = \lim_{T \to \infty} \frac{1}{\mathrm{T}} \int_{-T/2}^{T/2} q(t)^2 \, dt$$

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$$= \lim_{T \to \infty} \frac{1}{\mathrm{T}} \int_{-T/2}^{T/2} \left[a_1^2 + a_2^2 + a_3^2 + \cdots + 2a_1a_2 + 2a_1a_3 + 2a_1a_4 + \cdots \right] dt$$

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Quantization

Power or Mean square of Quantization noise,

$$\widetilde{\overline{q^2(t)}} = \lim_{T\to\infty}\frac{1}{\mathrm{T}}\int_{-T/2}^{T/2}\left[\sum_k q^2(kT_s)\operatorname{sinc}^2(2\pi Bt-k\pi)\right]dt$$

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We can prove that,

$$\int_{-\infty}^{\infty} \mathrm{sinc}(2\pi Bt - m\pi)\mathrm{sinc}(2\pi Bt - n\pi)dt = \begin{cases} 0 & m \neq n \\ \dfrac{1}{2B} & m = n \end{cases}$$

Quantization

Power or Mean square of Quantization noise,

$$\widetilde{q^2(t)} = \lim_{T \to \infty} \frac{1}{2BT} \sum_k q^2(kT_s)$$

Quantization

Power or Mean square of Quantization noise,

$$\overline{q^2(t)} = \lim_{T \to \infty} \frac{1}{2\mathrm{BT}} \sum_k q^2(kT_s)$$

As sampling frequency $f_s = 2B$,
$2BT$ = total no. of samples over averaging time T

RHS is the mean of the square of quantization error

Quantization

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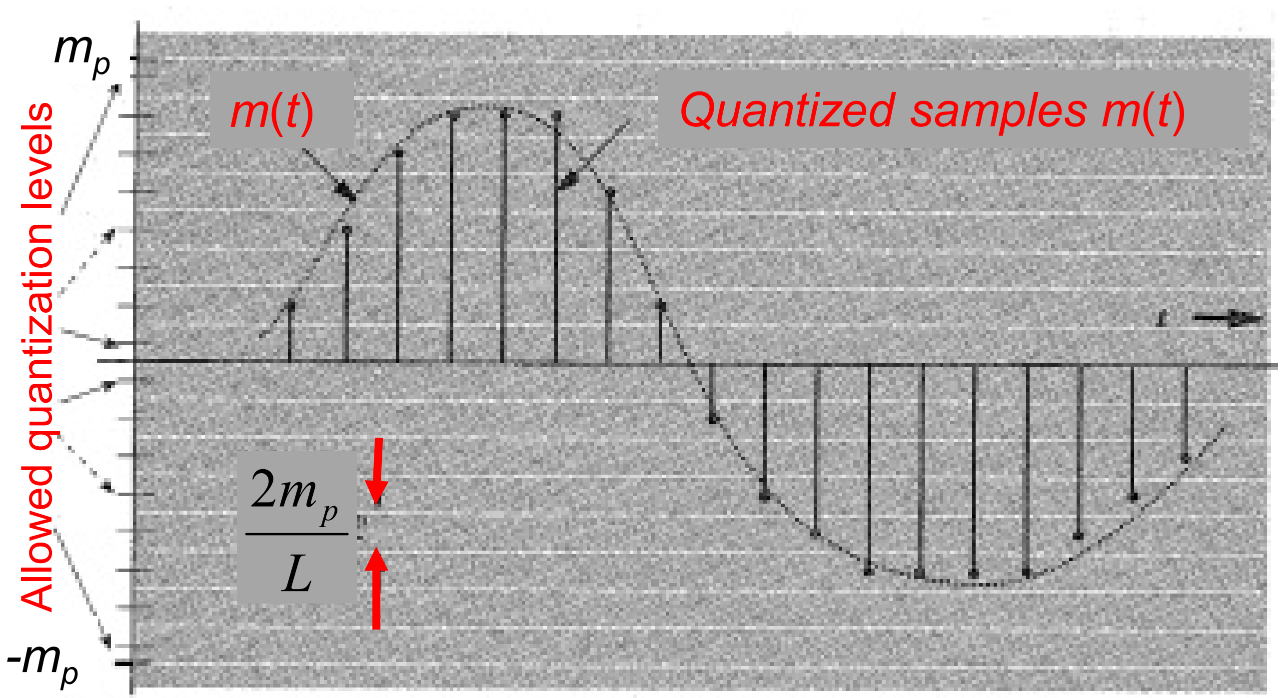
As sampling frequency $f_s = 2B$,
$2BT$ = total no. of samples over averaging time T

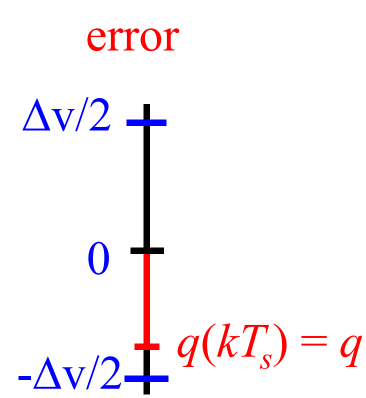
RHS is the mean of the square of quantization error

Therefore, power of quantization noise = mean square quantization error

Quantization

We know, quantization error q lies in $(-\Delta v/2,\ \Delta v/2)$



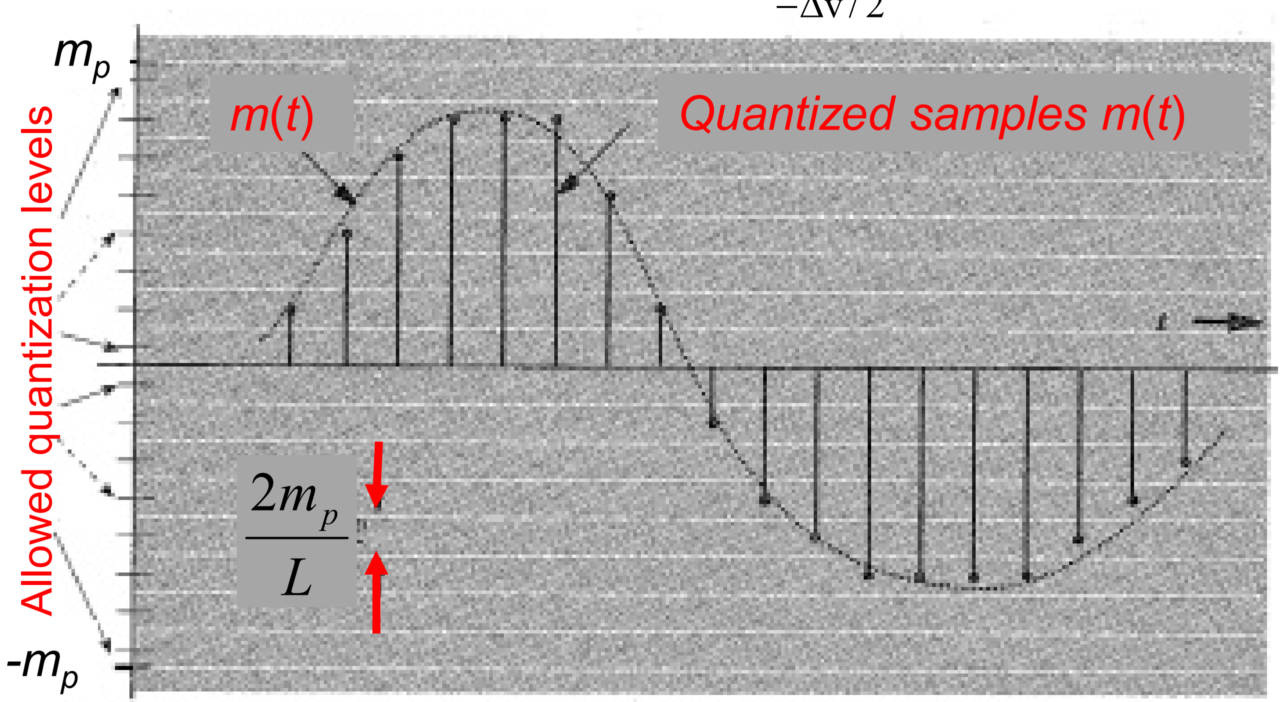


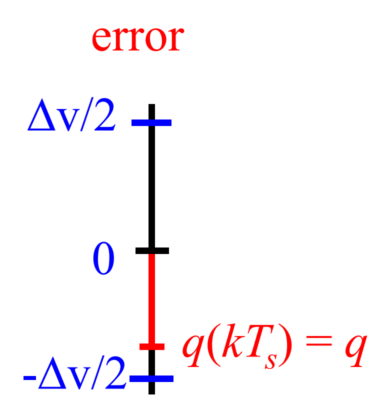
$$\Delta v = \frac{2m_p}{L}$$

Quantization

Mean square quantization error is given by

$$\widetilde{q^2} = \frac{1}{\Delta v}\int_{-\Delta v/2}^{\Delta v/2} q^2\, dq$$





$$\Delta v = \frac{2m_p}{L}$$

Quantization

Mean square quantization error is given by

$$\widetilde{q^2} = \frac{1}{\Delta v} \int_{-\Delta v/2}^{\Delta v/2} q^2 dq \quad = \frac{(\Delta v)^2}{12}$$

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$$\widetilde{q^2} = \frac{1}{\Delta v}\int_{-\Delta v/2}^{\Delta v/2} q^2\,dq = \frac{(\Delta v)^2}{12} = \frac{m_p^2}{3L^2}$$

where, $\Delta v = \dfrac{2m_p}{L}$

Quantization

Mean square quantization error is given by

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We proved, power of quantization noise (N_0) = mean square quantization error

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$$N_0 = \widetilde{q^2(t)} = \widetilde{q^2} = \frac{m_p^2}{3L^2}$$

Quantization

power of quantization noise (N_0) = mean square quantization error

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Assume, power of message signal (S_0) is given by $S_0 = \widetilde{m^2(t)}$

Quantization

power of quantization noise (N_0) = mean square quantization error

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Assume, power of message signal (S_0) is given by $S_0 = \overline{m^2(t)}$

Signal-to-noise ratio (SNR) is

$$SNR = \frac{S_0}{N_0} = 3L^2 \frac{\overline{m^2(t)}}{m_p^2}$$

Quantization

$$SNR = \frac{S_0}{N_0} = 3L^2 \frac{\widetilde{m^2(t)}}{m_p^2}$$

- Higher SNR means higher quality of received signal
- L increases SNR
- Higher limit of quantizer (m_p) decreases SNR

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S_0 varies

- from speaker to speaker
- due to different length of connecting circuits

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S_0 varies

- from speaker to speaker
- due to different length of connecting circuits

For these reasons,

- *SNR* varies widely
- Quality of received signal deteriorates remarkably for soft speakers

Quantization

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S_0 varies

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- due to different length of connecting circuits

For these reasons,

- SNR varies widely
- Quality of received signal deteriorates remarkably for soft speakers

However statistically,

- Small amplitudes (soft speakers) predominate in speech
- Larger amplitudes (loud speakers) are less frequent