

Sorting 1

"I'd rather attempt to do something great and fail, than to attempt nothing and succeed."

~ Robert H. Schuller



Good

Evening ☺

Content for Today's class

→ Basics of sorting → Stable / unstable
 └─ Inplace sorting

01. Find k^{th} smallest element

02. Selection sort

03. Bubble sort

04. Merge two sorted Arrays

05. Mergesort

06. Inversion count

Sorting: Arranging the data in specific order based on the parameter

1 2 3 5 7 → Increasing (parameter → values)

9 6 3 2 1 → Decreasing (parameter → values)

Quiz → 1 7 2 9 24 → Increasing (parameter → count of factors)
factors 1 2 2 3 8

Why sorting? → Searching of an ele becomes easy
→ accessing of ele is also easy

stable / unstable → Two data points have the same parameter value & the order is preserved before & after the sorting, then this is stable sorting.

sort data in increasing order based on marks

Name	Marks
Mohit	0
Aman	100
Shreesh	45
Kajal	32
Aayush	45
Aditya	92

Mohit
Kajal
Shreesh
Aayush
Aditya
Aman
↑
stable

Name	Marks
Mohit	0
Kajal	32
Aayush	45
Shreesh	45
Aditya	92
Aman	100

↑
unstable

$\text{arr}[] = 1 \ 5 \ 2 \ 6 \ 2$ \Rightarrow sort this array para = values

Sort 1 = 1 2 2 5 6 \rightarrow stable

Sort 2 = 1 2 2 5 6 \rightarrow unstable

Inplace sorting Algorithm \rightarrow Sorting in the same array without extra space. $SC = O(1)$

Q1. Given $\text{arr}[n]$ elements, find k^+ smallest element

\hookrightarrow in the sorted arr,
return $\text{arr}[k-1]$

$\text{arr}[8] = \boxed{2 \ 8 \ 4 \ -4 \ 6 \ 7 \ 5 \ 10 \ -1}$

$k = 3$

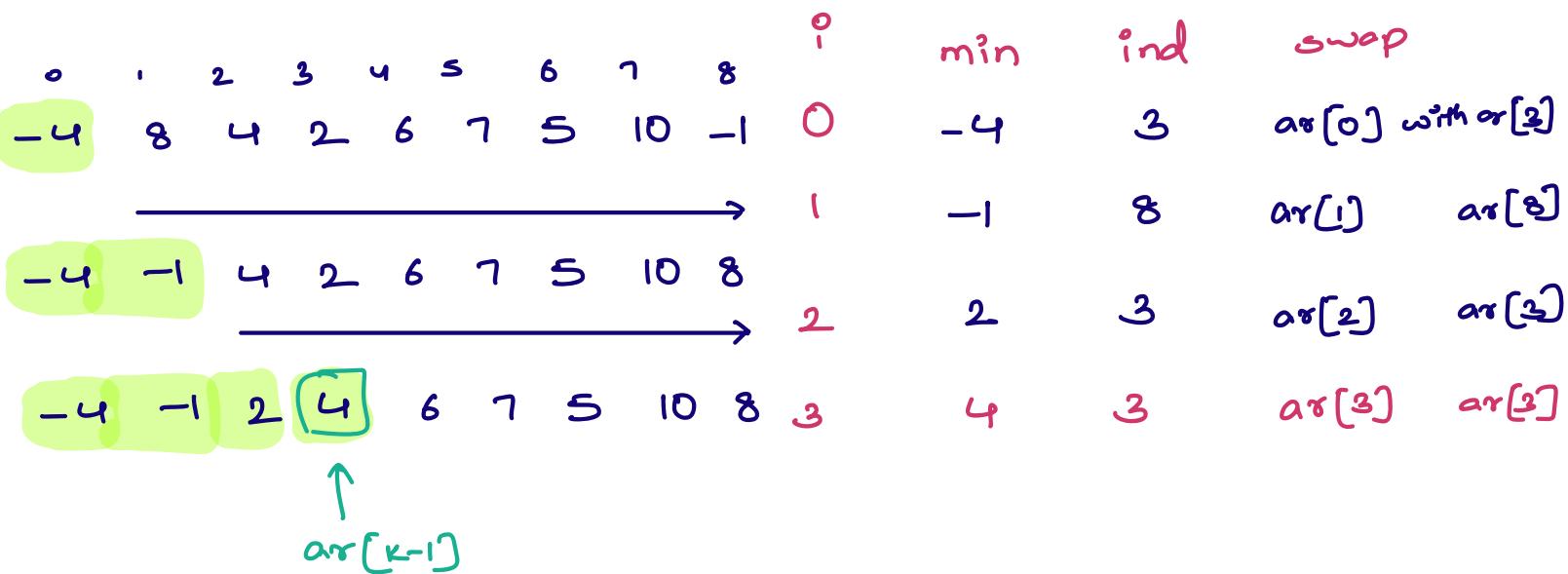
Idea 1 \rightarrow Sort the array & return $\text{arr}[k-1]$

TC: $O(n \log n)$

Idea 2 \rightarrow Iterate on arr, find min & keep it at its correct position. Repeat this process for k times

K=4

$ar[8] =$	2	8	4	-4	6	7	5	10	-1
	0	1	2	3	4	5	6	7	8



Pseudocode

k^{th} smallest ele (arr, K)

```
for( i=0 ; i < k ; i++ ) {
```

```
    min = ar[i]      minidx = i
```

```
    for( j=i ; j < n ; j++ )
```

```
        if ( ar[j] < min ) {
```

```
            min = ar[j]
```

```
            minidx = j
```

TC: $O(k+n)$

SC: $O(1)$

```
    swap( ar[i], ar[minidx] );
```

```
return ar[k-1];
```

Selection sort

selection sort (arr) {

 for (i=0 ; i < n ; i++) {

 min = arr[i] minidx = i

 for (j = i ; j < n ; j++)

 if (arr[j] < min) {

 min = arr[j]

 minidx = j

 swap (arr[i] , arr[minidx]) :

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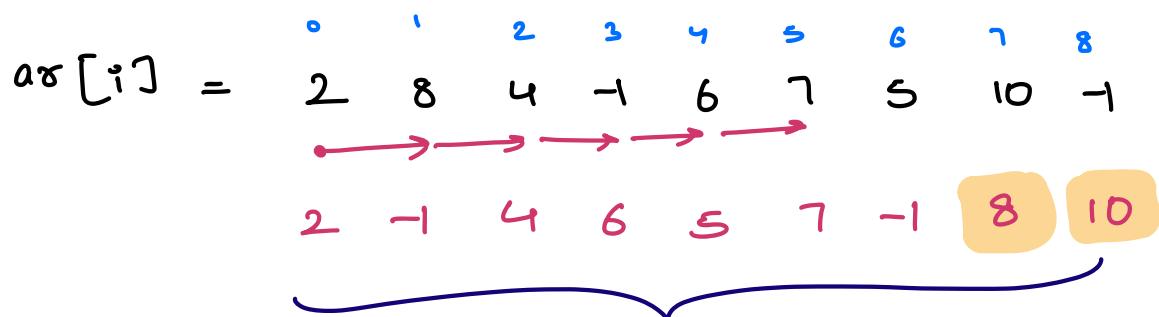
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Q2. Given $ar[N]$, we can only swap adjacent elements, sort the array in increasing order



Repeat this process

for $(n-1)$ time

bubble sort ([] arr) :

```
for (i=0; i<n; i++) {  
    for (j=0; j<n-1; j++) {  
        if (ar[j] > ar[j+1]) {  
            swap(ar[j], ar[j+1]);  
        }  
    }  
}
```

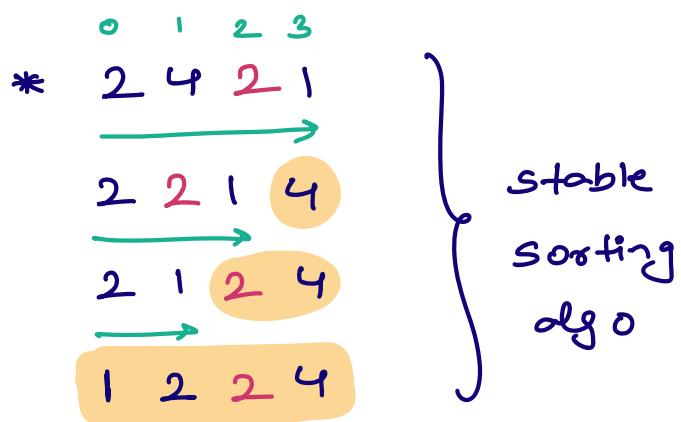
TC: $O(n^2)$

SC: $O(1)$

↳ *inplace*

sorting algo

$$\text{No. of swaps} = \frac{n * (n-1)}{2}$$



Q3. Given 2 sorted arrays . $a[N]$, $b[M]$, create $c[]$ which contains overall sorted data

Eg:- $a[4] : \{7 \ 10 \ 11 \ 14\}$

$$a[3] : \{3 \ 6 \ 10\}$$

$$b[3] : \{3 \ 8 \ 9\}$$

$$b[2] : \{5 \ 9\}$$

$$c[4+3] = \{3 \ 7 \ 8 \ 9 \ 10 \ 11 \ 14\}$$

$$c[3+2] = \{3 \ 5 \ 6 \ 9 \ 10\}$$

* Idea 1 \rightarrow Copy both the arrays in C & then sort it

$$TC: (n+m) \log(n+m)$$

Idea 2 two pointers to compare ele in both the arrays

$A[] = \{ ?_0, 10_1, 11_2, 14_3 \}$

P_1
↓

$B[] = \{ 3_0, 8_1, 9_2 \}$ out of arry
↑
 P_2

$C[] = \{ 3, 7, 8, 9, 10, 11, 14 \}$

Merge two sorted arrays

int [] merge (int [n] a , int [m] b)

int [] c = new int [n+m];

$P_1 = 0$

$P_2 = 0$

$K = 0$ // fill ele in C array

while ($P_1 < n$ && $P_2 < m$)

if ($a[P_1] \leq b[P_2]$) {

$c[K] = a[P_1]$

$P_1++;$

$K++;$

else {

c[k] = b[p₂]

p₂++;

k++;

2

3

TC: O(n+m)

SC: O(1)

while (p₁ < n) {

c[k] = a[p₁]

p₁++;

k++;

3

while (p₂ < m) {

c[k] = b[p₂]

p₂++;

k++;

3

return c

3

* If the question asks you to return an array
then SC: O(1)

04 Given arr [N] elements & 3 indices s, m, e &

subarray [s m] is sorted

subarray [m+1 e] is sorted

Sort the subarray from [s e]

$$\frac{s}{2} \frac{m}{6} \frac{e}{9}$$

P₁
↓

P₂
↓

arr[12] = { 4 8 -1 2 6 9 11 3 4 7 13 10 0 }
0 1 2 3 4 5 6 7 8 9 10 11
s m m+1 e

$$c[e-s+1] = \{-1 2 3 4 6 7 9 11\}$$

int [] merge (int a[], s, m, e)

int [] c = new int [e-s+1];

P₁ = s

P₂ = m+1

K = 0 // fill ele in C array

while ($P_1 \leq m$ && $P_2 \leq e$)

 if ($a[P_1] \leq a[P_2]$) {

$c[K] = a[P_1]$

$P_1++;$

$K++;$

 } else {

$c[K] = a[P_2]$

$P_2++;$

$K++;$

}

while ($P_1 \leq m$) {

$c[K] = a[P_1]$

$P_1++;$

$K++;$

}

while ($P_2 \leq e$) {

$c[K] = a[P_2]$

$P_2++;$

$K++;$

}

return c;

* Fill the original array.

3

* Given $ar[N]$, sort it without inbuilt function

$$\hookrightarrow N = 100$$

Case - I

$ar[N] =$



TC

iterations

$$O(n^2)$$

$$10^4$$

Case - II =



$$\left(\frac{n}{2}\right)^2 * 2 + n$$



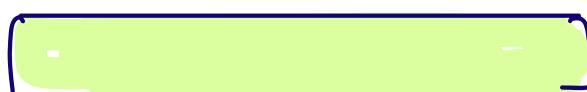
$$\gamma_2$$

$$\gamma_2$$

$$= \left(\frac{100}{2}\right)^2 * 2 + 100$$

$$= 5100$$

Case - III



$$= \left(\frac{n}{4}\right)^2 * 4 + 2n$$



$$\gamma_2$$

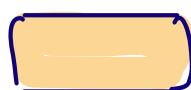


$$\gamma_2$$

$$\Rightarrow 2700$$



$$\gamma_4$$



$$\gamma_4$$



$$\gamma_4$$



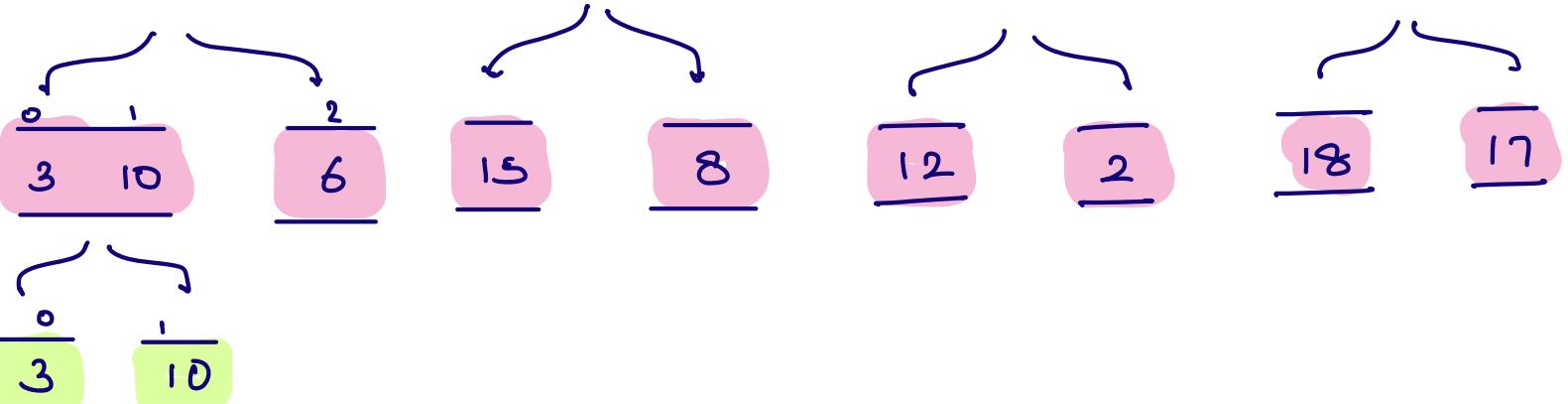
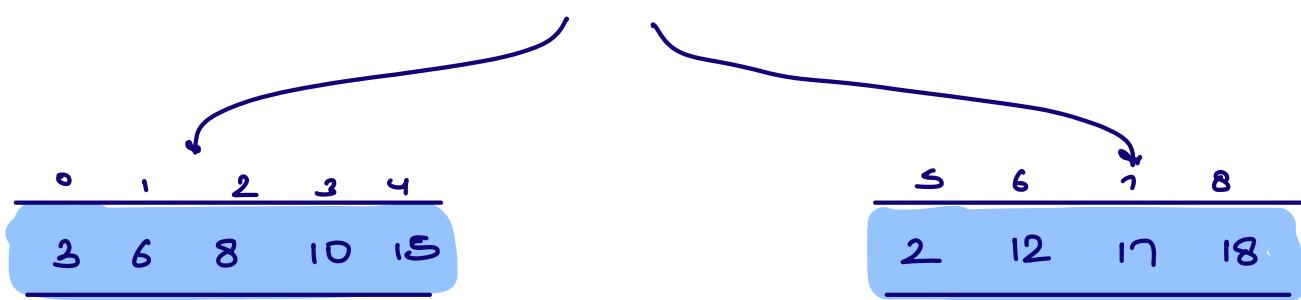
$$\gamma_4$$

Idea for Merge Sort

↳ Split the array until it gets reduced to one ele & then start merging those arrays

arr =

0	1	2	3	4	5	6	7	8
2	3	6	8	10	12	15	17	18



* Stable sorting algorithm

mergesort (arr, s, e)

if ($s == e$) return

int $m = \frac{s+e}{2}$

mergesort (arr, s, m)

mergesort (arr, m+1, e)

merge (arr, s, m, e)

TC: $O(n \log n)$
SC: $O(n)$

3

Recurrence relation

$$f(n) = f(n_2) * 2 + n \Rightarrow \text{Expand by yourself}$$

Inversion count

Given an integer array, count the number of inversion pairs in the array

Inversion pair $\rightarrow (i, j)$ where $(i < j \text{ } \&\& \text{ } a[i] > a[j])$

$$A = [4 \ 5 \ 1 \ 2 \ 6 \ 3]$$

0 1 2 3 4 5

No. of inversion pair = 7

(0,2) (0,3) (0,5)

(1,2) (1,3) (1,5)

(4,5)

$$A = [8, 3, 4]$$

0 1 2

No. of inversion pair = 2

(0,1) (0,2)

Brute force \rightarrow Check for all pairs (i, j) & $i < j$ and $a[i] > a[j]$, increase the count

TC: $O(n^2)$

SC: $O(1)$

Idea 2 \Rightarrow Use Merge sort

$$\text{ans} = 6 + 2 + 4 + 5 + 2 + 3 + 1 + 2 + 1 = 26$$

$$arr[] = \{ 10, 3, 8, 15, 6, 12, 2, 18, 7, 1 \}$$

$\overbrace{\hspace{10em}}$ $\overbrace{\hspace{10em}}$ $\overbrace{\hspace{10em}}$	$\overbrace{\hspace{10em}}$ $\overbrace{\hspace{10em}}$ $\overbrace{\hspace{10em}}$
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Sort : $\overbrace{\hspace{10em}}$
 $\overbrace{\hspace{10em}}$
 $\overbrace{\hspace{10em}}$

$\overbrace{\hspace{10em}}$
 $\overbrace{\hspace{10em}}$
 $\overbrace{\hspace{10em}}$

Merge = $\overbrace{\hspace{10em}}$

$ar[p_1] < ar[p_2] \rightarrow \text{no inversion}$

$ar[p_1] > ar[p_2] \rightarrow \text{inversion}$

$$\text{Inversion} = 5 + 5 + 3 + 1 = 14$$

$$\begin{array}{llll}
 (3, 1) & (3, 2) & (8, 7) & (15, 12) \\
 (6, 1) & (6, 2) & (10, 7) & \\
 (8, 1) & (8, 2) & (15, 7) & \\
 (10, 1) & (10, 2) & & \\
 (15, 1) & (15, 2) & &
 \end{array}$$

$$\text{Inversion count} = m - p_1 + 1$$

Code $\rightarrow \{ \text{TODO} \}$