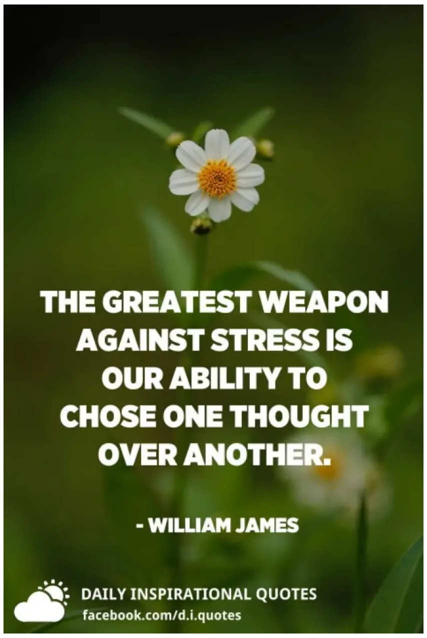
Maths : GCD



Easiest class of Advance module

Content

- → GCD intro
- → Properties of GCD
- → GCD function
- → PUBG
- → Delete one

GCD = Greatest common divisor

HCF = Highest common factor

GCD(A, B) = greatest factor that divides both a & b.

$$GCD(A,B) = x \quad \left.\begin{array}{l} a \,\%\, x = 0 \\ b \,\%\, x = 0 \end{array}\right\} \; x \rightarrow \text{highest factor}$$

GCD(15, 25) = 5

15	25
1	1
3	5
5	25
15	

GCD(12, 30) = 6

12	30
1	1
2	2
3	3
4	5
6	6
12	10
	15
	30

GCD(10, −25)

10	−25
1	−25
2	−5
5	−1
10	1
	5
	25

GCD(0, 4)

0	4
1	1
2	2
3	4
4	
⋮	
∞	

GCD(0, −10)

0	−10
1	−10
2	−5
3	−2
⋮	−1
10	1
⋮	2
∞	5
	10

GCD(0, 0)

0	0
1	1
2	2
3	3
⋮	⋮
∞	∞

* Properties of GCD

01. $GCD(A, B) = GCD(B, A)$

02. $GCD(A, B) = GCD(|A|, |B|)$ $|x|$ = absolute val

03. $GCD(0, A) = |A|$

04. $GCD(A, B, C) = GCD(A, GCD(B, C))$ ✓

$= GCD(B, GCD(A, C))$ ←

$= GCD(C, GCD(A, B))$

$GCD(2, 3, 4) = GCD(2, GCD(3, 4))$

$= GCD(2, 1) \Rightarrow \boxed{1}$

* $GCD(3, GCD(2, 4))$

$GCD(3, 2) \Rightarrow \boxed{1}$

$GCD(-2, -4)$

-2	-4
-1	-2
1	-1
2	1
	2
	4

* Special property of GCD

Given $A, B > 0$ && $A \geq B$ && $GCD(A, B) = x$

$GCD(A, B) = x$ $\quad A \% x = 0$ $\quad B \% x = 0$

x ✓

B ✓

$A-B$ ✓

$GCD(A-B, B) = x$

↓

$(A-B) \% x = 0$ & $B \% x = 0$

$\Rightarrow \{A \% x - B \% x + x\} \% x$

$\{0 - 0 + x\} \% x \Rightarrow x \% x = 0$

* Claim $\rightarrow$ $A, B > 0$ && $A \geq B$

$GCD(A, B) = GCD(A-B, B)$

* $GCD(23, 5) = GCD(18, 5) \rightarrow GCD(13, 5) \rightarrow GCD(8, 5)$

↓

$GCD(3, 5)$

$$GCD(23,5) = GCD(3,5)$$
$$= GCD(23 \% 5, 5)$$

$$GCD(A,B) = GCD(A-1B, B)$$
$$= GCD(A-2B, B)$$
$$= GCD(A-3B, B)$$
$$\vdots$$
$$= GCD(A-\underbrace{xB}, B)$$

greates multiple of B $\leq$ A

$$\boxed{GCD(A,B) = GCD(A \% B, B)}$$

* Write a function to find GCD(A, B)

Q: $GCD(24,16) \overset{A>B}{=} GCD(8,16) \overset{A<B}{=} GCD(8,16) \rightarrow$ Infinite loop

$$\boxed{GCD(A,B) = GCD(B, A \% B)}$$

* $GCD(24,16) = GCD(16,8) = GCD(8,0) = \boxed{Ans = 8}$

* $GCD(14,21) = GCD(21,14) = GCD(14,7) = GCD(7,0) \rightarrow \boxed{Ans = 7}$

* $GCD(3,5) = GCD(5,3) = GCD(3,2) = GCD(2,1)$

$\Downarrow$

$GCD(1,0) = \boxed{Ans = 1}$

* Given $a, b > 0$

```
int gcd(a,b){
    if (b==0) return a;
    return gcd(b, a%b)
}
```

TC: $O(\log_2 \max(a,b))$

```
main(){
    a = -ve
    b = -ve
    gcd(|a|, |b|);
}
```

* Given ar[ ], calculate GCD of entire array

ar[3] = {6, 12, 15}

ans = 6 → 6 → 3 Ans = 3

```
int gcdarr (int [] ar )
{
    ans = ar[0];

    for(i=1; i<n; i++){
        ans = gcd(ans, ar[i]);
    }
    return ans;
}
```

TC: O(n log max,...)

10:12 → 10:20 pm

* A = {5, 5} Ans = 5

Q = Let N players are playing PUBG &

$A[i]$ = health of i^{th} player

* If i^{th} player attacks j^{th} player →

(i) If $(A[i] \leq A[j]) \Rightarrow A[j] = A[j] - A[i]$

(ii) else $A[j] = 0$ (die)

Find min health of the last surviving person.

$A = \{6, 4\} = \{6, 0\}$ = last person health = 6

$A = \{6, 4\} = \{2, 4\} = \{2, 2\} = \{2, 0\}$

Ans = 2

* $A = \{6, 10, 15\} \Rightarrow$ 6 attack $= \{6, 4, 9\}$

$\Rightarrow$ 4 attack $= \{2, 4, 5\}$

$\Rightarrow$ 2 attack $= \{2, 2, 3\}$

$\Rightarrow$ 2 attack $= \{2, 0, 1\}$

$\Rightarrow$ 1 attack $= \{1, 0, 1\}$

$\Rightarrow$ 1 attack = $\{0, 0, 1\}$

Ans = 1

* $A = \{6, 10, 15\} = \{0, 0, 15\}$ Ans = 15

* $A = \{6, 10, 15\} = \{0, 10, 5\}$

$= \{0, 5, 5\}$

$= \{0, 0, 5\}$

* Observation → Always make sure that weak person attacks

$x \le y \le z$

x & y fight = $(x, y) = (x, y-x) = (x, y-2x) \ldots$

.... y becomes less than x

$y = y \% x$

→ GCD of x & y

GCD(x, y) will fight with z

Ans = GCD of complete array

* Delete one

Given ar[N] elements, we have to delete one element such that gcd of remaining ele becomes max.

$$ar[\,] = \{\overset{0}{24}\quad \overset{1}{16}\quad \overset{2}{18}\quad \overset{3}{30}\quad \overset{4}{15}\}$$

GCD

$\{\overset{0}{\cancel{24}}\quad \overset{1}{16}\quad \overset{2}{18}\quad \overset{3}{30}\quad \overset{4}{15}\}$ 1

$\{\underset{24}{\boxed{\overset{0}{24}}}\quad \overset{1}{\cancel{16}}\quad \overset{2}{18}\quad \overset{3}{30}\quad \overset{4}{15}\}$ 3 $\Rightarrow$ Ans GCD=24

$\{\underset{8}{\boxed{\overset{0}{24}\quad \overset{1}{16}}}\quad \overset{2}{\cancel{18}}\quad \overset{3}{30}\quad \overset{4}{15}\}$ 1

$\{\boxed{\overset{0}{24}\quad \overset{1}{16}\quad \overset{2}{18}}\quad \overset{3}{\cancel{30}}\quad \overset{4}{15}\}$ 1

$\{\overset{0}{24}\quad \overset{1}{16}\quad \overset{2}{18}\quad \overset{3}{30}\quad \overset{4}{\cancel{15}}\}$ 2

Brute force → Repeat this for all the arr elements

{ Delete ar[i], calculate the gcd for all the remaining elements }

TC : $O(n^2 \log max)$

* Idea 2 ⇒ prefix arrays

```
int deleteone ( int [] ar )
    ans = 0
    int [] pfgcd ;     pfgcd[i] = gcd of all ele from 0 to i
    int [] sfgcd       sfgcd[i] = gcd of all ele from i to n-1

    for ( i=0 ; i<n; i++)
        // delete ar[i] & find gcd of all other elem

        ar[] = a0 a1 a2 ... a[i] ai+1 ... an-1

        left = gcd of all ele from 0 to i-1

        left = 0
        if (i != 0) left = pfgcd[i-1]
```

```
        right = gcd of all ele from i+1 to n-1

        right = 0

        if (i != n-1) right = sfgcd[i+1]

        int val = gcd(left, right)

        ans = Math.max(ans, val);

    }
    return ans;
}
```

TC : O(nlogmax)

SC : O(n)

```
ans = 0

for (i=0; i<n; i++)
    ans = gcd(ans, ar[i]);
    pfgcd[i] = ans;
}
```

Time complexity

$$TC : O(\log \max(a,b))$$

$$N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \frac{N}{8} \ldots 1 \Rightarrow \boxed{TC: O(\log N)}$$

$$GCD(A,B) = GCD(\underbrace{a \% b}_{<b}, b)$$

$\Downarrow$

$b < a/2$	$b = a/2$	$b > a/2$
$a\%b < b < a/2$	$a \% b < b$	$2b > a$
$a\%b < a/2$	$b = a/2$	$2b - a > 0$
	$a\%b < a/2$	$a - 2b < 0$

Adding a on both sides

$$a + a - 2b < 0 + a$$

$$2a - 2b < a$$

$$2(a-b) < a$$

$$a-b < \frac{a}{2}$$

$$\boxed{a \% b < \frac{a}{2}}$$

$$\boxed{a \% b = a - xb}$$

$$a \% b = a - b$$
$$= a - 2b$$
$$= a - 3b$$
$$\vdots$$

Worst case scenario

$$a \% b = \frac{a}{2}$$

when $a > b$ TC : $O(\log a)$

when $b > a$ TC : $O(\log b)$

$$\boxed{TC : O(\log_2 \max(a, b))}$$