

①

	$x=0$	$x=1$
$y=0$	$\frac{1}{12}$	$\frac{1}{12}$
$y=1$	$\frac{1}{6}$	$\frac{1}{3}$

a)  $P_x(x=1) = \frac{1}{12} + \frac{1}{3} = \boxed{\frac{7}{12}}$

b)  $P(x=1 | Y=1) = \frac{P(x=1, Y=1)}{P(Y=1)} = \frac{\frac{1}{3}}{\frac{1}{12} + \frac{1}{3}} = \boxed{\frac{2}{3}}$

c)  $\text{Var}(x) = \mathbb{E}[x^2] - \mathbb{E}[x]^2$

$$\mathbb{E}[x] = 0 \cdot (\frac{1}{12} + \frac{1}{6}) + 1 \cdot (\frac{1}{12} + \frac{1}{3})$$

$$\mathbb{E}[x] = \frac{7}{12}$$

$$\mathbb{E}[x^2] = 0^2 \cdot (\frac{1}{12} + \frac{1}{6}) + 1^2 \cdot (\frac{1}{12} + \frac{1}{3})$$

$$\mathbb{E}[x^2] = \frac{7}{12}$$

$$\therefore \text{Var}(x) = \frac{7}{12} - \left(\frac{7}{12}\right)^2 = \boxed{\frac{35}{144}}$$

d)  $\text{Var}(z)$  when  $z = x | Y=1$

$$\text{Var}(z) = \mathbb{E}[x^2 | Y=1] - \mathbb{E}[x | Y=1]^2$$

$$\mathbb{E}[x | Y=1] = 0 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3} = \frac{2}{3}$$

$$\mathbb{E}[x^2 | Y=1] = 0^2 \cdot \frac{1}{3} + 1^2 \cdot \frac{2}{3} = \frac{2}{3}$$

$$\therefore \text{Var}(x | Y=1) = \frac{2}{3} - \frac{4}{9} = \boxed{\frac{2}{9}}$$

e) Find  $\mathbb{E}[x^3 + y^2 + 3y^7 | Y=1]$

$$= (0^3 + 0^2 + 3) \cdot \frac{1}{6} + (1^3 + 1^2 + 3) \cdot \frac{1}{3}$$

$$= \frac{3}{6} + \frac{5}{3}$$

$$= \frac{13}{6}$$