Lab 09: Graph Representation and Operations

1. Adjacency Matrix and Adjacency List

- Given an undirected graph, G = (V, E) in its adjacency matrix representation. Obtain the corresponding adjacency list representation.
- Given an undirected graph, G = (V, E) in its adjacency list representation. Obtain the corresponding adjacency matrix representation.
- Given a directed graph G = (V, E) in its adjacency matrix representation. Obtain the corresponding adjacency list representation.
- Given a directed graph G = (V, E) in its adjacency list representation. Obtain the corresponding adjacency matrix representation.

2. Check Graph Type Given an adjacency matrix, determine:

1. Whether the graph is directed or undirected.
2. Whether it is sparse or dense. (A graph is dense if $|E| > \frac{|V|(|V|-1)}{4}$.)

Test Case:

```
Input:
Number of nodes: 4
0 1 1 1
0 0 1 1
0 0 0 0
0 0 1 0
```

3. Transpose of a Directed Graph Given an adjacency matrix of a directed graph, construct its **transpose graph** by reversing all edges. Print the adjacency list of the transposed graph.

Test Case:

```
Input:
number of nodes: 4
number of edges: 4
Edges are as follows:
```

```
0 1
0 2
1 3
2 3

Output:
Transposed Graph (Adjacency List):
0:
1: 0
2: 0
3: 1 2
```

4. Path Counting Using Matrix Multiplication A *path* of length k in a directed graph $G = (V, E)$ is a sequence of vertices

$$v_0, v_1, v_2, \ldots, v_k$$

such that $(v_{i-1}, v_i) \in E$ for all $1 \leq i \leq k$. Note that in this sequence, a node is not repeated.

Objective: Given the adjacency matrix A of a directed graph G, compute the number of distinct paths of length 2 between every pair of vertices.

Hint: Use matrix multiplication

Example:

```
Input:
3
0 1 1
0 0 1
0 0 0

Explanation:
Vertices: 0, 1, 2
Edges: 0->1, 0->2, 1->2

Paths of length 2:
0 -> 1 -> 2
Only one path of length 2 exists (from 0 to 2).
```