Chemistry: A Molecular Approach, Global Edition | (4th Edition)

Problem

An 85.0-gram laboratory animal ingests 10.0 mg of a substance that contained 2.55% by mass Pu-239, an alpha emitter with a half-life of 24,110 years.

a. What is the animal's initial radiation exposure in Curies?

b. If all of the energy from the emitted alpha particles is absorbed by the animal's tissues, and if the energy of each emission is 7.77 × 10–12 J what is the dose in rads to the animal in the first 4.0 hours following the ingestion of the radioactive material? Assuming a biological effectiveness factor of 20, what is the 4.0-hour dose in rems?

Step-by-step solution

Step 1 of 11

a) A laboratory animal ingests $10.0\ \text{mg}$ of a substance that contains $2.55\ \%$ by mass of Pu-239. The Pu-239 is an alpha emitter with a half-life of $24{,}110\ \text{years}$. The mass of animal is $85.0\ \text{mg}$.

Calculate animal's initial radiation exposure in Curies.

Calculate the rate constant k using the half-life expression.

The expression for the half-life $t_{1/2}$ is as follows:

$$t_{1/2} = \frac{0.693}{k} \quad \ldots\ldots (1)$$

Comment

Step 2 of 11

Rearrange the equation (1) and calculate the rate constant as follows:

$$t_{1/2} = \frac{0.693}{k} \quad \ldots\ldots (1)$$

Therefore,

$$\begin{aligned} k &= \frac{0.693}{t_{1/2}} \\ &= \frac{0.693}{24{,}110\ \text{years}} \times \frac{1\ \cancel{\text{yr}}}{365\ \cancel{\text{d}}} \times \frac{1\ \cancel{\text{d}}}{24\ \cancel{\text{hr}}} \times \frac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}} \\ &= 9.1144 \times 10^{-13}\ \text{decay / atom} \cdot \text{s} \end{aligned}$$

Comment

Step 3 of 11

A substance that contains $2.55\ \%$ by mass of Pu-239. Calculate number of atoms in $10.0\ \text{mg}$ of a substance as follows:

$= 6.425 \times 10^{17}$ atoms

Comment

Step 4 of 11

Calculate the initial activity as follows:

$$9.1144 \times 10^{-13}\ \text{decay} / \cancel{\text{atom}} \cdot \text{s} \times 6.425 \times 10^{17}\ \cancel{\text{atoms}}$$
$$= 5.856 \times 10^{5}\ \text{decay/s}$$

The curie (Ci) is defined as 3.7×10^{10} decays per second.

The conversion factors are

$$\frac{1\ \text{Ci}}{3.7 \times 10^{10}\ \text{decay/s}} \text{ and } \frac{3.7 \times 10^{10}\ \text{decay/s}}{1\ \text{Ci}}$$

Therefore,

$$5.856 \times 10^{5}\ \cancel{\text{decay}} / \cancel{\text{s}} \times \frac{1\ \text{Ci}}{3.7 \times 10^{10}\ \cancel{\text{decay}} / \cancel{\text{s}}}$$
$$= 1.58 \times 10^{-5}\ \text{Ci}$$

Hence, the initial activity is $\boxed{1.58 \times 10^{-5}\ \text{Ci}}$.

Comment

Step 5 of 11

(b)The number of emissions is $5.856 \times 10^{5}\ \text{decay/s}$. Calculate the dose in rads to the animal in the first 4.0 hours.

The expression for the integrated rate law is as follows:

$$\ln\frac{N_t}{N_0} = -kt \ \ ...... (2)$$

Here, N_0 is the initial number of radioactive nuclei and N_t is the number of radioactive nuclei at time t.

Calculate the initial mass of Pu-239 as follows:

$$N_0 = 10.0\ \text{mg} \times \frac{2.55}{100}$$
$$= 0.255\ \text{mg}$$

Calculate the mass of Pu-239 that is left after 4.0 hr as follows:

$$\ln\frac{N_t}{N_0} = -kt \ \ ...... (2)$$

Comment

Step 6 of 11

Therefore,

$$\frac{N_t}{N_0} = e^{-kt}$$
$$N_t = N_0 e^{-kt}$$
$$= 0.255\ \text{mg} \times e^{\left[-9.1144 \times 10^{-13} / \cancel{\text{s}} \times 4\ \cancel{\text{hr}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right]}$$
$$= 0.254\ \text{mg}$$

Comment

The mass of Bi-210 that is decayed is calculated as follows:

Comment

Step 8 of 11

$$1.2\ \text{g} - 0.1847451\ \text{g} = 1.0152549\ \text{g}$$

Comment

Step 9 of 11

The atomic mass of Bi-210 is $209.984105\ \text{amu}$. Calculate the number of beta decays as follows:

$$1.0152549\ \cancel{\text{g}} \times \frac{1\ \cancel{\text{mol}}}{209.984105\ \cancel{\text{g}}} \times \frac{6.022 \times 10^{23}}{1\ \cancel{\text{mol}}}$$
$$= 2.9 \times 10^{21}$$

Therefore, $\boxed{2.9 \times 10^{21}}$ beta emissions occur $13.5\ \text{days}$.

Comment

Step 10 of 11

If a person's body intercepts $5.5\ \%$ of the total beta emissions, calculate the dose of radiation in curie (Ci).

There are 2.9×10^{21} beta emissions in $13.5\ \text{days}$.

Calculate number of beta emissions per second as follows:

$$2.9 \times 10^{21}\ \frac{\text{decay}}{13.5\ \cancel{\text{d}}} \times \frac{1\ \cancel{\text{d}}}{24\ \cancel{\text{hr}}} \times \frac{1\ \cancel{\text{hr}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}}$$
$$= 2.4862826 \times 10^{15}\ \text{decay/s}$$

A person's body intercepts $5.5\ \%$ of those emissions. Therefore, the radiation exposure is

$$2.4862826 \times 10^{15}\ \text{decay/s} \times \frac{5.5}{100}$$
$$= 1.367455 \times 10^{14}\ \text{decay/s}$$

Comment

Step 11 of 11

The curie (Ci) is defined as $3.7 \times 10^{10}\ \text{decays}$ per second.

The conversion factors are

$$\frac{1\ \text{Ci}}{3.7 \times 10^{10}\ \text{decay / s}}\ \text{and}\ \frac{3.7 \times 10^{10}\ \text{decay / s}}{1\ \text{Ci}}$$

Therefore,

$$1.367455 \times 10^{14}\ \cancel{\text{decay}} / \cancel{\text{s}} \times \frac{1\ \text{Ci}}{3.7 \times 10^{10}\ \cancel{\text{decay}} / \cancel{\text{s}}}$$
$$= 3{,}696\ \text{Ci}$$
$$\approx 3{,}700\ \text{Ci}$$

Hence, the dose of radiation is $\boxed{3{,}700\ \text{Ci}}$.

Was this solution helpful? 1 5

Recommended solutions for you in Chapter 20

Chapter 20, Problem 15E

Explain the main concepts behind the technique of radiocarbon dating. How can radiocarbon dating be corrected for changes in atmospheric concentrations of C-14? What range of ages can be reliably determined by C-14 dating?

See solution

Chapter 20, Problem 65E

If 1.0 g of matter were converted to energy, how much energy would be formed?

See solution

feedback