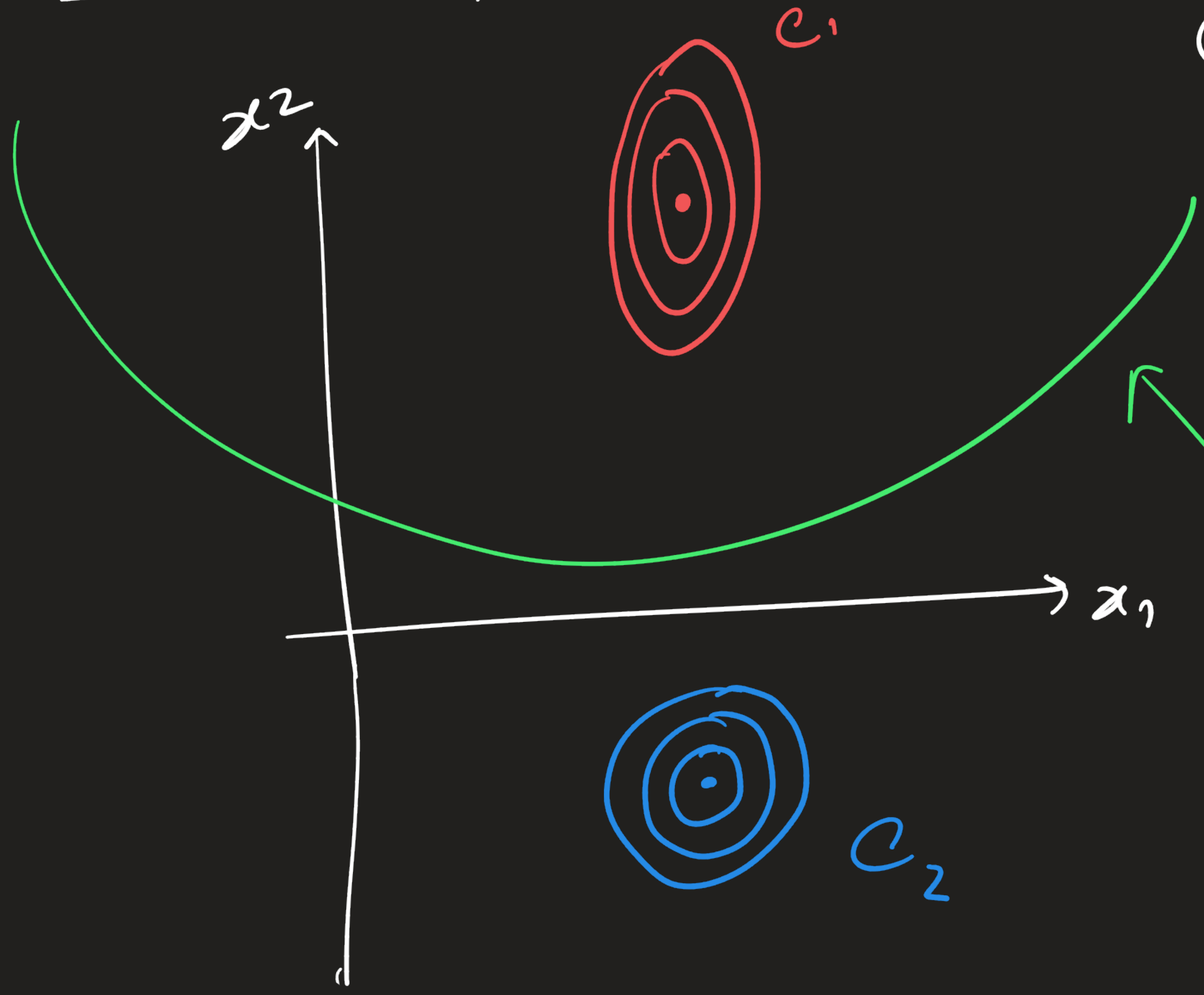
Two-class problem



$C_1 \sim G(\mu_1, \Sigma_1)$

$C_2 \sim G(\mu_2, \Sigma_2)$

$P(C_1) = P(C_2) = \frac{1}{2}$

discriminant function

$y(x)$

$$\frac{P(C_1|x)}{P(C_2|x)} > 1 \quad , \quad x \in C_1$$

otherwise $x \in C_2$

$$\frac{\dfrac{P(x|C_1) \cdot P(C_1)}{\cancel{P(x)}}}{\dfrac{P(x|C_2) \cdot P(C_2)}{\cancel{P(x)}}} > 1$$

$$\frac{P(x|C_1) \cdot P(C_1)}{P(x|C_2) \cdot P(C_2)} > 1$$

$$\underbrace{\ln\left(\frac{P(x|C_1) \cdot P(C_1)}{P(x|C_2) \cdot P(C_2)}\right)}_{y(x) \,\equiv\, \text{discriminant function.}} > \overset{0}{\overset{\shortparallel}{\ln(1)}}$$

$$\underbrace{\underbrace{\ln\left(P(x|C_1) \cdot P(C_1)\right)}_{g_1(x)} - \underbrace{\ln\left(P(x|C_2) \cdot P(C_2)\right)}_{g_2(x)}}_{y(x)} > 0$$

$$P(x|C_1) = \frac{1}{(2\pi)^{d/2} \cdot |\Sigma_1|} \cdot \exp\left\{-\frac{1}{2}(x-\mu_1)^T \cdot \Sigma_1^{-1} \cdot (x-\mu_1)\right\}$$

$$\mu_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}, \quad \Sigma_1 = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}, \quad |\Sigma_1| = \frac{1}{2} \times 2 = 1, \quad \Sigma_1^{-1} = \frac{1}{|\Sigma_1|}\begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$\mu_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad |\Sigma_2| = 4, \quad \Sigma_2^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$P(C_1) = P(C_2) = \frac{1}{2}$$

$$\boxed{g_1(x) = \ln\left(P(x|C_1) \cdot P(C_1)\right)}$$

$$= -\frac{d}{2} \cdot \ln(2\pi) - \frac{1}{2} \cdot 1 - \frac{1}{2}\left[\begin{bmatrix} x_1 - 3 \\ x_2 - 6 \end{bmatrix}^T \cdot \begin{bmatrix} 2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x_1 - 3 \\ x_2 - 6 \end{bmatrix}\right\} + \ln$$

$d = 2$
\# features

$$\boxed{= -\ln(2\pi) - \frac{1}{2} + \ln\left(\frac{1}{2}\right) - \frac{1}{2}\left(2(x_1-3)^2 + \frac{1}{2}(x_2-6)^2\right)}$$

Repeat for $g_2(x) = \ln\left(P(x|C_2) \cdot P(C_2)\right)$. EXERCISE

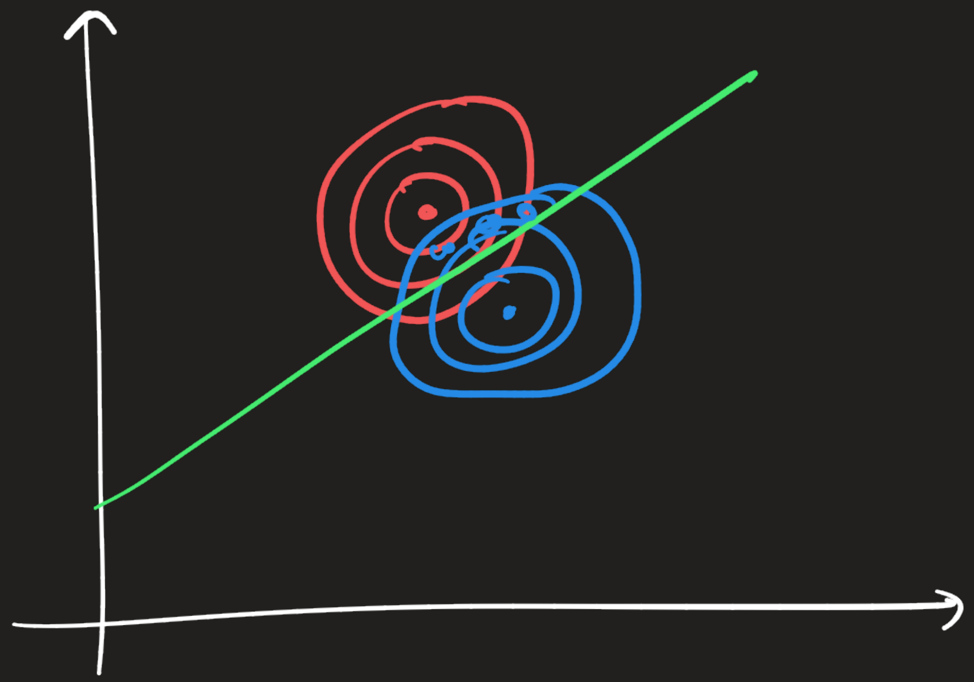
$$y(x) = g_1(x) - g_2(x)$$

Simplify to : (EXERCISE)

$$\boxed{x_2 - 0.1875 \cdot x_1^2 + 1.125 \cdot x_1 - 3.514 = 0}$$

$> \; c_1$

$< \; c_2$



$\Sigma_1 = \Sigma_2$

Mixture Model

hyperparameter

$$P(x \mid \theta) = \sum_{k=1}^{K} \pi_k \cdot P_k(x \mid \theta_k)$$

where $\sum_{k=1}^{K} \pi_k = 1$

and $0 \leq \pi_k \leq 1$

$\theta \equiv$ parameters of the mixture model

$K \equiv$ # components in the mixture

$\pi_k \equiv$ weight for k^{th} component

$P_k \equiv$ probabilistic model for k^{th} component

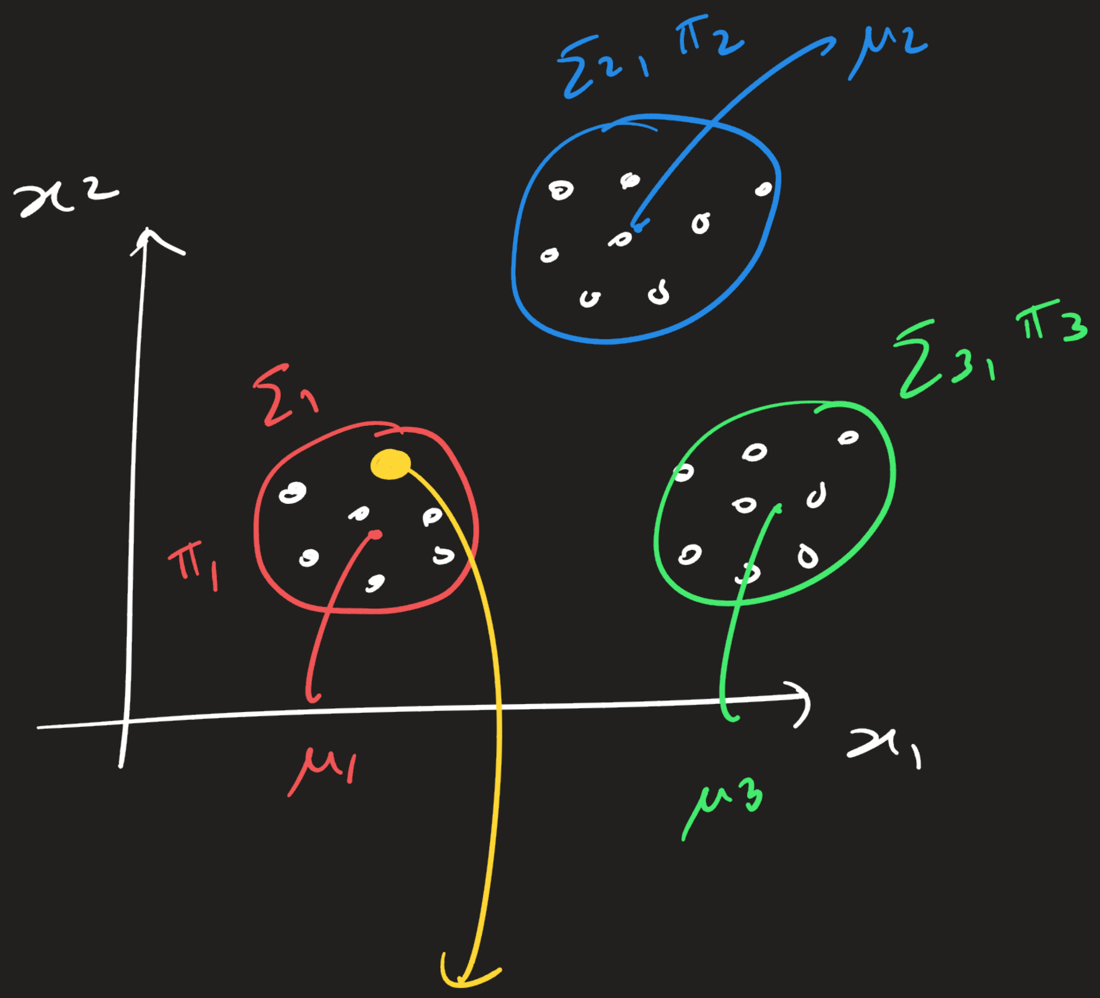
Gaussian Mixture Model (GMM)

$$P(x|\theta) = \sum_{k=1}^{K} \pi_k \cdot G(x|\mu_k, \Sigma_k)$$

$$\theta = \{\pi_k, \mu_k, \Sigma_k\}_{k=1}^{K}$$

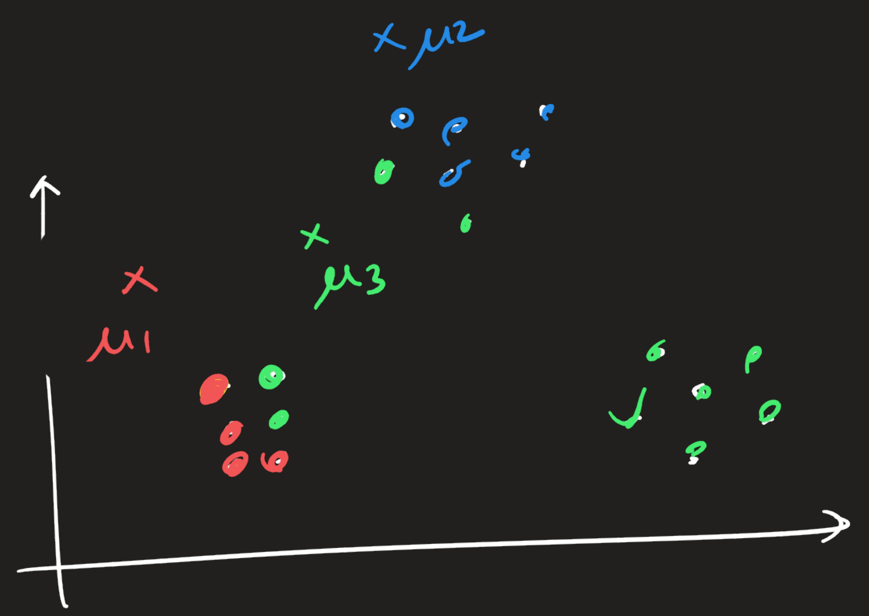
where $\sum_{k=1}^{K} \pi_k = 1$

and $0 \leq \pi_k \leq 1$



Each sample is assumed to have been modeled by <u>one</u> component.

x_i will be a member of component K.



① $\Sigma_1 = \Sigma_2 = \Sigma_3 = I$

$\pi_1 = \pi_2 = \pi_3 = \frac{1}{3}$

② Memberships for each sample

$$U = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{N \times K} \qquad \frac{P(x_1 | C_3) \cdot \pi_3}{\sum_{j=1}^{3} p(x_1 | C_j) \cdot \pi_j}$$

③ Now, fix U. Update the parameters,

$$\theta = \{\mu_K, \Sigma_K, \pi_K\}_{K=1}^{K}$$

④ go back to step ② until convergence.