Mathematical logic :-

* We communicate our ideas to others through sentences. In our day to day life following types of sentences are used.

(i) Assertive sentence [T/F]
(ii) Imperative sentence [Order/request]
(iii) Exclamatory sentence !
(iv) Interrogative sentence ?

* **Statement :**

(assertive)

A statement is a declarative sentence which is either true or false, but not both simultaneously.

Eg ① Earth revolve around the sun
→ The statement is TRUE.

② 6 is a prime number.
→ This is a statement and it is FALSE.

* **TRUTH VALUE of Statement :**

→ Every statement must be either true / false. If the statement is true, we say that its T.V. is TRUE (T) or else FALSE (F).

Ex ① → Delhi is a capital of India
→ True

(ii) Rome is in England
→ False

(iii) Two plus three is equal to six.

This statement may be written as $2+3=6$.

→ Even if it is written in this form, it is called a statement.

→ Sentences which are imperative, exclamatory or interrogative are not statements.

→ Therefore, these sentences are simple sentence.

* Open statement : (No Statement)

→ An open statement is a sentence whose truth can vary according to some condition which are not stated in sentence.

Ex (i) $x+2=7$; (ii) He is a musician.

⇒ (i) Depends upon the values of x

(ii) Depends upon the pronoun 'he'.

* Logical Connectives, Compound statement and Truth Table.

eg (i) All roses are red and violets are blue.

(ii) 5 is a real number or $5+i$ is a complex number.

The words or group of words like and, or, not, if ... then, if and only if ... can be used to join simple sentences.

→ Such words are called logical connective or connectives.

Sr.No	Connective	Symbol	Terms used
1.	and	$\wedge$	conjuction
2.	or	$\vee$	disjunction
3.	not	$\sim$	negation
4.	If ... then	($\rightarrow$ or $\Rightarrow$)	implication
5.	... if and only if	($\leftrightarrow$ or $\Leftrightarrow$)	bi-conditional

Eg ① $2+3<6$, $(a+b)^2 = a^2+2ab+b^2$

→ Their conjuction is
$2+3<6$ and $(a+b)^2 = a^2+2ab+b^2$

→ If $p \wedge q$ is defined to have truth value "true" if both p and q have the truth values "true". In all other cases, $p \wedge q$ is defined to have the truth value "false".

Truth Value of conjuction ($\wedge$)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction ($\vee$):

- A compound statement obtained by combining any two given simple statement 'or' is called disjunction

Truth Table for disjunction (v)

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

* Table for negotiation:-

p	~p
T	F
F	T

* Table for Conditional ($\rightarrow$)

→ If $p \rightarrow q$ defined to have the truth value 'false' if p has truth value 'true' and q has truth value 'false'.

→ In all other cases, $p \rightarrow q$ is defined to have the truth value 'true'.

Truth table for conditional ($\rightarrow$)

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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Eg (i) Transform the following statement in symbolic form
If the sky is clear, then the sun is shining.

(ii) Lata is hard work only if she is successful in life.

Sol^n → p: The sky is clear, q: The sun is shining.
→ Therefore, symbolic form will be
$p \rightarrow q$.

(ii) p: Lata is hard working
q: Lata is successful in life
→ Therefore, symbolic form will be $p \rightarrow q$.

* **Converse, Inverse and contrapositive:**

→ From the conditional (implication) $p \rightarrow q$, we can write down three additional conditional statement are as follows:

(i) Converse of $p \rightarrow q$ is $q \rightarrow p$

(ii) Inverse of $p \rightarrow q$

(iii) Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

Eg-1) If Amir is courageous, then he will join Indian Army
→ Let p: Amir is courageous q: Amir will join Indian Army.

* Converse: $q \rightarrow p$.

→ If Amir joins Indian Army, then he is courageous.

* Inverse :- $\sim p \rightarrow \sim q$

→ ~~If the women in a family are not literate then family.~~

If Amir is not courageous, then he will not join Indian Army.

* contraposition :- $\sim q \rightarrow \sim p$ i.e if Amir does not join Indian Army, then he is not corageous.

* Bicondition (Double Implication) ($\leftrightarrow$) or ($\Leftrightarrow$)

Truth Table for biconditional ($\leftrightarrow$)

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Eg(1) Hari is rich if and only if he owns a car.

→ p: Hari is rich   q: He owns a car.

→ ∴ The given statement in symbolic form is $p \Leftrightarrow q$.

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Q. ① Prepare the truth table.

① $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$p \vee q$	$p \vee r$	$(p \vee q) \vee (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	T	F	T	T
F	F	F	F	F	F	F	F

∴ 4th & 8th column are identical

∴ $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

H/P

② $\sim r \to \sim (p \wedge q) \equiv \sim (q \to r) \to \sim p$

p	q	r	$\sim p$	$\sim r$	$p \wedge q$	$\sim (p \wedge q)$	$\sim r \to \sim (p \wedge q)$
T	T	T	F	F	T	F	T
T	T	F	F	T	T	F	F
T	F	T	F	F	F	T	T
F	T	T	T	F	F	T	T
T	F	F	F	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	F	T	T
F	F	F	T	T	F	T	T

$\therefore$ 8th & 10th column are identical

$\therefore \sim p \to (\sim)(p \wedge q) \equiv \sim(q \to r) \to \sim p$.

* Tautology:

→ A statement pattern which always takes truth value T whatever be the assignment of the truth value to its component statement, is called Tautology.

p	~p	p ∨ ~p
T	F	T
F	T	T

* Contradiction (Fallacy):

→ A statement pattern which always takes truth value F, whatever be the assignment of truth value to its component statement is called a contradiction.

p	~p	p ∧ ~p
T	F	F
F	T	F

* Contingency:

→ A statement pattern which is neither a tautology nor a contradiction is called a contigency.

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p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Quantifiers and Quantified statements:-

(i) The symbol '∀' stands for 'all values of'. This is known as universal quantifier.

(ii) The symbol '∃' stands for "there exists". This symbol is existential quantifier.

→ There are two types of quantifiers

- Universal quantifier (∀)
- Existential quantifier (∃)

$p \vee p \equiv p$	$\leftarrow$ Idempotent Law $\rightarrow$	$p \wedge p \equiv p$
$p \vee q \equiv q \vee p$	$\leftarrow$ Commutative Law $\rightarrow$	$p \wedge q \equiv q \wedge p$
$(p \vee q) \vee r \equiv p \vee (q \vee r) \equiv p \vee q \vee r$	$\leftarrow$ Associative Law $\rightarrow$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r) \equiv p \wedge q \wedge r$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$\leftarrow$ Distributive Law $\rightarrow$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \vee \mathrm{F} \equiv p$ $p \wedge \mathrm{F} \equiv \mathrm{F}$	$\leftarrow$ Identity Law $\rightarrow$	$p \vee \mathrm{T} \equiv \mathrm{T}$ $p \wedge \mathrm{T} \equiv p$
$p \vee \sim p \equiv \mathrm{T}$	$\leftarrow$ Complement Law $\rightarrow$	$p \wedge \sim p \equiv \mathrm{F}$
$\sim(\sim p) \equiv p$	$\leftarrow$ Involution Law $\rightarrow$	$\sim \mathrm{T} \equiv \mathrm{F}, \sim \mathrm{F} \equiv \mathrm{T}$
$\sim(p \vee q) \equiv \sim p \wedge \sim q$	$\leftarrow$ DeMorgan's Law $\rightarrow$	$\sim(p \wedge q) \equiv \sim p \vee \sim q$
$p \vee (p \wedge q) \equiv p$	$\leftarrow$ Absorption Law $\rightarrow$	$p \wedge (p \vee q) \equiv p$
$p \rightarrow q \equiv \sim p \vee q$	$\leftarrow$ Conditional Law $\rightarrow$	$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

Table 1.26

Table 1.26 represents list of standard equivalent statements, most of them have been already proved.