CS 383 - Machine Learning

Assignment 3 - Closed Form Linear Regression
Summer 2017
Amir Omidi

1 Theory

The matrix we're dealing with in this question is the following:

$$\begin{bmatrix} -2 & 1 \\ -5 & -4 \\ -3 & 1 \\ 0 & 3 \\ -8 & 11 \\ -2 & 5 \\ 1 & 0 \\ 5 & -1 \\ -1 & -3 \\ 6 & 1 \end{bmatrix}$$

1.1 Standarization

This is the same matrix as our first assignment, so with reference to that assignment, we know the first row standardized is:

$$\begin{bmatrix} -0.2602 & 1 \\ -0.9697 & -4 \\ -0.4967 & 1 \\ 0.2129 & 3 \\ -1.6792 & 11 \\ -0.2602 & 5 \\ 0.4494 & 0 \\ 1.3954 & -1 \\ -0.0237 & -3 \\ 1.6319 & 1 \end{bmatrix}$$

1.2 Feature Addition

Let's add our additional feature of a 1×10 matrix:

$$\begin{bmatrix} 1 & -0.2602 & 1 \\ 1 & -0.9697 & -4 \\ 1 & -0.4967 & 1 \\ 1 & 0.2129 & 3 \\ 1 & -1.6792 & 11 \\ 1 & -0.2602 & 5 \\ 1 & 0.4494 & 0 \\ 1 & 1.3954 & -1 \\ 1 & -0.0237 & -3 \\ 1 & 1.6319 & 1 \end{bmatrix}$$

1.3 Weight Calculation

Let's compute the weight! We know the formula for this is:

$$\theta = \left(X^T X\right)^{-1} X^T Y$$

Plugging in the values, we get:

$$\left(X^T X\right) = \begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.00001.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ -0.2602 & -0.9697 & -0.4967 & 0.2129 & -1.6792 - 0.2602 & 0.4494 & 1.3954 & -0.0237 & 1.6319 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & -0.2602 \\ 1 & -0.9697 \\ 1 & -0.4967 \\ 1 & 0.2129 \\ 1 & -1.6792 \\ 1 & -0.2602 \\ 1 & 0.4494 \\ 1 & 1.3954 \\ 1 & -0.0237 \\ 1 & 1.6319 \end{bmatrix} = \begin{bmatrix} 10.0000 & 0.0000 \\ 0.0000 & 9.0000 \end{bmatrix}$$

$$det(\left(X^T X\right)) = 90$$

$$\left(X^T X\right)^{-1} = \frac{1}{90} \times \begin{bmatrix} 10.0000 & 0.0000 \\ 0.0000 & 9.0000 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1000 & 0.0000 \\ 0.0000 & 0.1111 \end{bmatrix}$$

$$\theta = \begin{bmatrix} 0.1000 & 0.0000 \\ 0.0000 & 0.1111 \end{bmatrix}$$

$$\times \begin{bmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.00001.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ -0.2602 & -0.9697 & -0.4967 & 0.2129 & -1.6792 - 0.2602 & 0.4494 & 1.3954 & -0.0237 & 1.6319 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.4000 \\ -1.7499 \end{bmatrix}$$

So with this we know:

$$\theta = \begin{bmatrix} 1.4000 & -1.7499 \end{bmatrix}^T$$

Therefore our model is:

$$y = 1.4 + 1.75x_1$$

2 Programming

2.1 Closed Form Linear Regression

2.1.1 Final Model

The final model calculated for the data is:

$$y = 3425.57 + 846.95x_1 - 369.22x_2$$

2.1.2 RMSE

The RMSE value calculated was 853.3806.

2.2 S-Folds Cross-Validation

2.2.1 RMSE

The RMSE value calculated was 632.3608.