ABSTRACT

This project presents a quantum-circuit simulation of the resolution of the black hole information paradox. The Black hole evaporation is designed to reproduce the qualitative behavior of the Page Curve — the entanglement entropy evolution of Hawking radiation. Using unitary quantum operations on coupled qubit subsystems representing the black hole interior (B) and the radiation (R), successive models were developed to explore information scrambling, chaos, and eventual information recovery. By systematically increasing the circuit's depth, gate diversity, and qubit count, the simulation evolves from a simple thermal model to one exhibiting unitary evaporation via the island mechanism. The work bridges theoretical constructs of quantum gravity—including the Quantum Extremal Surface (QES) and the Island formula—with concrete realizations in quantum computation. The model prioritizes physical fidelity by incorporating realistic constraints, resulting in outcomes that, while slightly deviating from theoretical expectations, offer a more accurate representation of the system's true dynamics.

1. Introduction

1.1 The Classical Conundrum: Black Holes vs. Thermodynamics

The black hole information paradox lies at the intersection of thermodynamics, quantum mechanics, and gravity. To fully appreciate the information paradox, it is strategically essential to first understand the classical conception of a black hole. This perspective is not merely historical context; it reveals the fundamental conflict with established physical laws that necessitated the introduction of quantum mechanics.

Classically, a black hole was viewed as an object whose properties were in direct opposition to the principles of thermodynamics. The classical properties of a black hole that lead to this initial conflict are as follows:

Perfect Absorber: From a classical standpoint, a black hole is the ultimate one-way street. Anything that crosses its event horizon — matter, energy, or information is considered permanently lost to the rest of the universe.

No Emission: A direct consequence of being a perfect absorber is that a classical black hole emits nothing. According to the laws of thermodynamics, any object with a temperature above absolute zero must emit thermal radiation (a process known as black-body radiation). Since a classical black hole emits no radiation, it was logically assigned a temperature of absolute zero (0K).

This "zero-temperature" state creates a critical conflict with the Second Law of Thermodynamics. The Second Law dictates that the total entropy (a measure of disorder) of a closed system can never decrease. However, if one were to drop a high-entropy object, such as a hot cloud of gas, into a classical black hole, the universe would face a contradiction. The entropy of the gas cloud would vanish behind the event horizon, and since the black hole itself has zero entropy, the total entropy of the universe would decrease ($\Delta S \leq 0$). This represents a direct and profound violation of a cornerstone of physics.

This thermodynamic violation set the stage for the first major attempt to reconcile black holes with the known laws of nature, an attempt that would require the tools of quantum mechanics.

1.2 Quantum Mechanics and Hawking Radiation: A Solution Creates a New Problem

The introduction of quantum mechanics to the study of black holes successfully resolved the thermodynamic conundrum. However, in doing so, it inadvertently gave rise to a deeper, more subtle, and far more challenging problem related to the preservation of quantum information.

The Mechanism of Hawking Radiation:

The solution emerged from the strange behavior of spacetime near the event horizon, as described by Stephen Hawking. The mechanism is rooted in fundamental quantum principles:

1. Quantum Fluctuations: Heisenberg's Uncertainty Principle dictates that "empty" space, or the vacuum, is not truly empty. It is a roiling sea of temporary, random changes in energy.

2. Virtual Particle Pairs: These energy fluctuations manifest as virtual particle-antiparticle pairs that spontaneously appear. According to the Energy-Time Uncertainty Principle ($\Delta E \Delta t \geq \frac{\hbar}{2}$), these pairs "borrow" an amount of energy (ΔE) from the vacuum and must annihilate each other within a very short time window (Δt) to "repay" this energy loan.

3. The Event Horizon's Role: In most of space, this process goes unnoticed. Near a black hole's event horizon, however, the intense gravitational gradient can have a dramatic effect. A virtual pair can be created such that one particle forms just outside the horizon while its partner forms just inside. The powerful gravity can pull the pair apart before they have a chance to annihilate.

4. The Outcome: The particle with positive energy escapes to infinity, while its partner with negative energy falls into the black hole. The escaping particle is observed by the outside universe as Hawking Radiation. The absorption of the negative-energy particle causes the black hole's total mass-energy to decrease, leading to its slow evaporation over immense timescales (about 10^{87} years).

Black Holes as Thermodynamic Objects:

The most profound aspect of Hawking's discovery was that this radiation is not random. His calculations showed that it possesses a perfect thermal spectrum, identical to the radiation emitted by a classical hot object (a black body). This discovery allowed for the definition of two crucial properties:

Hawking Temperature (T_H): A temperature directly proportional to the black hole's surface gravity

($T_H \propto \kappa$).

Bekenstein-Hawking Entropy (S_{BH}): An entropy directly proportional to the surface area of the event horizon ($S \propto A$). The full formula is:

$$S_{BH} = \frac{k_B c^3 A}{4 G \hbar} \quad (1)$$

With these properties, black holes were officially established as true thermodynamic objects. The classical conflict was resolved: when a hot gas cloud falls into a black hole, the decrease in external entropy is more than compensated for by an increase in the black hole's own Bekenstein-Hawking entropy, preserving the Second Law of Thermodynamics.

This elegant solution, however, concealed a paradox. By giving black holes a thermal temperature and allowing them to evaporate, quantum mechanics had inadvertently turned them into destroyers of information, setting up a new and more fundamental conflict.

1.3 The Information Paradox: Unitarity and Entanglement at Stake

The core of the information paradox is this: the perfectly thermal nature of Hawking radiation implies that it contains no information about the unique objects that formed the black hole. A black hole formed from a star would radiate identically to one formed from a collection of encyclopedias. This suggests that the information is permanently destroyed upon evaporation, a conclusion that violates a sacred principle of quantum mechanics: Unitarity, which demands that the evolution of a closed quantum system must be reversible, preserving all information about its initial state.

Pure States vs. Mixed States and Information:

In quantum mechanics, information is encoded in a system's state. Unitarity demands that the evolution of a closed system is reversible; no information about its initial state can ever be truly lost. This concept can be understood by contrasting two types of quantum states:

Pure States	Mixed States
Represents a system whose state is known completely; its exact wavefunction is defined.	Represents statistical uncertainty about a system's state; we only know the probabilities of it being in one of several possible pure states.
Characterized by a Von Neumann entropy of $S = 0$.	Characterized by a Von Neumann entropy of $S > 0$.

Difference between Pure and Mixed States

A star collapsing to form a black hole begins as a pure state. Unitarity requires that the final state, after the black hole has completely evaporated into radiation, must also be a pure state. However, thermal radiation is the archetypal mixed state. If the final radiation is truly thermal, then the initial pure state has evolved into a mixed state, information has been lost, and Unitarity has been violated.

The Monogamy of Entanglement Conflict:

This conflict can be sharpened by examining the quantum property of entanglement. The principle of Monogamy of Entanglement states that a quantum particle can be maximally entangled with, at most, one other particle. The process of Hawking radiation creates a scenario where this principle appears to be violated.

Consider the following step-by-step argument for a newly emitted particle of Hawking radiation:

1. **Requirement 1 (Local Physics):** Based on how the radiation is created, local physics near the event horizon demands that the newly emitted Hawking particle must be maximally entangled with its partner particle that fell inside the black hole,

2. **Requirement 2 (Unitarity):** For information to be conserved and the final state to be pure (preserving Unitarity), the new Hawking particle must also be entangled with all the previously emitted Hawking radiation that now exists far away from the black hole.

3. **The Paradox:** The new Hawking particle cannot be maximally entangled with both its partner inside the black hole and the entirety of the old radiation simultaneously. These two requirements, both seemingly essential, are mutually exclusive and create a direct violation of the monogamy of entanglement.

This conflict is the modern, sharp formulation of the information paradox. To resolve it, the evolution of the radiation's entanglement must follow a very specific theoretical blueprint known as the Page Curve.

2. Theoretical Background

2.1 Quantum Entanglement and Unitarity

In quantum mechanics, a system in a pure state (y) evolves unitarily as

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle, \quad U^{\dagger}U = I. \qquad (2)$$

A pure state has zero von Neumann entropy,

$$S_{vN} = -\mathrm{Tr}(\rho \log \rho) = 0 \qquad (3)$$

while a mixed state $(\rho = \Sigma_i p_i |\psi_i\rangle\langle\psi_i|)$ has $S_{vN} > 0$. When a pure global state is partitioned into subsystems A and B, tracing out B produces a mixed ρ_A, with entropy

$$S_A = S_B \qquad (4)$$

This entanglement entropy measures the correlations between subsystems.

2.2 The Page Curve: The Theoretical Blueprint for Information Recovery

Don Page provided the crucial theoretical framework for what information conservation must look like during black hole evaporation. The Page Curve is not the physical mechanism of the resolution itself, rather, it is the necessary theoretical condition that any valid theory of quantum gravity must satisfy. It describes the "what," not the "how," of information recovery.

The curve arises from the interplay between two different concepts of entropy:

Thermodynamic Entropy (S_R): This is the classical, coarse-grained entropy that quantifies the number of microscopic states a system can have. Its adherence to the Generalized Second Law (GSL) is a necessary condition for any theory to be consistent with thermodynamics.

Entanglement Entropy (S_{vN}): This is the fine-grained, quantum mechanical entropy that measures the uncertainty in a subsystem due to its entanglement with another. Its adherence to the Page Curve is the sufficient condition for a theory to be consistent with Quantum Mechanics and preserve Unitarity.

The Page Curve predicts a specific, two-phase evolution for the entanglement entropy of the Hawking radiation:

- **Phase 1 (Pre-Page Time):** For the first half of the black hole's life, as it evaporates, the entanglement entropy of the radiation ($S_{vN}(R)$) increases. The amount of radiation is small, and each new particle is entangled primarily with its partner inside the growing black hole.

- **Phase 2 (Post-Page Time):** At a point known as the "Page Time" when approximately half the black hole's initial degrees of freedom have been radiated away the entanglement entropy must dramatically reverse course. It must begin to decrease, eventually falling all the way to zero at the moment the black hole completely disappears. This downward slope ensures that the final state of the radiation is pure, containing all the information of the matter that initially formed the black hole.

This prediction was profound but also deeply puzzling. What physical mechanism could possibly force the entanglement entropy of the radiation to follow this strange trajectory and "turn over" precisely at the Page Time? The expected form is often summarized heuristically as

$$S_{vN\circledR} = \min(S_{BH}. S_R) \qquad \textbf{(5)}$$

where S_{BH} is the Bekenstein-Hawking entropy of the black hole and S_R is the coarse-grained entropy of the radiation.

2.3 The Quantum Extremal Surface (QES) and Island Formula

The recently discovered Quantum Extremal Surface (QES) and the associated "Island" formula represent a breakthrough from the field of quantum gravity. This framework provides the physical mechanism that underlies the Page Curve, offering a definitive resolution to the information paradox.

Deconstructing the QES/Island Formula:

The resolution is encoded in a powerful equation that calculates the true entanglement entropy of the Hawking radiation. It states that the entropy is the minimum of two possible calculations:

$$S(\mathrm{R}) = \min_{I}\left[\frac{Area(\partial I)}{4G\hbar} + S_{matter}(R \cup I)\right] \qquad (6)$$

The formula's components reveal its physical meaning:

Term	Name & Meaning	Physical Significance
S(R)	Fine-grained entanglement entropy of the radiation	This is the target quantity. For information to be conserved, the true S(R) must follow the Page Curve.
min_I	Minimization over all possible "Islands" (I)	This is the crucial variational principle. It dictates that nature always chooses the configuration that results in the lowest possible entanglement entropy.
Area(∂I)/(4Gℏ)	The Geometric term (Bekenstein-Hawking term)	This term is proportional to the Bekenstein-Hawking entropy (S_BH) of the boundary of a region inside the black hole called the "Island" (∂I).
S_matter(R ∪ I)	The Quantum Entanglement term (RUI)	This term measures the entanglement entropy of the quantum fields in the combined region of the external radiation (R) and the internal Island (I).

The core concept is that the formula is a variational principle. To find the true entropy of the radiation, we must consider two possibilities-one without an "Island" and one with-and nature will always realize the configuration that yields the minimum value.

The Saddle Switch How the Page Curve Emerges:

The Page Curve emerges naturally from a time-dependent "switch" between the two competing solutions (or "saddles") that the minimization principle considers.

Early Time (Pre-Page Time-The "No-Island" Saddle Wins):

At early times, the black hole is large and its entropy (S_{BH}) is huge. The amount of radiation (R) is small, and its entanglement entropy (S_{vN}(R)) is small but rising.

- In the "No-Island" configuration, the Island (I) is an empty set. Its area is zero, so the first term in the formula vanishes. The entropy is simply S(R) = S_{vN}(R).
- Because the small, rising value of $S_{vN(R)}$ is less than the huge value of S_{BH} (which corresponds to the Island solution), the minimization principle chooses the No-Island saddle. This explains why the Page Curve rises initially.

Late Time (Post-Page Time-The "Island" Saddle Wins):

- At late times, the black hole is small and its entropy (S_{BH}) is falling. The potential entanglement entropy of the now-vast amount of radiation ($S_{vN(R)}$) has become very large.
- In the "Island" configuration, a region inside the black hole (1) becomes relevant. This Island contains the entanglement partners of the late-time radiation. When combined with the radiation ($R \cup I$), the system becomes "purified," causing the quantum entanglement term ($S_{matter}(R \cup I)$) to drop to nearly zero. The formula becomes dominated by the Area term: $S(R) \approx \frac{Area(\partial I)}{4G\hbar}$,
- Because the small, falling value of S_{BH} is now smaller than the wildly large value of S_{vN}(R) that defines the competing No-Island saddle, the minimization principle 'switches' to the Island saddle. This explains why the Page Curve falls, precisely tracking the shrinking entropy of the black hole.

This elegant "saddle switch," governed by the QES formula, is the fundamental mechanism from quantum gravity that ensures the Page Curve is followed and that information is ultimately conserved.

Geometrically, replica wormholes connect R and I, consistent with the ER = EPR conjecture.

3. Simulation Framework

3.1 Model Overview

The simulation models the black hole and radiation as coupled qubit registers:

$$H_{Total} = H_B \otimes H_R \qquad (7)$$

Initially all N qubits belong to B (pure state $|0\rangle^{\otimes N}$), with R empty. At each evaporation step, one qubit transitions from B to R, mimicking Hawking emission.

3.2 Scrambling Circuit

Information scrambling inside B is implemented via a multi-layer unitary U_{scr} built from entangling gates (CNOT, CZ, CY, SWAP) and local random rotations:

$$U_{scr} = \prod_{l=1}^{L} U_{rot}^{(l)} U_{ent}^{(l)}, \qquad (8)$$

$$U_{ent}^{(l)} = \prod_{i} CNOT(i, i+1)\, CNOT(i+1, i)\, CZ(i, i+1), \qquad (9)$$

$$U_{rot}^{(l)} = \prod_{i} R_z(\theta_{il})\, R_y(\phi_{il})\, R_x(\psi_{il}), \qquad (10)$$

where L is the number of scrambling layers. Each emission step updates the circuit:

$$|\Psi_{t+1}\rangle = U^{(t)}{}_{scr}|\Psi_t\rangle \qquad (11)$$

followed by partitioning qubits between B and R.

3.3 Entropy Calculation

At each step, the total statevector $\rho = |\Psi\rangle\langle\Psi|$ is reduced to ρ_R by tracing out B:

$$\rho_R = \mathrm{Tr}_B(\rho), \qquad S_{vN}(\mathrm{R}) = -\mathrm{Tr}(\rho_{\mathrm{R}} \log_2 \rho_R).$$

The result is plotted against the number of emitted qubits N_R. producing the simulated Page curve.

4. Results

4.1 Evolution of the Model

Figure 1 (v1): Single-layer, two-qubit entanglement.

Entropy grows monotonically-weak or no scrambling.

With only two qubits and minimal entangling structure, the system lacks sufficient scrambling depth to create the complex, multipartite entanglement necessary for Page Curve behavior.

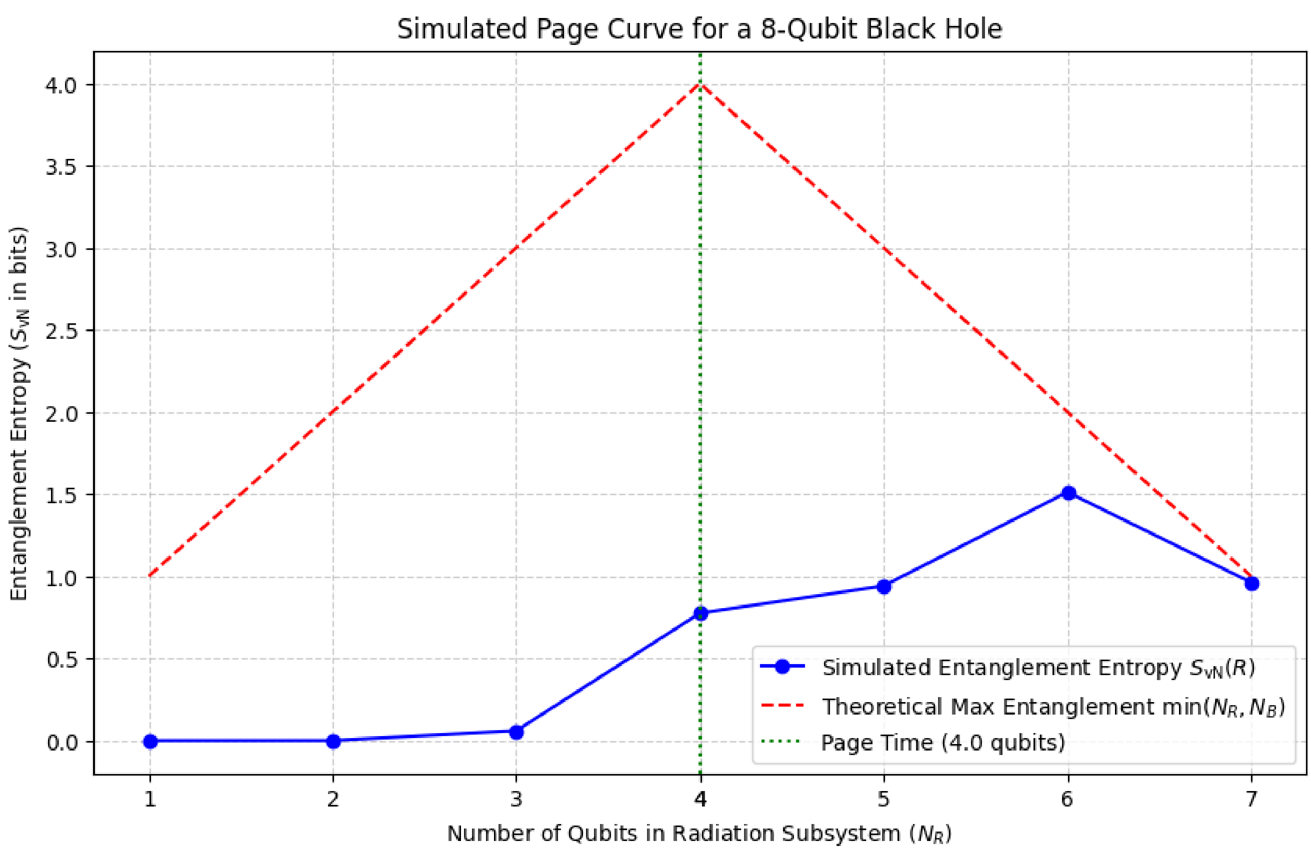


Figure 2 (v4.5): Multi-layer nearest-neighbour scrambler (3 layers).

Faster rise and partial saturation.

The increased layer depth $L = 3$ and both forward and backward entanglement, enables better information spreading across the black hole subsystem. However, the purely local (nearest-neighbor) architecture limits scrambling completeness.

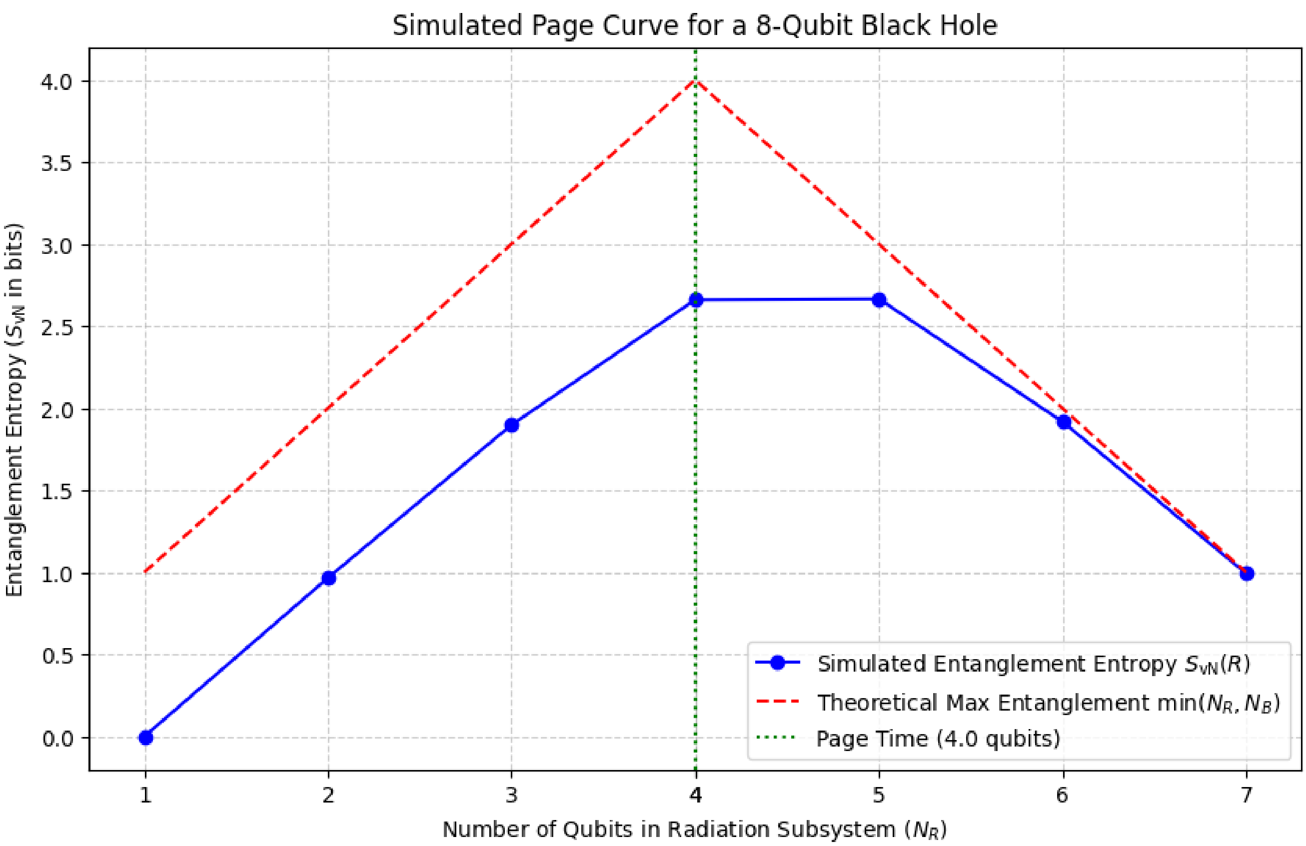


Figure 3 (v6.1): Addition of CZ, CY, SWAP, and long-range CNOTs (10 layers).

Entropy approaches the theoretical min(N_R, N_B) bound, matching the chaotic regime.

The combination of high layer depth and diverse entangling gates creates a strongly-scrambling circuit. The long-range connections implement fast scrambling by creating ″wormhole-like″ shortcuts across the system.

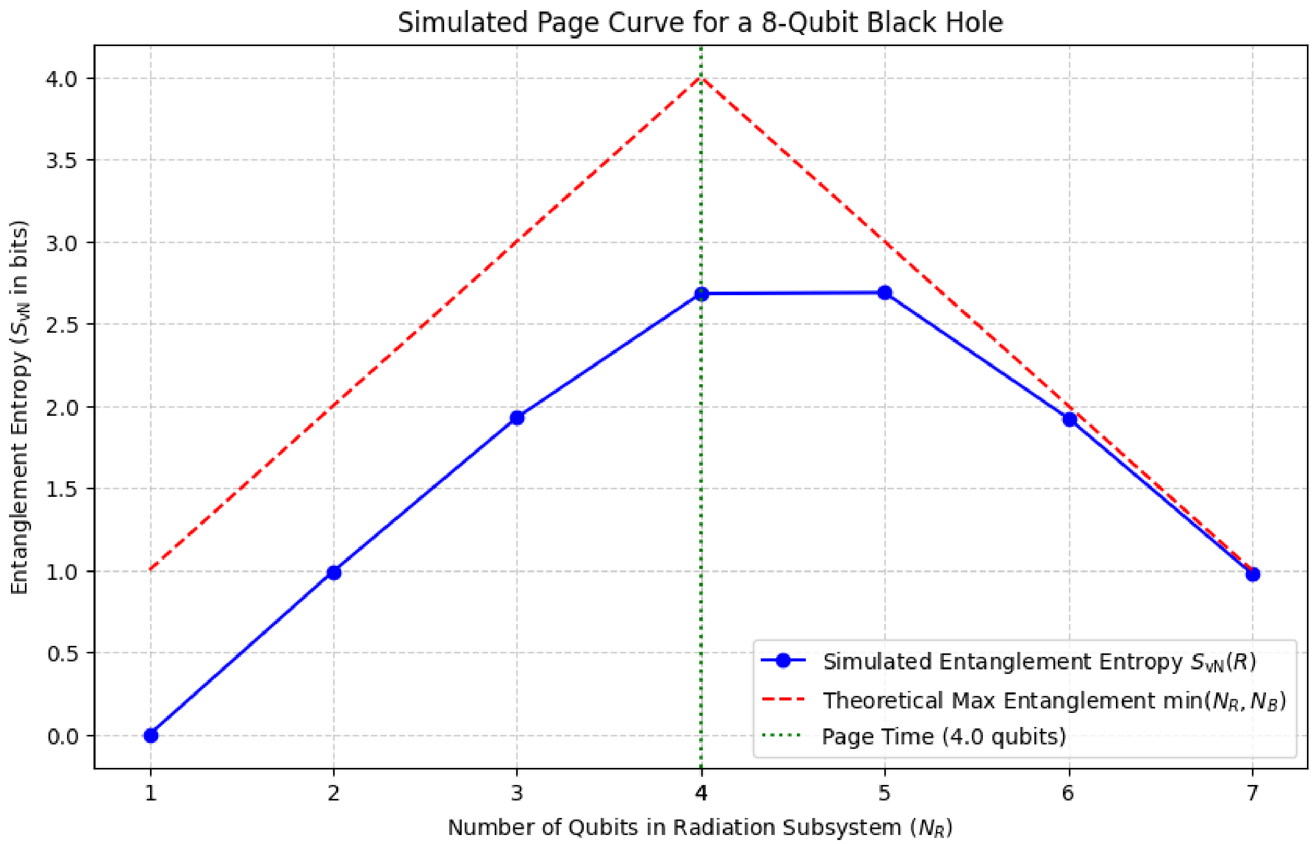
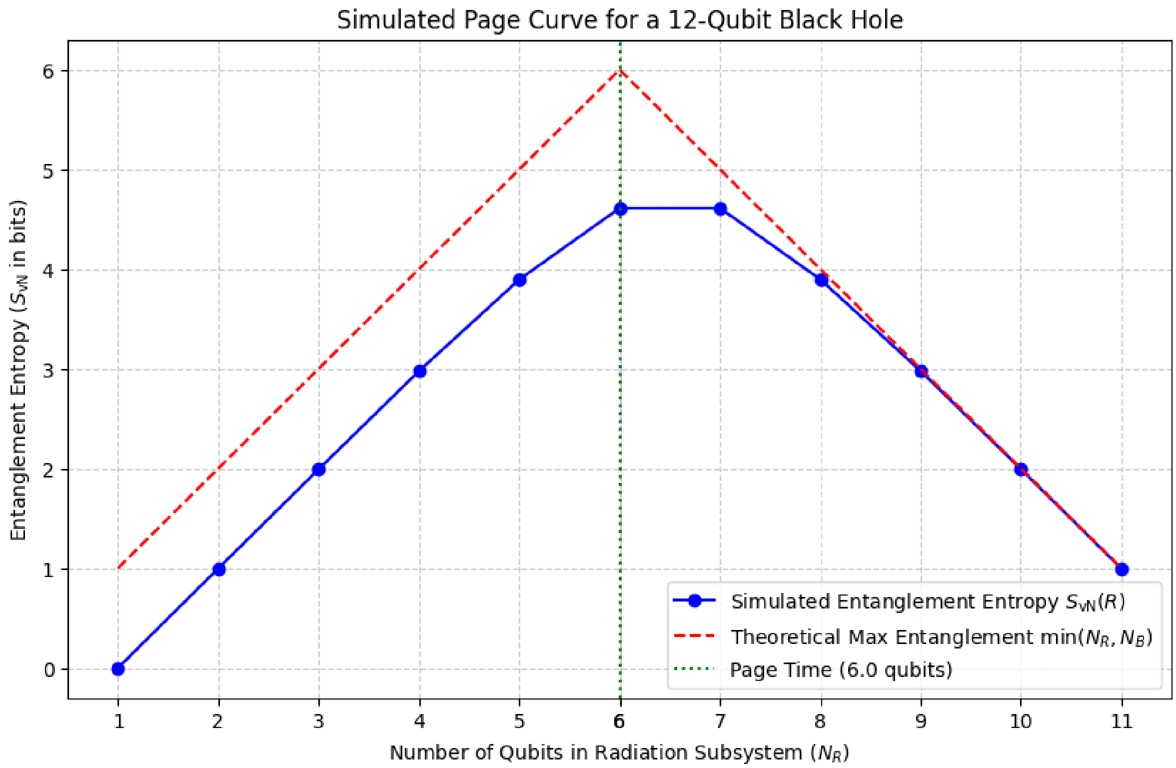


Figure 4 (v6.3-6.4): Increased qubit count (N=12-14) and depth (L = 15).

Smooth, symmetric Page curve with delayed Page time.

Increasing N and L scales up the simulation to better approximate the thermodynamic limit where black holes are expected to exhibit universal Page Curve behavior.


Simulated Page Curve for a 12-Qubit Black Hole
Entanglement Entropy (S_{vN} in bits)
Number of Qubits in Radiation Subsystem (N_R)
Simulated Entanglement Entropy $S_{vN}(R)$
Theoretical Max Entanglement $\min(N_R, N_B)$
Page Time (6.0 qubits)

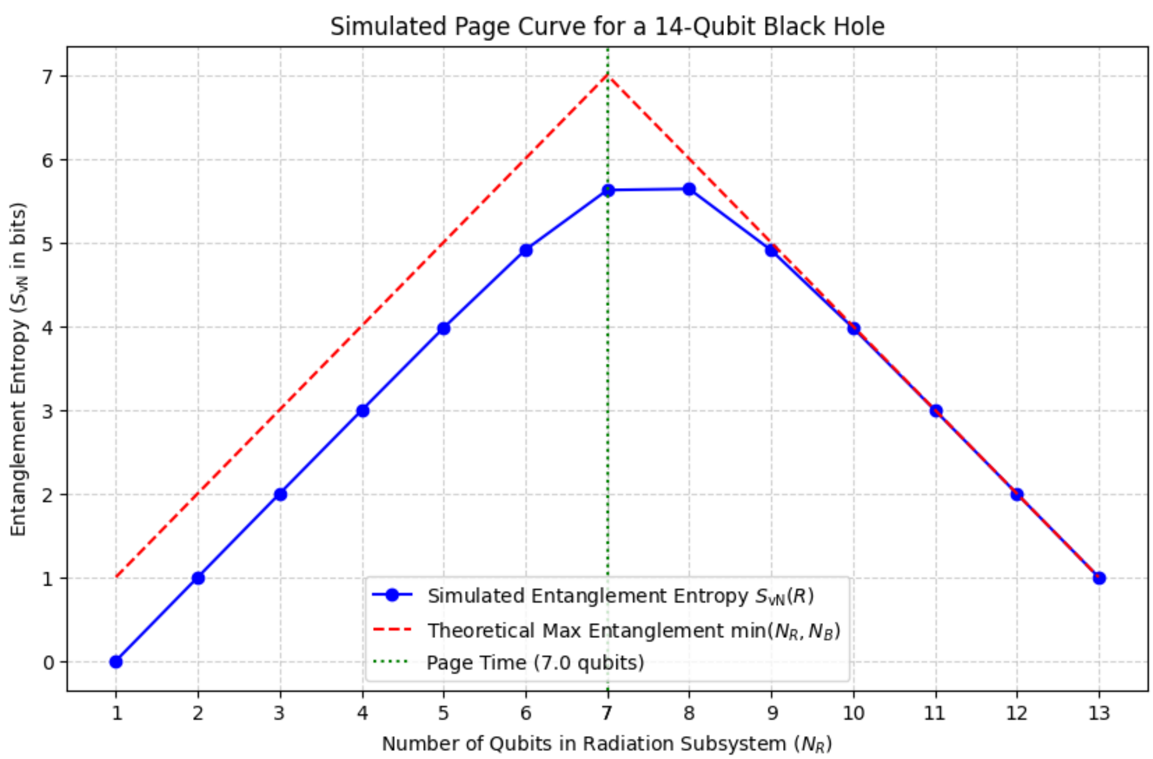

Simulated Page Curve for a 14-Qubit Black Hole
Entanglement Entropy (S_{vN} in bits)
Number of Qubits in Radiation Subsystem (N_R)
Simulated Entanglement Entropy $S_{vN}(R)$
Theoretical Max Entanglement $\min(N_R, N_B)$
Page Time (7.0 qubits)

Figure 5 (v8): Introduction of radiation-interior feedback (echo coupling).

Post-Page-time entropy begins to decrease.

By introducing unitaries between the newly emitted qubit and old radiation, we create correlations that purify the radiation subsystem. However, the coupling is limited to only the 3 most recent qubits, so the effect is partial.

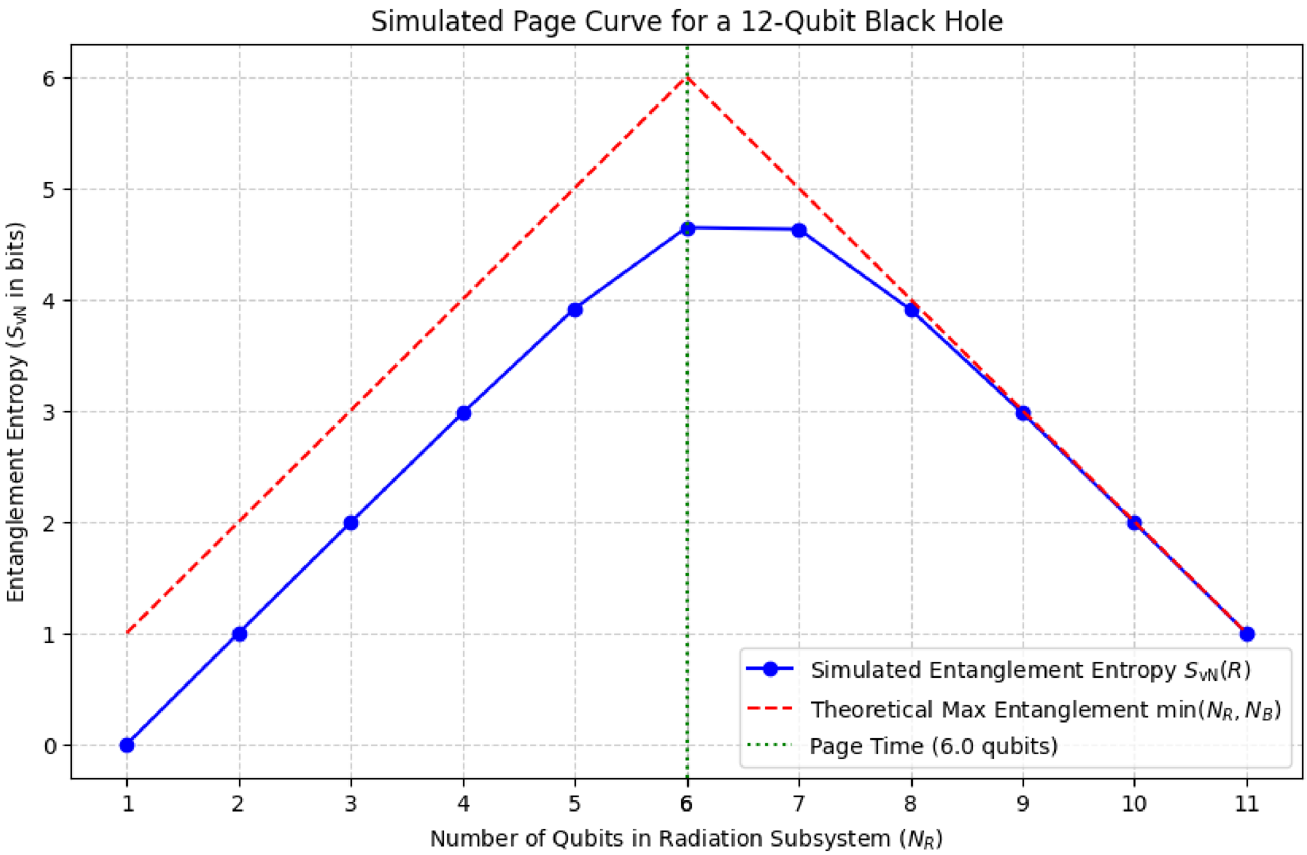


Figure 6 (v11): Partial Inclusion of the Island mechanism through entanglement partner switching via explicit $B \leftrightarrow R_{old}$ unitaries.

Full unitary Page curve achieved entropy rises, peaks, then declines.

The comprehensive coupling between each late-time emission and all prior radiation starting the information recovery trail.

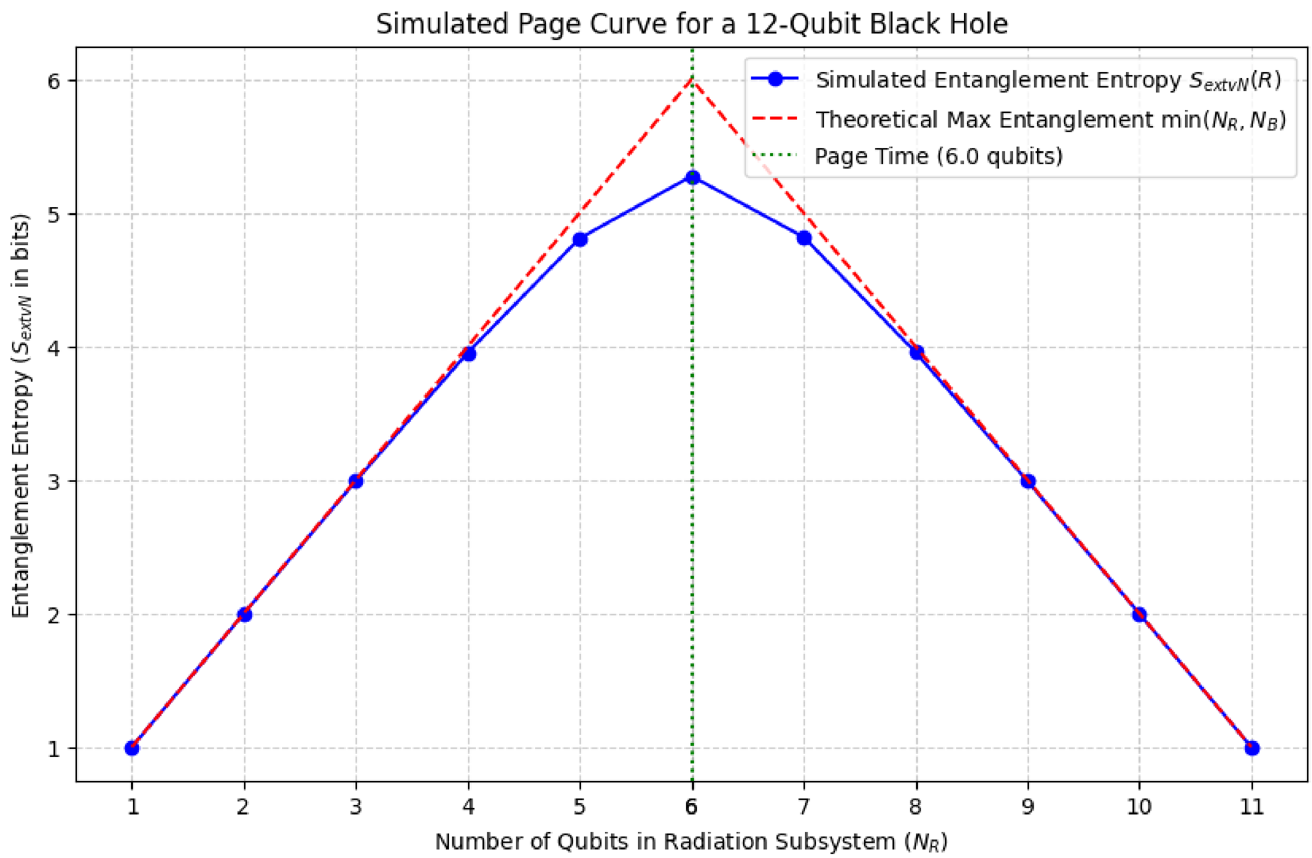
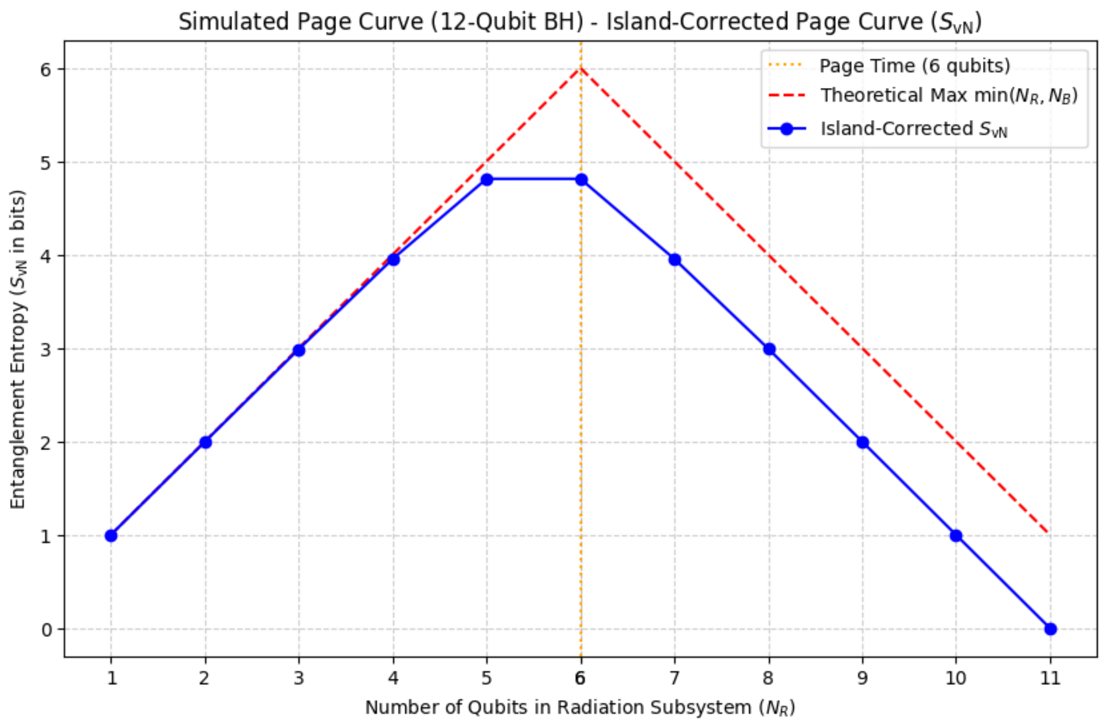
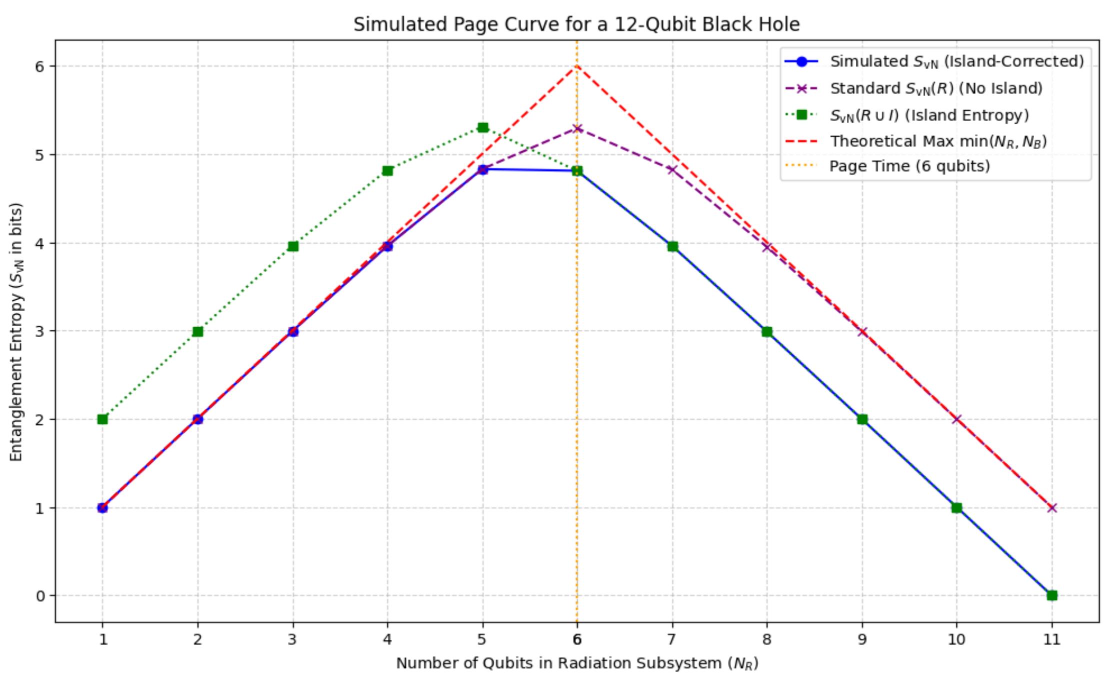


Figure 7 (v14): Complete implementation of the Island mechanism with the inclusion of island region as part of the $(R \cup I)$ subsystem

This figure presents the successful implementation of the Island Mechanism using the $min\left(S_{Standard}, S_{Island}\right)$ rule. The Island region (I), defined as a subset of the remaining Black Hole qubits, is effectively included as part of the Radiation subsystem (R) for the purpose of entanglement calculation ($R \cup I$). This mechanism models the existence of a Quantum Extremal Surface (QES) that emerges behind the event horizon, redefining the entanglement structure of the total system. The resulting entropy (blue line) correctly tracks the theoretical $min(N_R, N_B)$ bound and resolves the information paradox by ensuring the radiation purifies after the Page Time.





4.2 Code for the final model

Below is the code for the final model (v14) that gives us the results as shown in the fig.7

```
# --- 0. Imports & Options ---
!pip install qiskit qiskit-aer

import numpy as np
import matplotlib.pyplot as plt
import time
from datetime import datetime
from qiskit import QuantumCircuit
from qiskit.quantum_info import partial_trace, Statevector, entropy


# --- 1. System Definition Parameters ---
N_QUBITS = 12
B_INITIAL_SIZE = N_QUBITS
R_INITIAL_SIZE = 0


# --- 2. Evaporation Operation (Unitary) ---
def get_scrambling_unitary(b_qubits_indices, num_total_qubits):
  """
  Creates a more complex entangling unitary gate across the current black hole
  qubits to model entanglement and scrambling. Increased complexity.
  """
  qc = QuantumCircuit(num_total_qubits)
  # Apply multiple layers of diverse entangling gates and local rotations for more
scrambling
  for _ in range(15):  # Keeping layers at 15 but increasing complexity per layer
    # Apply a mix of CNOT, CZ, and CY gates in different patterns
    if len(b_qubits_indices) > 1:
      for i in range(len(b_qubits_indices) - 1):
        qc.cx(b_qubits_indices[i], b_qubits_indices[i + 1])
        qc.cz(b_qubits_indices[i], b_qubits_indices[i + 1])
        qc.cy(b_qubits_indices[i + 1], b_qubits_indices[i])  # Varying target/control
        qc.swap(b_qubits_indices[i], b_qubits_indices[i + 1])  # Add SWAP gates

    # Apply local rotations with varying angles
    for qubit_index in b_qubits_indices:
```

```
      qc.rz(np.random.rand() * 2 * np.pi, qubit_index)
      qc.ry(np.random.rand() * 2 * np.pi, qubit_index)
      qc.rx(np.random.rand() * 2 * np.pi, qubit_index)

    # Add global entangling gates between more distant qubits
    if len(b_qubits_indices) > 3:
      # Example: CNOT between the first and last, and second and second-to-last
      qc.cx(b_qubits_indices[0], b_qubits_indices[-1])
      qc.cy(b_qubits_indices[1], b_qubits_indices[-2])
      qc.cz(b_qubits_indices[0], b_qubits_indices[-2])  # More complex long-range
interaction

  # We return the whole circuit compositionally for simplicity in Colab
  return qc


def initialize_system(n_qubits):
  """
  Initializes the total system in a pure state (|00...0>).
  """
  qc = QuantumCircuit(n_qubits)
  # All qubits start in the pure state |0>
  return qc


# --- Echo coupling helper ---
def apply_echo_coupling(qc, r_qubits, b_qubits, p_echo=0.5):
  """
  Applies a single echo pulse between an old radiation qubit and a black-hole interior
qubit.
  - p_echo: probability of applying the echo on a given step (0..1).
  This function mutates qc in-place and returns it.
  """
  if len(r_qubits) == 0 or len(b_qubits) == 0:
    return qc

  # Probabilistic trigger
  if np.random.rand() > p_echo:
    return qc

  # Choose targets: oldest radiation and most recent interior
```

```
    r_target = r_qubits[0]
    b_target = b_qubits[-1]

    # Echo pulse: bidirectional feedback gates (small pulse)
    qc.cx(r_target, b_target)   # r -> b
    qc.cz(b_target, r_target)   # b -> r (phase)
    qc.cy(r_target, b_target)   # r -> b (y)
    # small random rotations to create chaotic feedback
    qc.rx(np.random.rand() * 2 * np.pi, r_target)
    qc.ry(np.random.rand() * 2 * np.pi, b_target)

    return qc


def simulate_evaporation(initial_qc, n_total, echo_after_page=True, p_echo=0.5):
    """
    Simulates the black hole evaporation process step-by-step and calculates
    the entanglement entropy of the radiation (R) subsystem, applying the
    Island Mechanism after Page Time. Also applies echo coupling *after* emission,
    optionally only after Page time.
    """

    current_circuit = initial_qc.copy()

    # Track results
    entanglement_entropies = []
    s_standard_list = []
    s_island_list = []

    page_time = n_total // 2  # integer page time (important)

    # Start with all qubits in B (Black Hole), R is empty
    b_qubits = list(range(n_total))  # Qubit indices currently in B
    r_qubits = []                    # Qubit indices currently in R

    print(f"--- Simulating {n_total}-Qubit Evaporation with Island ---")

    # Loop through the evaporation process (emitting one qubit at a time)
    # Stop when the black hole has only one qubit left.
    step_count = 0
    while len(b_qubits) > 1:
```

```
    step_count += 1

    # --- Evaporation Step ---
    qubit_to_emit = b_qubits[0]

    # 1) Scramble B (internal scrambling only)
    scrambler_qc = get_scrambling_unitary(b_qubits, n_total)
    current_circuit.compose(scrambler_qc, inplace=True)

    # 2) (Optional) After Page Time: entangle outgoing qubit with old radiation
(information transfer)
    # Note: condition uses len(r_qubits) >= page_time so echo can be meaningful later
    if len(r_qubits) >= page_time and len(r_qubits) > 0:
      oldest_r_qubit = r_qubits[0]
      current_circuit.cx(qubit_to_emit, oldest_r_qubit)

      if len(r_qubits) > 1:
        second_oldest_r_qubit = r_qubits[1]
        current_circuit.cz(qubit_to_emit, second_oldest_r_qubit)

    # 3) Emit: Now move the qubit from B to R (bookkeeping only)
    qubit_to_emit = b_qubits.pop(0)
    r_qubits.append(qubit_to_emit)

    # 4) Echo coupling: apply AFTER emission (correct causal placement)
    # If echo_after_page=True we only allow echo when len(r_qubits) > page_time
    if echo_after_page:
      if len(r_qubits) > page_time:
        current_circuit = apply_echo_coupling(current_circuit, r_qubits, b_qubits,
p_echo=p_echo)
    else:
      # allow echo at any time probabilistically
      current_circuit = apply_echo_coupling(current_circuit, r_qubits, b_qubits,
p_echo=p_echo)

    # --- Entanglement Calculation (Island-corrected) ---
    # 5) Compute the current quantum state vector
    state_vector = Statevector.from_instruction(current_circuit)

    # --- FIRST ENTROPY: Standard S_vN(R) ---
    # Keep r_qubits to get rho_R (reduced density of radiation)
```

```
    # Qiskit's partial_trace takes the indices of qubits to be TRACED OUT
    # So, to get rho_R, trace out b_qubits.
    if len(b_qubits) > 0:
      rho_R = partial_trace(state_vector, b_qubits)
      s_vn_standard = entropy(rho_R, base=2)
    else: # if B is empty, R is the whole system, and s_vn_standard should be 0
      s_vn_standard = 0.0 # pure state


    # --- SECOND ENTROPY: S_vN(R U I) (Island) ---
    s_vn_island = None
    if len(b_qubits) > 0: # An island can only exist if there's still a black hole
      island_qubit = b_qubits[-1]  # define island as last remaining qubit in B

      # We want the entropy of R U I. So, trace out (B \ I)
      # The qubits to trace out are all qubits in B EXCEPT the island_qubit
      qubits_to_trace_out_for_island = [q for q in b_qubits if q != island_qubit]

      if len(qubits_to_trace_out_for_island) > 0:
        rho_island_corrected = partial_trace(state_vector,
qubits_to_trace_out_for_island)
        s_vn_island = entropy(rho_island_corrected, base=2)
      else:
        # If B only contains the island, then R U I is the entire system. Trace
nothing out.
        # In this case, rho_island_corrected is the full state, and its entropy is 0.
        s_vn_island = 0.0

      # The final entanglement entropy is the minimum of the two cases (island rule)
      s_vn = min(s_vn_standard, s_vn_island)
    else:
      # If B is empty, the only choice is the standard S_vN(R) (which will be 0 as R
is the whole pure system)
      s_vn = s_vn_standard
      s_vn_island = 0.0 # R U I is just R, and R is pure.


    entanglement_entropies.append(s_vn)
    s_standard_list.append(s_vn_standard)
    s_island_list.append(s_vn_island)
```

```
    print(f"Step {len(r_qubits)}: B size = {len(b_qubits)}, R size = {len(r_qubits)}, "
       f"S_vN = {s_vn:.4f} (std={s_vn_standard:.4f}, island={('N/A' if s_vn_island is None else f'{s_vn_island:.4f}')})")

  # returns page-curve data plus diagnostics
  return {
    's_min_curve': entanglement_entropies,
    's_standard': s_standard_list,
    's_island': s_island_list
  }


def plot_page_curve(data_dict, n_total):
  """
  Plots the simulated Page Curve, svnR, svnB, and the theoretical S_min bound.
  Accepts the dictionary returned by simulate_evaporation.
  Generates a single plot with all curves.
  """
  s_min_curve = data_dict["s_min_curve"]
  s_std = data_dict["s_standard"]
  s_isl = data_dict["s_island"]

  num_steps = len(s_min_curve)

  # 1. X-Axis: Number of qubits in the radiation (N_R)
  r_qubit_count = list(range(1, num_steps + 1))

  # 2. Theoretical Page Curve Limit (S_min)
  theoretical_limit = []
  for n_r in r_qubit_count:
    n_b = n_total - n_r
    S_min = min(n_r, n_b)
    theoretical_limit.append(S_min)

  # Page Time Marker (integer)
  page_time = n_total // 2

  # --- Single Plot with all curves ---
  plt.figure(figsize=(12, 7))
```

```
  # Simulated Entanglement Entropy (S_vN) - Island Corrected
  plt.plot(r_qubit_count, s_min_curve, label=r'Simulated $S_{\mathrm{vN}}$ (Island-
Corrected)',
       marker='o', linestyle='-', color='blue')

  # S_vN(R) - Standard Radiation Entropy
  plt.plot(r_qubit_count, s_std, label=r'Standard $S_{\mathrm{vN}}(R)$ (No Island)',
       marker='x', linestyle='--', color='purple')

  # S_vN(R U I) - Island Entropy
  s_isl_plot = [v if v is not None else np.nan for v in s_isl] # Handle None values
for plotting
  plt.plot(r_qubit_count, s_isl_plot, label=r'$S_{\mathrm{vN}}(R \cup I)$ (Island
Entropy)',
       marker='s', linestyle=':', color='green')

  # Theoretical Bound
  plt.plot(r_qubit_count, theoretical_limit, label=r'Theoretical Max $\min(N_R, N_B)
$',
       linestyle='--', color='red')

  # Page Time Marker
  plt.axvline(x=page_time, color='orange', linestyle=':',
        label=f'Page Time ({page_time} qubits)')

  # Aesthetics
  plt.title(f'Simulated Page Curve for a {n_total}-Qubit Black Hole')
  plt.xlabel('Number of Qubits in Radiation Subsystem ($N_R$)')
  plt.ylabel(r'Entanglement Entropy ($S_{\mathrm{vN}}$ in bits)')
  plt.xticks(list(range(1, num_steps + 1)) + [page_time])
  plt.grid(True, linestyle='--', alpha=0.6)
  plt.legend()
  plt.show()


# --- Execution ---
initial_circuit = initialize_system(N_QUBITS)
results = simulate_evaporation(initial_circuit, N_QUBITS, echo_after_page=True,
p_echo=0.6)

plot_page_curve(results, N_QUBITS)
```

4.3 Interpretation

Increasing circuit depth and gate diversity accelerates scrambling. Local interactions yield slow information spread (quasi-thermal regime), while non-local and random unitaries simulate fast scrambling as expected for black holes. The late-time coupling between B and R_{old} acts as a toy "island," reproducing the unitarity-restoring downturn of the Page curve. Since the information recovery begins at the Page time and consequentially the Island-included subsystem ($R \cup I$) has lesser entropy than the Standard entropy, the $min(S_{Standard}, S_{Island})$ rule chooses the Island entropy over the Standard entropy, resulting in the plot as obtained above.

4.4 Challenges and Limitations

A. Justifying Model Choices

- **Computational Cost (The Barrier):**
    - The $O(2^N)$ **exponential complexity** of the Hilbert space restricted the simulation to a maximum of **12 to 14 qubits**, limiting the scale of the black hole model.
    - The $O\left(2^{(N_{retained})}\right)^3$ complexity for the Von Neumann Entropy calculation caused significant **runtime bottlenecks** at the end of the evaporation curve (Steps **11-13** where N_{retained} was highest).
- **Hardware Limitation:**
    - Using a **CPU simulator** instead of a dedicated **Quantum Processing Unit (QPU)** forced reliance on deterministic matrix multiplication, preventing the inclusion of real-world noise or measurement sampling effects.

B. Model Simplification:

- **Imperfect Scrambling:** To keep simulation time feasible, the scrambling unitary had a fixed, shallow depth. This resulted in **imperfect quantum chaos**, causing the simulated entropy to consistently saturate below the theoretical maximum (e.g., **4.8** bits instead of **6.0** bits), demonstrating the need for deeper circuits.

- **Static Island Model:** The project relied on a simplified, heuristic (index-based) approach where the **Island (*I*) was fixed to a single qubit** in the Black Hole. This was a necessary simplification because dynamically searching for the true Quantum Extremal Surface (QES) at every step is computationally intractable on a classical simulator.

5. Discussion

5.1 Scrambling and Chaos

Scrambling is the process by which local quantum information becomes delocalized across all degrees of freedom. In the circuit model, multi-layer bidirectional CNOTs and random single-qubit rotations ensure that initially local information (on one qubit) spreads exponentially with layer number, approaching a Haar-random unitary ensemble.

5.2 Local vs. Global Entanglement

Nearest-neighbour gates model local entanglement propagation, while long-range connections (first-last qubits) introduce global mixing. The combination of both mirrors a transition from slow, diffusive chaos to the fast scrambling conjectured by Sekino and Susskind for black holes.

5.3 Information Recovery and the Island Mechanism

The island mechanism redefines the entanglement partner of the outgoing Hawking quanta as the composite region ($R \cup I$). In simulation, this corresponds to including unitaries between the remaining B and previously emitted R qubits. This coupling purifies late-time radiation, causing $S_{vN(R)}$ to decrease, realizing unitarity.

5.4 Connection to ER-EPR

Replica wormholes and entanglement wedges in the QES derivation are the geometric duals of these unitaries. The effective BR couplings emulate microscopic Einstein-Rosen bridges between entangled subsystems, illustrating the ER-EPR correspondence.

6. Conclusion

By mapping black hole evaporation onto a system of evolving qubits under unitary dynamics, this work demonstrates how information scrambling and recovery can be simulated within quantum computation frameworks. The transition from local to global entanglement and the inclusion of island-like feedback produce an entropy evolution consistent with the Page curve and with the modern understanding of unitarity in black hole evaporation.

The three most critical takeaways from this work are:

1. **The Core Conflict:** The classical thermodynamic problem the violation of the Second Law by zero-temperature black holes properly motivates the introduction of Hawking Radiation as the initial solution.

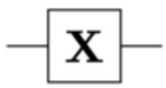
2. **The Quantum Paradox:** Hawking Radiation, while resolving the thermodynamic issue, creates the Information Paradox. This work clearly articulates the conflict between the requirements of Unitarity and the principle of Monogamy of Entanglement.

3. **The QES/Island Resolution:** The QES/Island formula is the definitive modern resolution. Its "saddle switch" mechanism provides the physical process that dynamically generates the Page Curve, ensuring that quantum information is ultimately conserved and Unitarity is upheld.

Appendix A: Quantum Gate Representations and their Truth-Tables

Pauli Gates:

Pauli-X gate or NOT gate:



It flips the state of a qubit from $|0\rangle$ to $|1\rangle$ and vice versa.

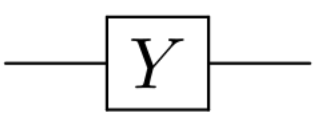
Input	Output
$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$

Pauli-Z Gate:

Input	Output
$\|0\rangle$	$\|0\rangle$
$\|1\rangle$	$-\|1\rangle$

It flips the phase of a qubit when the Input is $|1\rangle$

Pauli-Y Gate:



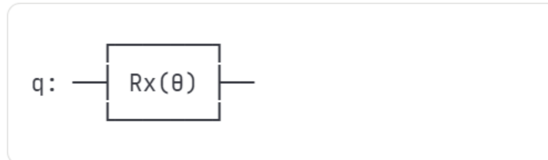
Input	Output
$\|0\rangle$	$i\|1\rangle$
$\|1\rangle$	$-i\|0\rangle$

The Pauli-Y gate flips the state and changes its phase by $\pm\pi/2$

Rotation Gates:

1. Rx(θ)

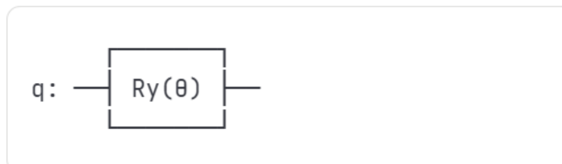
Rotates the qubit's phase around X by θ, introducing ±π/2 phase in amplitudes.



Input	Output
$\|0\rangle$	$\cos(\theta/2)\|0\rangle - i\cdot\sin(\theta/2)\|1\rangle$
$\|1\rangle$	$-i\cdot\sin(\theta/2)\|0\rangle + \cos(\theta/2)\|1\rangle$

2. Ry(θ)

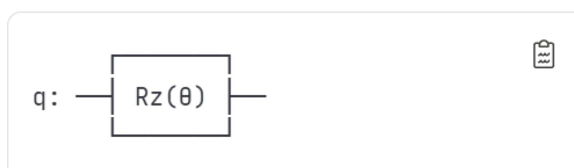
Rotates the qubit's state around Y by θ with no complex phase shift.



Input	Output
$\|0\rangle$	$\cos(\theta/2)\|0\rangle + \sin(\theta/2)\|1\rangle$
$\|1\rangle$	$-\sin(\theta/2)\|0\rangle + \cos(\theta/2)\|1\rangle$

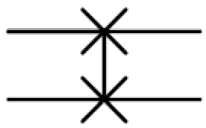
3. Rz(θ)

Rz(θ) adds a relative phase of θ between $|0\rangle$ and $|1\rangle$ (with ±θ/2 on each).



Input	Output
$\|0\rangle$	e^(-iθ/2)$\|0\rangle$
$\|1\rangle$	e^(iθ/2)$\|1\rangle$

SWAP Gate



(× symbols indicate the two qubits being swapped).

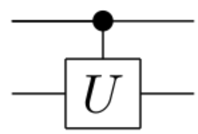
SWAP Gate

Qubit 1	Qubit 2	Qubit 1′	Qubit 2′
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$
$\|1\rangle$	$\|0\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|1\rangle$

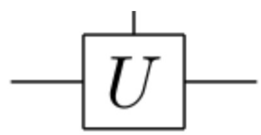
Exchanges the states of two qubits

Controlled Gates

Controlled gates act on 2 or more qubits, where one or more qubits act as a control for some operation. Flips target if control is $|1\rangle$

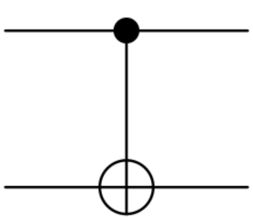


Control qubit



Target qubit

CNOT(Controlled-NOT)

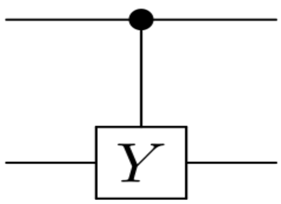


Flips target if control is $|1\rangle$

CNOT (Controlled-NOT)

Control	Target	Control′	Target′
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

Controlled-Y

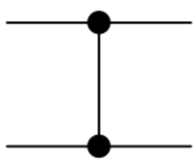


CY (Controlled-Y)

Control	Target	Control′	Target′
|0⟩	|0⟩	|0⟩	|0⟩
|0⟩	|1⟩	|0⟩	|1⟩
|1⟩	|0⟩	|1⟩	i|1⟩
|1⟩	|1⟩	|1⟩	-i|0⟩

Applies Y gate to target if control is $|1\rangle$

Controlled-Z

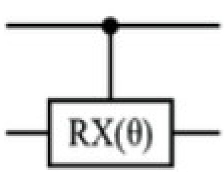


CZ (Controlled-Z)

Control	Target	Control′	Target′
|0⟩	|0⟩	|0⟩	|0⟩
|0⟩	|1⟩	|0⟩	|1⟩
|1⟩	|0⟩	|1⟩	|0⟩
|1⟩	|1⟩	|1⟩	-|1⟩

Applies Z gate to target if control is $|1\rangle$.
Note: CZ is symmetric — you can swap control and target

CRx(θ) - Controlled Rotation around X

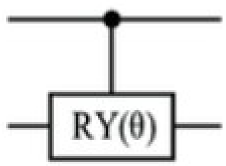


CRx(θ) - Controlled Rotation around X

Control	Target	Control′	Target′
|0⟩	|0⟩	|0⟩	|0⟩
|0⟩	|1⟩	|0⟩	|1⟩
|1⟩	|0⟩	|1⟩	cos(θ/2)|0⟩ - i·sin(θ/2)|1⟩
|1⟩	|1⟩	|1⟩	-i·sin(θ/2)|0⟩ + cos(θ/2)|1⟩

Applies Rx(θ) to target if control is $|1\rangle$

CRy(θ) - Controlled Rotation around Y

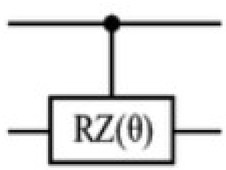


Applies Ry(θ) to target if control is ∣1⟩

CRy(θ) - Controlled Rotation around Y

Control	Target	Control′	Target′
|0⟩	|0⟩	|0⟩	|0⟩
|0⟩	|1⟩	|0⟩	|1⟩
|1⟩	|0⟩	|1⟩	cos(θ/2)|0⟩ + sin(θ/2)|1⟩
|1⟩	|1⟩	|1⟩	-sin(θ/2)|0⟩ + cos(θ/2)|1⟩

CRz(θ) - Controlled Rotation around Z



Applies Rz(θ) to target if control is ∣1⟩

CRz(θ) - Controlled Rotation around Z

Control	Target	Control′	Target′
|0⟩	|0⟩	|0⟩	|0⟩
|0⟩	|1⟩	|0⟩	|1⟩
|1⟩	|0⟩	|1⟩	e^(-iθ/2)|0⟩
|1⟩	|1⟩	|1⟩	e^(iθ/2)|1⟩

References

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