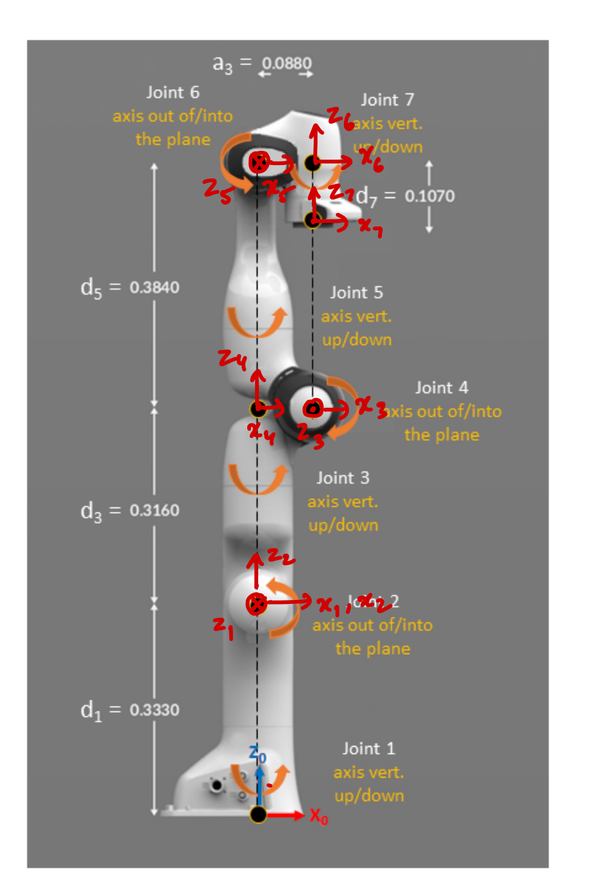
MODELING
HW5

1.

⊗ - out of plane

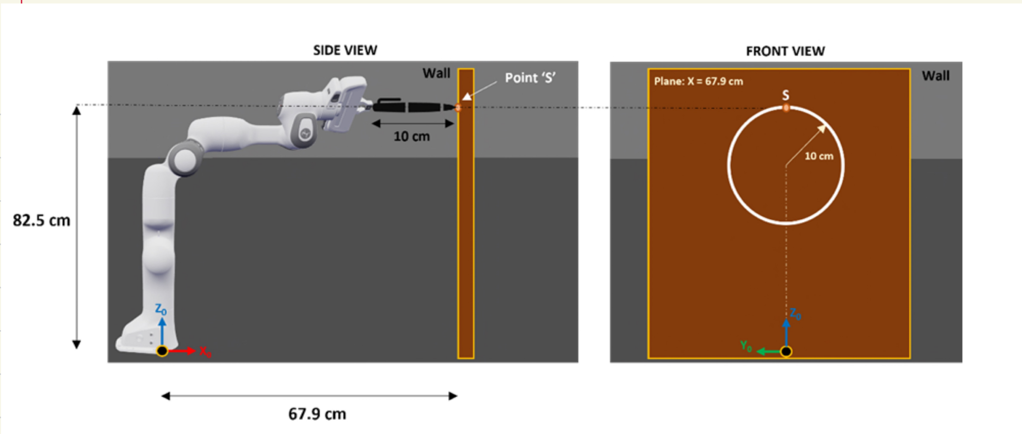
⊙ - inside the plane



Franka Emika Panda Cobot

D-H Parameter Table

	α	θ	d	a
$0 \to 1$	$\pi/2$	θ_1	d_1	0
$1 \to 2$	$-\pi/2$	θ_2	0	0
$2 \to 3$	$-\pi/2$	θ_3	d_3	a_3
$3 \to 4$	$\pi/2$	θ_4	0	$-a_3$
$4 \to 5$	$\pi/2$	θ_5	d_5	0
$5 \to 6$	$-\pi/2$	θ_6	0	a_3
$6 \to 7$	0	θ_7	$-d_7$	0



$\theta_3 = 0$ since Joint 3 is fixed

D-H Parameter Table

	α	θ	d	a
$0 \to 1$	$\pi/2$	0	d_1	0
$1 \to D$	$-\pi/2$	0	0	0
$D \to 3$	$-\pi/2$	0	d_3	a_3
$3 \to 4$	$\pi/2$	$\pi/2$	0	$-a_3$
$4 \to 5$	$\pi/2$	0	d_5	0
$5 \to 6$	$-\pi/2$	π	0	a_3
$6 \to 7$	0	0	$-d_7$	0

$d_7 = 0.107 + \underbrace{0.1}$ $\rightarrow$ Length of pen
$= 0.207$

$$q(\text{at } t=0) = [q_1, q_2, q_3, q_4, q_5, q_6, q_7]^T$$
$$= [0, 0, 0, \pi/2, 0, \pi, 0]^T$$

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rot_{x,\alpha_i}$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Position of end-effector is given by :

$$T_7^0 = A_1 A_2 A_D A_4 A_5 A_6 A_7$$

Now, For Jacobian to be a square matrix of size 6×6, we take six homogeneous transformation matrix as follows :

$$M_1 = T_1^0$$

$$M_2 = T_1^0 T_D^1$$

$$M_3 = T_1^0 T_D^1 T_3^D T_4^3$$

$$M_4 = T_1^0 T_D^1 T_3^D T_4^3 T_5^4$$

$$M_5 = T_1^0 T_D^1 T_3^D T_4^3 T_5^4 T_6^5$$

$$M_8 = T_1^0 T_D^1 T_3^D T_4^3 T_5^4 T_6^5 T_7^6$$

We extract the z-components from each transformation matrix and the end-effector position (x_p) from the last transformation matrix for the calculation of Jacobian.

$$X_p(q_1, q_2 \cdots q_7) = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix}$$

For calculating the components of the Jacobian using second method, we use the formula:

$$J_i = \left[\begin{array}{c} \dfrac{dX_p}{dq_i} \\ \hdashline Z_i \end{array}\right]$$

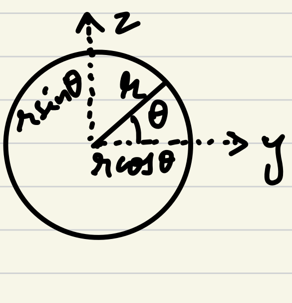
where Z_i is the z-component from each transformation matrix

$\therefore$ The final Jacobian matrix is then given by

$$J = \begin{bmatrix} J_1 & J_2 & J_3 & J_4 & J_5 & J_6 \end{bmatrix}$$

Now, radius of circle = 10 cm = 0.1 m
Coordinates of point S = (0.679, 0, 0.825)
Center of circle = (0.679, 0, 0.725)

$\therefore$ Circle equation in parametric form is given by:



$$x = 0.679$$
$$y = r\cos\theta = 0.1\cos\theta$$
$$z = r\sin\theta = 0.1\sin\theta + 0.725$$

Differentiating with respect to time, we get velocity in the x, y and z direction

$$V_x = \dot{x} = 0$$

$$V_y = \dot{y} = -0.1 \sin\theta \cdot \dot{\theta}$$

$$V_z = \dot{z} = 0.1 \cos\theta \cdot \dot{\theta}$$

Here, $\dot{\theta} = \dfrac{d\theta}{dt} = \dfrac{2\pi}{5}$ $\left[\dfrac{\text{Change in angle}}{\text{Change in time}}\right]$

Now, $q = \begin{bmatrix} q_1 \\ q_2 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \end{bmatrix}$; q_3 is fixed

$q_i = \theta_i$

$$\dot{x} = \begin{bmatrix} V_x \\ V_y \\ V_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 \\ -0.1 \sin\theta \cdot \dot{\theta} \\ 0.1 \cos\theta \cdot \dot{\theta} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\omega_x, \omega_y, \omega_z$ is 0 because the end-effector is fixed and has no angular velocity

$$\dot{X} = J \cdot \dot{q}$$

$$\therefore \dot{q} = J^{-1} \dot{X}$$

where $\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \\ \dot{q}_7 \end{bmatrix}$

To obtain new joint angles we use the formula:

$$q_{next} = q_{current} + \dot{q}_{next} \Delta t$$

where $\Delta t = \frac{T}{N}$; T is total time
N is the no. of data points

Finally, to plot the circle, we use the x, y and z components from X_p matrix which we derive from H_6 matrix.

The dynamic equation of the robot is given by:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + J^T(q)F \quad - (1)$$

Assuming the robot motion is quasi-static

$$\therefore \dot{q} \cong 0 \; , \; \ddot{q} \cong 0$$

$\Rightarrow$ Dynamic Equation (1) reduces to :-

$$g(q) = \tau + J^T(q)F \quad - (2)$$

To find torques, we rearrange equation (2)

$$\tau = g(q) - J^T(q)F \quad - (3)$$

where $\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_4 \\ \tau_5 \\ \tau_6 \\ \tau_7 \end{bmatrix}$; $F = \begin{bmatrix} -5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

We can calculate potential energy of each link by:

$$P_i = m_i g^T r_{c_i} \, ;$$

where
- P_i is P.E of i^{th} link
- g is gravity vector
- r_{c_i} is the center of mass coordinates of i^{th} link

$$P = \begin{bmatrix} P_1 \\ P_2 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix}$$

Differentiating P matrix with q_k gives us the gravity matrix:

$$g_k = \frac{\delta P}{\delta q_k}$$

using eq^n 3

Finally, after calculating torques of each joint, we plot it with respect to time (200s)