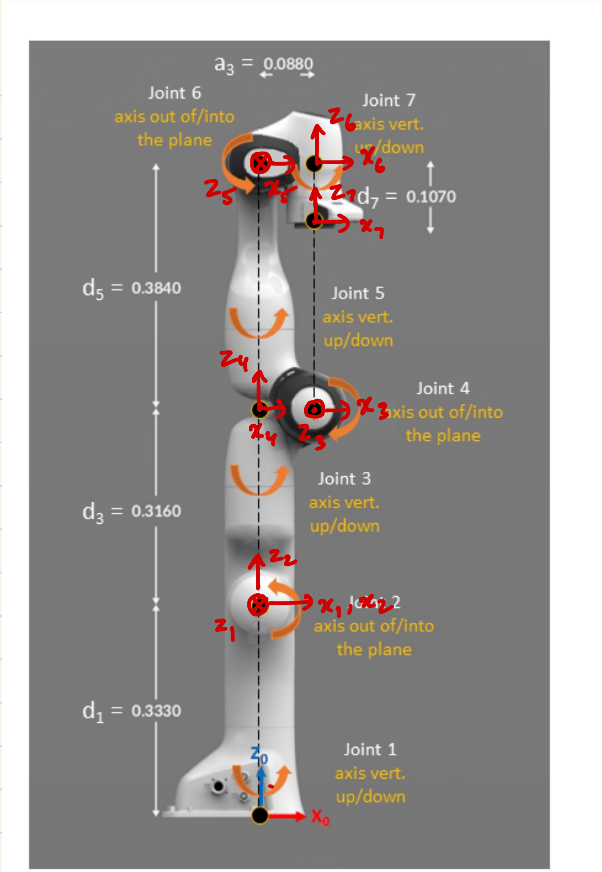
MODELING
HW4

1.

$\otimes$ - out of plane

$\odot$ - inside the plane



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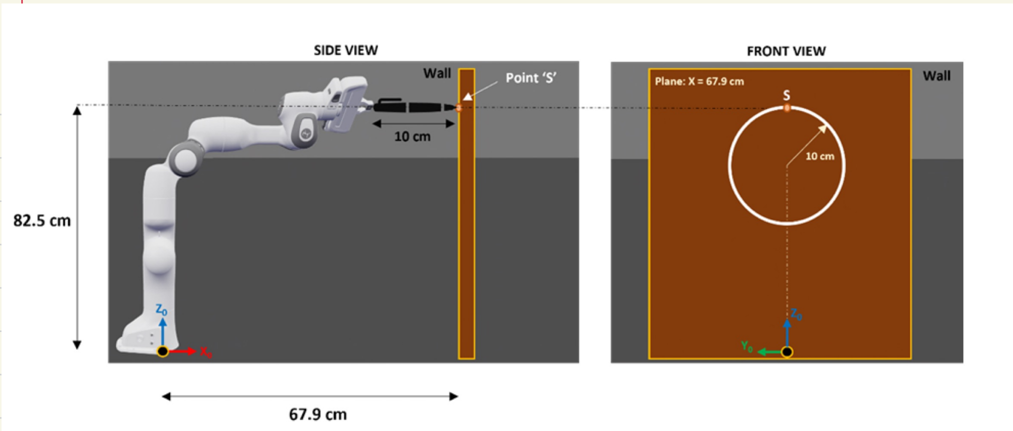
D-H Parameter Table

	α	θ	d	a
$0 \to 1$	$\pi/2$	θ_1	d_1	0
$1 \to 2$	$-\pi/2$	θ_2	0	0
$2 \to 3$	$-\pi/2$	θ_3	d_3	a_3
$3 \to 4$	$\pi/2$	θ_4	0	$-a_3$
$4 \to 5$	$\pi/2$	θ_5	d_5	0
$5 \to 6$	$-\pi/2$	θ_6	0	a_3
$6 \to 7$	0	θ_7	$-d_7$	0



$\theta_3 = 0$ since Joint 3 is fixed

D-H Parameter Table

	α	θ	d	a
$0 \to 1$	$\pi/2$	0	d_1	0
$1 \to D$	$-\pi/2$	0	0	0
$D \to 3$	$-\pi/2$	0	d_3	a_3
$3 \to 4$	$\pi/2$	$\pi/2$	0	$-a_3$
$4 \to 5$	$\pi/2$	0	d_5	0
$5 \to 6$	$-\pi/2$	π	0	a_3
$6 \to 7$	0	0	$-d_7$	0

$d_7 = 0.107 + \underset{\sim\sim}{0.1} \longrightarrow$ Length of pen
$= 0.207$

$$q(\text{at } t=0) = [q_1, q_2, q_3, q_4, q_5, q_6, q_7]^T$$
$$= [0, 0, 0, \pi/2, 0, \pi, 0]^T$$

$$A_i = Rot_{z,\theta_i}\, Trans_{z,d_i}\, Trans_{x,a_i}\, Rot_{x,\alpha_i}$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Position of end-effector is given by :

$$T_7^0 = A_1 A_2 A_D A_4 A_5 A_6 A_7$$

Now, For Jacobian to be a square matrix of size 6×6, we take six homogeneous transformation matrix as follows:

$$M_1 = T_1^0$$

$$M_2 = T_1^0\, T_D^1$$

$$M_3 = T_1^0\, T_D^1\, T_3^D\, T_4^3$$

$$M_4 = T_1^0\, T_D^1\, T_3^D\, T_4^3\, T_5^4$$

$$M_5 = T_1^0\, T_D^1\, T_3^D\, T_4^3\, T_5^4\, T_6^5$$

$$M_8 = T_1^0\, T_D^1\, T_3^D\, T_4^3\, T_5^4\, T_6^5\, T_7^6$$

We extract the z-components from each transformation matrix and the end-effector position (x_p) from the last transformation matrix for the calculation of Jacobian.

$$X_p(q_1, q_2 \dots q_7) = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix}$$

For calculating the components of the Jacobian using second method, we use the formula:

$$J_i = \left[\begin{array}{c} \frac{dX_p}{dq_i} \\ \hdashline Z_i \end{array}\right]$$

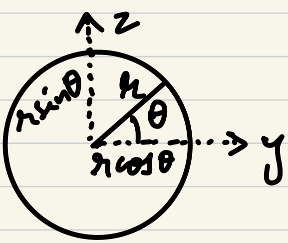
where Z_i is the z-component from each transformation matrix

$\therefore$ The final Jacobian matrix is then given by

$$J = [J_1 \; J_2 \; J_3 \; J_4 \; J_5 \; J_6]$$

Now, radius of circle = 10cm = 0.1m
Coordinates of point S = (0.679, 0, 0.825)
Center of circle = (0.679, 0, 0.725)

$\therefore$ Circle equation in parametric form is given by:



$$x = 0.679$$
$$y = r\cos\theta = 0.1\cos\theta$$
$$z = r\sin\theta = 0.1\sin\theta + 0.725$$

Differentiating with respect to time, we get velocity in the x, y and z direction

$$V_x = \dot{x} = 0$$

$$V_y = \dot{y} = -0.1 \sin\theta . \dot{\theta}$$

$$V_z = \dot{z} = 0.1 \cos\theta . \dot{\theta}$$

Here, $\dot{\theta} = \frac{d\theta}{dt} = \frac{2\pi}{5}$ $\left[\frac{\text{Change in angle}}{\text{Change in time}}\right]$

Now, $q = \begin{bmatrix} q_1 \\ q_2 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \end{bmatrix}$; q_3 is fixed

$q_i = \theta_i$

$$\dot{x} = \begin{bmatrix} V_x \\ V_y \\ V_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 \\ -0.1 \sin\theta . \dot{\theta} \\ 0.1 \cos\theta . \dot{\theta} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\omega_x, \omega_y, \omega_z$ is 0 because the end-effector is fixed and has no angular velocity

$\dot{X} = J . \dot{q}$

$\therefore \dot{q} = J^{-1} \dot{X}$

where $\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \\ \dot{q}_7 \end{bmatrix}$

To obtain new joint angles we use the formula:

$$q_{next} = q_{current} + \dot{q}_{next} \Delta t$$

where $\Delta t = \frac{T}{N}$; T is total time
N is the no. of data points

Finally, to plot the circle, we use the x, y and z components from X_p matrix which we derive from H_6 matrix.