Implementing Informed RRT and Comparing Results with RRT and RRT**

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***Abstract*—This paper discusses motion planning using Informed RRT* and compares with optimal Rapidly-exploring random trees (RRT*s). Though optimal RRTs (RRT*s) are used to find the optimal solution, they are inefficient and inconsistent with their single-query nature. Informed RRT on the other hand directly samples a subset of states that can improve a solution, which is described by a prolate hyperspheroid. This sampling technique improves the convergence rate and final solution quality while retaining the same guarantees on completeness and optimality as RRT*. The algorithm is a simple modification to RRT* yet outperforms it in rate of convergence, final solution cost, and ability to find difficult passages while being less dependent on the state dimension and range of the planning problem. The two algorithms are then compared to verify the optimality of the solution obtained.**

I. Introduction

In motion planning, two commonly used approaches are graph-based searches and stochastic incremental searches. Graph-based searches like A* discretize the continuous state space using a grid and are often resolution complete and resolution optimal. They guarantee finding the optimal solution and return failure otherwise, but they do not scale well with the problem size. On the other hand, stochastic searches use sampling-based methods to avoid discretization, allowing for more effective scaling with problem size and direct consideration of kinodynamic constraints. Stochastic searches, such as RRTs, Probabilistic Roadmaps (PRMs), and Expansive Space Trees (ESTs), have a less strict completeness guarantee, but are probabilistically complete. RRTs guarantee a high probability of finding a solution, if one exists, as the number of iterations approaches infinity.

However, the issue with RRTs is that they are not asymptotically optimal since the current state graph can create bias in future expansion. RRT* addresses this problem by introducing incremental rewiring of the graph. This means that new states are not only added to the tree, but they are also considered as replacement parents for nearby existing states in the tree. By using uniform global sampling, RRT* becomes an algorithm that asymptotically finds the optimal solution to the planning problem by finding the optimal paths from the initial state to every state in the problem domain. However, this method is not consistent with their single-query nature and can be costly in high dimensions.

The paper introduces the focused optimal planning problem for minimizing path length in R^n. It is shown that adding states from an ellipsoidal subset of the planning domain is necessary to improve the solution at any iteration. However, the probability of adding such states through uniform sampling becomes arbitrarily small as the size of the planning problem increases or the solution approaches the theoretical minimum. To address this, the paper presents an exact method to sample the ellipsoidal subset directly, which allows for the creation of informed-sampling planners. The Informed RRT* planner is introduced as an example, which behaves as RRT* until a first solution is found, after which it only samples from the subset of states defined by an admissible heuristic to possibly improve the solution. This results in a more focused search that can find better topologically distinct paths sooner and can find solutions within tighter tolerances of the optimum than RRT*. The algorithm also has less dependence on the dimension and domain of the planning problem and can be used in combination with other algorithms to further reduce the search space. Simulation results demonstrate its effectiveness on both simple and more difficult configurations.

II. Literature Review

Previous efforts to refine RRT and RRT* algorithms have utilized a variety of techniques, including sample biasing, heuristic-based sample rejection, heuristic-based graph pruning, and iterative searches. *Sample biasing*, for example, aims to increase the frequency of sampling from the region X_f by adjusting the distribution of samples drawn from X. While this can add states from outside X_f, it also introduces states that cannot improve the solution, leading to a nonuniform density over the search problem and violating a key assumption of RRT*.

A. Rapid-exploring Random Trees (RRT)

RRT is a path planning algorithm in navigation that crates the graph and find the path from one point to another in the space. The path created by RRT is often not optimal, given that new points are randomly generated which adds a bias from the very start of the graph generation.

B. Rapid-exploring Random Trees Star (RRT*)

RRT* is an optimized and modified alogirhtm of RRT that achieves the optmial path/shortest path between the start and the goal position. RRT* is guranteed to find a path, if exists, between the start and the goal position in the infintie time frame. This algorithm, however, unrealistic due to its time consuming nature, still finds the best possible solution.

This guarantee of finding solution with RRT* lies in its node rewiring phase, where after generating a random point in space and finding a parent node to connect to – same that of RRT, it rewires other nodes in a specified neighbourhood region to see if the better parent in form of newly generated node exists for the other nodes or not. RRT* takes into the consideration the cost-to-come to decide the parent node. This part of the algorithm is computationally expensive and time consuming.

Heuristic-biases sampling:

Heuristic-biased sampling is a type of sample biasing that seeks to increase the probability of sampling from X_f by weighting the sampling of X with a heuristic estimate of each state. Informed RRT* uses this sampling biases to converge to the solution faster.

Initially, a coarse abstraction of the planning problem is solved to provide a heuristic cost for each discrete state. RRT* then samples new states by randomly selecting a discrete state and sampling within it using a continuous uniform distribution. The discrete sampling is biased to favor states belonging to the abstracted solution. While this technique provides a heuristic bias throughout the RRT* algorithm, it maintains a nonzero probability of selecting every state to account for the discrete abstraction. Therefore, it still samples states that cannot improve the current solution.

Path-biased sampling is another technique used to increase the frequency of sampling from X_f by sampling around the current solution path. This approach assumes that the current solution is either homotopic to the optimum or separated only by small obstacles. However, since this assumption is not always true, path-biasing algorithms also need to continue to sample globally to avoid local optima. The ratio between these two sampling methods is often a user-defined parameter.

Heuristic-based sample rejection is a technique that aims to increase the real-time rate of sampling X_f by using rejection sampling on X to sample $X_{\hat{f}}$. Samples drawn from a larger distribution are either kept or rejected based on their heuristic value. Akgun and Stilman [13] use this technique in their algorithm. Although this approach is computationally inexpensive for a single iteration, the number of iterations required to find a single state in $X_{\hat{f}}$ is proportional to its size relative to the sampling domain. This becomes more challenging as the solution approaches the theoretical minimum or the planning domain grows.

C. Informed – RRT*

Infomred RRT* is one of the most recent alogorithms in the field of path planning that provides optimized path with considerable computational efficiency over RRT*.

Informed RRT* is an extension of RRT*. When the first between the start and the goal position is found using RRT*, all the future points in the space are generated using *Ellipsoidal Heuristic* that puts a constraints on a randomly generated node that do not benefit the existing solution. The nodes in the region are rewired similarly as RRT*.

The size of the *Ellipsoidal Heuristic* keeps shrinking as more optimal solutions are found due the nodes generated in the *Ellipsoidal Heuristic* itself. In the absence of any obstacle between the start and the end position, Informed RRT* provides a straight line as a solution. This is possible due to the constant rewiring of the nodes in the *Ellipsoidal region.*

III. METHODOLOGY

The methodology of Informed RRT* is the same as RRT* until the first solution is found. After this Informed RRT* restricts random nodes that are generated by using *Ellipsoidal Heuristic.*

The *Ellipsoidal* is defined by a major and a minor axis that are directly related to the current cost to reach the goal and the best cost to reach the goal – given there's no obstacle between the start and the goal node, respectively.

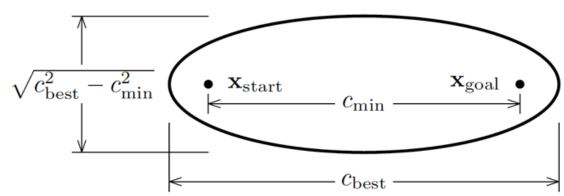

Fig 1. Ellipsoidal Heuristic

Here,

C_{min} is the cost-to-go between the start and the goal node in the space in the absence of obstacle(s).

C_{best} is the current best cost that from the sampling technique.

Euclidean distance is an admissible heuristic for both terms when minimizing the path length in R^n. This informed subset of states that may improve the path, $X_{\hat{f}} \supseteq X_f$, can be written in the closed form in terms of cost of the current best solution, c_{best}, as:

$$X_{\hat{f}} = \left\{ x \in X \,\middle|\, \|x_{start} - \mathrm{x}\|_2 + \left\|x_{goal} - \mathrm{x}\right\|_2 \leq c_{best} \right\}$$

This is the general equation of an n-dimensional prolate hyperspheroid. This hyperspheroid has focal points, x_{start} and x_{goal}, transverse diameter as c_{best} and the conjugate diameters as $\sqrt{c_{best}{}^2 - c_{min}{}^2}$.

As the new nodes are rewired and the and the costs are updated the C_{best} keeps shrinking. Every iteration, the best cost-to-come in the goal region is checked and the ellipsoid is reduced accordingly.

The method proposed in this paper directly calculates and samples from $X_{\hat{f}}$ without making any assumptions about the homotopy class of the optimum or exploring irrelevant states. As it is based on RRT*, it is able to retain all states found in $X_{\hat{f}}$ throughout the search, unlike Anytime RRTs. By sampling directly from $X_{\hat{f}}$, it always explores potential improvements, regardless of the relative size of $X_{\hat{f}}$ to X. Contrary to sample rejection and graph pruning methods, it can function well regardless of the complexity of the planning problem or the cost of the present solution in comparison to the theoretical minimum. In cases where the heuristic does not provide additional information, this method performs similarly to RRT*.

The Direct sampling of an Ellipsoidal subset provides the optimal soultion to this path planning proble.

By transforming uniformly distributed samples from the unit n-ball, $x_{ball} \sim \mathcal{U}(X_{ball})$,uniformly distributed samples in a hyperellipsoid, $x_{ellipse} \sim \mathcal{U}(X_{ellipse})$ can be produced.

$$x_{ellipse} = \boldsymbol{L}x_{ball} + x_{centre}$$

Where x_{centre} is the centre of the hyperellipsoid given in terms of x_{f1} and x_{f2}, shown below:

$$x_{centre} = (x_{f1} + x_{f2})/2, \text{ and}$$

$$X_{ball} = \{x \in X \mid \|x\|_2 \leq 1\}[21]$$

By utilizing the Cholesky decomposition of the hypererllipsoid matrix, $S \in R^{n \times n}$, this transformation can be calculated.

$$LL^T \equiv S$$

$$\text{Where } (x - x_{centre})^T S (x - x_{centre}) = 1.$$

Here, S has eigenvalues corresponding to squares of the hyperellipsoid's radii, $\{r_i^2\}$ and eigenvectors corresponding to the hyperellipsoid's axes, $\{a_i\}$. The uniform distribution in $X_{ellipse}$ defines the transformation, L [22]. This transformation can then be calculated using the radii and transverse axis of the prolate hyperellipsoid.

In a coordinate system aligned with transverse axis, the diagonal matrix gives the hyperellipsoid matrix.

$$S = diag\left\{\frac{c_{best}{}^2}{4}, \frac{c_{best}{}^2 - c_{min}{}^2}{4}, \ldots, \frac{c_{best}{}^2 - c_{min}{}^2}{4}\right\}$$

With resulting decomposition,

$$L = diag\left\{\frac{c_{best}}{2}, \frac{\sqrt{c_{best}{}^2 - c_{min}{}^2}}{2}, \ldots, \frac{\sqrt{c_{best}{}^2 - c_{min}{}^2}}{2}\right\}$$

The rotation from the hyperellipsoid frame to the world frame, $C \in SO(n)$ can be calculated by solving it as a general Wahba problem [23]. Even if the problem may be underspecified, it has been proved that a valid solution exists [24].

The rotation matrix is given by:

$$C = U \, diag\,\{1, \ldots, 1, det(U) det(V)\} V^T$$

Where U and V are unitary matrices such that $U \Sigma V^T \equiv M$ obtained via singular value decomposition. The matrix M is given by:

$$M = a_1 1_1^T$$

Where a_1 is the outer product of the transverse axis in the world frame, 1_1 is the first column of the identity matrix.

$$a_1 = \frac{(x_{goal} - x_{start})}{\|x_{goal} - x_{start}\|_2}$$

A sample uniformly drawn from a unit n-ball, $x_{ball} \sim \mathcal{U}(X_{ball})$, can thus help to calculate a state uniformly distributed in the informed subset, $x_{\hat{f}} \sim \mathcal{U}(X_{\hat{f}})$, through a transformation (5), rotation (6) and translation,

$$x_{\hat{f}} = CLx_{ball} + x_{centre}$$

Direct sampling in the ellipsoid heuristic in python is performed by finding C_{min} and C_{best} first. Then at each iteration the ellipsoid with these parameters is constructed at the origin of the graph.

Based on the angle between the line passing from the goal and the start node and the x-axis of the space, the ellipsoid is rotated accordingly at the origin.

And then the entire ellipsoid is translated to the center of the start and goal node for direct biased sampling.

The rotation of ellipsoid by Ø is done using below rotation matrix:

$$\begin{pmatrix} \cos Ø & -\sin Ø \\ \sin Ø & \cos Ø \end{pmatrix}$$

Below is the sampling algorithm in the ellipsoid heuristic.

if $c_{\max} < \infty$ **then**
 $c_{\min} \leftarrow ||\mathbf{x}_{\text{goal}} - \mathbf{x}_{\text{start}}||_2$;
 $\mathbf{x}_{\text{centre}} \leftarrow (\mathbf{x}_{\text{start}} + \mathbf{x}_{\text{goal}})/2$;
 $\mathbf{C} \leftarrow \texttt{RotationToWorldFrame}(\mathbf{x}_{\text{start}}, \mathbf{x}_{\text{goal}})$;
 $r_1 \leftarrow c_{\max}/2$;
 $\{r_i\}_{i=2,\ldots,n} \leftarrow \left(\sqrt{c_{\max}^2 - c_{\min}^2}\right)/2$;
 $\mathbf{L} \leftarrow \text{diag}\{r_1, r_2, \ldots, r_n\}$;
 $\mathbf{x}_{\text{ball}} \leftarrow \texttt{SampleUnitNBall}$;
 $\mathbf{x}_{\text{rand}} \leftarrow (\mathbf{C}\mathbf{L}\mathbf{x}_{\text{ball}} + \mathbf{x}_{\text{centre}}) \cap X$;
else
 $\mathbf{x}_{\text{rand}} \sim \mathcal{U}(X)$;
return $\mathbf{x}_{\text{rand}}$;

Fig 2. Algorithm for sampling in Ellipsoid Heuristic

IV. Implementaion, Simulation and Results

Informed RRT* was compared with RRT* in two different obstacle spaces. Their time to reach their goal and find the optimal path was compared. It was concluded that Informed RRT* finds the shortest and optimal path in less time than RRT* in the absence of obstacle panned across the minor axis of the ellipsoid heuristic at any given iteration. There are scenarios where both the path planning algorithms give similar results.

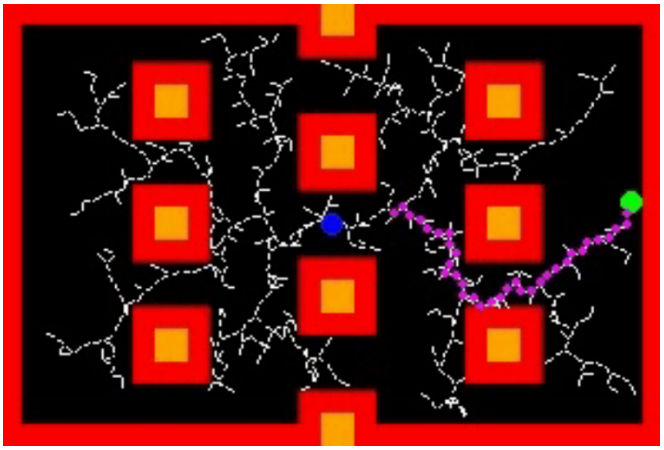

Fig 3. RRT path planning algorithm (0.21 s)

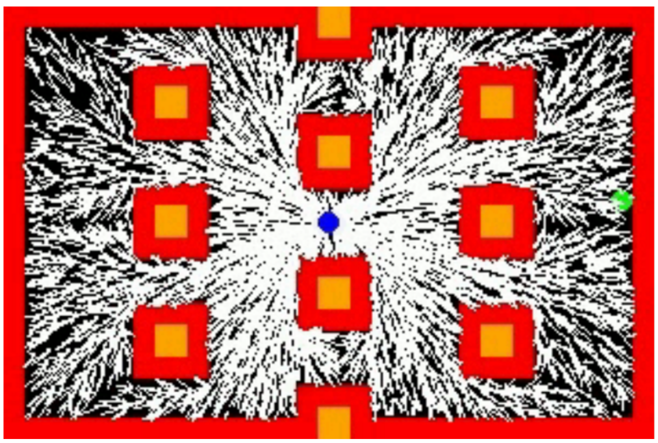

Fig 4. Exploration of RRT for 10,000 iterations*

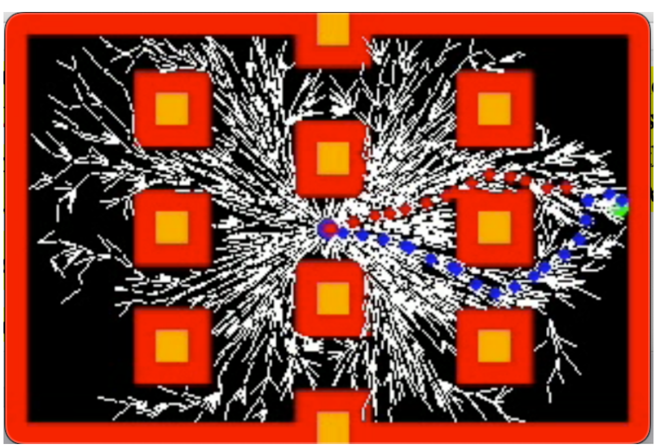

Fig 5. Comparison of optimal path generated by Informed RRT algorithm (red) (2.02 s) and RRT* (blue)(3.59 s)*

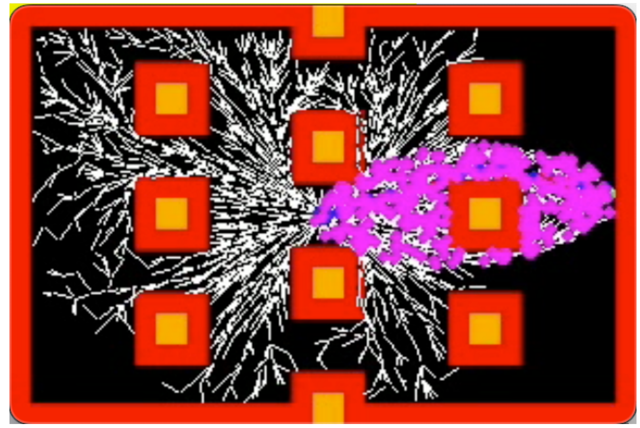

Fig 6. Node sampling in Informed RRT Ellipsoid Heuristic*

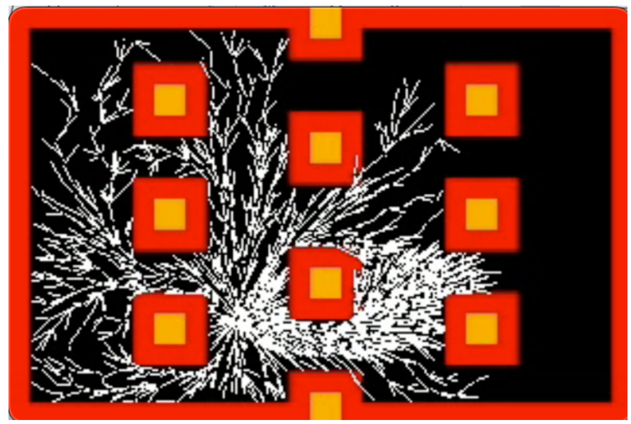

Fig 7. Visualization of Node sampling in Informed RRT Ellipsoid Heuristic*

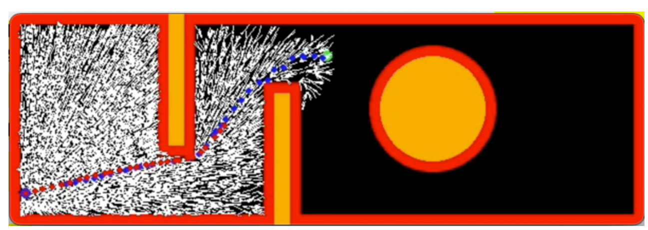

*Fig 8. Limitations of Informed RRT**

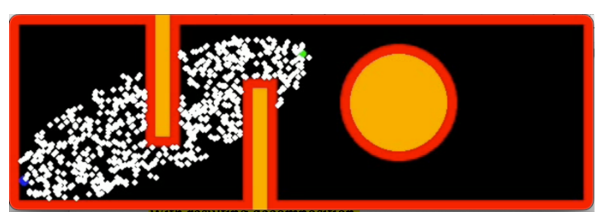

Fig 9. Node sampling in Informed RRT Ellipsoid Heuristic*

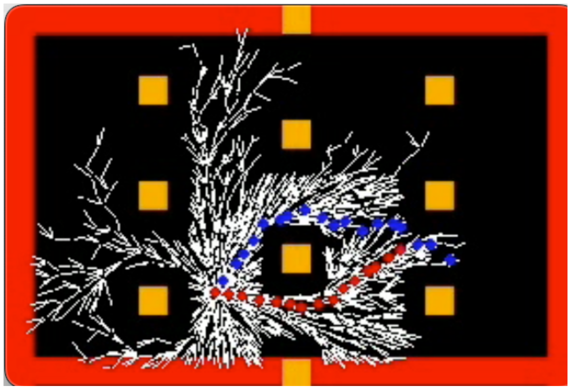

Fig 10. Comparison of optimal path generated by Informed RRT (red) and RRT* (blue)*

Conclusion

Comparing Informed RRT* with RRT and RRT* in two different obstacle spaces gave strong evidence that in the absence of obstacle covering the minor axis of ellipsoid heuristic during any given iteration guarantees the optimized path than RRT and is less computationally expensive than RRT*.

Comparison (s)			
	RRT	RRT*	Informed RRT*
Competition Arena	0.21 s	3.59 s	2.02 s
Proejct3-Phase2 Arena	0.45 s	15.38 s	14.62 s

Table 1. Comparison b/w algorithms (in seconds)

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